

### Solution for Saxon “Advanced Math” Lesson 30 Problem #27

The key to solving this problem is to remember that an inscribed angle is half the intercepted arc (see lesson 11.B). You need to use some other facts about angles as well, and solve parts until you have enough information to solve it. Basically, keep finding out more facts, given your current facts, until you solve the whole thing. There are actually *many* ways to solve this; here’s one (in this table,  $m\widehat{CD}$  means the length of the arc from C to D):

Step	Statement	Justification
1	$m\angle BAC=40^\circ$ , $m\angle BCA=30^\circ$ , $m\widehat{CD}=70^\circ$	Given
2	$m\angle ABC=180^\circ-40^\circ-30^\circ=110^\circ$	Sum of angles in a triangle is $180^\circ$
3	$m\angle CAD= m\widehat{CD}/2 = 70^\circ/2 = 35^\circ$	An inscribed angle is $1/2$ the intercepted arc. See lesson 11.B. We were given the intercepted arc length, so we can find the inscribed angle.
4	$m\angle ABC= m\widehat{ADC}/2$ so $110^\circ = m\widehat{ADC}/2$ so $m\widehat{ADC} = 220^\circ$	An inscribed angle is $1/2$ the intercepted arc. This time we're going the other way; we know the inscribed angle, so we can find the intercepted arc length.
5	$m\widehat{ABC} = 360^\circ - m\widehat{ADC} = 360^\circ - 220^\circ = 140^\circ$	Total arc around a circle is $360^\circ$ . We already know $m\widehat{ADC}$ from the previous step, and since $m\widehat{ABC}$ is what is left in the circle, we can find its value.
6	$x = m\angle ADC = m\widehat{ABC}/2 = 140^\circ/2 = 70^\circ$	An inscribed angle is $1/2$ the intercepted arc; we know the intercepted arc from the last step, so we can now solve for $x$ .
7	$360^\circ = x + y + 30^\circ + 110^\circ + 40^\circ + 35^\circ$	Sum of interior angles of a quadrilateral is $360^\circ$ ; this is the set of angles in the quadrilateral ABCD.
8	$360^\circ = y + 285^\circ$	Simplify step 7 by substituting $x=70^\circ$ (see step 6) and adding up the rest
9	$y = 75^\circ$	Simplify step 8 by solving for $y$ .

So  $x=70^\circ$ ,  $y = 75^\circ$ .