thus far we have dealt almost exclusively with static situations. In fact, we've had no time dependence at all even when current was involved.
→ current was steady.

Let's talk a bit more about current as we begin to bring in time dependent features.

Current

What makes current flow?

- Inside a material (wire, other circuit elements), there will be resistance to the motion of charges (they scatter, heat up - thermal motion, etc.)
- You need some force to maintain the motion - the current. (like friction in 183)

With free electrons → constant push means accelerating charges. and thus increasing current!

In most materials → constant push means a constant flow!

Model of this is known as Ohm's Law,

$$\vec{J} = \sigma \vec{f}$$

current density ∝ force per unit charge.

The constant of proportionality, σ, is a material dependendent constant — conductivity.

(It is not surface charge!)

We can also rewrite this,

$$\vec{f} = \frac{1}{\sigma}\vec{J} = \rho\vec{J}$$

ρ is the resitivity $= 1/\sigma$

(not charge density!)

It's often the case that the force responsible for this motion is the Lorentz force,

$$\vec{f} = \frac{\vec{F}_{Lorentz}}{q} = \vec{E} + \vec{v} \times \vec{B}$$

And typically $\vec{v}$ & $\vec{B}$ are small enough where only $\vec{E}$ really affects the charges * (more later on this)

So,

$$\vec{J} = \sigma \vec{f} = \sigma(\vec{E} + \vec{v} \times \vec{B}) \simeq \sigma \vec{E}$$

this is Ohm's Law, which is really a Model that many materials seem to be able to be modeled by.

<u>Note:</u> $\vec{F} = m\vec{a}$ does not imply an increasing current even though $\vec{J} \propto \vec{v}$ (remember this?)

If there were no damping (collision, thermal losses), it would. But electrons in real materials are like a gas, they have large random $\vec{v}$'s depending on temperature. So applying a (small) force causes a drift, but the collisions tend to still randomize the motion (called thermalization).

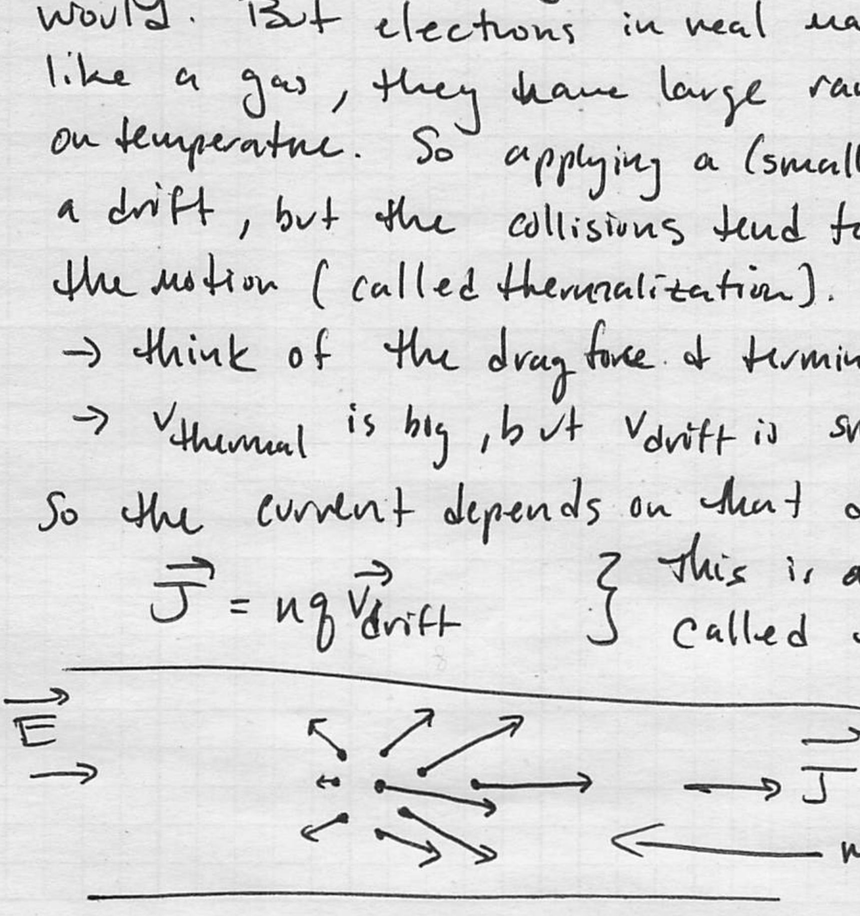
→ think of the drag force & terminal velocity

→ $v_{thermal}$ is big, but v_{drift} is small

So the current depends on that drift velocity

$$\vec{J} = nq\vec{v}_{drift}$$

} This is a classical model called the Drude model



(Lorentz averaging remember?)

net effect is a current to the right.

Comment! σ depends on the material

Materials w/ large conductivity are good conductors. (you only need a small force to get a large flow)

- Copper is used in most household wiring

$$\underbrace{\sigma_{cu} \simeq 6 \cdot 10^{7} \frac{C/s \cdot m^2}{N/C}} = \frac{C^2 s}{kg\, m^3} = \frac{1}{\text{Ohm}\, m} = \frac{1}{\Omega m}$$

- This is a huge conductivity. By contrast, wood (an insulator) has $\sigma_{wood} \simeq 10^{-8}$ to $10^{-11} \frac{1}{\Omega m}$
- A resistor in a circuit would be more like 10^{+3} or $10^{+2}\, 1/\Omega m$ ("mid range")

Comment: I thought $E=0$ in metals!

For static situations, yes that's true, $\vec{J} = \sigma \vec{E}$ so if $\vec{J}=0$ then $\vec{E}=0$.

For a metal σ is very large ($\sigma \to \infty$), so that $\vec{E} = \vec{J}/\sigma \to 0$ even if there's finite current.

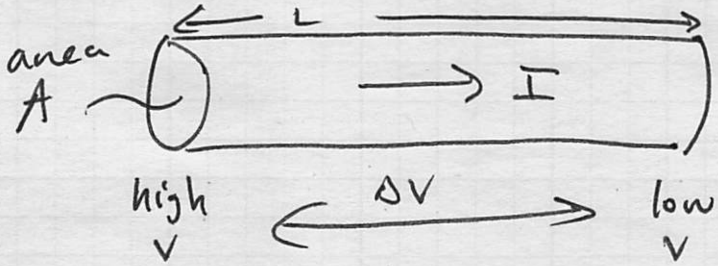
That is, very small $\vec{E}$ fields are needed to drive currents in metals. and in our approximation that $\sigma \to \infty$, $\vec{E} \to 0$ still in this case.

Final Comment: As there are collisions and thermal losses when driving current, the power dissipated in the system must be

$$P = \Delta V\, I = \frac{\text{work}}{\text{charge}} \cdot \frac{\text{charge}}{\text{second}}$$

Example : Uniform Conducting Wire

Here's a bit of wire,



We can use Ohm's Law,

$$\vec{J} = \sigma \vec{E}$$

to find 184's Ohm's.

the current density is uniform: $J = I/A$

* here the electric field is also uniform : $E = \dfrac{\Delta V}{L}$

(* we will come back to this)

so,

$$\frac{I}{A} = \sigma \frac{\Delta V}{L} \Rightarrow \Delta V = \frac{L}{\sigma A} I$$

We can call $\dfrac{L}{\sigma A} = R$ the resistance of the material

R depends on the geometry and the resistivity of the material. In this case,

$$R = \frac{L}{\sigma A} = \rho \frac{L}{A} \quad \text{where } \rho = 1/\sigma \text{ (remember!)}$$

$[R] = \text{Ohms} = [\Omega]$ so $\underline{\Delta V = RI}$ (like 184)

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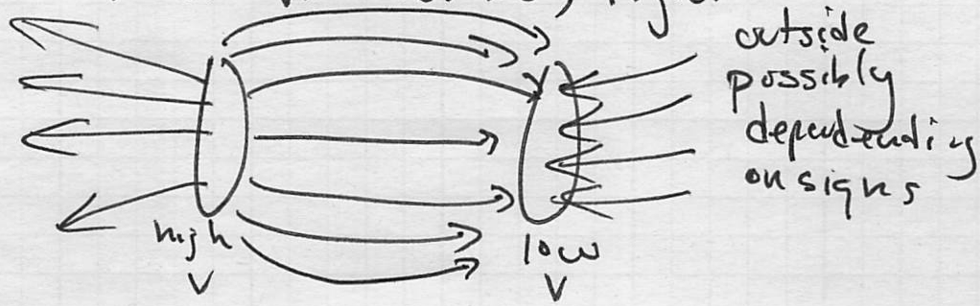
Real wires have small E-fields in them and thus small ΔV's. They are measurable, too!

But, big ΔV's occur across resistive elements; hence, we often focus on them!

In the previous example, why was E uniform?

If there was no material, just two plates, then it wouldn't be, right?

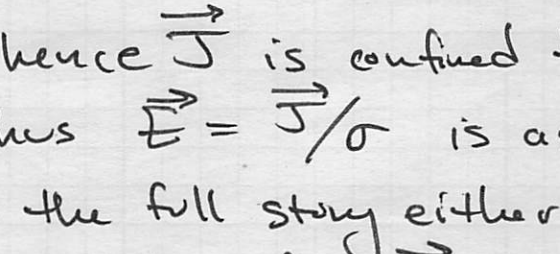


The cylindrical conductor changes things. the current can't leave the conductor for free space, hence $\vec{J}$ is confined to the conductor, thus $\vec{E} = \vec{J}/\sigma$ is as well.*

(this isn't the full story either! It's the $\vec{E}$ responsible for $\vec{J}$ that doesn't leave, but the charges generating that $\vec{E}$ produce external fields!) we'll come back to this!

$\vec{E}$ must be parallel to the edges! $\vec{E}\cdot\hat{n} = 0$

(this is in steady state; when the "switch" is closed charges quickly accumulate to make this $\vec{E}$.)



"Rings" of surface charge accumulate to shape the field!

Another explanation,

$\nabla\cdot\vec{E} = 0 \rightarrow \nabla^2 V = 0$ Laplace's Eqn is satisfied in steady state.

$V = V_0\left(1 - \frac{x}{L}\right)$ solves this w/ $\partial V/\partial n = 0$

Uniqueness guarantees that $\vec{E} = -\nabla V = \frac{+V_0}{L}\hat{x}$

Going back to conservation of current,

$$\frac{d\rho}{dt} + \nabla \cdot \vec{J} = 0$$ is a local statement

that is, it holds in the bulk ~~[illegible]~~

and outside but might have different

ρ's & $\vec{J}$'s depending on where you are.

In the bulk, in steady state, $d\rho/dt = 0$

the distribution of charge is (roughly) unchanged

even though charges are moving!

So, $\nabla \cdot \vec{J} = 0$ again locally (at every pt.)

Because $\vec{J} = \sigma \vec{E}$ in the bulk,

$\nabla \cdot \vec{E} = 0$ (thus, $\nabla^2 V = 0$ Laplace's eqn can be used. Ch 3 tools are OK!)

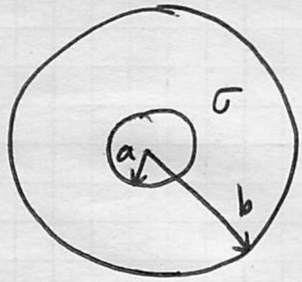
Even funny shaped resistors will have

$\nabla \cdot \vec{E} = 0$ and $\vec{E} \cdot \hat{n} = 0$ at the edges.

Charge built up on the surface to shape the field.

Example: Two Concentric Spheres

Two spheres (radii a & b, b>a) are constructed so the larger one contains the smaller one. There is a material of conductivity, σ, between them. A potential of V is maintained between them with the smaller sphere at higher potential. What's I?



Assume they are metal spheres so that any charges are on the surfaces.

Can also assume Q distributed over the sphere inside the larger one as long as we solve for Q in terms of known variables.

$$\vec{E} = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r} \quad \text{so that,}$$

$$\vec{J} = \sigma\vec{E} = \frac{Q\sigma}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r}$$

the total current (in terms of Q) is,

$$I = \int \vec{J}\cdot d\vec{a} = \sigma\int\vec{E}\cdot d\vec{a} = \sigma\frac{Q}{\epsilon_0} \quad \leftarrow \text{Gauss' Law}$$

We need to relate this to V so we compute the potential between the spheres.

$$V = -\int_a^b \vec{E}\cdot d\vec{l} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right) < 0 \quad \text{right?}$$

$\underline{\underline{V_b < V_a}}$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{a-b}{ab}\right) < 0 \qquad b>a \text{ remember?}$$

so with $I = \sigma \dfrac{Q}{\epsilon_0}$ and $V = \dfrac{Q}{4\pi\epsilon_0}\dfrac{(a-b)}{ab}$,

$Q = \dfrac{\epsilon_0 I}{\sigma}$ so that,

$$V = \frac{(\epsilon_0 I/\sigma)}{4\pi\epsilon_0}\frac{(a-b)}{ab} = I\left(\frac{1}{4\pi\sigma}\right)\left(\frac{a-b}{ab}\right)$$

or

$$V = I\frac{\rho}{4\pi}\left(\frac{a-b}{ab}\right) = IR \text{ so}$$

$R = \dfrac{\rho}{4\pi}\left(\dfrac{a-b}{ab}\right)$ and depends only on geometry (a,b) & ρ.