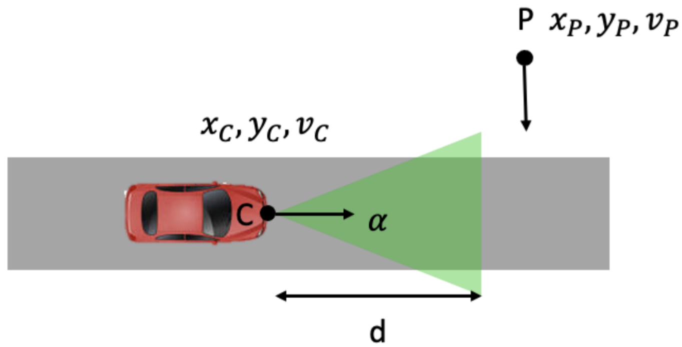
The Problem



We consider a *scenario* with two agents:

1. **Car Agent.** A car C moving down a road, with parameters initial position (x_C, y_C), initial velocity v_C, and double integrator dynamics where its decision logic (DL) somehow sets the acceleration), and sensor parameters α and d that define a cone extending from (x_C, y_C) which describes the car's line of sight. Also are defined the maximum braking and acceleration values a_{min} and a_{max} respectively. In this problem the road is a straight one-lane map, but this problem can be extended to different maps.

2. **Ped Agent.** A pedestrian P moving across the road, with parameters initial position (x_P, y_P) and initial velocity v_P. We assume that the pedestrian is in uniform motion, but the problem can be extended to consider a time-varying v_p.

We also have a *sensor model* that detects the pedestrian when it is inside the cone that defines the car's line of sight. Detection means C gets the values of relative position and relative velocity of P with respect to C. The problem can be extended to consider a much more complex sensor function such as a neural network model.

Consider a meaningful range R of initial values of C and P and a time horizon T_S. Given a particular deterministic DL and a particular initial condition in R, the execution of the system is *safe* if there is never a collision between the C and P within T_S.

The problem is to design a DL such that:

- all executions in R are safe, and
- it maximizes the average speed of the car over T_S over a set of critically chosen test executions.

Proposed Solution

We first consider a simplified version of the problem, where the car's line of sight is defined by a circle of radius r instead of a cone with parameters α and d.

Dynamics of the Agents

- **Car Agent.**

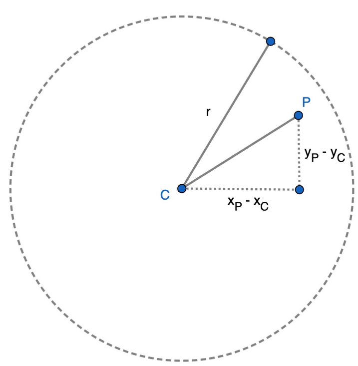
$$\dot{x_C} = v_C$$
$$\dot{v_C} = a_C$$
$$\dot{a_C} = 0$$

- **Ped Agent.**

$$\dot{x_P} = v_P$$
$$\dot{v_P} = 0$$

Decision Logic of Car Agent

Detecting the Pedestrian



If the pedestrian is within the car's line of sight and the car is moving towards the pedestrian, then the car should implement a braking or accelerating system to avoid collision. This detection is done following the conditions below:

- The pedestrian is within the car's line of sight which is a circle of radius r, that is,

$$\sqrt{(x_P - x_C)^2 + (y_P - y_C)^2} < r$$

.

- The car is moving towards the pedestrian, that is, either $(x_C < x_P) \wedge (v_C > 0)$, or $(x_C > x_P) \wedge (v_C < 0)$. These two constraints can be summarised by a single constraint

$$x_C.v_C < x_P.v_C$$

.

Consider the timestep at which the sensor detects that the pedestrian P, at position (x_P, y_P) is within the car's line of sight. The car C is at position (x_C, y_C). In succeeding arguments, we consider that $v_C > 0$, that is, the car is moving along the X-axis in the positive direction. This implies that $x_P > x_C$, and removes any confusion regarding the signing conventions of acceleration and deceleration.

Determining the Action

Let the time taken for the pedestrian to reach (x_P, y_C) be T_P. As the pedestrian is moving in uniform motion, we have

- $T_P = -\frac{(y_P - y_C)}{v_P}$ if $v_P < 0$, that is, $y_P > y_C$
- $T_P = \frac{(y_C - y_P)}{v_P}$ if $v_P > 0$, that is, $y_P < y_C$

In other words,

$$T_P = \frac{(y_C - y_P)}{v_P}$$

Let the time taken for the car to reach (x_P, y_C) be T_C. Now, in order for the car to avoid crashing into the pedestrian, we must ensure that $T_C \neq T_P$. Assuming that we do not brake or accelerate, that is, the car remains in uniform motion, we have

$$T_C = \frac{(x_P - x_C)}{v_C}$$

Hence, if $\frac{(x_P - x_C)}{v_C} = \frac{(y_C - y_P)}{v_P}$, we can say that a collision occurs, therefore, the system is not safe. So, in order to avoid this, we must ensure that either

- **Case 1.** $T_C < T_P$, that is, $\frac{(x_P - x_C)}{v_C} < \frac{(y_C - y_P)}{v_P}$, in which case we can accelerate (thereby further reducing T_C) at $a = a_{max}$ until a threshold velocity v_{max} (defining the speed limit of the road) is reached, without worrying about the possibility of collision, or
- **Case 2.** $T_C > T_P$, that is, $\frac{(x_P - x_C)}{v_C} > \frac{(y_C - y_P)}{v_P}$, in which case we can decelerate (thereby further increasing T_C) without worrying about the possibility of collision. But our objective is to maximize the average speed of the car, so instead of decelerating we can opt for continuing to move with uniform velocity until the car crosses (x_P, y_C) after which we can accelerate at $a = a_{max}$ until the threshold velocity v_{max} is reached, without worrying about the possibility of collision.

However, in the case where $T_C = T_P$, that is, $\frac{(x_P - x_C)}{v_C} = \frac{(y_C - y_P)}{v_P}$, the car must either accelerate with the highest possible magnitude so that it crosses (x_P, y_C) within some time $t < T_P$ (*Speeding*) or decelerate at the least possible magnitude (without stopping, so as to not reduce the average speed of the car) so that it crosses (x_P, y_C) after some time $t > T_P$ (*Deceleration without Braking*).

Speeding

The third equation of motion holds that

$$s = u.t + \frac{1}{2}a.t^2$$

Given $s = (x_P - x_C)$ and $u = v_C$, our aim is to set an acceleration a such that $t < T_P$. Let the acceleration for $t = T_P$ be $a_{speed} > 0$. Then,

$$(x_P - x_C) = v_C.T_P + \frac{1}{2}a_{speed}.T_P^2$$

That is,

$$\frac{1}{2}a_{speed}.T_P^2 = (x_P - x_C) - v_C.T_P$$

Or,

$$a_{speed} = \frac{2(x_P - x_C)}{T_P^2} - \frac{2v_C}{T_P}$$

To ensure $t < T_P$, the DL must set an acceleration $a > a_{speed}$. Intuitively, we understand that the average speed of the car can be improved by opting for speeding. Hence, we can set $a = a_{max}$ (if $a_{speed} < a_{max}$) and continue to accelerate at this rate until either the threshold velocity v_{max} is reached or the car crosses (x_P, y_C), preferring the latter over the former (it is acceptable to break a few traffic laws if it means saving a life). However, if $a_{speed} \geq a_{max}$, this implies that $a > a_{max}$ which is a violation of the constraints set by the problem. In such cases, we must opt for decelerating.

Deceleration without Braking

The third equation of motion holds that

$$s = u.t + \frac{1}{2}a.t^2$$

Given $s = (x_P - x_C)$ and $u = v_C$, our aim is to set a deceleration a such that $t > T_P$. Let the deceleration for $t = T_P$ be $a_{decel} < 0$. Then,

$$(x_P - x_C) = v_C.T_P + \frac{1}{2}a_{decel}.T_P^2$$

That is,

$$\frac{1}{2}a_{decel}.T_P^2 = (x_P - x_C) - v_C.T_P$$

Or,

$$a_{decel} = \frac{2(x_P - x_C)}{T_P^2} - \frac{2v_C}{T_P}$$

To ensure $t > T_P$, the DL must set a deceleration $a < a_{decel}$. We can set $a = 1.01 * a_{decel}$ (if $a_{decel} > a_{min}$) and continue to decelerate at this rate until the car crosses (x_P, y_C), upon which we can accelerate at $a = a_{max}$ until the threshold velocity v_{max} is reached. However, if $a_{decel} \le a_{min}$, this implies that $a < a_{min}$ which is a violation of the constraints set by the problem. In such cases, the safety of the system cannot be guaranteed by our DL unless we introduce some mechanism for emergency braking.

Emergency Braking

If we have both $a_{speed} \ge a_{max}$ and $a_{decel} \le a_{min}$, the DL must implement an emergency braking system. This can be done by setting $v_C = 0$ when the pedestrian gets too close to the car, say, $\sqrt{(x_P - x_C)^2 + (y_P - y_C)^2} < 1$ and resuming the car with an acceleration $a = a_{max}$ when the pedestrian has crossed the axis along which the car is moving. This can happen if either of the two conditions hold:

- The pedestrian is moving along negative Y-axis, that is, $v_P < 0$. In this case, if $y_P < y_C$, then the pedestrian has crossed the car's axis.
- The pedestrian is moving along positive Y-axis, that is, $v_P > 0$. In this case, if $y_P > y_C$, then the pedestrian has crossed the car's axis.

These two constraints can be summarised by a single constraint

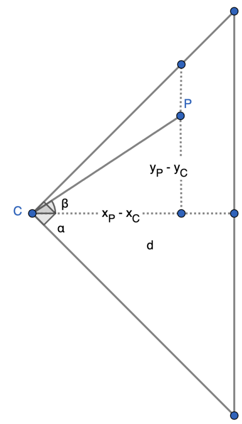
$$y_P.v_P > y_c.v_P$$

Safety Assertion

We define an error parameter δ such that collision is detected within this δ-bound. That is, we must assert that the following condition (corresponding to collision within this δ-bound) does not hold:

$$|x_P - x_C| < \delta \wedge |y_P - y_C| < \delta$$

Extension



We now modify our proposed DL such that the car's line of sight is defined by a cone with parameters α and d. In order to do so, we only need to change the condition for detecting the pedestrian. The rest of the DL remains unaffected. Here, again, we consider that $v_C > 0$, that is, the car is moving along the X-axis in the positive direction. This implies that $x_P > x_C$. We also assume that $0 < \alpha < \pi$, hence $0 < \alpha/2 < \pi/2$ and therefore, $\tan(\alpha/2)$ is a monotonic function.

For P to lie inside the cone that defines the car's line of sight, the first criteria is that $(x_P - x_C) < d$. The second criteria is that $0 < \beta < \alpha/2$, hence $\tan(\beta) < \tan(\alpha/2)$. As α is known, $\tan(\alpha/2)$ can be calculated. Moreover, $\tan(\beta)$ can be given as

$$\tan(\beta) = \frac{|y_P - y_C|}{(x_P - x_C)}$$

Therefore, the condition for detection is given as:

$$(x_P - x_C) < d \wedge \frac{|y_P - y_C|}{(x_P - x_C)} < \tan(\alpha/2)$$

Sarthak Das,
JRF, IACS Kolkata.