



**FACULTY
OF MATHEMATICS
AND PHYSICS**
Charles University

MASTER THESIS

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**Bayesian Optimization of Hyperparameters
Using Gaussian Processes**

Institute of Formal and Applied Linguistics

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Dedication.

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Introduction

0.1 Bayesian Optimization

Consider the problem of optimizing an arbitrary continuous function $f : \mathcal{X} \rightarrow \mathbb{R}$ where $\mathcal{X} \subset \mathbb{R}^d, d \in \mathbb{N}$. We call f the *objective function* and treat it as a black box, making no assumption on its analytical form, or on our ability to compute its derivatives. Our goal is to find the global minimum \mathbf{x}_{opt} over the set \mathcal{X} , that is

$$\mathbf{x}_{\text{opt}} = \arg \min_{\mathbf{x} \in \mathcal{X}} f(\mathbf{x}).$$

We also assume that the evaluation of f is expensive, as the goal of Bayesian optimization is to find the optimum as quickly as possible. If the function can be evaluated cheaply, other global optimization approaches such as simulated annealing or evolution strategies could potentially yield better results (TODO ref).

Consider the case when evaluating f means performing a computation that is not only time consuming, but for example also costs a lot of money. We might only have a fixed budget which puts a hard limit on the number of evaluations we can perform.

Bayesian optimization techniques are some of the most efficient approaches in terms of the number of function evaluations required. Much of the efficiency stems from the ability to incorporate prior belief about the problem and to trade of exploration and exploitation of the search space. [Nando 2012] It is called Bayesian because it combines the prior knowledge $p(f)$ about the function together with the data in the form of the likelihood $p(x|f)$ to formulate a posterior distribution on the set of possible functions $p(f|x)$. We will use the posterior distribution to figure out which point should be evaluated next to give a likely improvement over the currently obtained maximum.

Let $\mathcal{D}_n = \{(\mathbf{x}_i, y_i), i \in 1 : n\}$ denote a set of n samples (evaluations) of the function f , that is $y_i = f(\mathbf{x}_i)$. Our goal is to pick the next \mathbf{x}_{n+1} to maximize our chance of finding the optimum quickly.

Consider the set of all continuous functions $f \in \mathcal{F}$ with a prior distribution $p(f)$. Conditioning on our samples gives us a posterior distribution over possible functions $p(f|\mathcal{D})$.

Conclusion

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A. Attachments

A.1 First Attachment