

Homework 6

Due on Gradescope 10/25/2016 at 4:00PM (Before Lecture)

The first two problems should look familiar!

1. Deriving Parameters, to appear in TMD: The goal of this problem is for you to use the definitions and notation from class in familiar contexts. For each random variable, derive the expectation $E(X)$, the variance $\text{Var}(X)$, and the standard deviation $SD(X)$ from their definitions (i.e., work out the sum).

- (a) X is the number of spots that show on one roll of a fair six-sided die.
- (b) X is an “indicator” random variable; it has the value 1 with probability p , and the value 0 with probability $1 - p$. This random variable is a Boolean, that is, it can only be 0 or 1. Just as 0s and 1s are powerful in computing, so also indicators are powerful in probability theory. You’ll see how next week.
- (c) X is the number of heads in one toss of a fair coin.
- (d) X is the number of heads in two tosses of a fair coin.
- (e) X is the number of red cards among two cards picked at random without replacement from a standard deck (52 cards of which 26 are red).

2. Stock Price: (**Note:** In this problem, you may use these facts presented in class: If X is a binomial random variable with size n and probability p , $E(X) = np$ and $\text{Var}(X) = np(1 - p)$.)

Suppose that TechCo is one of San Francisco’s hottest publicly traded tech startups, and its stock price moves in the following way: every day, it either increases by \$1 with probability p or decreases by \$1 with probability $1 - p$, and the change on each day is independent. Let Z be the change in the price of TechCo’s stock over two weeks; that is, Z is the price of TechCo’s stock on Oct 19 minus the price of TechCo’s stock today, Oct 5.

- (a) What is $E(Z)$?
- (b) What is $SD(Z)$?
- (c) Suppose we redefine Z so that it is the change in the stock price over n days, and that p is 0.51, so the stock price has a very slight upward drift. How many days would it take for $E(Z)$ to be more than 2 standard deviations ($SD(Z)$) away from 0?

3. Polling Margin of Error: (**Note:** In this problem, you may use these facts presented in class: If X is a binomial random variable with size n and probability p , $E(X) = np$ and $\text{Var}(X) = np(1 - p)$.)

When pollsters conduct surveys of political opinions, they draw a sample of size n without replacement from a population of size N , where N is much larger than n . Nonetheless, they often model the number of people supporting a particular candidate (for the purposes of this question, let’s say Clinton), X , as a binomial random variable with size n and probability p . In this case, the pollster does not know the true value p , but we can write down expectations and variances of important quantities in terms of p .

- (a) Technically, what are the requirements for a random variable X to be binomially distributed? Which of these requirements are violated in this case? Intuitively, why is the binomial model approximately correct here? (Hint: Sizes of N and n matter).
- (b) Define Y as the *proportion* of respondents who say they will vote for Clinton, $\frac{X}{n}$. What is $E(Y)$?
- (c) Using the same definition of Y as above, what is $\text{Var}(Y)$?
- (d) For what value of p is $\text{Var}(Y)$ maximized? Use calculus or an algebraic argument.
- (e) Simple polls will report Y as their estimate of the unknown proportion of Clinton supporters p . They will also report a “margin of error”, which is 2 times the worst case standard deviation of Y (that is, the largest SD among all potential parameters p). For a poll with n respondents, what is the margin of error?