1 Introduction

Some basics to linear algebra.

2 Vectors

2.1 A simple vector

$$\vec{x} = \begin{pmatrix} 8 \\ 6 \\ 7 \\ 5 \\ 3 \end{pmatrix}$$

2.2 Vector addition

$$A = [a_1, a_2, \ldots, a_n] \tag{1}$$
$$B = [b_1, b_2, \ldots, b_n] \tag{2}$$
$$A + B = [a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n] \tag{3}$$

2.3 Scalar multiplication

$$\vec{v} = [3, 6, 8, 4] \times 1.5 = [4.5, 9, 12, 6]$$

2.4 Inner product

$$(\vec{x}, \vec{y}) = \vec{x} \cdot \vec{y} = \sum_{i=1}^{n} x_i y_i$$

2.5 Orthogonality

Two vectors are orthogonal if their inner product is zero.

$$[2, 1, -2, 4] \cdot [3, -6, 4, 2] = 2(3) + 1(-6) - 2(4) + 4(2) = 0$$

2.6 Calculating vector length

$$|\vec{v}| = \sqrt{\sum_{i=1}^{n} v_i^2}$$

2.7 Normal vector

A normal vector (or unit vector) is a vector of length 1

$$|\vec{u}| = \sqrt{(2/5)^2 + (4/5)^2 + (1/5)^2 + (2/5)^2} = \sqrt{25/25} = 1$$

2.8 Orthonormal vectors

Vectors of unit length that are orthogonal to each other are said to be orthonormal.

$$\vec{u} = [2/5, 1/5, -2/5, 4/5] \tag{4}$$

$$\vec{v} = [3/\sqrt{65}, -6/\sqrt{65}, 4/\sqrt{65}, 2/\sqrt{65}] \tag{5}$$

$$|\vec{u}| = \sqrt{(2/5)^2 + (1/5)^2 + (-2/5)^2 + (4/5)^2} = 1 \tag{6}$$

$$|\vec{v}| = \sqrt{(3/\sqrt{65})^2 + (-6/\sqrt{65})^2 + (4/\sqrt{65})^2 + (2/\sqrt{65})^2} = 1 \tag{7}$$

$$\vec{u} \cdot \vec{v} = \frac{6}{5\sqrt{65}} - \frac{6}{5\sqrt{65}} - \frac{8}{5\sqrt{65}} + \frac{8}{5\sqrt{65}} = 0 \tag{8}$$

3 Matrices

3.1 Matrix of numbers

$$\begin{bmatrix} 17 & 18 & 5 & 5 & 45 & 1 \\ 42 & 28 & 30 & 15 & 115 & 3 \\ 10 & 10 & 10 & 21 & 51 & 2 \\ 28 & 5 & 65 & 39 & 132 & 5 \\ 24 & 26 & 45 & 21 & 116 & 4 \end{bmatrix}$$

3.2 Matrix with subscripts and dots

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}$$

3.3 Square matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

3.4 Transpose

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \tag{9}$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \tag{10}$$

3.5 Matrix multiplication

$$AB = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 5 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & 26 \end{bmatrix}$$

3.6 Identity matrix

$$AI = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix}$$

3.7 Orthogonal matrix

$$AA^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3/5 & -4/5 \\ 0 & 4/5 & 3/5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3/5 & 4/5 \\ 0 & -4/5 & 3/5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3.8 Diagonal matrix

$$A = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{mm} \end{bmatrix}$$

3.9 Determinant of a 2x2 matrix

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$