

AsymmeTree User Manual

David Schaller

sdavid@bioinf.uni-leipzig.de

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1 Introduction

AsymmeTree is an open-source Python library for the simulation and analysis of phylogenetic scenarios. It includes a simulator for species and gene trees with asymmetric evolution rates, tools for the inference and analysis of phylogenetic best matches [15, 16] (resp. best hits) from known gene trees or evolutionary distances. Moreover, it includes tools for the analysis of horizontal gene transfer (HGT) events and a method to estimate rooted species trees from an ensemble of orthology/paralogy relations [21].

The library, and especially the simulator, is primarily designed to explore and validate mathematical concepts, and to test inference methods for various steps on the way to more realistically available data, i.e., dated gene trees, additive distances of gene sets, noisy distances and finally sequences. Both nucleotide and amino acid sequence simulation with or without indels are supported. In both cases, several substitution models are available.

The software is hosted on GitHub and also available via The Python Package Index (PyPI). Please feel free to report bugs or make suggestions for improvement in the Issues section of the GitHub repository.

If you use AsymmeTree in your project or code from it, please cite:

Peter F. Stadler, Manuela Geiß, David Schaller, Alitzel López Sánchez, Marcos González Laffitte, Dulce I. Valdivia, Marc Hellmuth, Maribel Hernández Rosales (2020). From pairs of most similar sequences to phylogenetic best matches. Algorithms for Molecular Biology. doi: 10.1186/s13015-020-00165-2. [38].

2 Installation

AsymmeTree requires Python 3.7 or higher. Python 2 is not supported.

2.1 Easy Installation with pip

The asymmetree package is available on The Python Package Index (PyPI):

```
pip install asymmetree
```

For details about how to install Python packages see here.

2.2 Installation with the setup file

Alternatively, you can download or clone the repo, go to the root folder of package and install it using the command:

```
python setup.py install
```

2.3 Dependencies

AssymmeTree has several dependencies (which are installed automatically when using pip or the setup.py):

NetworkX

- SciPy and NumPy
- Matplotlib
- tralda

The simulation of phylogenetic scenarios and sequences, as well as most functions for their analysis, do not have any other dependencies.

However, to use the tree reconstruction method for best match inference and the C++ implementation of the quartet method [38], the software RapidNJ [37], resp., qinfer must be installed. I recommend that you compile these tools on your machine, place the binaries into a persistent location and add this location to your PATH environment variable.

3 Usage

AsymmeTree is divided into several subpackages and modules, an overview of which is given in Appendix A. The package uses many classes and functions from the tralda library for tree algorithms and data structures. Therefore, an overview of this package is also given in Appendix A. The API functions are described in the following sections and can be imported directly from the respective subpackage (see examples).

The term 'color' regularly appears in the library and refers to the reconciliation of gene trees with species trees. In particular, the terms 'color' and 'species'/'genome' in which a gene resides are used interchangeably. The reason for this is that the information in which species/genomes the genes reside is usually modeled as a (vertex) coloring, such as e.g. in (colored) best match graphs [15].

3.1 Tree Data Structures

The two classes Tree (in tralda) and PhyloTree (inherits from Tree) implement tree data structures which are essential for most of the modules in the package. The latter contains converters and parsers for the Newick format and a NetworkX graph format.

The vertices of a PhyloTree instance are of type PhyloTreeNode and contain the following attributes:

ID	vertex ID (int)
label	label (str), in gene trees: 'S' for speciation, 'D' for duplication, 'H' for horizontal gene transfer, '*' for loss
color	only gene trees; species in which the gene resides, i.e., ID of some vertex in a species tree, int for extant genes, can be of type tuple (of two ints) for inner and loss vertices
tstamp	time stamp of the event (double)
dist	evolutionary distance from the parent vertex (double); if no evolution rates (see below) were simulated yet, then this value corresponds to the divergence time between the vertex and its parent
transferred	only gene trees; indicates whether the edge from the parent is the transfer edge from an HGT event; ${\tt 1}$ if yes and ${\tt 0}$ otherwise

Both species and gene trees can be converted into Newick format using the function to_newick() of the PhyloTree class. In case of a gene tree, the color is represented in brackets, e.g.

```
>>> '(3<1>:0.534,2<2>:0.762)S<0>:0.273'
```

To suppress this, use to_newick(color=False). Likewise, to suppress the distances, you can use to_newick(distance=False). The function PhyloTree.parse_newick() can handle this customized format as well as the standard Newick format.

Moreover, phylogenetic trees can easily be serialized in json or pickle (Python's serialization library) format:

```
# tree is of type 'PhyloTree'
tree.serialize('tree.pickle')
# or
tree.serialize('tree.json')
```

By default, the serialization format is inferred from the file extension. Alternatively, it can be specified as keyword argument, e.g. mode='json'. To load a tree that was serialized this way, use:

```
from asymmetree.datastructures import PhyloTree
tree = PhyloTree.load('tree.json')
```

An overview over selected functions of the tree classes is given in Appendix B.

The class LCA (in tralda) can be initiated with an instance of type Tree or its inheriting classes and then provides functions for efficient last common ancestor queries in that tree.

```
from tralda.datastructures import LCA

# tree is of type 'Tree' (or an inheriting class)
lca_T = LCA(tree)

# l.c.a. queries via 'TreeNode' instances or IDs (int)
print( lca_T(4, 7) )

# triple queries (e.g. is 3 5 / 2 displayed?)
print( lca_T.displays_triple(3, 5, 2) )
```

All such queries take instance of type TreeNode (or inheriting classes) as input or ints, and raise a KeyError if this node or ID, resp., is not in the tree.

3.2 Simulation of Species and Gene Trees

The subpackage asymmetree.treeevolve contains modules for the simulation of dated species and gene trees. In terms of divergence time, these trees define an ultrametric on the set of their (extant) leaves. Gene trees, furthermore, can be manipulated with a realistic rate heterogeneity among their branches resulting in general additive distances (but no longer ultrametric).

3.2.1 Species Trees

The function simulate_species_tree(N) simulates a dated species tree with N leaves (i.e. recent species) using the specified model. The following models are available:

'innovation'	Innovation model Keller-Schmidt and Klemm [28], if not specified the divergence time between the (planted) root and the leaves will be normalized to unity
'yule'	standard Yule model [46], default birth rate is 1.0
'BDP'	constant-rate birth-death process [see e.g. 29, 19], default birth rate is 1.0 and death rate is 0.0
'EBDP'	episodic birth-death process, algorithm of [39]

The following keyword parameters (with their default value) are available:

model='innovation'	model for the species tree simulation, currently only the 'innovation model' is available
non_binary_prop=0.0	probability that an inner edge is contracted, results in a non-binary tree
planted=True	add a planted root that has the first true speciation node as its single neighbor, this way duplication (and loss) events can occur before the first speciation event in a subsequent gene tree simulation
remove_extinct= False	remove all branches leading to losses, only relevant for models with death events
rescale_to_height= None	specify the divergence time between the (planted) root and the leaves i.e. the final height of the dated tree

For any model, the root of the resulting tree has the maximal time stamp and all (extant) species have time stamp 0.0. The episodes of the 'EBDP' model must be supplied as a list of tuples/lists where each episode has the structure

```
(birthrate, deathrate, proportion_of_survivors, time_stamp).
```

Note that the first elements in this list correspond to the most recent ones, and that the first episode should have a time stamp of 0.0. Example usage:

3.2.2 Gene Trees

Dated gene trees are simulated along a given species tree S using a birth-death process [29, 18] with speciation events as additional branching events (fixed time points given by the species tree). At each time point, the total event rate is given by the sum of the event rates over all branches that are currently active (not extinct). Thus, the total event rate in general increases during the simulation if the loss rate does not dominate the rates of the

branching events. To simulate gene tree, use the class <code>GeneTreeSimulator</code> or the function <code>simulate_dated_gene_tree(S, **kwargs)</code> with a species tree of type <code>PhyloTree</code>. The following parameters are available:

```
dupl_rate=0.0
                            duplication rate (float)
loss_rate=0.0
                            loss rate (float)
hgt_rate=0.0
                            horizontal gene transfer rate (float)
dupl_polytomy=0.0
                            allows non-binary duplication events by specifying the pa-
                            rameter \lambda of a Poisson distribution (copy number = 2 +
                            drawn number)
prohibit_extinction=
                            avoid loss events for genes that are the last survivor in their
'per_species'
                            species branch ('per_species'), the last survivor of the
                            whole family ('per_family'); or no constraints (False)
replace_prob=0.0
                            probability that a a transferred copy replaces one (ran-
                            domly chosen) homolog in the receiving branch; default is
                            0.0 in which case all HGT events are additive
```

For the constraints to avoid extinction, the loss rate in the respective branches are temporarily set to zero. Note that if replacing HGT is enabled (replace_prob > 0.0), then the resulting tree may contain loss leaves even if the loss rate is zero. Hence, constructing the observable tree (see below) also becomes relevant in the latter setting.

Example usage:

The function observable_tree(tree) returns the observable part of a gene tree, i.e., it copies the tree, removes all branches that lead to loss events only and suppresses all inner nodes with only one child. It also removes the planted root. Example usage:

```
# observable gene tree
ogt = ts.observable_tree(tree)
```

3.2.3 Assignment of Variable Evolution Rates

The module EvolutionRates contains functions to model realistic (asymmetric) evolution rates for a given gene tree. Moreover, correlation of the evolution rates between genes of the same (and closely related) species is introduced (autocorrelation, [31]). The function assign_rates(T, S) takes a gene tree T and the corresponding species tree S as input, and manipulates the branch length of the gene tree. The following keyword parameters (with their default values) are available:

```
starting value for the substitution rate (per time unit) and
base_rate=1.0
                            expected value for conserved genes
autocorr factors=None
                            a dictionary containing rate factors for the edges of the
                            species tree
                            variance factor for a lognormal distribution that controls
autocorr_variance=0.0
                            autocorrelation between genes of the same (and closely re-
                            lated) species, the higher the lower the autocorrelation; only
                            relevant if autocorr_factors is not directly supplied
rate_increase=
                            distribution of the (relative) rate increase (w.r.t. the base
('gamma', 0.5, 2.2)
                            rate) for divergent genes, i.e. to a factor 1 + x, the param-
                            eters the for default Gamma distribution are chosen to fit
                            observed asymmetries between paralogs in yeast data [3]
CSN_weights=(1, 1, 1)
                            weights for choice between conservation, subfunctionaliza-
                            tion and neofunctionalization after a duplication event
inplace=True
                            manipulate edge lengths (dist) of the gene tree in-place,
                            otherwise copy the tree
```

It is recommended to apply the rate assignment to the true gene tree that still contains loss events. Note that the rates are used to manipulate the dist attributes in the gene tree and not returned explicitly. Example usage:

```
import asymmetree.treeevolve as te
S = te.simulate_species_tree(10)
# true gene tree (with losses)
tree = te.simulate_dated_gene_tree(S, dupl_rate=1.0, loss_rate=0.5, hgt_rate=0.1)
te.assign_rates(tree, S, base_rate=1,
               autocorr_variance=0.2,
               rate_increase=('gamma', 0.5, 2.2))
# or in a single step
tree = simulate_gene_trees(S, N=1,
                           dupl_rate=('gamma', 1.0, 0.8), # distribution
                           loss_rate=0.5,
                                                            # or constant value
                           hgt_rate=0.1,
                           base_rate=('constant', 1.0),
                           autocorr_variance=0.2,
                           rate_increase=('gamma', 0.5, 2.2))
# observable gene tree
ogt = te.observable_tree(tree)
```

The function simulate_gene_trees(S) combines the simulation of dated gene trees and the rate assignment into one step. If N=1, a single gene tree is returned. Otherwise, a list of gene trees is returned that shared the same rate factors for the branches in the species tree (autocorrelation factors) in the rate assignment procedure. Moreover, distribution for the base rate (assigned the planted edge of the gene tree) and for the event rates can be specified with the parameters base_rate, dupl_rate, loss_rate and hgt_rate. For available distributions and their syntax see Appendix C.

3.2.4 Distance Matrix and Noise

Distances derived from (real-life) gene or protein sequences are always burdened with noise. Such data can either be modeled by simulating sequences, or by disturbing the distances specified by a given tree directly. The latter alternatively is described briefly in this section.

The additive (i.e. noiseless) distance from an **observable** gene tree can be computed using the function **distance_matrix()** of a PhyloTree instance. It returns a tuple containing a list of leaves in the tree (corresponding to the row/column order) and the distance matrix as a 2-dimensional numpy array.

```
# tree is an observable gene tree
leaves, D = tree.distance_matrix()
leaf_index_dict = {leaf: i for i, leaf in enumerate(leaves)}
```

In the next step, noise can be introduced into a distance matrix using the NoisyMatrix module. Random noise can be simulated with the function noisy_matrix(orig_matrix, sd). The following parameters are available (keyword arguments are indicated by their default value):

orig_matrix	original matrix to be disturbed
sd	standard deviation of a normal distribution with mean 1 from which noise factors are drawn
metric_repair='reject'	method to ensure that the resulting distance matrix is still a metric, available are the rejection of noise steps that violate the metric property ('reject'), the decrease-only metric repair ('DOMR') and the general metric repair ('general') algorithm

Alternatively, the function <code>convex_linear_comb(D1, D2)</code> can be used to simulate systematically biased noise by computing a linear convex combination with a disturbance matrix. The function thus takes two distance matrices (numpy arrays) not necessarily of the same size as input and disturbs them with one another. The contribution of the respective disturbance matrix is controlled by the keyword parameter <code>alpha</code> (default is <code>0.5</code>). If the keyword parameter <code>first_only</code> is <code>True</code>, only the first disturbed matrix is returned. Otherwise, both are returned in a tuple.

3.3 Simulation of Sequences

AsymmeTree supports the simulation of nucleic and amino acid sequences using time-continuous Markov models, as usually applied for this purpose [for textbooks, see e.g. 12, 44, 45]. The subpackage asymmetree.seqevolve contains the modules and functions for this task.

The class Evolver takes several model as parameters for its initialization:

- a subtitution model (SubstModel, required) for the substitution of single bases or amino acids,
- an **indel** model (IndelModel, optional) for the simulation of insertions and/or deletion, and

• a model for rate **heterogeneity** (HetModel, optional) among the sites of the sequence under evolution

3.3.1 Substitution Model

A substitution model usually comprises an exchangeability matrix S and a vector π containing the equilibrium frequencies of the alphabet A of nucleobases, amino acids et cetera. From this, the rate matrix Q can be computed as $S\Pi$ where $\Pi = \text{diag}\{\pi_1, \ldots, \pi_{|A|}\}$ [44]. The substitution probability matrix, in turn, is given by

$$P - e^{Qt}$$

which is computed efficiently by AsymmeTree using matrix diagonalization.

The following models for nucleotide (n) and amino acid (a) substitution are currently available (codon models are not supported at the moment):

model	type	reference	required parameters (kwargs)
JC96	n/a	Jukes & Cantor 1969 [27]	-
K80	n	Kimura 1980 [30]	kappa (transition/transversion rate ratio)
GTR	n	general time-reversable model (GTR) 1986 [40]	abcdef (list of rates (a) $C \leftrightarrow T$, (b) $A \leftrightarrow T$, (c) $G \leftrightarrow T$, (d) $A \leftrightarrow C$, (e) $C \leftrightarrow G$, (f) $A \leftrightarrow G$); f (list of equilibrium frequencies $A/C/G/T$)
DAYHOFF	a	Dayhoff 1978 [10]	-
BLOSUM62	a	BLOSUM62 1992 [23]	-
JTT	a	Jones, Taylor & Thornton 1992 [26]	-
WAG	a	Whelan & Goldman 2001 [41]	-
LG	a	Le & Gascuel 2008 [32]	-
CUSTOM	n/a	-	filename (path to a file with a model in PAML [43] format)

Note that a custom substitution model can be specified via $model_name='CUSTOM'$. In this case, the path to the model in PAML [43] format must be supplied. Moreover, the model type (n/a) must fit this model. Example usage:

```
from asymmetree.seqevolve import SubstModel

# non-empirical model
subst_model_jc69 = SubstModel('n', 'K80', kappa=2.0)

# empirical model
subst_model_jtt = SubstModel('a', 'JTT')

# custom model
subst_model_custom = SubstModel('a', 'CUSTOM', filename='path/to/custom.paml')
```

3.3.2 Indel Model

Insertions and deletions are modeled based on Dawg [4]. An indel model requires sitewise rates for insertion insertion_rate and deletion deletion_rate. Moreover, the following parameters are available (with default values):

<pre>length_distr= ('zipf', 1.821)</pre>	distribution of indel length, default value for zipf distribution cf. [5]
min_length=1	integer min. value at which the specified distribution is truncated, must be less than the expected value of the distribution, None means no limit
max_length=None	integer max. value at which the specified distribution is truncated, must be greater than the expected value of the distribution, None means no limit

For available length distributions and their syntax see Appx. C. A zipf or negative binomial distribution are typically used for this purpose [4, 9]. Example usage:

```
from asymmetree.seqevolve import IndelModel

# sitewise insertion rate 0.01 and deletion rate 0.008
indel_model = IndelModel(0.01, 0.008, length_distr=('zipf', 2.0))
```

3.3.3 Heterogeneity Model

Selective pressure usually varies among the sites of a sequence under evolution. To model this, rate factors r for single sites or groups of sites are commonly drawn from a Gamma distribution ('+ Γ ') with mean 1 and parameter α [4, 13, 9]. The rate matrix rQ is then used instead of Q. Note that smaller values for α correspond to higher heterogeneity.

AsymmeTree supports two modes of the ' $+\Gamma$ '-model. You can specify a number of classes to which the sites are assigned randomly and uniformly distributed. Sites of the same class share a common factor r. The other possibility is a sitewise heterogeneity, i.e., every site has its own rate. In both cases, the rate or class membership is inherited from the parent sites during the evolution along a tree. Note that the sitewise mode is expected to have a longer running time.

An other aspect of among site heterogeneity is the modeling of invariant sites ('+I'), i.e., sites that never mutate at all as a result of very strong selective pressure. The (expected) proportion p of invariant sites can be specified by the user, and sites are assigned as 'invariant' with probability p. Note that p > 0 affects the overall substitution rate. In other words, the rates of the non-invariant sites are **not** adjusted to compensate the decreased number of expected substitution over all sites.

To summarize, the following parameters are available for the class HetModel (keyword arguments are indicated by their default value):

alpha	parameter α of the Gamma distribution
classes=5	number of classes; sites in the same class share the same
	rate factor

sitewise=False	if True, factors are drawn sitewise; the number of classes is ignored in this case
invariant=0.0	(expected) proportion p of invariant sites

Note that the '+I'-model can be used without the ' $+\Gamma$ '-model by setting classes=1 (the single class will have a factor of 1) and invariant to some proportion greater than 0. Example usage:

```
from asymmetree.seqevolve import HetModel
# +G
het_model_g = HetModel(1.5, classes=10)
# +G +I
het_model_gi = HetModel(2.0, sitewise=True, invariant=0.2)
# +I (value of alpha is irrelevant)
het_model_i = HetModel(1.0, classes=1, invariant=0.25)
```

3.3.4 The Class Evolver

The class Evolver evolves a sequence according to the specified models (see previous sections) along a phylogenetic tree. In AsymmeTree, the **dist** attribute of the vertex v in an edge uv of the tree (u is closer to the root) is always used as the **expected number of substitutions** along this edge. Thus, PAM distances as e.g. used optionally in [9] are not supported. The **dist** attribute is also used as the duration of the Markov process in which insertions and deletions are drawn.

The following parameters are available for the initialization of an Evolver instance (keyword arguments are indicated by their default value):

subst_model	substitution model; instance of SubstModel
indel_model=None	model for insertions and deletions; instance of ${\tt IndelModel}$
het_model=None	model for among site heterogeneity and invariant sites; instance of ${\tt HetModel}$
jump_chain=False	if True, an alternative Gillespie-like [18] algorithm is applied for the substitution process instead of the computation of $P=e^{Qt}$

Once the Evolver is initialized, its function evolve_along_tree() can be called to evolve a sequence along a tree. The following parameters are available for this function (keyword arguments are indicated by their default value):

tree	phylogenetic tree; instance of PhyloTree
start_length=200	length of the root sequence which is randomly drawn from the equilibrium frequencies in the specified substitution model

The sequences of a simulation run are returned by this function (and also available via the attribute sequences as long as the function has not been called again), which is a dict containing the nodes (inner and leaf nodes) as keys and instances of type EvoSeq as values. The latter can be converted into str using subst_model.to_sequence(evo_seq).

The function true_alignment() can be used to compute (and optionally write into a file) the 'true' alignment of a simulation run. The following keyword parameters are available:

Example usage of the class Evolver:

3.4 Simulation of Genomes

The class GenomeSimulator combines multiple steps described in the previous section in order to conveniently simulate whole genomes/proteomes. An instance of this class is initialized with a species tree (of type PhyloTree), and optionally the path to an output directory (outdir=...) if the user wants to save the results. The gene trees and the sequences are simulated in subsequent steps using the classes' function

```
(i) simulate_gene_trees(N, **kwargs), and
```

(ii) simulate_sequences(subst_model, **kwargs).

The first step (i) takes the same keyword parameters as input as the function simulate_gene_trees() in Section 3.2.3, where N is the number of gene families to be simulated. Thus, rates for the three event types (dupl_rate, loss_rate, hgt_rate), autocorrelation (autocorr_variance), the distribution of base rates (base_rate_distr) etc. can be specified.

The second step (ii) simulates the sequences along the observable part (without loss branches) of the simulated gene trees. The function takes the following parameters as input (keyword arguments are indicated by their default value):

subst_model	substitution model; instance of SubstModel
indel_model=None	$model \ for \ insertions \ and \ deletions; \ instance \ of \ {\tt IndelModel}$
het_model=None	model for among site heterogeneity and invariant sites; instance of ${\tt HetModel}$
root_genome=None	list of sequences for the roots of the gene trees; must contain the same number of str sequences as trees were simulated in step (i) i.e. N; sequences must be compatible with the specified substitution model (model_type='n'/'a')
<pre>length_distr= ('constant', 200)</pre>	distribution of the length of the root sequences if root_genome is not supplied; see Appx. C
min_length=10	minimal length at which the distribution of lengths is truncated; must be less than the mean of this distribution
max_length=None	maximal length at which the distribution of lengths is truncated; must be greater than the mean of this distribution
write_fastas=True	if True and an output directory was specified, write the sequences (one file per species) into the directory fasta_files in the output directory
write_alignments=True	if True and an output directory was specified, write the true alignments (one file per gene tree) into the directory alignments in the output directory

After step (i), the lists of full and observable gene trees are accessible via the attributes true_gene_trees and observable_gene_trees, respectively. Moreover, the full gene trees are serialized into the directory true_gene_trees if an output directory was specified. After step (ii), the lists of sequence dictionaries are accessible via the attribute sequence_dicts.

Example usage:

```
from asymmetree.genome import GenomeSimulator
from asymmetree.treeevolve import simulate_species_tree
from asymmetree.seqevolve import SubstModel, IndelModel

# simulate a species tree
species_tree = simulate_species_tree(10, model='yule')

# specify models (heterogeneity omitted)
subst_model = SubstModel('n', 'JC69')
indel_model = IndelModel(0.015, 0.012)

# simulate 50 gene families incl. sequences
```

3.5 Best Match Inference

Phylogenetic best matches of a gene x of species X are defined as those genes y of another species $Y \neq X$ that share the lowest common ancestor with x in the gene tree among all genes in that species [15, 17, 16]. In contrast, two genes are orthologs if their last common ancestor was a speciation event. Orthology and reciprocal best matches are closely related [16].

The subpackage asymmetree.best_matches contains functions to compute both relations from a given gene tree or to estimate them from distance data on a set of genes [38]. If the true (observable) gene tree is known (as e.g. the case in simulations), best matches and orthologs can be computed using the module TrueBMG. The functions bmg_from_tree() and orthology_from_tree() return the respective graph representation as NetworkX (Di)Graphs:

```
from asymmetree.best_matches import orthology_from_tree, bmg_from_tree
# tree is an observable gene tree
orthology_graph = orthology_from_tree(tree)
# Best Match Graph and Reciprocal Best Match Graph as a tuple
bmg, rbmg = bmg_from_tree(tree, supply_rbmg=True)
```

If only distance data is available, best matches have to be estimated. AsymmeTree currently implements three different methods that are described by Stadler et al. [38]:

- Extended Best Hits (module ExtBestHits)
- Neighborjoining [35] and midpoint rooting (module TreeReconstruction, requires the installation and accessibility of RapidNJ [37])
- Quartet method (module Quartets, Python implementation and wrapper for the C++ tool qinfer)

Please see the file examples/best_match_infer.py in the GitHub repo for an example usage of these modules following the simulation of gene tree scenarios.

3.6 Analysis of Horizontal Gene Transfer

The subpackage hgt contains several functions for the analysis of horizontal gene transfer events in the simulated scenarios. In particular, transfer edges, the directed and undirected Fitch graph can be extracted, as well as the pairs of genes that diverged later than the respective species in which they reside, i.e. the so-called later-divergence-time (LDT) graph. The latter situation is indicative for the presence of HGT events in the scenario.

An edge (u, v) in a gene tree is a "true" transfer edge if an HGT event happened on the path from u to v. In the simulated trees, this is indicated by the attribute transferred of PhyloTreeNode v which is set to 1. An edge (u, v) in the gene tree is an rs-transfer edge (named after the relaxed scenario concept) if the colors of u and v are incomparable in the corresponding species tree S. True and rs-transfer edges may not be equivalent, e.g. when a transfer from branch A to B is followed by a transfer from B to A and this gene lineage does not survive in branch B.

```
from asymmetree.hgt import true_transfer_edges, rs_transfer_edges
# gene tree T and species tree S
transfer_edges1 = true_transfer_edges(T)
transfer_edges2 = rs_transfer_edges(T, S)
```

The directed Fitch graph of a tree T has as vertex set the leaves of T and a directed edge (x, y) when the path from the last common ancestor of x and y to the leaf y contains a transfer edge [14]. The undirected Fitch graph of a tree T also has as vertex set the leaves of T and an undirected edge xy when the path from x to y contains a transfer edge [22].

```
from asymmetree.hgt import fitch, undirected_fitch
# gene tree 'T', and precomputed 'transfer_edges'
dfg = fitch(T, transfer_edges)
ufg = undirected_fitch(T, transfer_edges)
```

As mentioned above, the situation in which two genes diverged later that their corresponding species witnesses HGT events. The graph that contains edges for any such gene pair has been termed the later-divergence-time (LDT) graph.

```
from asymmetree.hgt import ldt_graph
# gene tree 'T', CORRESPONDING species tree 'S'
ldt = ldt_graph(T, S, lca_T=None, lca_S=None)
```

In order to reduce runtime, precomputed instances of the class LCA for T and S can be supplied using the keyword parameters lca_T and lca_S, respectively. Otherwise, such instances are initiated within the function.

3.7 ParaPhylo

The subpackage asymmetree.paraphylo contains a method to compute rooted species tree from orthology/paralogy relations. This is a re-implementation of ParaPhylo [21] which uses heuristics for the NP-hard optimization steps instead of exact ILP solutions. For cograph editing, the $\mathcal{O}(n^2)$ algorithm (where n is the number of vertices in a connected graph) by Crespelle [7] is applied (implemented in tralda). For the Maximum Consistent Triple Set problem, tree different heuristics are available:

BPMF	Best-Pair-Merge-First [42] (modified for weighted triples)
MINCUT	Aho's BUILD with weighted MinCut $[1, 2]$
GREEDY	a greedy approch based on Aho's BUILD

The class TreeReconstructor in the module SpeciesTreeFromParalogs computes a species tree after it is provides with one or more NetworkX graphs that represent (estimated) orthology relations. To this end, the nodes in these graph must have a 'color' attribute, since these will be the leaf labels in the reconstructed species tree. Example usage:

The module SpeciesTreeFromProteinOrtho contains functions to estimate a species tree from a ProteinOrtho [33] output file. For example, the function reconstruct_from_proteinortho(filename, triple_mode='BPMF') takes the filename to the output file and optionally the triple heuristic as input, and returns a tuple consisting of the estimated species tree (PhyloTree) and a Newick representation (str) containing support values for the inner nodes.

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A Subpackages and Modules

A.1 asymmetree

Packages and Modules	Description			
datastructures				
PhyloTree	Includes the class PhyloTree for phylogenetic trees (inherits from Tree), provides a Newick parser, etc.			
treeevolve				
SpeciesTree	Simulator for dated species trees with different models.			
GeneTree	Simulator for dated gene trees, construction of the observable gene tree.			
EvolutionRates	Simulation of evolution rate asymmetries, autocorrelation between ancestors and descendants as well as correlation between genes in the same species.			
NoisyMatrix	Generation of a noisy matrix (random perturbation or wrong topology noise).			
seqevolve				
Evolver	Includes the class Evolver for the simulation of sequences along a phylogenetic tree.			
EvolvingSequence	Data structure for sequences that are under evolution based on a doubly-linked list.			
SubstModel	Substitution model for the sequence simulation.			
IndelModel	Insertion/deletion (indel) model.			
HetModel	Model for rate heterogeneity among the sites of an evolving sequence and for invariant sites.			
Alignment	Construction of the true multiple sequence alignment after the simulation of sequences along a tree.			
EmpiricalModels	Contains rate matrices and equilibrium frequencies for the empirical substitution models, taken from [8].			
genome				
GenomeSimulator	Includes the class GenomeSimulator for the simulation of multiple gene families along a common species tree.			
tools				
DistanceCalculation	Calculation of maximum likelihood distances of pairs of aligned sequences.			
Sampling	Includes the class Sampler which support drawing numbers from various distributions.			
best_matches				
TrueBMG	Computation of the true (R)BMG from a gene tree as well as the true orthology relation.			
LeastResolvedTree	Construction of a least resolved tree (LRT) from a BMG via <i>informative</i> triples (optionally uses minimal edge cuts) or from a leaf-colored tree.			
Scenario	Wrapper class for species and gene tree scenarios, computation of the (R)BMG as well as event counts and some statistics.			
ExtBestHits	Implementation of the Extended Best Hits method, optionally uses qinfer.			
TreeReconstruction	Reconstruction of the gene tree with RapidNJ [37] and midpoint rooting.			

 $Table\ continued\ on\ next\ page$

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Quartets	Implementation of <i>Quartet</i> approach with two different methods for outgroup selection, optionally uses qinfer.	
Augmentation	Augmentation of the least resolved tree (w.r.t. some BMG) in order to identify all unambiguously false orthology assignments [36].	
hgt		
Fitch	Extraction of transfer edges from a gene tree (together with a species tree). Construction of the directed and undirected Fitch graph.	
TimeComparison	Comparison of the divergence time of genes with the divergence time of their respective species.	
paraphylo		
SpeciesTreeFromParalogs	Species tree reconstruction from orthology/paralogy relations. Heuris tic version of ParaPhylo [21].	
SpeciesTreeFrom ProteinOrtho	Species tree reconstruction from ProteinOrtho [33, 34] output.	
visualize		
GeneTreeVis	Visualization of simulated gene trees (of type PhyloTree), experimental.	

A.2 tralda

Packages and Modules	Description		
datastructures			
Tree	Includes the basic class Tree, provides functions for tree traversals Newick parser, etc. The class LCA provides efficient last common ancestor queries.		
LinkedList	Implementation of a linked list.		
DoublyLinkedList	Implementation of a doubly-linked list.		
AVLTree	Implementation of an ordered set (TreeSet) and an ordered dictionary (TreeDict) as a balanced binary search tree (AVL tree).		
Partition	Dynamic partition that supports efficient merge operations.		
hdtgraph.DynamicGraph	Dynamic graph data structure described by Holm et al. [25].		
supertree			
Build	Includes the classes Build and MTT for triple consistency tests and tree construction [1, 20].		
BuildST	Includes the class BuildST that computes a supertree from a given list of tress (with overlapping labels) [11].		
CommonRefinement	Includes the class LinCR and function linear_common_refinement() for computing the common refinement for trees on the same leaf set in linear time.		
tools			
GraphTools	Miscellaneous functions for graphs, e.g. check for graph equality.		
cograph			
Cograph	Includes the classes Cotree and CotreeNode as well as a generator for random cotrees/cographs. The class LinearCographDetector implements an $\mathcal{O}(V + E)$ algorithm for cograph detection and cotree construction [6].		

 $Table\ continued\ on\ next\ page$

CographEditor Implements a heuristic for cograph editing [7].

B Tree functions

The following table contains an overview over selected functions of the classes that inherit for Tree.

Tree (corresponding node class: TreeNode)			
leaves()	Generator for the leaf nodes.		
preorder()	Generator for preorder traversal.		
postorder()	Generator for postorder traversal.		
inner_vertices()	Generator for inner nodes/vertices.		
edges()	Generator for the edges of the tree.		
euler_generator()	Generator for an Euler tour.		
supply_leaves()	Add a list of leaf nodes in the subtree of each node as attribute leaves to each respective node, and return the full list (the root's list).		
contract(edges)	Contract all edges in the collection edges.		
<pre>get_triples()</pre>	Return a list of all triples that are displayed by the tree.		
to_newick()	Return a str representation of the tree in Newick format. Inheriting classes implement their own version of this function.		
<pre>random_tree(N, binary=False)</pre>	Return a random tree with N that is optionally forced to be binary. Stepwise, a new child is attached to a randomly selected node until N are reached.		
PhyloTree (corresponding no	de class: PhyloTreeNode)		
sorted_nodes(oldest_to_youngest=True)	Return a list of nodes sorted by timestamp (default is from oldest, which should be the root, to youngest).		
<pre>distance_matrix(leaf_order=None)</pre>	Return a list of nodes and a distance matrix on the leaves, optionally takes a list of the leaves that defines their indices in the matrix.		
parse_newick(newick)	Parse a Newick str.		
to_nx()	Return a NetworkX DiGraph version of the tree and its root.		
parse_nx(G, root)	Convert a tree encoded as a NetworkX DiGraph (together with the root) back into a PhyloTree.		
<pre>serialize(filename, mode=None)</pre>	Serialize a tree in JSON or pickle format specified by mode. Default is None, in which case the mode is inferred from the filename ending.		
<pre>load(filename, mode=None)</pre>	Load a tree from a file in JSON or pickle format specified by mode. Default is None, in which case the mode is inferred from the filename ending.		
copy()	Return a copy of the tree.		
Cotree (corresponding node	class: CotreeNode)		
to_cograph()	Return the corresponding cograph as a NetworkX Graph.		
cotree()	Convert a cograph into a cotree.		
<pre>complement(inplace=False)</pre>	Return the cotree of the complement cograph.		
copy()	Return a copy of the cotree.		
<pre>random_cotree(N, force_series_root=False)</pre>	Returns a random cotree with N leaves. Optionally forced to be connected (= root is a series node).		

C Distributions for sampling

The following distributions are available for sampling:

distribution	syntax	parameters
constant	x ('constant', x)	x must be a number
uniform (continuous)	('uniform', a, b)	$a \le b$ must be numbers
uniform (discrete)	('discrete_uniform', a, b)	$a \le b$ must be integers
gamma	('gamma', shape, scale)	shape and scale must be floats > 0
gamma (mean)	('gamma_mean', mean)	$\begin{array}{lll} \text{mean must be a number} > 0, \\ \text{shape= 1 and scale=mean/shape} \end{array}$
exponential	('exponential', rate)	rate must be a float $\geq 0 > 0$
Zipf	('zipf', a)	a > 1 must be a float value
negative binomial	('negative_binomial', r, q)	$r \ge 1$ must be an integer, $0 < q < 1$ a float value