ClimODE: Climate Forecasting With Physics-informed Neural ODEs

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Problem Definition and Contribution

Goal: Enhancing climate forecasting by integrating physics-informed neural ordinary differential equations (ODEs) with uncertainty quantification.

Motivations:

- Existing models neglect the underlying physics, lack of uncertainty quantification and are computationally intensive.
- Enhance efficiency and effectiveness in global and regional weather prediction tasks.

Problem Formulation

Statistical Mechanics: Weather can be described as a spatial movement of quantities over time, governed by the partial differential continuity equation:

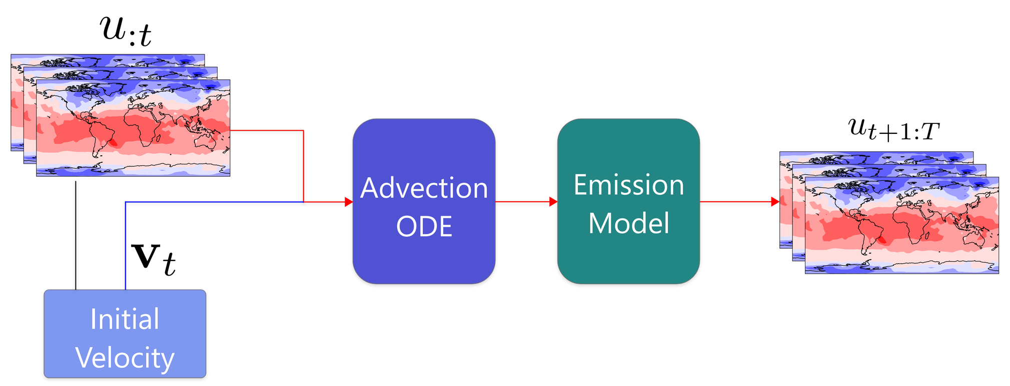
$$\underbrace{\frac{du}{dt}}_{\text{time evolution } \dot{u}} + \underbrace{\overbrace{\mathbf{v}\cdot\nabla u}^{\text{transport}} + \overbrace{u\nabla\cdot\mathbf{v}}^{\text{compression}}}_{\text{advection}} = \underbrace{s}_{\text{sources}},$$

where $u(x,t)$ is a quantity evolving over space $\mathbf{x}$ and time t driven by a flow's velocity $\mathbf{v}(\mathbf{x},t)$.

Main Idea: We solve the continuity equation over entire Earth as a system of neural ODEs.

Method

Network Architecture:



Initial Velocity Inference:

$$\hat{\mathbf{v}}_k(t) = \underset{\hat{\mathbf{v}}_k(t)}{\arg\min} \left\{ \left\| \tilde{\dot{u}}_k(t) + \mathbf{v}_k(t)\cdot\tilde{\nabla} u_k(t) + u_k(t)\tilde{\nabla}\cdot\mathbf{v}_k(x,t) \right\|_2^2 + \alpha \left\| \mathbf{v}_k(t) \right\| \right\}$$

Flow Velocity:

$$\dot{\mathbf{v}}_k(x,t) = f_{\text{conv}}\left(\mathbf{u}(t), \nabla\mathbf{u}(t), \mathbf{v}(t), \psi\right) + \gamma f_{\text{att}}\left(\mathbf{u}(t), \nabla\mathbf{u}(t), \mathbf{v}(t), \psi\right)$$

Advection ODE:

$$\begin{bmatrix}\mathbf{u}(t)\\ \mathbf{v}(t)\end{bmatrix} = \begin{bmatrix}\{u_k(t_0)\}_k\\ \{\mathbf{v_k}(t_0)\}_k\end{bmatrix} + \int_{t_0}^{t} \begin{bmatrix}\{-\nabla\cdot(u_k(\tau)\mathbf{v}_k(\tau))\}_k\\ \{f_\theta(\mathbf{u}(\tau), \nabla\mathbf{u}(\tau), \tau(t), \psi)_k\}_k\end{bmatrix} d\tau$$

Emission Model:

$$u_k^{\text{obs}}(\mathbf{x},t) \sim \mathcal{N}\left(u_k(\mathbf{x},t) + \mu_k(\mathbf{x},t), \sigma_k^2(\mathbf{x},t)\right), \; \mu_k(\mathbf{x},t), \sigma_k(\mathbf{x},t) = g_k\left(\mathbf{u}(\mathbf{x},t), \psi\right)$$

Loss Function:

$$\mathcal{L}_\theta = -\frac{1}{NKHW}\sum_{i=1}^{N}\left(\log\mathcal{N}\left(\mathbf{y}_i|\mathbf{u}(t_i) + \boldsymbol{\mu}(t_i), \operatorname{diag}\boldsymbol{\sigma}^2(t_i)\right) + \log\mathcal{N}_+\left(\boldsymbol{\sigma}(t_i)|\mathbf{0}, \lambda_\sigma^2 I\right)\right)$$

Experiments & Results

Critics