Analysis on Manifolds and Differential Geometry

Lecture Notes

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April 7, 2021

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Contents

1.0 Algebra and Topology of $\mathbb{R}$

1.1 Linear Algebra

Suppose a set V of *vectors*. We define two operations. The first, called *vector addition*, denotes the sum of vectors $\mathbf{x}$ and $\mathbf{y}$ as $\mathbf{x} + \mathbf{y}$. The second, called *scalar multiplication*, denotes the product of a scalar $c \in \mathbb{R}$ and a vector $\mathbf{x}$ as $c\mathbf{x}$ as $c\mathbf{x}$.

Definition 1.1. If the following properties hold for all vectors $\mathbf{x}, \mathbf{y}, \mathbf{z}$ and scalars c, d, then we call the set V with these operations a *vector space*.

1. $\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$ (Additive commutativity)
2. $\mathbf{x} + (\mathbf{y} + \mathbf{z}) = (\mathbf{x} + \mathbf{y}) + \mathbf{z}$
3. $\exists! \mathbf{0} (\mathbf{x} + \mathbf{0} = \mathbf{x} \forall \mathbf{x})$
4. $\mathbf{x} + (-1)\mathbf{x} = 0$