

## Problem Definition and Contribution

**Goal:** Enhancing climate forecasting by integrating physics-informed neural ordinary differential equations (ODEs) with uncertainty quantification.

### Motivations:

- Existing models neglect the underlying physics, lack of uncertainty quantification and are computationally intensive.
- Enhance efficiency and effectiveness in global and regional weather prediction tasks.

## Problem Formulation

**Statistical Mechanics:** Weather can be described as a spatial movement of quantities over time, governed by the partial differential continuity equation:

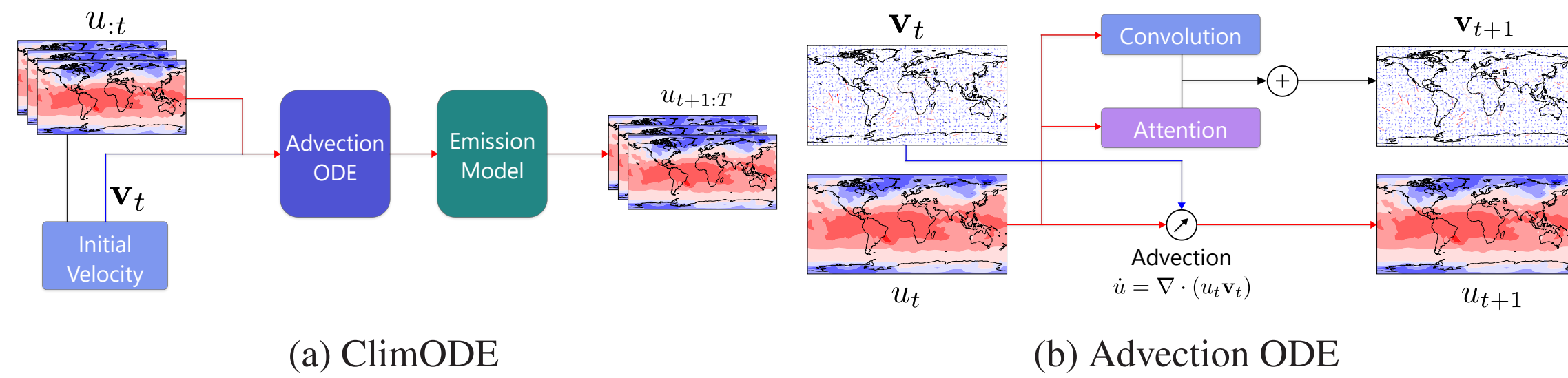
$$\underbrace{\frac{du}{dt}}_{\text{time evolution } \dot{u}} + \underbrace{\mathbf{v} \cdot \nabla u}_{\text{advection}} + \underbrace{u \nabla \cdot \mathbf{v}}_{\text{compression}} = \underbrace{s}_{\text{sources}},$$

where  $u(x, t)$  is a quantity evolving over space  $\mathbf{x}$  and time  $t$  driven by a flow's velocity  $\mathbf{v}(\mathbf{x}, t)$ .

**Main Idea:** We solve the continuity equation over entire Earth as a system of neural ODEs.

## Method

**Network Architecture:** Spatiotemporal embedding  $\psi$  encodes time and location information to capture cyclical patterns of day and season.



### Initial Velocity Inference:

$$\hat{\mathbf{v}}_k(t) = \arg \min_{\hat{\mathbf{v}}_k(t)} \left\{ \left\| \dot{u}_k(t) + \mathbf{v}_k(t) \cdot \nabla u_k(t) + u_k(t) \nabla \cdot \mathbf{v}_k(\mathbf{x}, t) \right\|_2^2 + \alpha \left\| \mathbf{v}_k(t) \right\| \right\}$$

**Advection ODE:** Modeling the dynamic evolution of quantities at specific locations by transforming second-order PDEs into first-order ODEs:

$$\begin{bmatrix} \mathbf{u}(t) \\ \mathbf{v}(t) \end{bmatrix} = \begin{bmatrix} \{u_k(t_0)\}_k \\ \{\mathbf{v}_k(t_0)\}_k \end{bmatrix} + \int_{t_0}^t \begin{bmatrix} \{-\nabla \cdot (u_k(\tau) \mathbf{v}_k(\tau))\}_k \\ \{f_\theta(\mathbf{u}(\tau), \nabla \mathbf{u}(\tau), \tau(t), \psi)_k\}_k \end{bmatrix} d\tau$$

**Flow Velocity:** Modeling local and global effects using a hybrid network:

$$\dot{\mathbf{v}}_k(\mathbf{x}, t) = f_{\text{conv}}(\mathbf{u}(t), \nabla \mathbf{u}(t), \mathbf{v}(t), \psi) + \gamma f_{\text{att}}(\mathbf{u}(t), \nabla \mathbf{u}(t), \mathbf{v}(t), \psi)$$

**Emission Model:** Accounting for uncertainty and value changes in climate estimates:

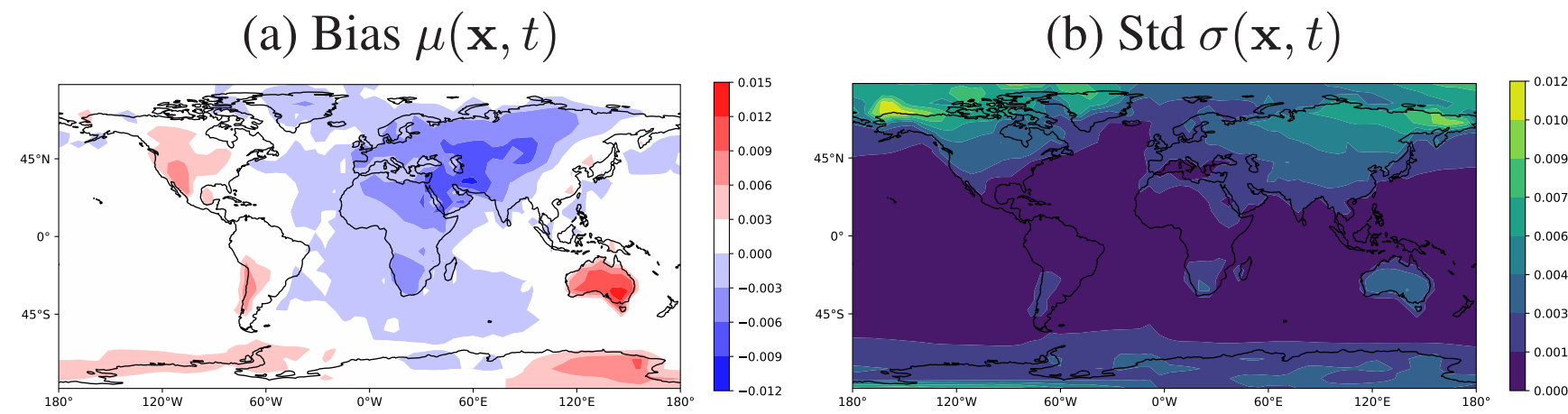
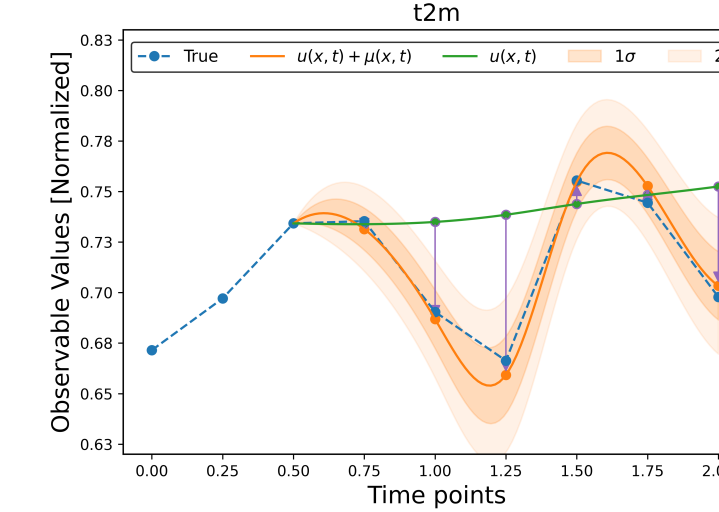
$$u_k^{\text{obs}}(\mathbf{x}, t) \sim \mathcal{N}(u_k(\mathbf{x}, t) + \mu_k(\mathbf{x}, t), \sigma_k^2(\mathbf{x}, t)), \quad \mu_k(\mathbf{x}, t), \sigma_k(\mathbf{x}, t) = g_k(\mathbf{u}(\mathbf{x}, t), \psi)$$

**Loss Function:** Gaussian negative log-likelihood:

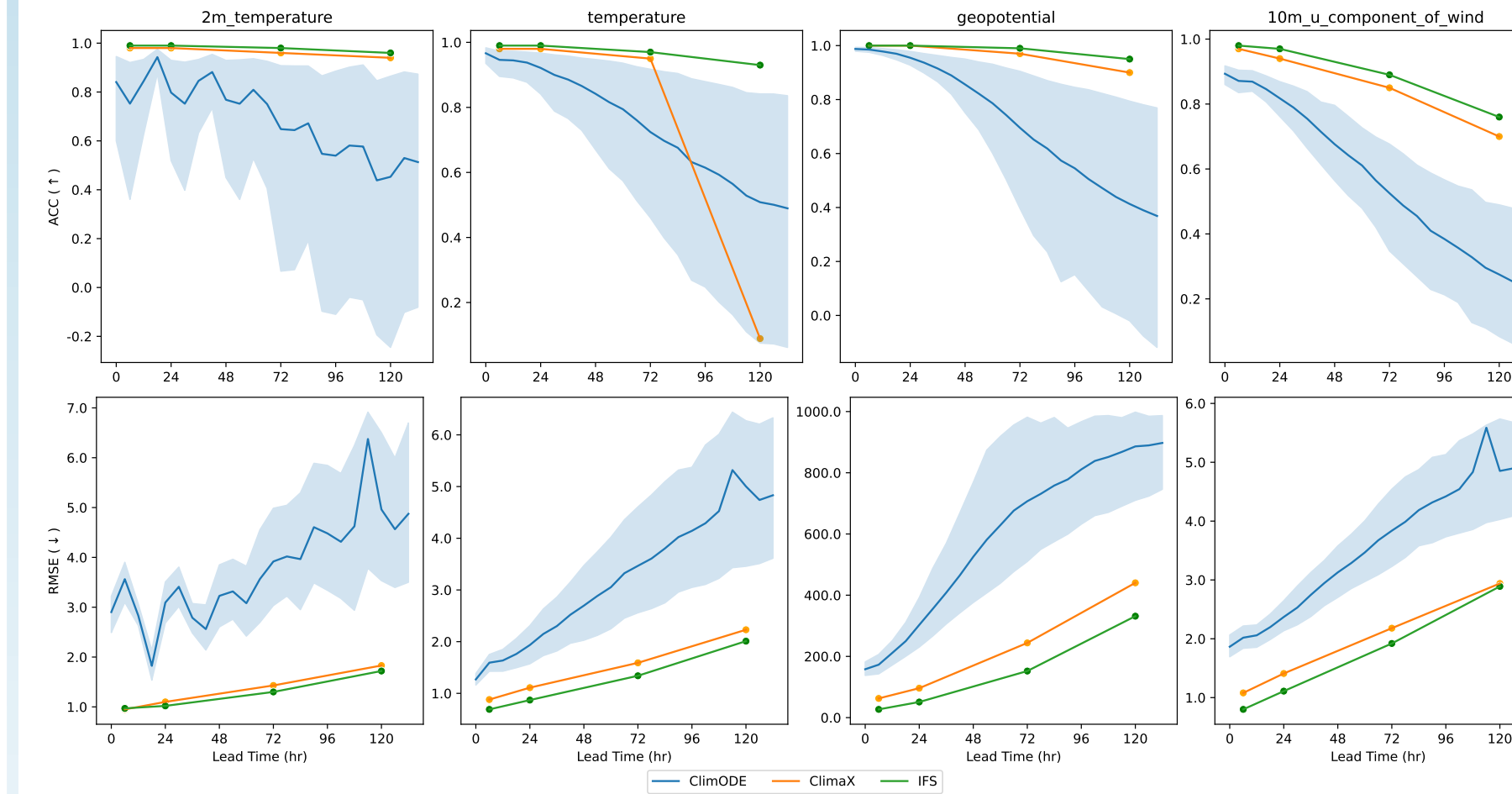
$$\mathcal{L}_\theta = -\frac{1}{NKH W} \sum_{i=1}^N \left( \log \mathcal{N}(\mathbf{y}_i | \mathbf{u}(t_i) + \boldsymbol{\mu}(t_i), \text{diag } \boldsymbol{\sigma}^2(t_i)) + \log \mathcal{N}_+(\boldsymbol{\sigma}(t_i) | \mathbf{0}, \lambda_\sigma^2 I) \right)$$

## Experiments & Results

### Effect of Emission Model:



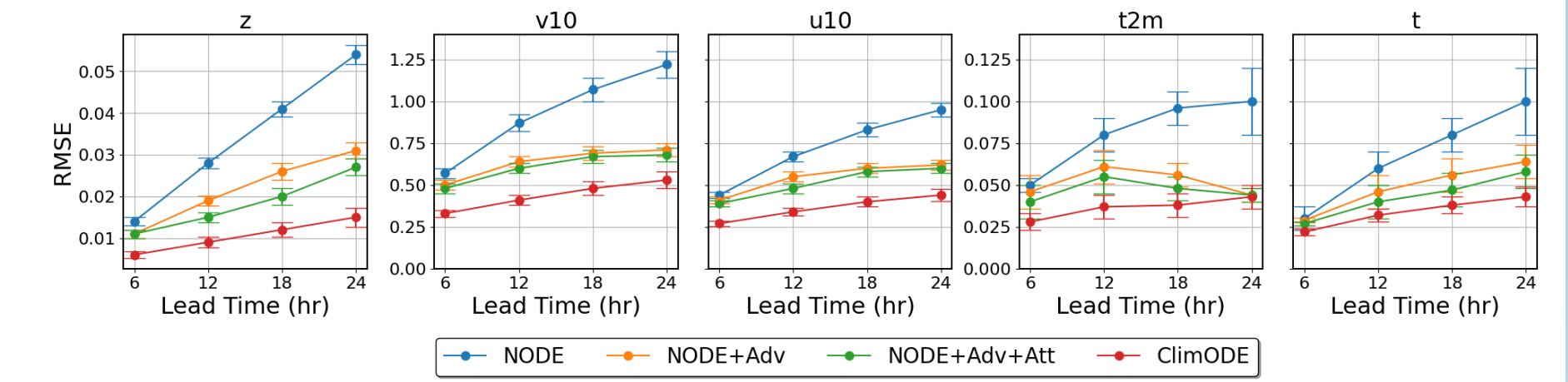
### Quantitative Results and Comparison on ERA5:



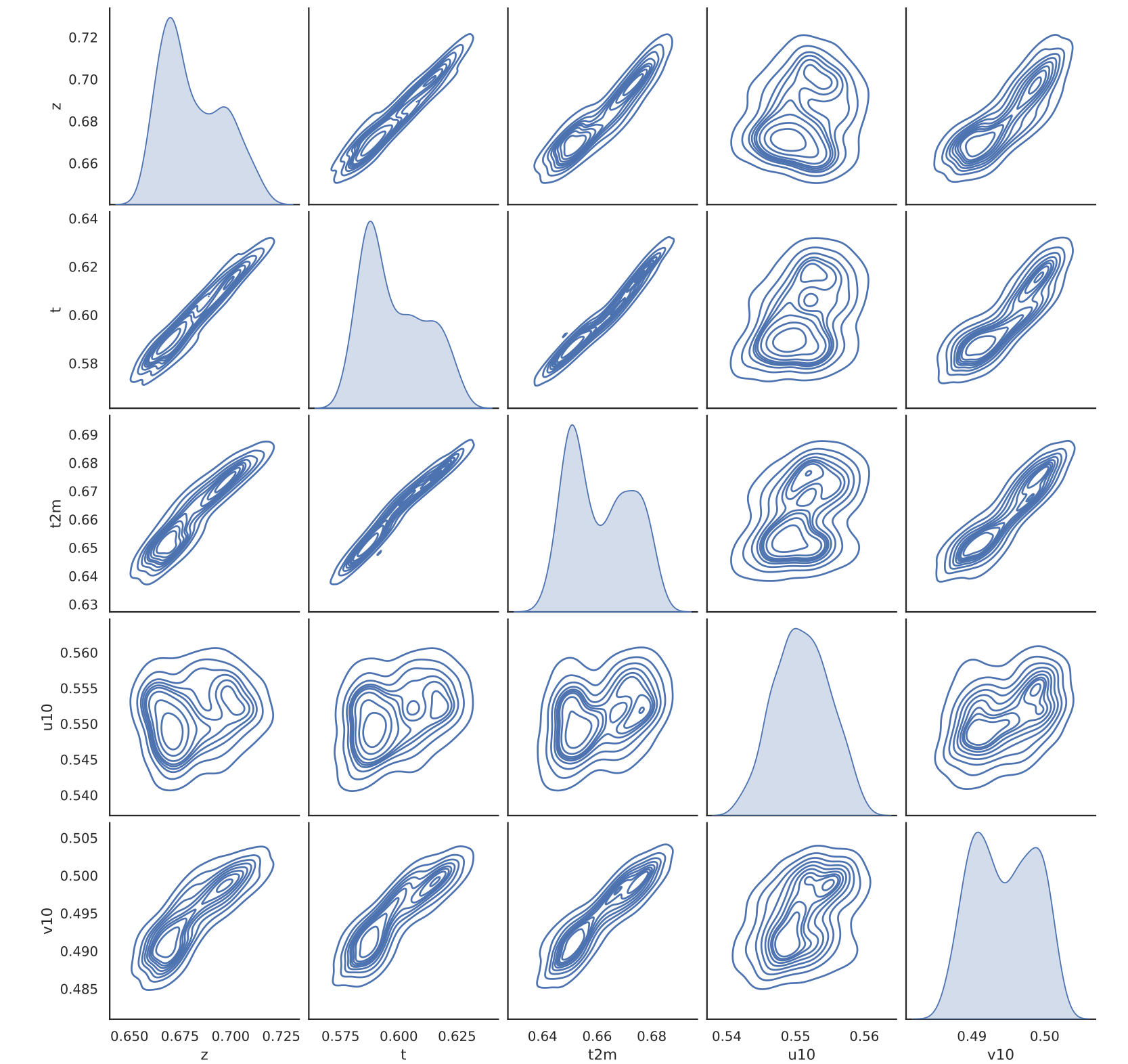
### Effect of ODE Solver (per batch, on a single A100):

Solver	Euler		RK4		Implicit Adams	
Adjoint	w	w/o	w	w/o	w	w/o
Time (s)	0.27	0.18	0.34	0.31	0.69	0.45
VRAM (Gb)	10.2	15.7	11.8	17.8	13.7	30.1

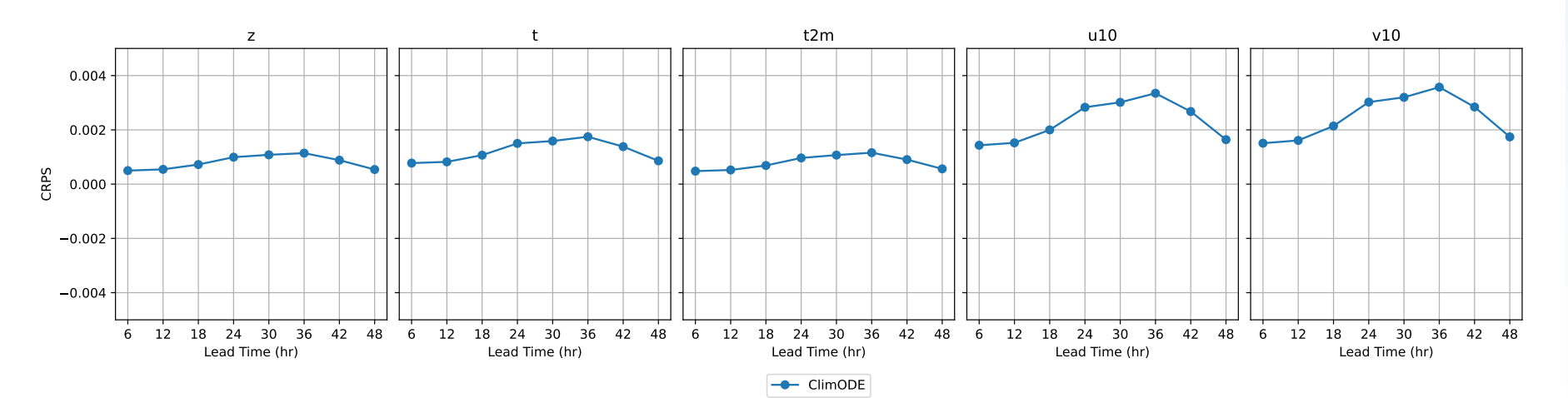
### Effect of Individual Components:



### Quantifying Climate Interdependencies:



### Validity of Mass Conservation:



## Critics

### Paper:

- Lacks comparison to other models, such as FourCastNet, FuXi or Pangu-Weather (WeatherBench2)
- Is the comparison against Climax fair?
- Benchmark only up to 36 hours while state-of-the-art methods reported results for up to 10 days ahead
- Why choose ResNet over U-Net in the Emission Model?

### Implementation (original code):

- Concerns about data leakage
- Absence of shuffling during training: impact?
- Unclear handling of residual architecture and dropout
- Code quality critique: Concerning coding standards
- Using Euler scheme to solve the ODE is known to be unstable but is a good compromise in terms of accuracy and efficiency