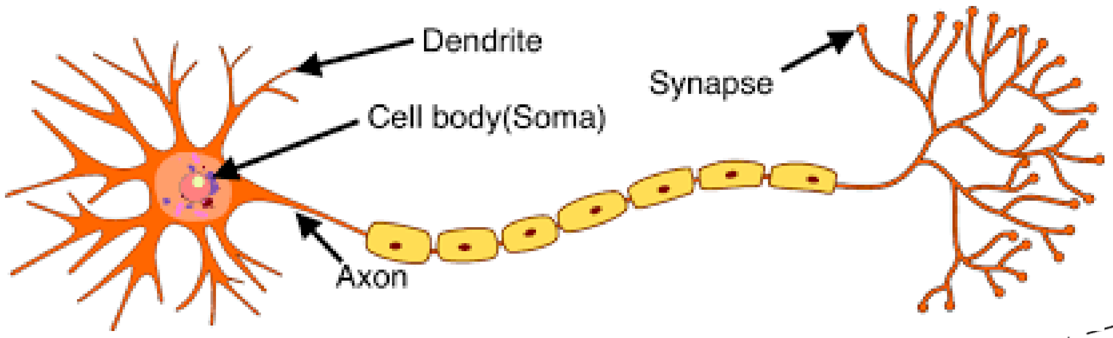
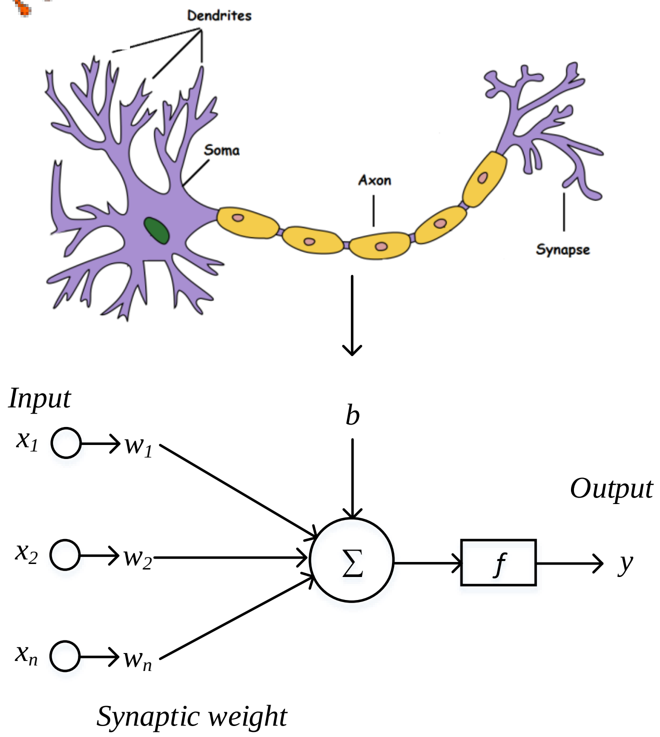
AI - FOUNDATION AND APPLICATION

Instructor:
Assoc. Prof. Dr. Truong Ngoc Son

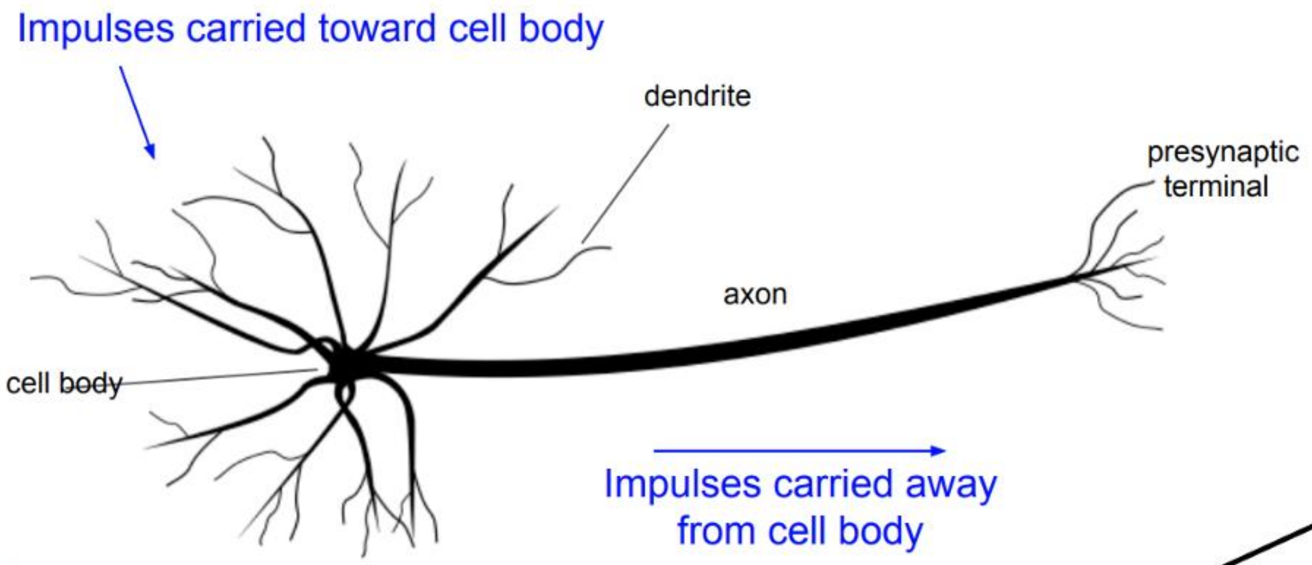
Chapter 1
Introduction of Neural network

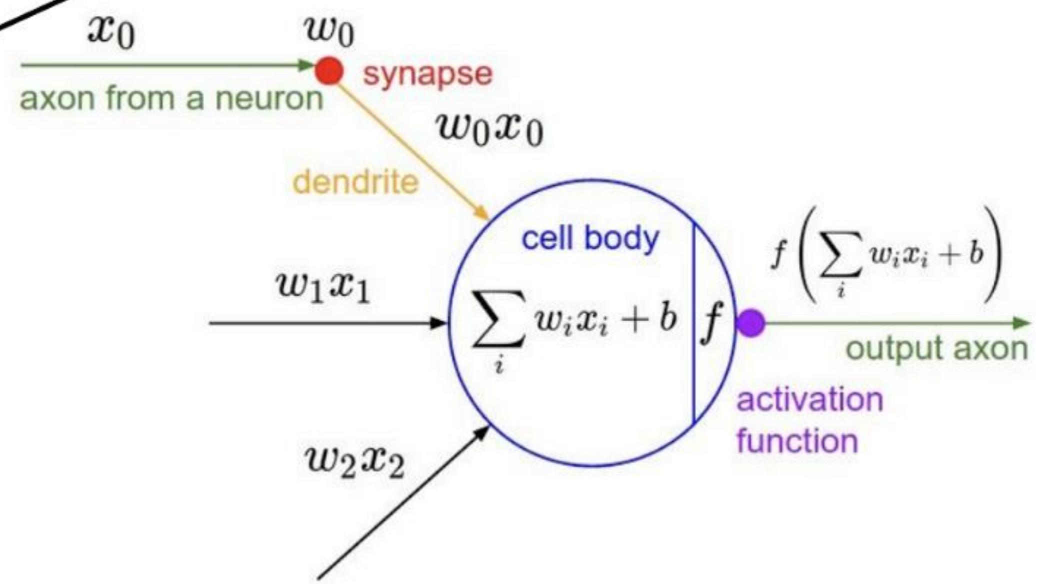
How a neuron is modelled?





How a neuron is modelled ?



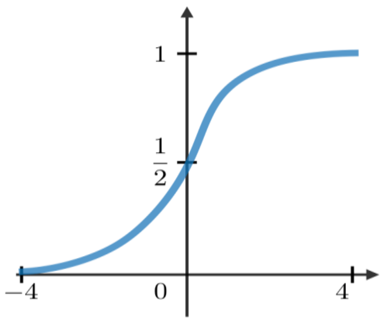


Training a network – Optimization method

Activation functions

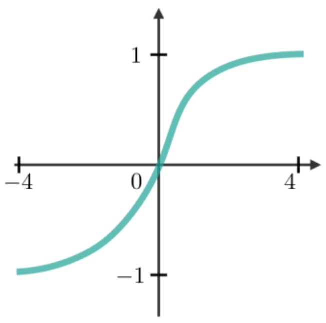
Sigmoid function

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$



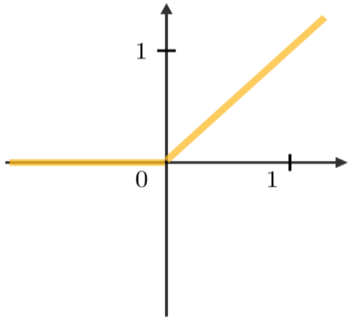
Tanh function

$$f(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

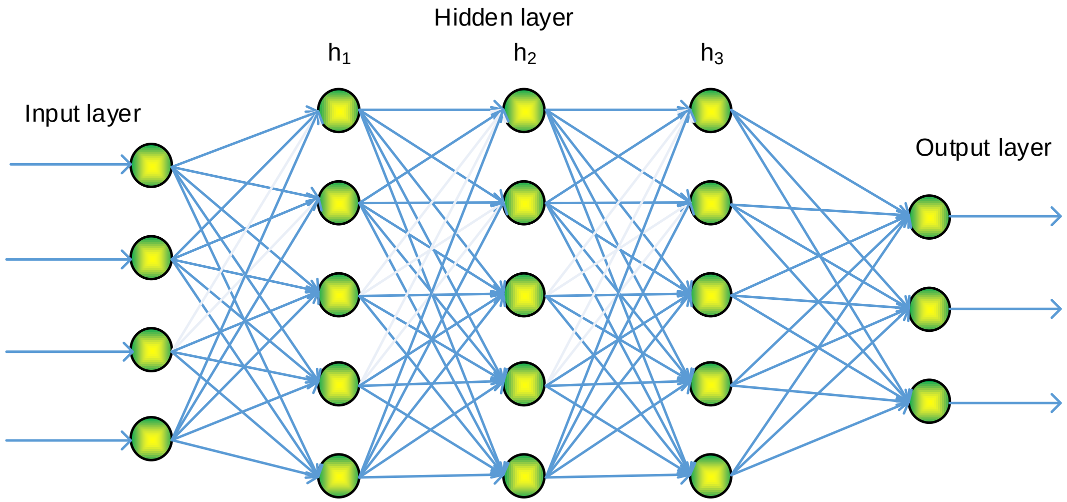


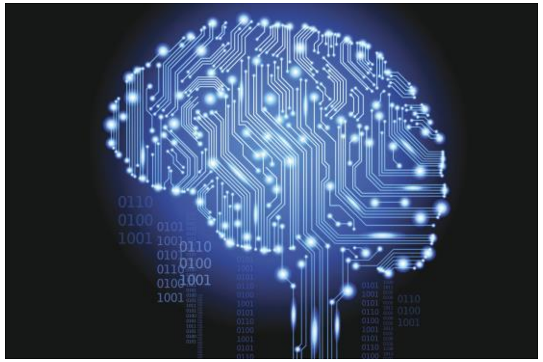
ReLU function

$$f(x) = \max(x, 0)$$

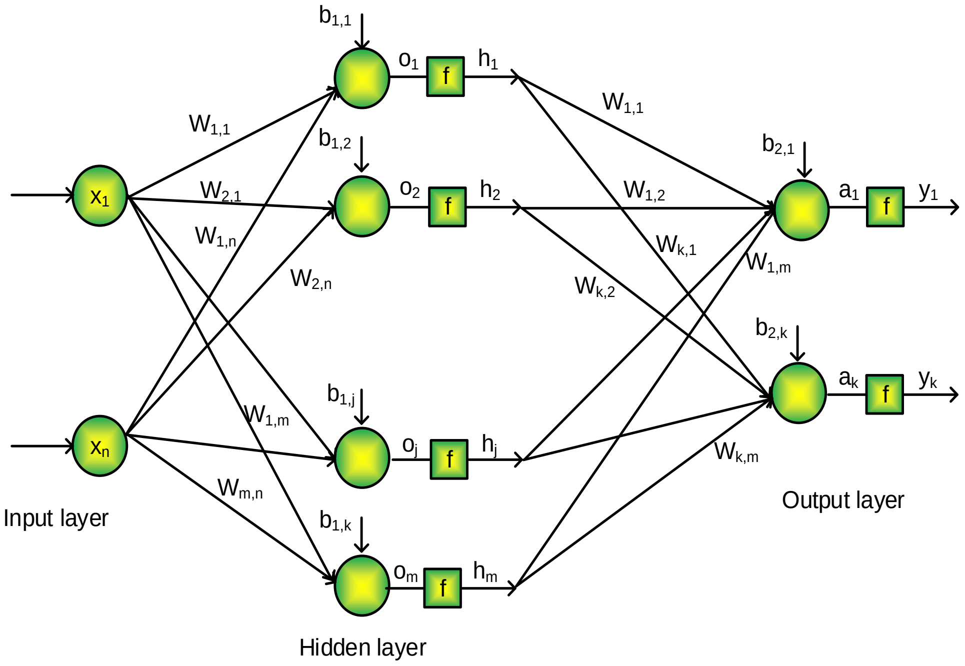


Neural network





Artificial neuron network

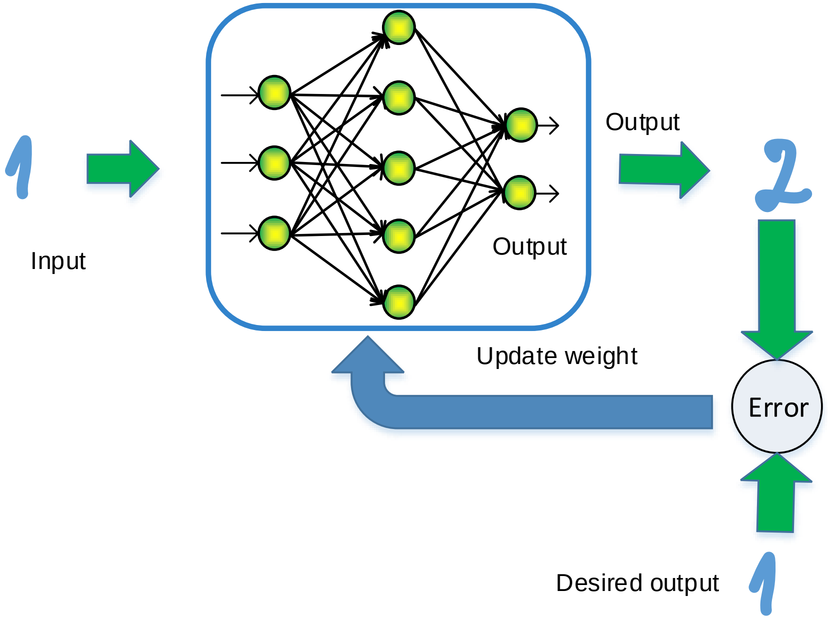


$$o_1 = x_1 W_{1,1} + \ldots + x_n W_{1,n} + b_{1,1}$$
$$h_1 = f(o_1)$$
$$o_2 = x_1 W_{2,1} + \ldots + x_n W_{2,n} + b_{1,2}$$
$$h_2 = f(o_2)$$
$$a_1 = h_1 W_{1,1} + \ldots + h_m W_{1,m} + b_{2,1}$$
$$y_1 = f(a_1)$$
$$a_k = h_1 W_{k,1} + \ldots + h_m W_{k,m} + b_{2,k}$$
$$y_k = f(a_k)$$

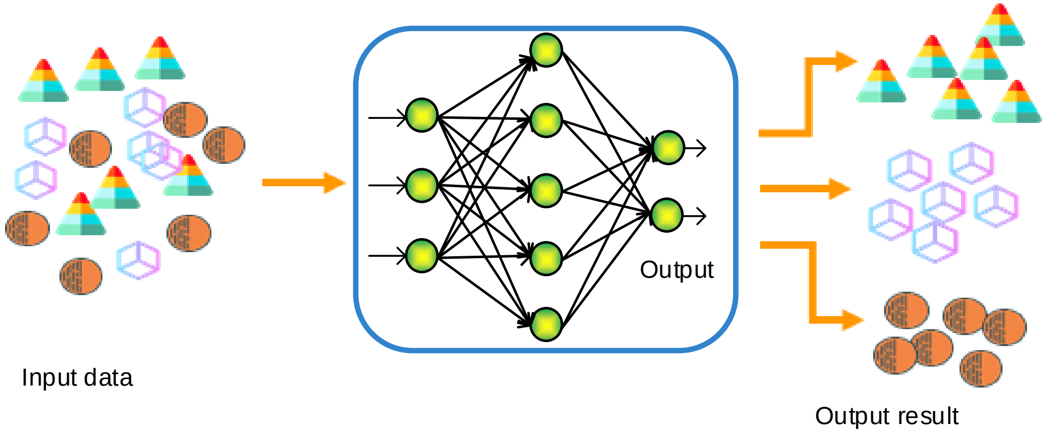
TRAINING NEURAL NETWORK

Supervised learning vs. unsupervised learning

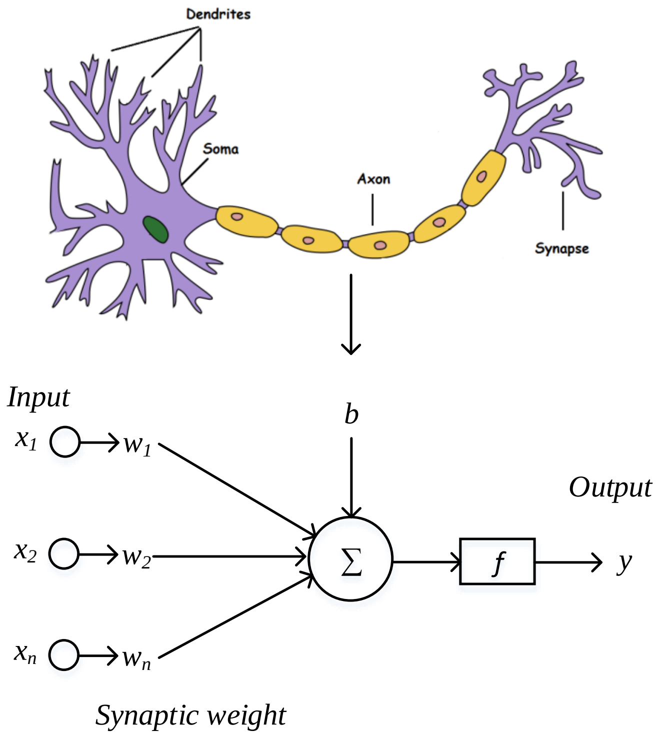
Training an artificial neural network
Supervised learning

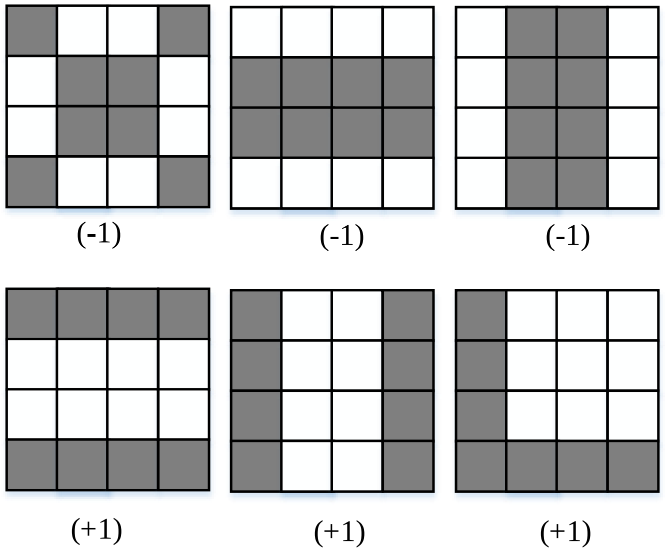


Training an artificial neural network
Unsupervised learning



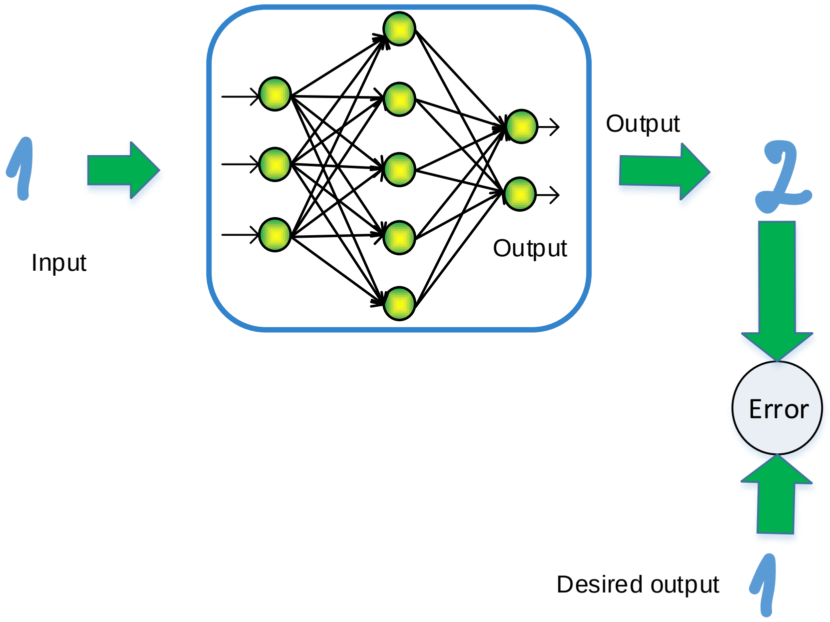
Simple neural network: Understanding of neuron network learning





Quantifying the loss

The loss of a network measure the cost incurred from incorrect prediction



$$\mathcal{L}\big(\underbrace{f\left(x^{(i)}; \boldsymbol{W}\right)}_{\text{Predicted}}, \underbrace{y^{(i)}}_{\text{Actual}}\big)$$

MSE: mean squared error

$$MSE = \frac{1}{n} \Sigma \underbrace{\left(y - \widehat{y} \right)^2}_{\text{The square of the difference between actual and predicted}}$$

Cross-entropy loss

$$\text{Loss} = -\sum_{i=1}^{\substack{\text{output}\\ \text{size}}} y_i \cdot \log \hat{y}_i$$

Training a network

Training a neural network is a process of **using an optimization algorithm** to find a set of weights to best map inputs to outputs.

In other word, this is the way to minimize the loss

$$W^* = \underset{w}{\operatorname{argmin}} \frac{1}{n} \sum_{i=1}^{n} \mathcal{L}(f(x^i, W), y^i)$$

So hard? Don't worry, we will dive into the detail later

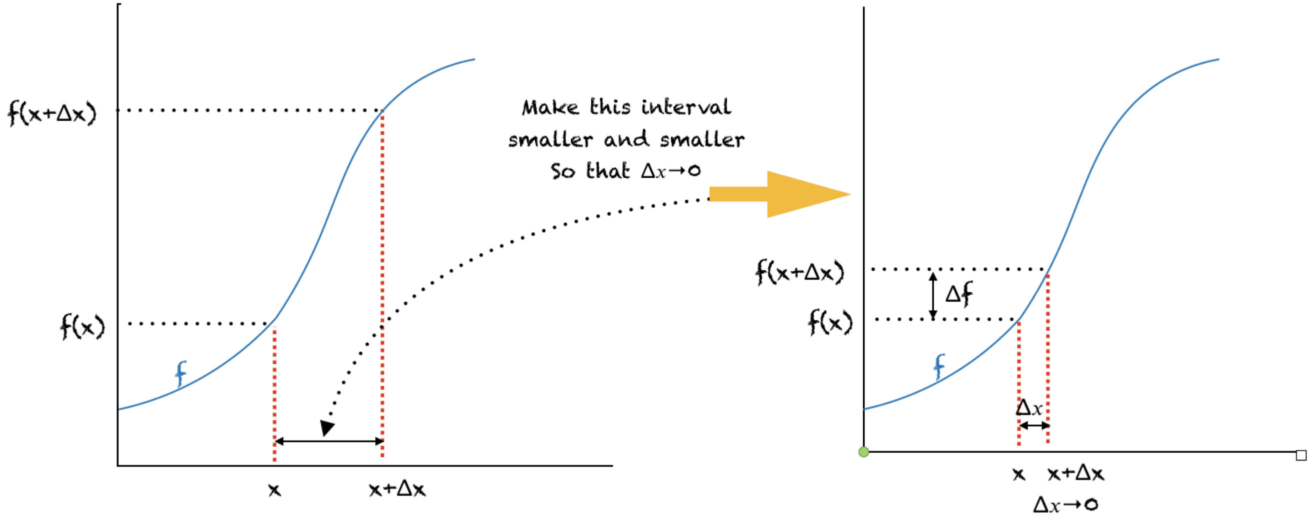
GRADIENT DESCENT

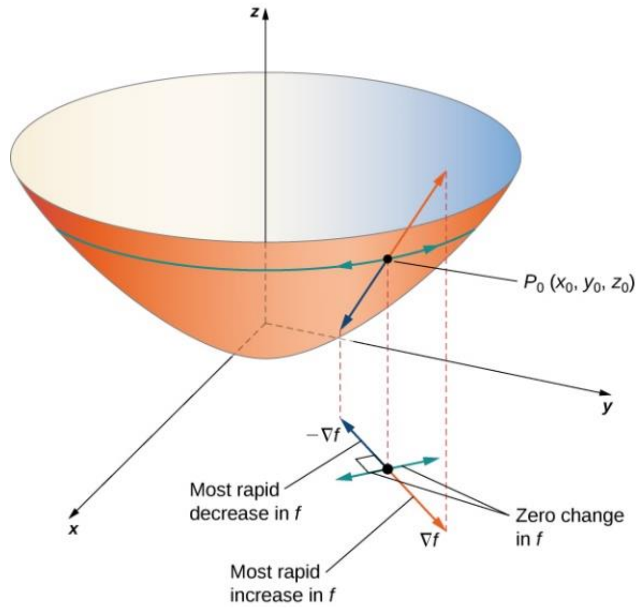
Training Neural Networks– Optimization of the loss

What is Gradient ?

Derivative of f is the rate of change of f

$$f'(x) = \frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$





$f(x, y, z)$

Gradient of f

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right]$$

$$\nabla f = \left[\frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k\right]$$

f

$$[x, y, z] = \left[x - \frac{\partial f}{\partial x}, y - \frac{\partial f}{\partial y}, z - \frac{\partial f}{\partial z}\right]$$

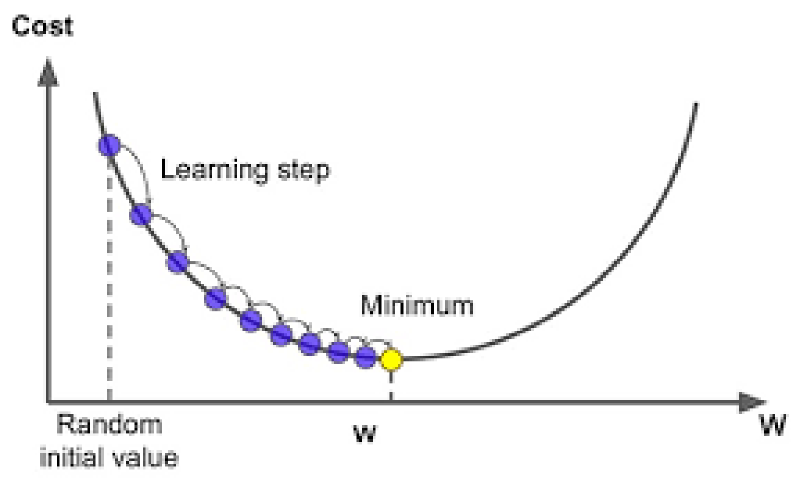
Training a network – Optimization of the loss

Gradient Descent

Gradient descent is an optimization algorithm used to find the values of parameters (coefficients) of a function (f) that minimizes a cost function (loss). This can be done by iteratively moving in the direction of steepest descent as defined by the negative of the gradient

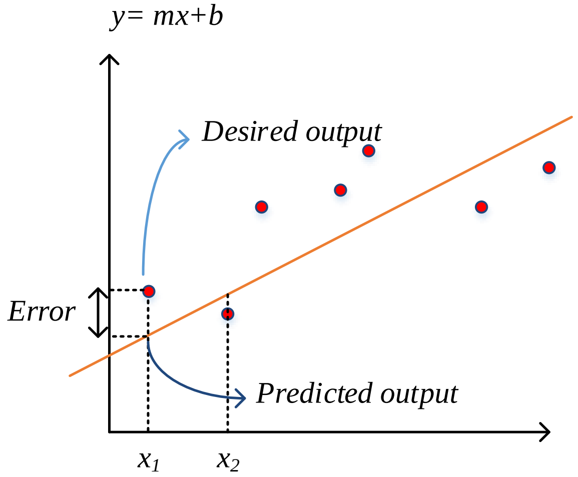
$$\vec{x}_0 = (x_0, y_0, z_0)$$
$$\vec{x}_{n+1} = \vec{x}_n - \eta \nabla f(\vec{x}_n)$$

$$[x, y, z] = \left[x - \eta \frac{\partial f}{\partial x}, y - \eta \frac{\partial f}{\partial y}, z - \eta \frac{\partial f}{\partial z}\right]$$



Optimization of the loss with gradient descent

Example: Linear Regression



$$loss = \sum_{i=1}^{n} (o_i - y_i)^2$$

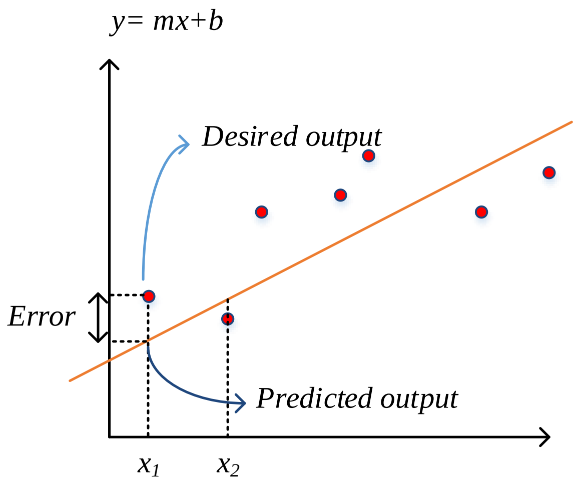
$m = m + \Delta m$

$b = b + \Delta b$

(loss)↘

Optimization of the loss with gradient descent

Assignment 01: Logistic Regresstion



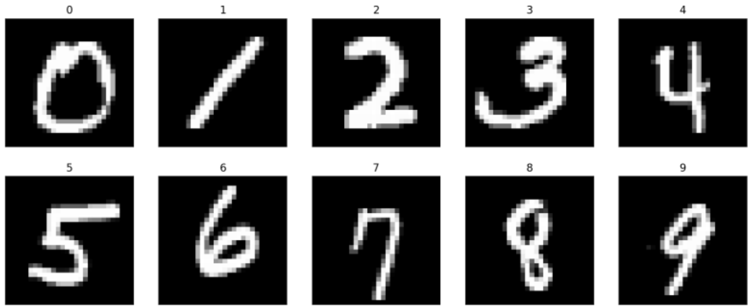
$$loss = \sum_{i=1}^{n} (o_i - y_i)^2$$

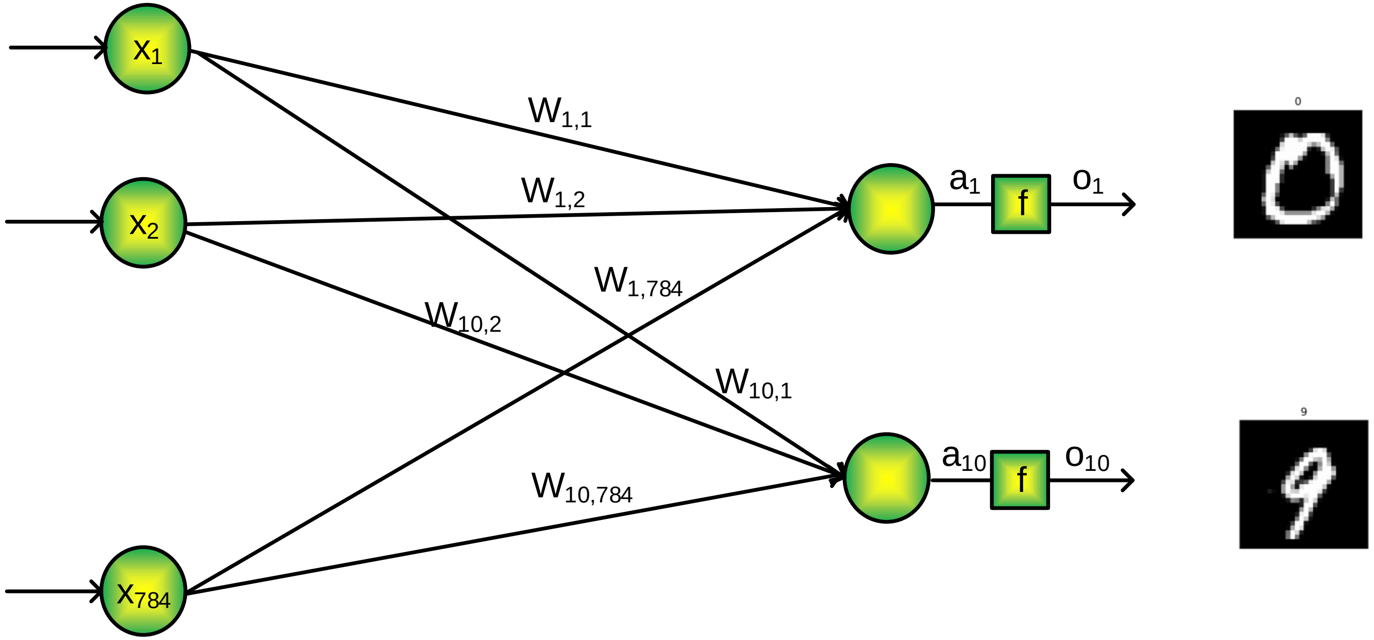
$m = m + \Delta m$

$b = b + \Delta b$

(loss)

Training Neural networks – Optimization method

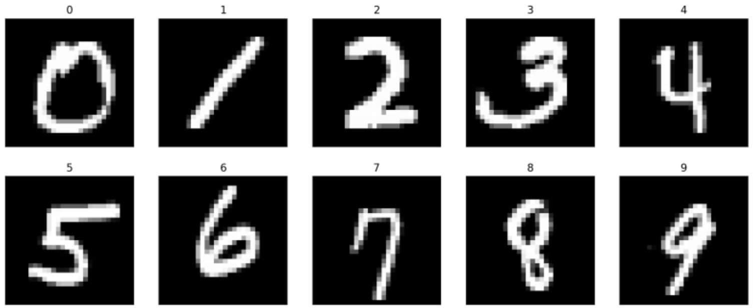


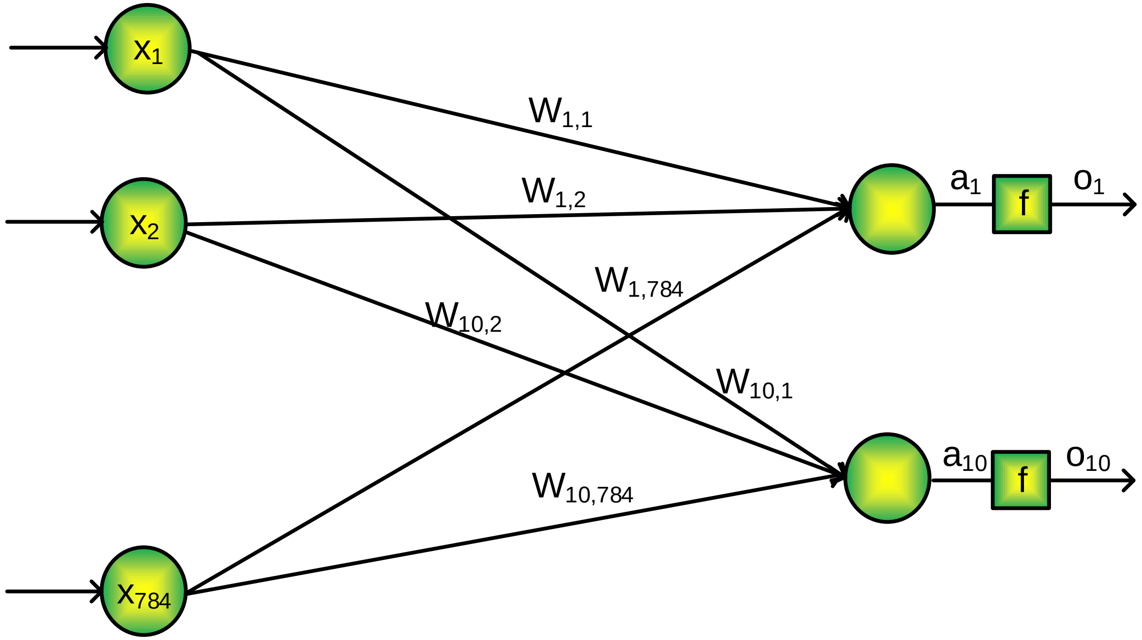


LOSS OPTIMIZATION WITH GRADIENT DESCENT

Mathematical modeling of Training Process

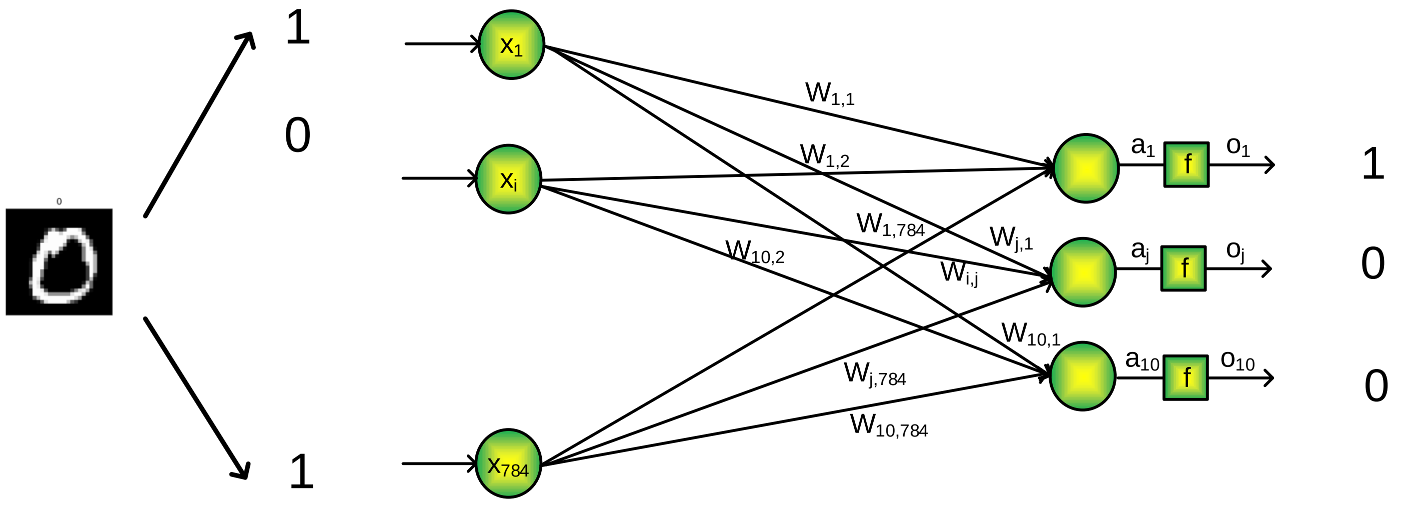
Example



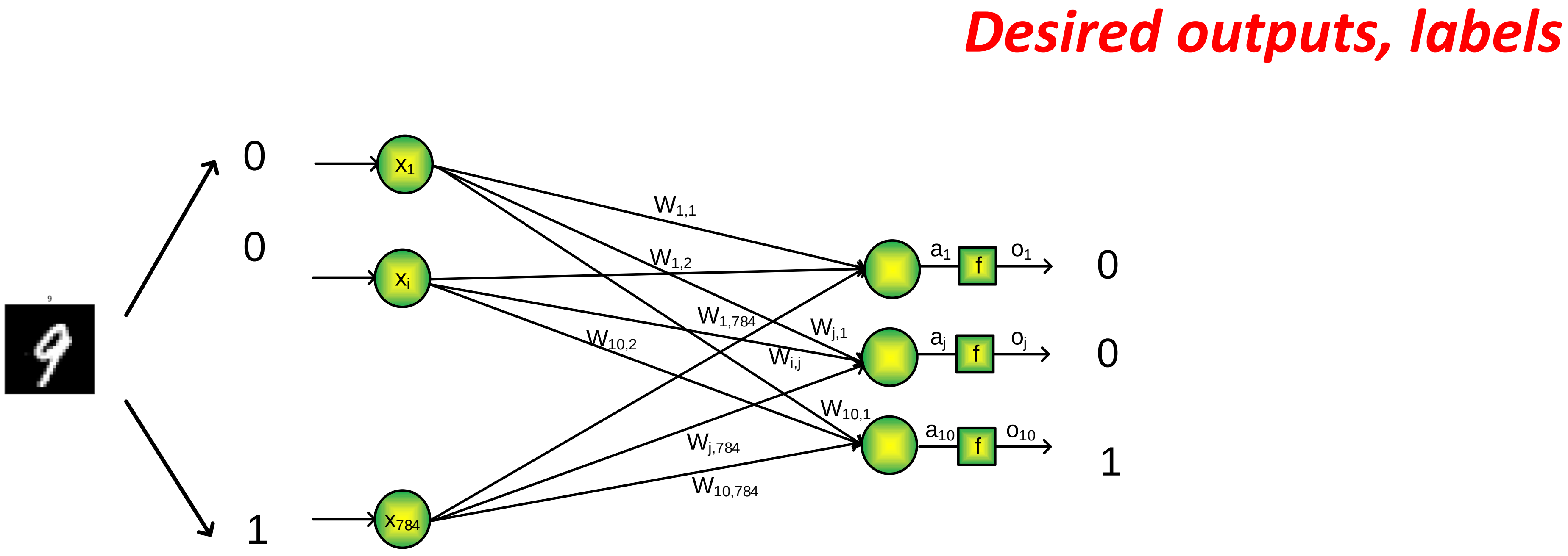


Mathematical modeling of Training Process

Desired outputs, labels



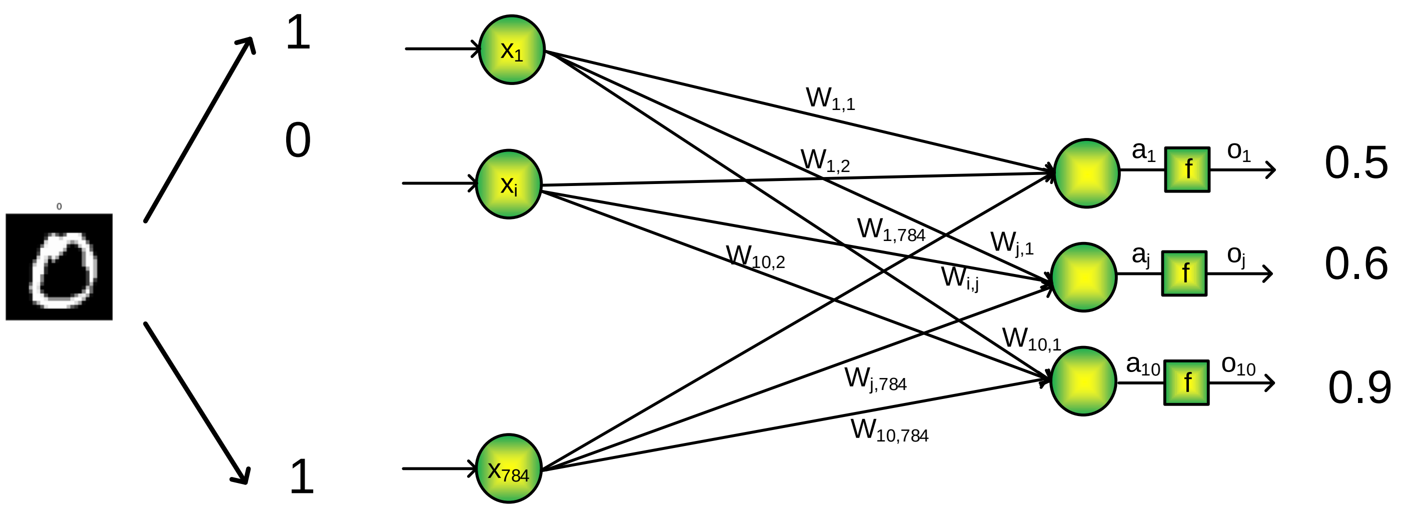
Mathematical modeling of Training Process



Mathematical modeling of Training Process

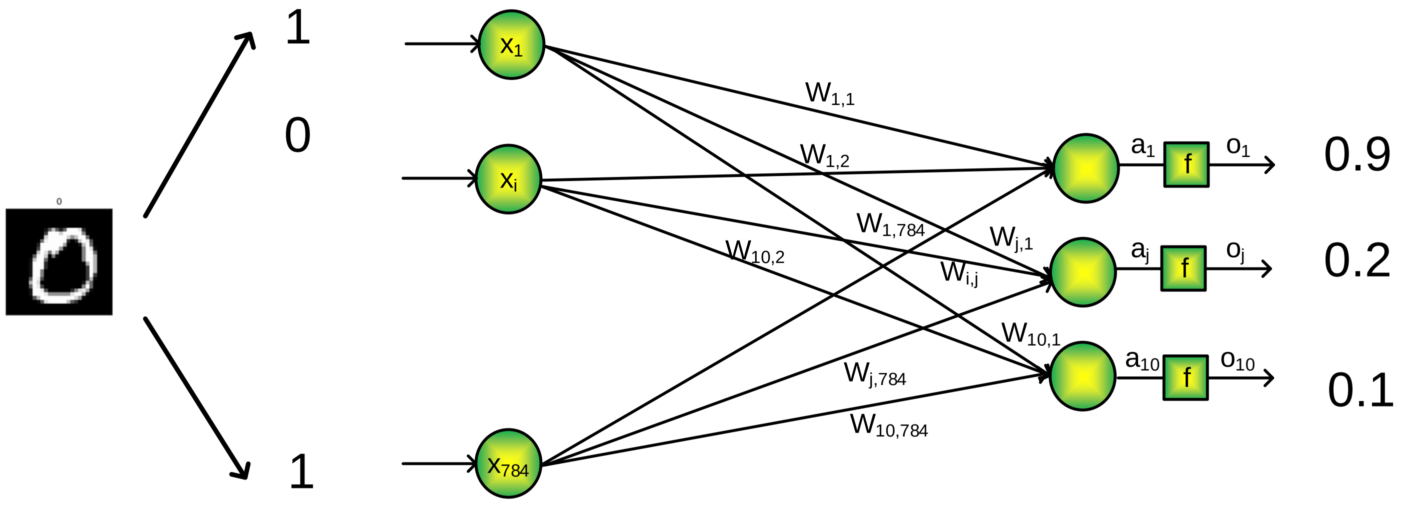
Randomly initialize

Weights, W



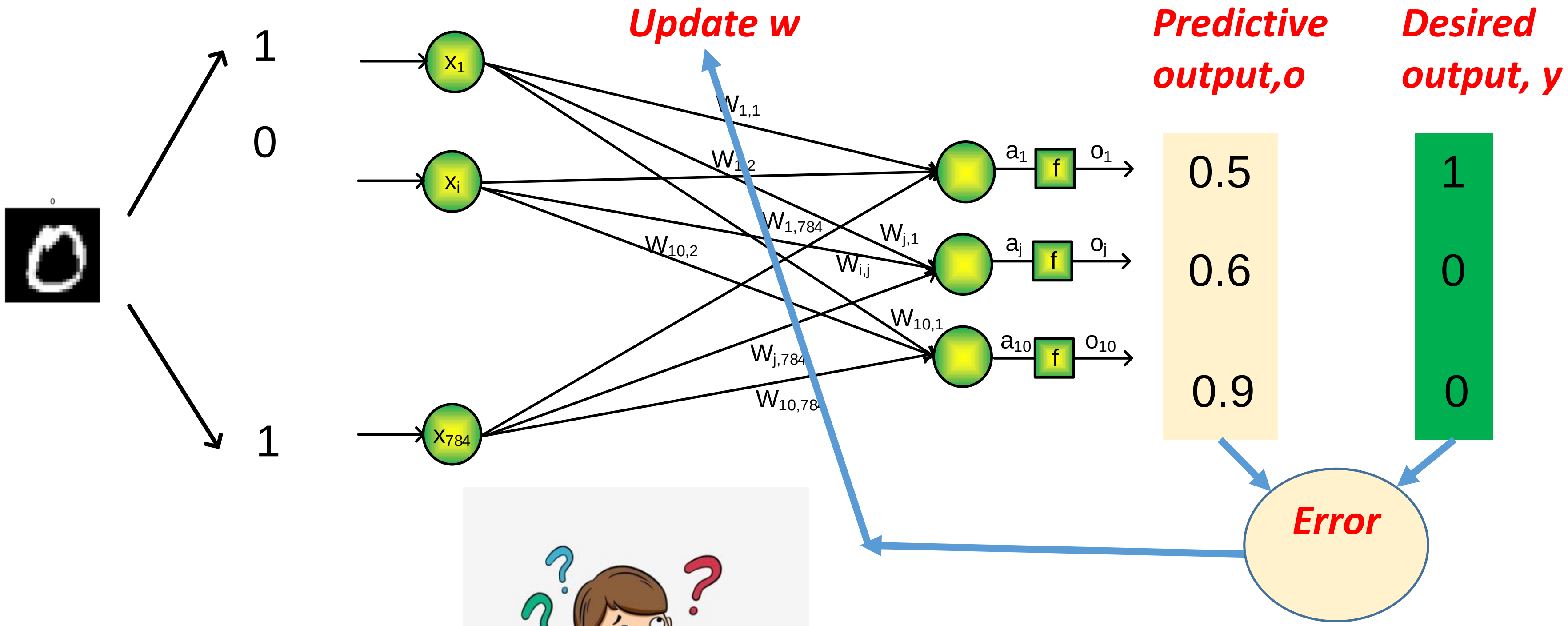
Mathematical modeling of Training Process

W = ArgMin (Loss)

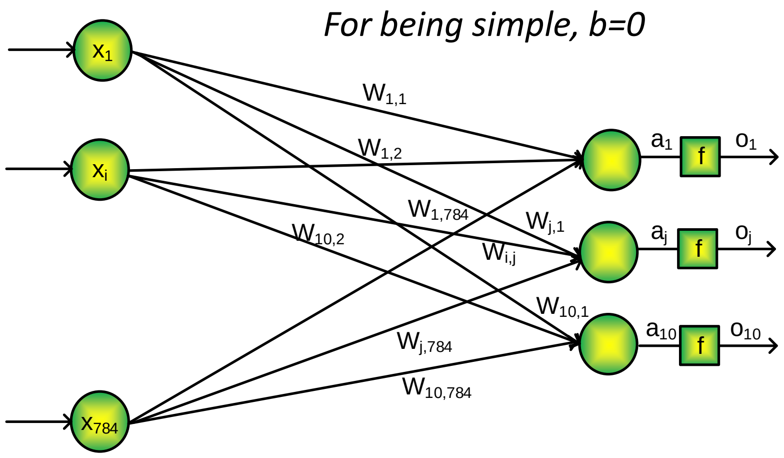


Mathematical modeling of Training Process

Training process



Mathematical modeling of Training Process



Formulate the output

$$a_j = \sum_{i=1}^{784} x_i w_{j,i}$$

$$o_j = \sigma(a_j) = \frac{1}{1+e^{-a_j}}$$

For j^{th} output

loss

$$L = \frac{1}{10}\sum_{t=1}^{10}\sum_{j=1}^{10}\left(y_j^t - o_j^t\right)^2$$

For j^{th} output

$$L = \frac{1}{10}\sum_{t=1}^{10}\left(y_j^t - o_j^t\right)^2$$

For j^{th} output

$$a_j = \sum_{i=1}^{784} x_i w_{j,i}$$

$$o_j = \sigma(a_j) = \frac{1}{1+e^{-a_j}}$$

Gradient descent

$$w_{j,i} \leftarrow w_{j,i} - \eta \frac{\partial L}{\partial w_{j,i}}$$

$$\frac{\partial L}{\partial w_{j,i}} = -\frac{2}{10}\sum_{t=1}^{10}\left(y_j^t - o_j^t\right)\left(\frac{\partial o_j^t}{\partial w_{j,i}}\right)$$

$$\frac{\partial L}{\partial w_{j,i}} = -\frac{2}{10}\sum_{t=1}^{10}\left(y_j^t - o_j^t\right)\left(\frac{\partial o_j^t}{\partial a_j^t}\frac{\partial a_j^t}{\partial w_{j,i}}\right)$$

$$o_j^t = \sigma(a_j^t) = \frac{1}{1+e^{-a_j^t}} \qquad \frac{\partial o_j^t}{\partial a_j^t} = o_j^t(1-o_j^t)$$

$$\frac{\partial L}{\partial w_{j,i}} = -\frac{2}{10}\sum_{t=1}^{10}\left(y_j^t - o_j^t\right)o_j^t(1-o_j^t)x_i^t$$

$$w_{j,i} = w_{j,i} + \Delta w_{j,i}$$

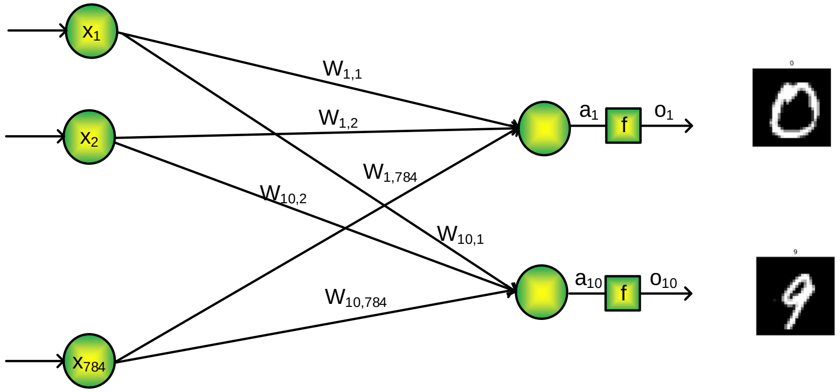
$$\Delta w_{j,i} = \eta\frac{2}{10}\sum_{t=1}^{10}\left(y_j^t - o_j^t\right)o_j^t(1-o_j^t)x_i^t$$

PYTHON CODE

Translating mathematics into code

Translating mathematics into code

MNIST Dataset



60,000 training samples
10,000 testing samples

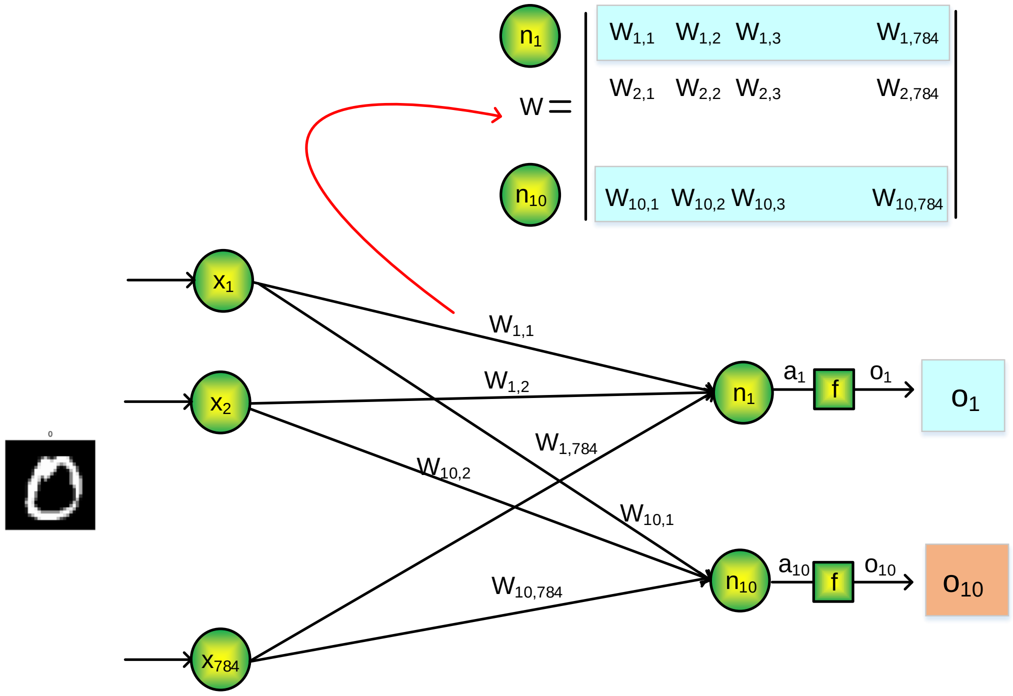
Gradient descent

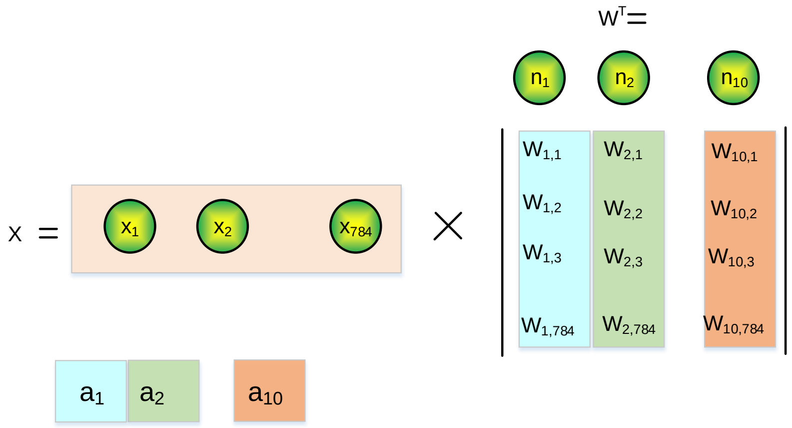
$$\frac{\partial L}{\partial w_{j,i}} = -\frac{2}{10}\sum_{t=1}^{10}(y_j^t - o_j^t)o_j^t(1 - o_j^t)x_i^t$$

$$w_{j,i} = w_{j,i} + \Delta w_{j,i}$$

$$\Delta w_{j,i} = \eta\frac{2}{10}\sum_{t=1}^{10}(y_j^t - o_j^t)o_j^t(1 - o_j^t)x_i^t$$

Neuron’s output





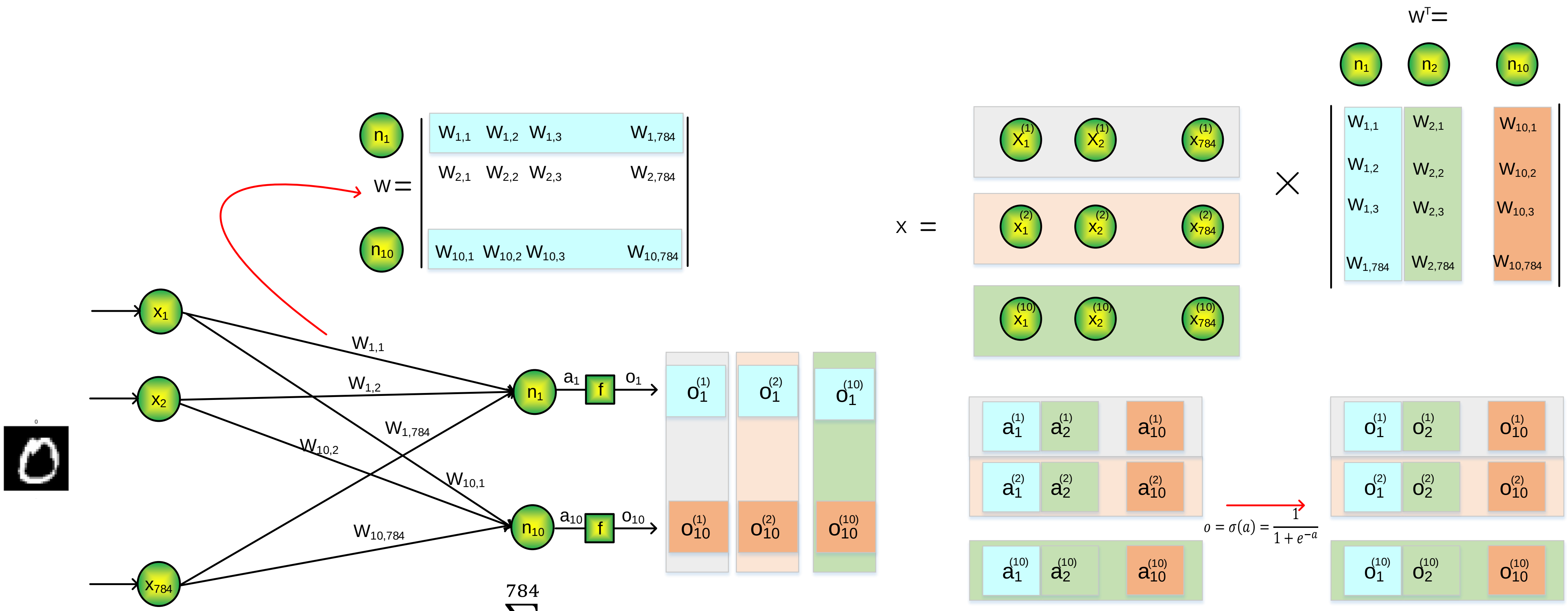
$$a_j = \sum_{i=1}^{784} x_i w_{j,i}$$

$$o_j = \sigma(a_j) = \frac{1}{1 + e^{-a_j}}$$

$$a = XW^T$$

$$o = \sigma(a) = \frac{1}{1 + e^{-a}}$$

Neuron’s output - batch of neurons



$$a_j = \sum_{i=1}^{784} x_i w_{j,i}$$

$$o_j = \sigma(a_j) = \frac{1}{1+e^{-a_j}}$$

$$a = XW^T$$

$$o = \sigma(a) = \frac{1}{1+e^{-a}}$$

Gradient calculating

$$\frac{\partial L}{\partial w_{j,i}} = -\frac{2}{10}\sum_{t=1}^{10}(y_j^t - o_j^t)o_j^t(1-o_j^t)x_i^t$$

$$w_{j,i} = w_{j,i} + \Delta w_{j,i}$$

$$\Delta w_{j,i} = \eta\frac{2}{10}\sum_{t=1}^{10}(y_j^t - o_j^t)o_j^t(1-o_j^t)x_i^t$$

$$d = \begin{bmatrix} d_1^{(1)} & d_2^{(1)} & \cdots & d_{10}^{(1)} \\ d_1^{(2)} & d_2^{(2)} & \cdots & d_{10}^{(2)} \\ \vdots & \vdots & & \vdots \\ d_1^{(10)} & d_2^{(10)} & \cdots & d_{10}^{(10)} \end{bmatrix}$$

$$d_j^t = (y_j^t - o_j^t)o_j^t(1-o_j^t)$$

(element-wise product)

$$d^T = \begin{bmatrix} d_1^{(1)} & d_1^{(2)} & \cdots & d_1^{(10)} \\ d_2^{(1)} & d_2^{(2)} & \cdots & d_2^{(10)} \\ \vdots & \vdots & & \vdots \\ d_{10}^{(1)} & d_{10}^{(2)} & \cdots & d_{10}^{(10)} \end{bmatrix} \qquad X = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_{784}^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_{784}^{(2)} \\ \vdots & \vdots & & \vdots \\ x_1^{(10)} & x_2^{(10)} & \cdots & x_{784}^{(10)} \end{bmatrix}$$

$$\eta\frac{2}{10}\, d^T \times X$$

$$\Delta w_{j,i} = \eta\frac{2}{10}\sum_{t=1}^{10}(y_j^t - o_j^t)o_j^t(1-o_j^t)x_i^t$$

$$\Delta W = \eta\frac{2}{10}\begin{vmatrix} \Delta W_{1,1} & \Delta W_{1,2} & \Delta W_{1,3} & \cdots & \Delta W_{1,784} \\ \Delta W_{2,1} & \Delta W_{2,2} & \Delta W_{2,3} & \cdots & \Delta W_{2,784} \\ \vdots & \vdots & \vdots & & \vdots \\ \Delta W_{10,1} & \Delta W_{10,2} & \Delta W_{10,3} & \cdots & \Delta W_{10,784} \end{vmatrix}$$

$$d = (y-o)o(1-o) \longleftarrow$$ (element-wise product)

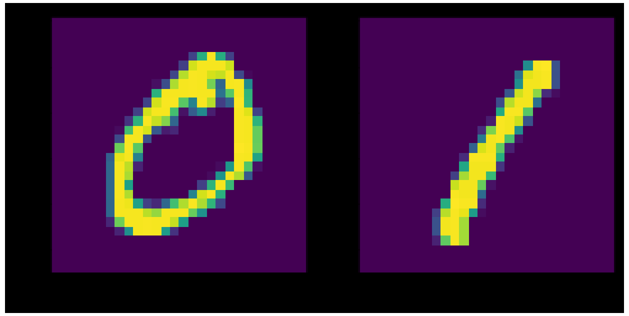
$$\Delta w = \eta\frac{2}{10}d^T X$$

Load data set, pick out 10 images

```
import numpy as np
import tensorflow as tf
import matplotlib.pyplot as plt
print('load data from MNIST')
mnist = tf.keras.datasets.mnist
(x_train,y_train), (x_test,y_test) = mnist.load_data()
dig = np.array([1,3,5,7,9,11,13,15,17,19]) # get the digit 0 - 9
x = x_train[dig,:,:]
y = np.eye(10,10)
plt.subplot(121)
plt.imshow(x[0])
plt.subplot(122)
plt.imshow(x[1])
x = np.reshape(x,(-1,784))/255
```

$$d = (y - o)o(1 - o)$$

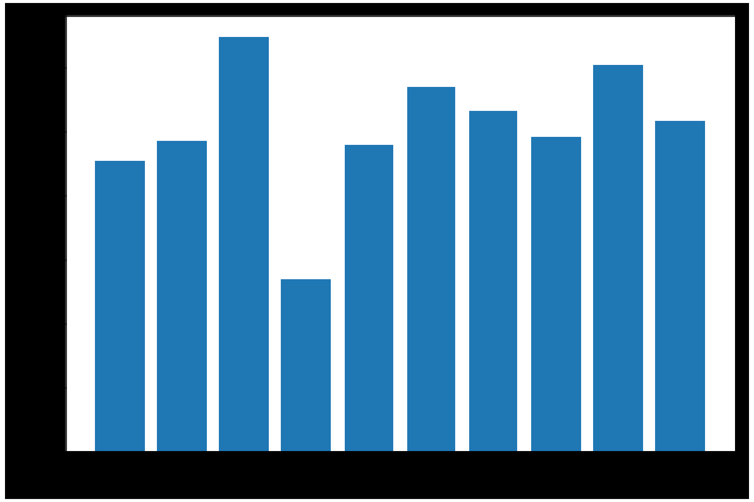
$$\Delta w = \eta \frac{2}{10} d^T X$$



Define parameters and functions

```
def sigmoid(x):
  return 1./(1.+np.exp(-x))

W = np.random.uniform(-0.1,0.1,(10,784))
o = sigmoid(np.matmul(x,W.transpose())) # matrix multiplication
print('output of first neuron with 10 digits ', o[:,0])
fig = plt.figure()
plt.bar([i for i, _ in enumerate(o)],o[:,0])
plt.show()
```



Training

```
#training process
n = 0.05
num_epoch  = 10
for epoch in range(num_epoch):
  o = sigmoid(np.matmul(x,W.transpose()))
  loss =np.power(o-y,2).mean()
  #calculate update for all wegihts in matrix
  dW =np.transpose((y-o)*o*(1-o))@x
  #update
  W=W+n*dW
  print(loss)

o = sigmoid(np.matmul(x,W.transpose()))
print('output of the first neuron  with 10 input digits ', o[:,0])

fig = plt.figure()
plt.bar([i for i, _ in enumerate(o)],o[:,0])
plt.show()
```

$$\Delta w = \eta \frac{2}{10} d^T X$$

$$d = (y-o)o(1-o)$$

This is just a simple example to intuitively understand how to translate math into python code