Algorithms 2019 Q7

a) i) We pick a stamp of value v where $1 \leq v \leq n$ and then the optimal solution will be v + the optimal solution for $n-v$. (min)
Say opt(x) is the optimal ~~sol~~ number of stamps for a cents value of x, then

$$opt(x) = \min_{1 \leq v \leq x} (1 + opt(x-v))$$

We need to check every v since we don't know which one is optimal.
Also $opt(1) = 1$

ii) Store the number opt(x), and the actual solution sol(x) (specific stamps), in an array

iii) Space complexity is $\Theta(n)$ since we store the optimal solutions for 1 to n in an array.
Time complexity is quadratic because at each of the n steps we have to iterate over the stamps to find the minimum so $\Theta(n^2)$

(I found this really hard so I basically just copied the solution)

~~b) i) Say stations(x) is the optimal set of stations for distance l. The optimal solution will have a station S_x plus the optimal solution for stations(d_x)~~

~~$$|\text{stations}(n)| = \min_{1 \le x \le l}(1 + |\text{stations}(d_x)|)$$~~

~~So check all the stations that car can reach and then use the one that gives the .~~

If B is less than l away then no stops. Otherwise say s_i is the first stop in the optimal solution. Let opt(i) be the minimum stops to reach B from s_i. Let stations(i) store the indices in this optimal solution.

$$\text{opt}(i) = \begin{cases} 0 & \text{if } d_n - d_i \le l \\ 1 + \min\limits_{i<j<n} \text{opt}(j) \end{cases}$$

At each step when the min is found add that station to stations(i).

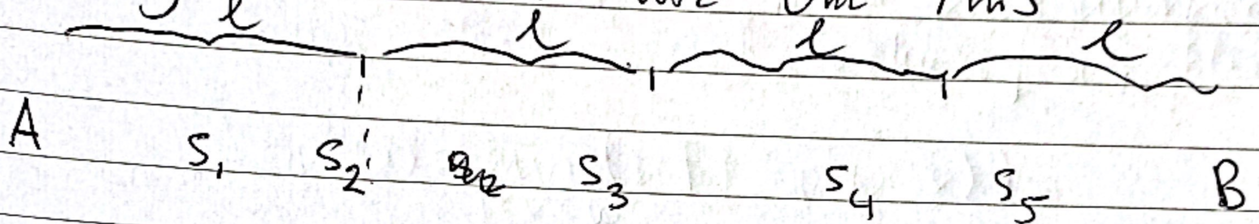
ii) Same array structure as previous question storing (opt(i), stations(i)) for each stop.

iii) Similarly to last question, space complexity is $\Theta(n)$ and time complexity is $\Theta(n^2)$.

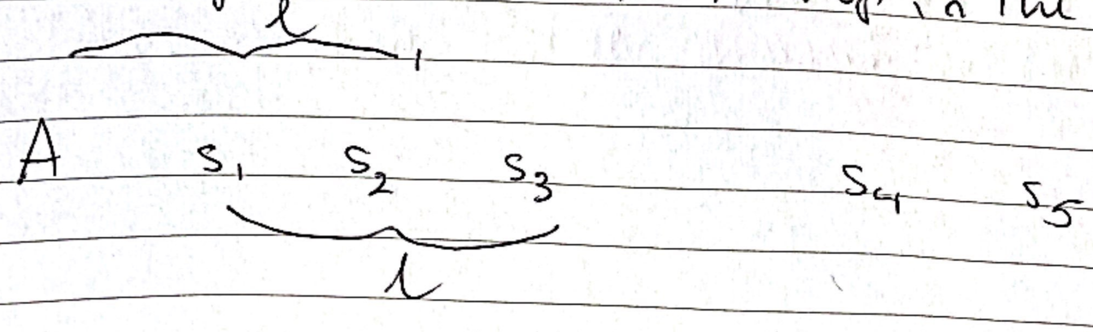
(Again I mainly used the solutions, I really don't know how to tackle these problems)

c) A greedy algorithm would be better for (b) since we can always take the furthest stop that is ~~less than~~ $\le l$ distance away.

Say our stations look like this



Say we have an optimal solution where the first stop s_i is not the furthest stop that could be reached by the car on a full tank.
Then in the optimal solution we go from s_i to ~~either~~ s_j.
s_j is either the furthest stop we could have reached or eventually we will either skip that stop or go to it.
Therefore since adjacent stops always have a distance $\le l$ we could have gone straight to the furthest stop and still been able to get to the next stop in the solution



If s_1, s_2, s_4 was optimal we could also have done s_1, s_3, s_4 since $d_4 - d_3 \le d_4 - d_2$.

c) At each step we find the furthest station we can reach which is linear $\Theta(n)$