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Exam FM

INTEREST MEASUREMENT

Effective Rate of Interest

$$i_t = \frac{a(t) - a(t-1)}{a(t-1)}$$

Effective Rate of Discount

$$d_t = \frac{a(t) - a(t-1)}{a(t)}$$

Simple Interest

$a(t) = 1 + it$

Force of Interest

$$\delta_t = \frac{a'(t)}{a(t)}$$

$$a(t) = \exp\left(\int_0^t \delta_r \, dr\right)$$

Compound Interest

$a(t) = (1+i)^t$

Relationships

$$v = \frac{1}{1+i} = 1 - d$$

$$d = \frac{i}{1+i} = iv$$

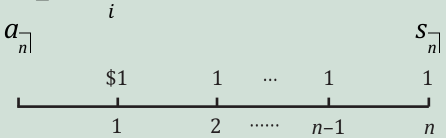
$$1 + i = \left(1 + \frac{i^{(m)}}{m}\right)^m = (1-d)^{-1} = \left(1 - \frac{d^{(m)}}{m}\right)^{-m} = e^{\delta}$$

ANNUITIES

Annuity-Immediate

$$PV = a_{\overline{n|}} = v + v^2 + \cdots + v^n = \frac{1-v^n}{i}$$

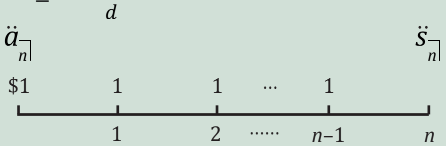
$$AV = s_{\overline{n|}} = 1 + (1+i) + \cdots + (1+i)^{n-1} = \frac{(1+i)^n - 1}{i}$$



Annuity-Due

$$PV = \ddot{a}_{\overline{n|}} = 1 + v + v^2 + \cdots + v^{n-1} = \frac{1-v^n}{d}$$

$$AV = \ddot{s}_{\overline{n|}} = (1+i) + (1+i)^2 + \cdots + (1+i)^n = \frac{(1+i)^n - 1}{d}$$



Immediate vs. Due

$\ddot{a}_{\overline{n|}} = a_{\overline{n|}}(1+i) = 1 + a_{\overline{n-1|}}$

$\ddot{s}_{\overline{n|}} = s_{\overline{n|}}(1+i) = s_{\overline{n+1|}} - 1$

Deferred Annuity

m-year deferred n-year annuity-immediate:

$PV = {}_{m|}a_{\overline{n|}} = v^m \cdot a_{\overline{n|}} = a_{\overline{m+n|}} - a_{\overline{m|}}$

Perpetuity

Perpetuity-immediate

$$PV = a_{\overline{\infty|}} = v + v^2 + \cdots = \frac{1}{i}$$

Perpetuity-due

$$PV = \ddot{a}_{\overline{\infty|}} = 1 + v + v^2 + \cdots = \frac{1}{d}$$

$\ddot{a}_{\overline{\infty|}} = 1 + a_{\overline{\infty|}}$

MORE GENERAL ANNUITIES

j-effective method is used when payments are more or less frequent than the interest period.

"j-effective" Method

Convert the given interest rate to the equivalent effective interest rate for the period between each payment.

Example: To find the present value of n monthly payments given annual effective rate of i, define j as the monthly effective rate where $j = (1+i)^{1/12} - 1$. Then apply $PV = a_{\overline{n|}}$ using j.

Payments in Arithmetic Progression

PV of n-year annuity-immediate with payments of P, $P+Q$, $P+2Q, \ldots, P+(n-1)Q$

$$Pa_{\overline{n|}} + Q\frac{a_{\overline{n|}} - nv^n}{i}$$

Calculator-friendly version:

$$\left(P + \frac{Q}{i}\right)a_{\overline{n|}} + \left(-\frac{Qn}{i}\right)v^n$$

$\text{N} = n, \text{I/Y} = i \text{ (in \%)}, \text{PMT} = P + \frac{Q}{i}, \text{FV} = -\frac{Qn}{i}$

PV of n-year annuity-immediate with payments of 1, 2, 3, …, n

Textbook formula: $(Ia)_{\overline{n|}} = \frac{\ddot{a}_{\overline{n|}} - nv^n}{i}$

P&Q version: $P = 1, Q = 1, N = n$

PV of n-year annuity-immediate with payments of n, $n-1$, $n-2, \ldots, 1$

Textbook formula: $(Da)_{\overline{n|}} = \frac{n - a_{\overline{n|}}}{i}$

P&Q version: $P = n, Q = -1, N = n$

PV of perpetuity-immediate and perpetuity-due with payments of: 1, 2, 3, …

$$(Ia)_{\overline{\infty|}} = \frac{1}{id} = \frac{1}{i} + \frac{1}{i^2} \qquad (I\ddot{a})_{\overline{\infty|}} = \frac{1}{d^2}$$

PV of an n-year annuity-immediate with payments of: $1, (1+k), (1+k)^2, \ldots, (1+k)^{n-1}$

$$\frac{1 - \left(\frac{1+k}{1+i}\right)^n}{i-k}$$

Level and Increasing Continuous Annuity

$$\bar{a}_{\overline{n|}} = \int_0^n v^t \, dt = \frac{1-v^n}{\delta} = \frac{i}{\delta}a_{\overline{n|}}$$

$$(\bar{I}\bar{a})_{\overline{n|}} = \int_0^n tv^t \, dt = \frac{\bar{a}_{\overline{n|}} - nv^n}{\delta}$$

YIELD RATES

Two methods for comparing investments:

- *Net Present Value (NPV):* Sum the present value of cash inflows and cash outflows. Choose investment with greatest positive NPV.
- *Internal Rate of Return (IRR):* The rate such that the present value of cash inflows is equal to the present value of cash outflows.

Choose investment with greatest IRR.

INTEREST MEASUREMENT OF A FUND

Dollar-weighted Interest Rate

The yield rate computation depends on the amount invested.

Method:

- Calculate amount of interest: $I = B - A - C$
  - A: Amount at the beginning of period
  - B: Amount at the end of period
  - C: Deposit/withdrawal
- Calculate the dollar-weighted interest rate:

$$i_{DW} = \frac{I}{A + \sum C_t(1-t)}$$

Time-weighted Interest Rate

The yield rate computation depends on successive sub-intervals of the year each time a deposit or withdrawal is made.

Method:

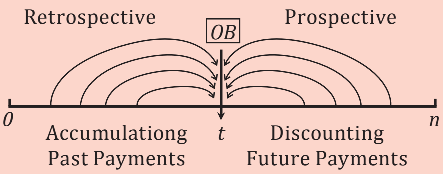
$$1 + i_{TW} = \left(\frac{A_2}{B_1}\right)\cdot\left(\frac{A_3}{B_2}\right)\cdot\left(\frac{A_4}{B_3}\right)\cdot\ldots\cdot\left(\frac{A_n}{B_{n-1}}\right)$$

	Date 1	Date 2
Account Before CF	A_1	A_2
Cash Flow (CF)	C_1	C_2
Account After CF	$B_1 = A_1 + C_1$	$B_2 = A_2 + C_2$

LOAN AMORTIZATION AND SINKING FUNDS

Outstanding Balance Calculation

- *Prospective:* $B_t = Ra_{\overline{n-t|}}$, R = level payments
  Present value of future payments.
- *Retrospective:* $B_t = R\left[a_{\overline{n|}}(1+i)^t - s_{\overline{t|}}\right]$
  Current value of all payments minus accumulated value of all past payments.



Loan Amortization

For a loan of $a_{\overline{n|}}$ repaid with n payments of 1:

	Period t	Total till time t
Interest (I_t)	$1 - v^{n-t+1}$	$t - \left(a_{\overline{n|}} - a_{\overline{n-t|}}\right)$
Principal repaid (P_t)	v^{n-t+1}	$a_{\overline{n|}} - a_{\overline{n-t|}}$
Total	1	t

General Formulas for Amortized Loan with Non-Level Payments

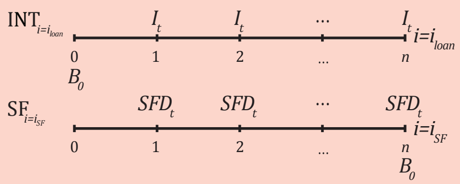
$I_t = i \cdot B_{t-1}$

$B_t = B_{t-1}(1+i) - R_t = B_{t-1} - P_t$

Sinking Fund

Sinking fund payments consist of interest paid to the lender (I_t) based on the total loan amount (B_0) and (ii) the sinking fund deposit (SFD_t).

$I_t = i_{\text{loan}} \cdot B_0 \qquad SFD_t = \dfrac{B_0}{s_{\overline{n}|\, i_{SF}}}$



BONDS

Price of Bond Formulas

P	Price of bond
F	Par value (face amount) of bond
r	Coupon rate per payment period
Fr	Amount of each coupon payment
C	Redemption value of bond
i	Interest rate per payment period
n	Number of payments

Basic Formula

$P = Fra_{\overline{n}|i} + Cv^n$

Premium/Discount Formula:

$P = C + (Fr - Ci)a_{\overline{n}|i}$

Premium vs. Discount

	Premium	Discount
Condition	$P > C$	$P < C$
Amortization Process	Write-Down	Write-Up
Amount	$(Fr - Ci) \cdot v^{n-t+1}$	$(Ci - Fr) \cdot v^{n-t+1}$

Price Between Coupon Dates

Full price (price including accrued interest)

Price $= B_{t+k} = B_t(1+i)^k$

Clean price (price excluding accrued interest)

Price $= B_{t+k} - kFr = B_t(1+i)^k - kFr$

Callable Bonds

Calculate the lowest price for all possible redemption dates at a certain yield rate. This is the highest price that guarantees this yield rate.

- Premium bond - earliest redemption date is the worst and therefore produces lowest price.
- Discount bond - latest redemption date is the worst and therefore produces lowest price.

FINANCIAL INSTRUMENTS

Price of Preferred Stock

$P = \dfrac{Fr}{i}$

F = par value, $\qquad r$ = fixed dividend rate

Theoretical Price of Common Stock

$P = \dfrac{D}{i-k}$

D = expected first dividend, $\qquad k$ = growth rate

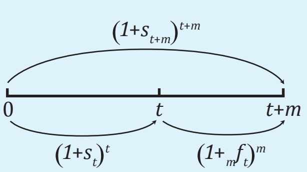
DURATION, CONVEXITY, IMMUNIZATION

Spot Rates and Forward Rates

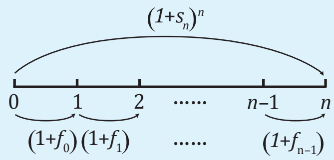
s_t is the spot rate for a term of length t.

${}_mf_t$ is the m-year forward rate which applies over the period from time t to time $t+m$.

$(1+s_t)^t \cdot (1+{}_mf_t)^m = (1+s_{t+m})^{t+m}$



$(1+s_n)^n = (1+f_0)\cdot(1+f_1)\cdots(1+f_{n-1})$



Duration and Convexity

$MacD = -\dfrac{P'(\delta)}{P(\delta)} = \dfrac{\sum_{t=1}^{n} t \cdot v^t \cdot CF_t}{\sum_{t=1}^{n} v^t \cdot CF_t}$

$ModD = -\dfrac{P'(i)}{P(i)} = \dfrac{\sum_{t=1}^{n} t \cdot v^{t+1} \cdot CF_t}{\sum_{t=1}^{n} v^t \cdot CF_t}$

$ModD = MacD \cdot v$

$MacD(n\text{-year zero-coupon bond}) = n$

$MacD_{t=n} = MacD_{t=0} - n$

$ModC = \dfrac{P''(i)}{P(i)} = \dfrac{\sum_{t=1}^{n} t \cdot (t+1) \cdot v^{t+2} \cdot CF_t}{\sum_{t=1}^{n} v^t \cdot CF_t}$

$MacC = \dfrac{P''(\delta)}{P(\delta)} = \dfrac{\sum_{t=1}^{n} t^2 \cdot v^t \cdot CF_t}{\sum_{t=1}^{n} v^t \cdot CF_t}$

$MacC(n\text{-year zero-coupon bond}) = n^2$

Duration of a portfolio

For a portfolio of m securities where invested amount $P = P_1 + P_2 + \cdots + P_m$ at time 0,

$MacD_P = \dfrac{P_1}{P}\, MacD_1 + \dfrac{P_2}{P}\, MacD_2 + \cdots + \dfrac{P_m}{P}\, MacD_m$

Approximate change in the price of an asset or liability for a change in interest

$\Delta P \approx P(i) \cdot [-(\Delta i)(ModD)]$

$\Delta P \approx P(i) \cdot \left[-(\Delta i)(ModD) + \dfrac{1}{2}(\Delta i)^2 \cdot ModC\right]$

Redington and Full Immunization

Redington	*Full*
$PV_{Assets} = PV_{Liabilities}$	
$MacD_A = MacD_L$ or $P'_A = P'_L$	
$C_A > C_L$ or $P''_A > P''_L$	There has to be asset cash flows before and after each liability cash flow.
Immunizes against small changes in i	Immunizes against any changes in i

BA-II PLUS CALCULATOR GUIDELINE

Basic Operations

ENTER (SET): Send value to a variable (option)

↑ ↓ : Navigate through variables

2ND: Access secondary functions (yellow)

STO + 0~9 : Send on-screen value into memory

RCL + 0~9 : Recall value from a memory

Time Value of Money (TVM)

Good for handling annuities, loans and bonds.

Note: Be careful with signs of cash flows.

N: Number of periods

I/Y: Effective interest rate per period (in %)

PV: Present value

PMT: Amount of each payment of an annuity

FV: Future value

CPT + (one of above): Solve for unknown

2ND + BGN: Switch between annuity immediate and annuity due

2ND + P/Y: *Please keep P/Y and C/Y as 1*

2ND + CLR TVM: Clear TVM worksheet

2ND + AMORT: Amortization (See Below)

For bonds ($P = Fra_{\overline{n}|i} + Cv^n$):

N $= n$; I/Y $= i$; PV $= -P$; PMT $= Fr$; FV $= C$.

Cash Flow Worksheet (CF , NPV , IRR)

Good for non-level series of payments.

Input (CF)

CF_0: Cash flow at time 0

C_n: n^{th} cash flow

F_n: Frequency of the cash flow

Output (NPV , IRR)

I: Effective interest rate (in %)

NPV + CPT: Solve for net present value

IRR + CPT: Solve for internal rate of return

Amortization Schedule (2ND + AMORT)

Good for finding outstanding balance of the loan and interest/principal portion of certain payments.

Note: BA-II Plus requires computing the unknown TVM variable before entering into AMORT function.

P1: Starting period

P2: Ending period

BAL: Remaining balance of the loan after P2

PRN: Sum of the principal repaid from P1 to P2

INT: Sum of the interest paid from P1 to P2

INTRODUCTION

Reasons For Using Derivatives

- Risk management – hedging
- Speculation – investment vehicle
- Reducing transaction cost
- Regulatory arbitrage

Bid-ask Spread

Bid price: The price at which brokers will buy and end-users will sell at.

Ask/Offer price: The price at which brokers will sell and end-users will buy at.

Bid-ask spread = Ask price – Bid price

Long vs. Short

A *long* position in an asset benefits from an *increase* in the price of the asset.

A *short* position in an asset benefits from a *decrease* in the price of the asset.

Short-Selling

- Borrow an asset from a lender.
- Immediately sell the asset and receive the current market value (usually kept by lender).
- Make deposits into margin account and refill the account during a margin call if needed.
- Buy the asset at a later date to repay the lender (close/cover the short position).

Haircut: Additional collateral/margin placed with lender by short-seller

Short rebate: Interest rate paid on haircut

Reasons for short-selling assets:

- Speculation – To speculate that the price of a particular asset will decline.
- Financing – To borrow money for additional financing of a corporation.
- Hedging – To hedge the risk of owning an asset or a derivative on the asset.

- **Option Moneyness**
- *In-the-money:* Produce a *positive* payoff if the option is exercised immediately
- *At-the-money:* The spot price is approximately *equal* to the exercise price
- *Out-of-the-money:* Produce a *negative* payoff if the option is exercised immediately

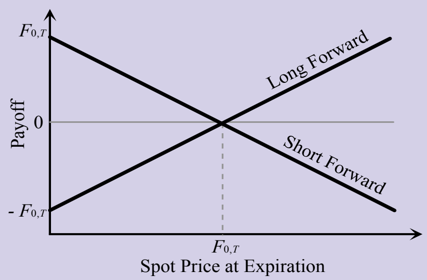
Option Style

- *European-style options* can only be exercised at *expiration.*
- *American-style options* can be exercised *at any time* during the life of the option.
- *Bermudan-style options* can be exercised during *specified periods* during the life of the option.

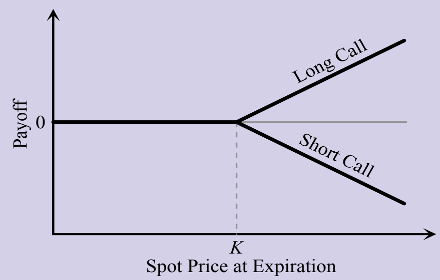
FORWARD CONTRACTS, CALL OPTIONS, AND PUT OPTIONS

Contract	Position in Contract	Description	Position in Underlying	Payoff	Profit	Maximum Loss	Maximum Gain	Strategy
Forward	Long Forward	Obligation to *buy* at the forward price	Long	$S_T - F_{0,T}$	$S_T - F_{0,T}$	$-F_{0,T}$	∞	Guarantee/lock in purchase price of underlying
	Short Forward	Obligation to *sell* at the forward price	Short	$F_{0,T} - S_T$	$F_{0,T} - S_T$	$-\infty$	$F_{0,T}$	Guarantee/lock in sale price of underlying
Call	Long Call	Right (but not obligation) to *buy* at the strike price	Long	$\max[0, S_T - K]$	$\max[0, S_T - K] - FV(\text{Prem.})$	$-FV(\text{Prem.})$	∞	Insurance against high underlying price
	Short Call	Obligation to *sell* at the strike price if the call is exercised	Short	$-\max[0, S_T - K]$	$-\max[0, S_T - K] + FV(\text{Prem.})$	$-\infty$	$FV(\text{Prem.})$	Sells insurance against high underlying price
Put	Long Put	Right (but not obligation) to *sell* at the strike price	Short	$\max[0, K - S_T]$	$\max[0, K - S_T] - FV(\text{Prem.})$	$-FV(\text{Prem.})$	$K - FV(\text{Prem.})$	Insurance against low underlying price
	Short Put	Obligation to *buy* at the strike price if the put is exercised	Long	$-\max[0, K - S_T]$	$-\max[0, K - S_T] + FV(\text{Prem.})$	$FV(\text{Prem.}) - K$	$FV(\text{Prem.})$	Sells insurance against low underlying price

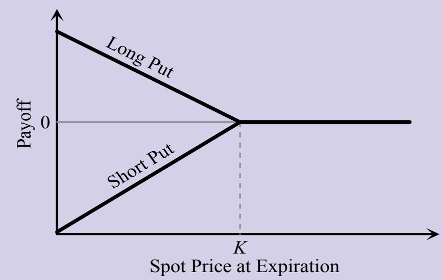
Forward



Call



Put

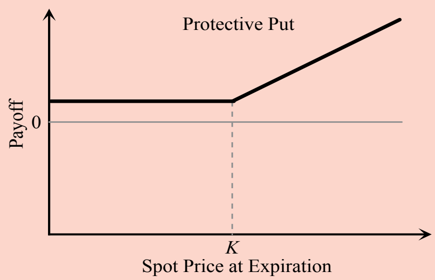


INSURING POSITIONS

Insuring a Long Position in an Asset	Insuring a Short Position in an Asset
Long asset + Long put = Long call + lending Buying zero-coupon bond = lending	Short asset + Long call = Long put + borrowing Selling zero-coupon bond = borrowing

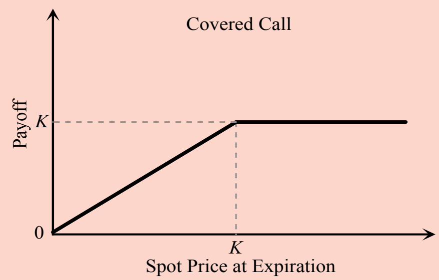
Protective Put

Long asset + Long put



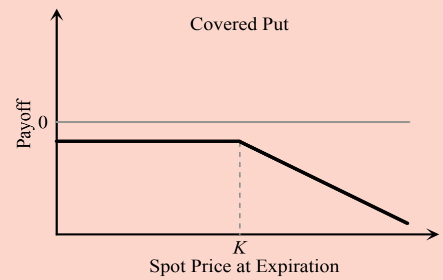
Covered Call

Long asset + Short call



Covered Put

Short asset + Short put



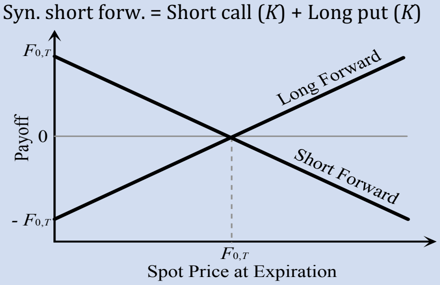
COMBINING OPTIONS

Put-Call Parity: The net cost of buying an asset using long call and short put equals the nest cost of buying the asset using a forward contract.

Formula: $C(K,T) - P(K,T) + PV(K) = PV(F_{0,T}) \rightarrow C(K,T) - P(K,T) = PV(F_{0,T}) - PV(K)$

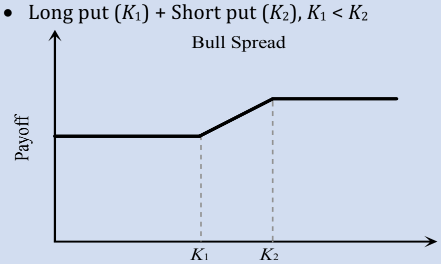
Synthetic Forward

Syn. long forw. = Long call (K) + Short put (K)
Syn. short forw. = Short call (K) + Long put (K)



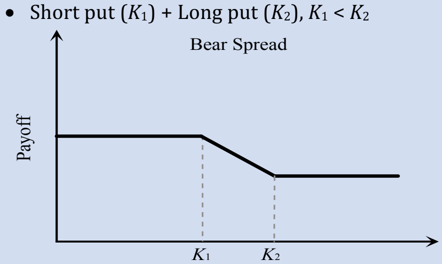
Bull Spread

- Long call (K_1) + Short call (K_2), $K_1 < K_2$
- Long put (K_1) + Short put (K_2), $K_1 < K_2$



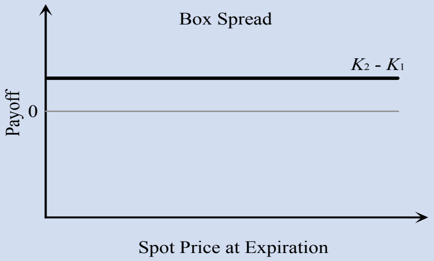
Bear Spread

- Short call (K_1) + Long call (K_2), $K_1 < K_2$
- Short put (K_1) + Long put (K_2), $K_1 < K_2$



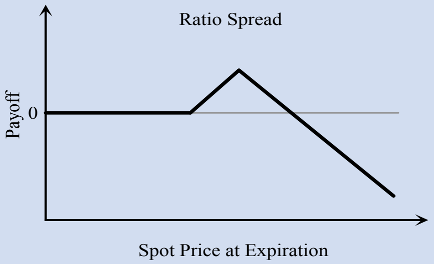
Box Spread

Synthetic long forward (K_1) + Synthetic short forward (K_2), $K_1 < K_2$



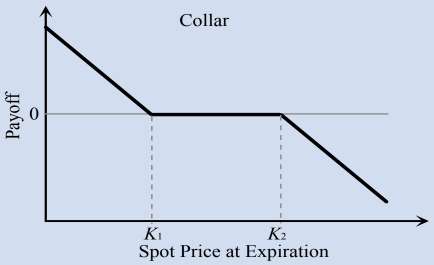
Ratio Spread

Long and short an unequal number of calls/puts with different strike prices



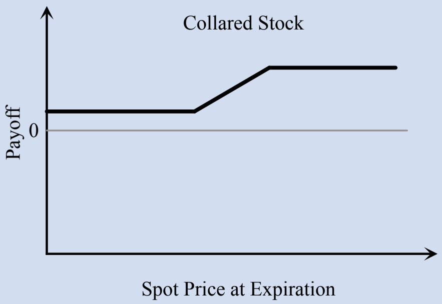
Collar

Long put (K_1) + Short call (K_2), $K_1 < K_2$



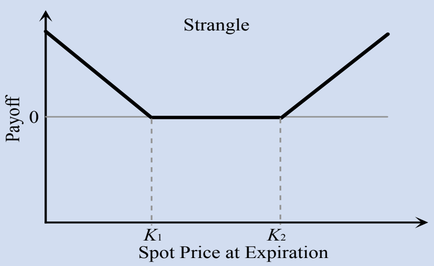
Collared Stock

Long collar + Long stock



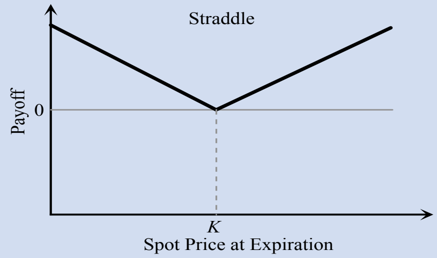
Strangle

Long put (K_1) + Long call (K_2), $K_1 < K_2$



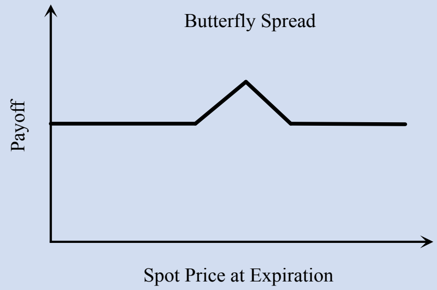
Straddle

Long put (K) + Long call (K)



Butterfly Spread

- 1 * Long call (K_1) + 2 * Short call (K_2) + 1 * Long call (K_3), $K_1 < K_2 < K_3$
- 1 * Long put (K_1) + 2 * Short put (K_2) + 1 * Long put (K_3), $K_1 < K_2 < K_3$



RISK MANAGEMENT

Hedging

Actions taken to reduce the risk of loss

Reasons to Hedge

- Decrease taxes
- Reduce probability of bankruptcy/distress
- Too expensive to raise funds externally
- Increase debt capacity
- Managerial risk aversion
- Manage non-financial risks

Reasons Not to Hedge

- Transaction costs
- Potentially costly expertise
- Monitoring and control capabilities
- Financial reporting, accounting, and tax considerations

FINANCIAL FORWARDS AND FUTURES

4 Ways to Buy a Share of Stock

Ways	Pay At Time	Receive Stock at Time	Payment
Outright purchase	0	0	S_0
Fully leveraged purchase	T	0	S_0e^{rT}
Prepaid forward contract	0	T	$F^P_{t,T}(S)$
Forward contract	T	T	$F_{t,T}(S)$

Relationship between $F_{t,T}(S)$ and $F^P_{t,T}(S)$

$F_{t,T}(S)$ = Accumulated Value of $F^P_{t,T}(S)$

$= F^P_{t,T}(S) \cdot e^{r(T-t)}$

Dividend Structure	$F^P_{t,T}(S)$
None	S_t
Discrete	S_t − PV(Divs)
Continuous	$S_te^{-\delta(T-t)}$

Dividend Structure	$F_{t,T}(S)$
None	$S_te^{r(T-t)}$
Discrete	$S_te^{r(T-t)}$ − AV(Divs)
Continuous	$S_te^{(r-\delta)(T-t)}$

Arbitrage

A transaction which generates a positive cash flow either today or in the future by simultaneous buying and selling of related assets, with no net investment or risk.

Arbitrage strategy: "Buy Low, Sell High."

Cash-And-Carry Arbitrage

Good when forward is overpriced.

Short forward + Borrow money + Long asset

Reverse Cash-And-Carry Arbitrage

Good when forward is underpriced.

Long forward + Lend money + Short asset

Futures Compared To Forward

- Traded on an exchange
- Standardized
- More liquid
- Marked-to-market and settled daily

SWAPS

Swap Types

Commodity Swap: Parties exchange fixed and variable commodity prices.

Interest Rate Swap: Parties exchange fixed and floating interest rate.

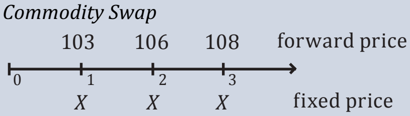
Market Value of a Swap

- Market value of a swap is 0 when first entered.
- The market value will generally no longer be zero as the forward prices or interest rates will change over time; even if they do not change, the swap value would change after first swap payment is made.

Assume 1-year spot rate = 4%, 2-year spot rate = 5% and 3-year spot rate = 6% for the following:

Finding Swap Price/Rate

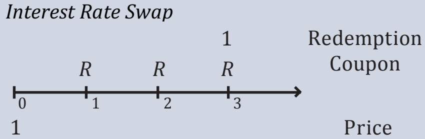
Commodity Swap



$$\frac{103}{1.04}+\frac{106}{1.05^2}+\frac{108}{1.06^3}=\frac{X}{1.04}+\frac{X}{1.05^2}+\frac{X}{1.06^3}$$

Fixed swap price is a zero-coupon-bond-price-weighted average forward prices.

Interest Rate Swap



Since an interest rate swap is equivalent to borrowing at a floating rate to buy a fixed-rate bond, fixed swap rate is the coupon rate on a par coupon bond.

$$\frac{R}{1.04}+\frac{R}{1.05^2}+\frac{R+1}{1.06^3}=1$$