<div align="center">

Homework 5

Due on Gradescope 10/11/2016 at 4:00PM (Before Lecture)

</div>

1. <u>Theory Meets Data, Section 5.4, Problem 5</u>: A fair 6-sided die will be rolled 10 times. Consider the smallest of the 10 numbers rolled.

   (a) For each k in the range 1,2,3,4,5,6, find the chance that the smallest number rolled is larger than k. (It may help to derive a formula that works for any k first).

   (b) For each k in the range 1,2,3,4,5,6, let p_k be the probability that the smallest number rolled is equal to k. Use your answers from the last part to find p_k.

   (c) Find the sum of your answers from the last part. Do the sum and get a numerical answer; dont just explain what the sum ought to be.

2. <u>Deriving Distributions, to appear in TMD</u>: The goal of this problem is for you to use the definitions and notation from class in familiar contexts. For each random variable, describe the sample space Ω (no need to calculate size), write the set of values that X can take, and derive the probability mass function.

   (a) X is the number of spots that show on one roll of a fair six-sided die.

   (b) X is an indicator random variable; it has the value 1 with probability p, and the value 0 with probability 1 p. This random variable is a Boolean, that is, it can only be 0 or 1. Just as 0s and 1s are powerful in computing, so also indicators are powerful in probability theory. Youll see how next week.

   (c) X is the number of heads in one toss of a fair coin.

   (d) X is the number of heads in two tosses of a fair coin.

   (e) X is the number of red cards among two cards picked at random without replacement from a standard deck (52 cards of which 26 are red).

3. <u>Stock Price</u>: Suppose that TechCo is one of San Francisco's hottest publicly traded tech startups, and its stock price moves in the following way: every day, it either increases by \$1 with probability p or decreases by \$1 with probability $1 - p$, and the change on each day is independent. Let Z be the change in the price of TechCo's stock over two weeks; that is, Z is the price of TechCo's stock on Oct 19 minus the price of TechCo's stock today, Oct 5.

   (a) Let X be the number of times in 14 days that TechCo's stock increased. What is the probability that the stock price increased $0 \le k \le 14$ times in this period?

   (b) Write Z, the change in price, as a function of X, the number of times the price increased.

   (c) What is the probability that the stock price rose overall during the 14 day period?

   (d) You make a bet with your friend that works like this: You will pay your friend \$5 on Oct 5, and on Oct 19 your friend will pay you Z if Z is positive, and nothing if Z is zero or negative[1]. What is the probability that you will make an overall profit on this agreement?

---

[1]For the curious, this sort of agreement is called a *European call option*. Here, the *strike price* is set to today's price.