PAM 006

Pulse Amplitude Modulation 006, with different pulse shapes p(t) and noise added. Added raised cosine in frequency ('rcf'), manchester ('man'), and "manchester sine" ('msin') pulse.

Thus version uses matched filter receiver.

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

In [2]:
```
%matplotlib notebook
fsz = (7, 4)
fsz1 = (fsz[0], 1.4*fsz[1])
fsz2 = (fsz[0], fsz[1]/2.0)
```

In [3]:
```
def asc2bin(txt, bits):
    """
    ASCII character sequence to binary string conversion.
    >>>>> dn = asc2bin(txt, bits) <<<<<
    where  dn        binary output string
           txt       text input (ASCII)
           bits<0    MSB first conversion
           bits>=0   LSB first conversion
           |bits|    number of bits per character
    """
    txt_10 = list(ord(chr) for chr in txt)
    if (bits < 0):
        pow2 = list(2**(i+1) for i in range(bits,0))
    else:
        pow2 = list((2**-i) for i in range(bits))
    B = np.array(np.outer(txt_10, pow2), int)
    B = np.mod(B, 2)
    dn = np.reshape(B, -1)
    return dn
```

In [4]:
```
def bin2asc(dn, bits):
    """
    Binary string to ASCII character string conversion.
    >>>>> txt = bin2asc(dn, bits) <<<<<
    where  txt       output string (ASCII)
           dn        binary string
           bits<0    MSB first conversion
           bits>=0   LSB first conversion
           |bits|    number of bits per character
    """
    Lb = int(np.floor(len(dn)/abs(bits)))  # length in multiples of 'bits'
    dn = dn[:Lb*abs(bits)]
    B = np.reshape(dn, (-1, abs(bits)))
    if (bits < 0):
        pow2 = list(2**(i-1) for i in range(abs(bits),0,-1))
    else:
        pow2 = list(2**i for i in range(bits))
    txt_10 = np.inner(B, pow2)
    return ''.join(chr(i) for i in txt_10)
```

In [5]:
```python
def pam_pt(FB, Fs, ptype, pparms=[]):
    """
    Generate PAM pulse p(t)
    >>>>> ttp, pt = pam_pt(FB, Fs, ptype, pparms) <<<<<
    where  ttp:   time axis for p(t)
           pt:    PAM pulse p(t)
           FB:    Baud rate  (Fs/FB=sps)
           Fs:    sampling rate of p(t)
           ptype: pulse type from list
                  ('man', 'msin', rcf', 'rect', 'rrcf', 'sinc', 'tri')
           pparms not used for 'rect','tri'
           pparms = [k, alfa]  for 'rcf'
           pparms = [k, beta]  for 'sinc'
           k:     "tail" truncation parameter for 'sinc'
                  (truncates p(t) to -k*TB <= t < k*TB)
           beta:  Kaiser window parameter for 'sinc'
           alfa: Rolloff parameter for 'rcf', 0<=alfa<=1
    """
    ptyp = ptype.lower()
    if (ptyp=='rect' or ptyp=='man' or ptyp=='msin'):
        kR = 0.5; kL = -kR
    elif ptyp=='tri':
        kR = 1.0; kL = -kR
    elif (ptyp=='rcf' or ptyp=='rrcf' or ptyp=='sinc'):
        kR = pparms[0]; kL = -kR
    else:
        kR = 0.5; kL = -kR
    tpL, tpR = kL/float(FB), kR/float(FB)
    ixpL, ixpR = int(np.ceil(tpL*Fs)), int(np.ceil(tpR*Fs))
    ttp = np.arange(ixpL, ixpR)/float(Fs)  # time axis for p(t)
    pt = np.zeros(ttp.size)
    if ptyp=='man':
        pt = -np.ones(ttp.size)
        ixp = np.where(ttp>=0)
        pt[ixp] = 1
    elif ptyp=='msin':
        pt = np.sin(2*np.pi*FB*ttp)
    elif ptyp=='rcf':
        pt = np.sinc(FB*ttp)
        if pparms[1] != 0:
            p2t = np.pi/4.0*np.ones(ttp.size)
            ix = np.where(np.power(2*pparms[1]*FB*ttp, 2.0) != 1)[0]
            p2t[ix] = np.cos(np.pi*pparms[1]*FB*ttp[ix])
            p2t[ix] = p2t[ix]/(1-np.power(2*pparms[1]*FB*ttp[ix],2.0))
            pt = pt*p2t
    elif ptyp=='rect':
        ixp = np.where(np.logical_and(ttp>=tpL,ttp<tpR))[0]
        pt[ixp] = 1    # rectangular pulse p(t)
    elif (ptype=='rrcf'):       # Root raised cosine in freq
        alfa = pparms[1]        # Rolloff parameter
        falf = 4*alfa*FB
        pt = (1-alfa+4*alfa/np.pi)*np.ones(len(ttp))
        ix = np.where(np.logical_and(ttp!=0,np.power(falf*ttp,2.0)!=1.0))[0]
        pt[ix] = np.sin((1-alfa)*np.pi*FB*ttp[ix])
        pt[ix] = pt[ix]+falf*ttp[ix]*np.cos((1+alfa)*np.pi*FB*ttp[ix])
        pt[ix] = 1.0/(FB*np.pi)*pt[ix]/((1-np.power(falf*ttp[ix],2.0))*ttp[ix])
        ix = np.where(np.power(falf*ttp,2.0)==1.0)[0]
        pt[ix] = (1+2/np.pi)*np.sin(np.pi/(4*alfa))+(1-2/np.pi)*np.cos(np.pi/(4*alf
a))
        pt[ix] = alfa/np.sqrt(2.0)*pt[ix]
    elif ptyp=='sinc':
        pt = np.sinc(FB*ttp)
        if len(pparms) > 1:        # Apply Kaiser window
            pt = pt*np.kaiser(len(pt),pparms[1])
```

In [6]:
```
def pam15(an, FB, Fs, ptype, pparms=[]):
    """
    Pulse amplitude modulation: a_n -> s(t), -TB/2<=t<(N-1/2)*TB,
    V1.5 for 'man', 'msin', 'rcf', 'rect', 'rectx', 'rrcf', 'sinc', and
    'tri' pulse types.
    >>>>> tt, st = pam15(an, FB, Fs, ptype, pparms) <<<<<
    where  tt:    time axis for PAM signal s(t) (starting at -TB/2)
           st:    PAM signal s(t)
           an:    N-symbol DT input sequence a_n
           FB:    baud rate of a_n, TB=1/FB
           Fs:    sampling rate of s(t)
           ptype: pulse type from list
                  ('man','rcf','rect','rrcf','sinc','tri')
           pparms not used for 'man','rect','tri'
           pparms = [k, alpha] for 'rcf','rrcf'
           pparms = [k, beta]  for 'sinc'
           k:     "tail" truncation parameter for 'rcf','rrcf','sinc'
                  (truncates p(t) to -k*TB <= t < k*TB)
           alpha: Rolloff parameter for 'rcf','rrcf', 0<=alpha<=1
           beta:  Kaiser window parameter for 'sinc'
    """
    N = len(an)
    ixL = round(-0.5*Fs/float(FB))     # Left index for time axis
    tlen = N/float(FB)    # duration of PAM signal in sec
    tt = np.arange(round(Fs*tlen))/float(Fs)
    tt = tt + ixL/float(Fs)    # shift time axis left by TB/2
    if ptype.lower() == 'rectx':
        ixa = np.array(np.round(Fs/float(FB)*np.arange(N)),np.int64)
        st = np.zeros(tt.size)
        st[ixa] = Fs*np.diff(np.hstack((0, an)));    # place transitions in s(t)
        st = np.cumsum(st)/float(Fs)
    else:
        ixa = np.array(np.round(Fs/float(FB)*(0.5+np.arange(N))),np.int64)
        ast = np.zeros(tt.size)
        ast[ixa] = Fs*an    # as(t) is CT version of an
        ttp, pt = pam_pt(FB, Fs, ptype, pparms)
        # Convolution  as(t)*p(t)
        st = np.convolve(ast, pt)/float(Fs)   # s(t) = a_s(t)*p(t)
        ixttp0 = np.argmin(np.abs(ttp))   # index for t=0 on ttp
        st = st[ixttp0:]  # trim after convolution
        st = st[:tt.size]  # PAM signal s(t)
    return tt, st
```

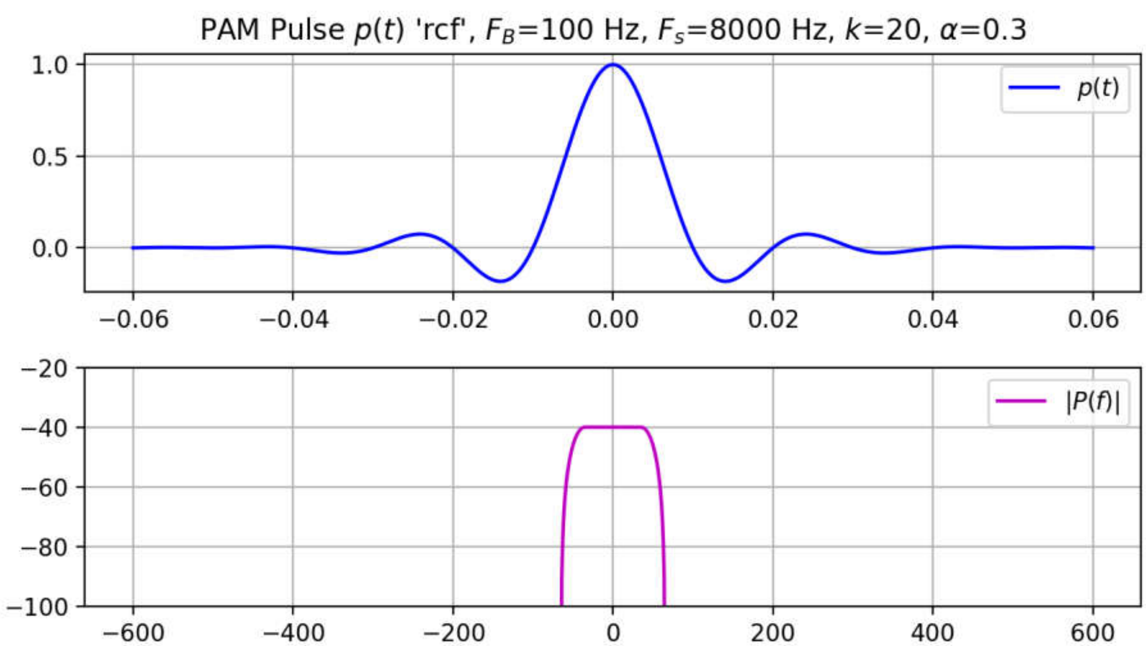
In [7]:
```
def pamrcvr15(tt, rt, FBparms, ptype, pparms=[]):
    """
    Pulse amplitude modulation receiver with matched filter:
    r(t) -> b(t) -> bn.
    V1.5 for 'man', 'msin', 'rcf', 'rect', 'rrcf', 'sinc', and 'tri'
    pulse types.
    >>>>> bn, bt, ixn = pamrcvr15(tt, rt, FBparms, ptype, pparms) <<<<<
    where  tt:     time axis for r(t)
           rt:     received (noisy) PAM signal r(t)
           FBparms: = [FB, dly]
           FB:     Baud rate of PAM signal, TB=1/FB
           dly:    sampling delay for b(t) -> b_n as a fraction of TB
                   sampling times are t=n*TB+t0 where t0 = dly*TB
           ptype: pulse type from list
                   ('man','msin','rcf','rect','rrcf','sinc','tri')
           pparms not used for 'man','msin','rect','tri'
           pparms = [k, alpha]  for 'rcf','rrcf'
           pparms = [k, beta]  for 'sinc'
           k:      "tail" truncation parameter for 'rcf','rrcf','sinc'
                   (truncates p(t) to -k*TB <= t < k*TB)
           alpha: rolloff parameter for ('rcf','rrcf'), 0<=alpha<=1
           beta:  Kaiser window parameter for 'sinc'
           bn:     received DT sequence after sampling at t=n*TB+t0
           bt:     received PAM signal b(t) at output of matched filter
           ixn:    indexes where b(t) is sampled to obtain b_n
    """
    if type(FBparms)==int:
        FB, t0 = FBparms, 0
    else:
        FB, t0 = FBparms[0], 0
        if len(FBparms) > 1:
            t0 = FBparms[1]
    Fs = (len(tt)-1)/(tt[-1]-tt[0])
    # ***** Set up matched filter response h_R(t) *****
    ttp, pt = pam_pt(FB, Fs, ptype, pparms)
    hRt = pt[::-1]              # h_R(t) = p(-t)
    hRt = Fs/np.sum(np.power(pt,2.0))*hRt  # h_R(t) normalized
    # Convolution  r(t)*h_R(t)
    bt = np.convolve(rt, hRt)/float(Fs)  # b(t) = r(t)*h_R(t)
    ixttp0 = np.argmin(np.abs(ttp))  # index for t=0 on ttp
    bt = bt[ixttp0:]  # trim after convolution
    bt = bt[:tt.size]  # PAM signal b(t) after matched filter
    N = np.ceil(FB*(tt[-1]-tt[0]))    # Number of symbols
    ixn = np.array(np.around((np.arange(N)+0.5+t0)*Fs/FB),int)
                                # Sampling indexes
    ix = np.where(np.logical_and(ixn>=0,ixn<len(tt)))[0]
    ixn = ixn[ix]               # Trim to existing indexes
    bn = bt[ixn]                # DT sequence sampled at t=n*TB+t0
    return bn, bt, ixn
```

In [8]:
```
# Parameters
Fs = 8000     # sampling rate
FB = 100      # Baud rate FB = 1/TB
bits = 8      # bits per char, LSB first conversion
polar = 1     # polar or unipolar
SNRdB = 10     # signal-to-noise ratio
#ptype, pparms = 'man', []
#ptype, pparms = 'msin', []
#ptype, pparms = 'rect', []
#ptype, pparms = 'tri', []
#ptype, pparms = 'sinc', [10, 6]
#ptype, pparms = 'sinc', [10, 0]
#ptype, pparms = 'rcf', [20, 0.1]
ptype, pparms = 'rcf', [20, 0.3]
#ptype, pparms = 'rcf', [20, 0.5]
#ptype, pparms = 'rrcf', [20, 0.3]
txt = 'AXOLOTL, also known as the Mexican Walking Fish,\n'
txt = txt + 'this amphibious salamander is critically endangered,\n'
txt = txt + 'and nearly went extinct in 2010.\n'
txt = txt + 'Although the axolotl is colloquially known as a "walking fish",\n'
txt = txt + 'it is not a fish, but an amphibian. The species was originally found\n
'
txt = txt + 'in several lakes, such as Lake Xochimilco underlying Mexico City.\n'
txt = txt + 'Axolotls are unusual among amphibians in that they reach adulthood\n'
txt = txt + 'without undergoing metamorphosis. Instead of developing lungs and\n'
txt = txt + 'taking to the land, adults remain aquatic and gilled.'
Ltxt = len(txt)
print(Ltxt)
dn = asc2bin(txt, bits)
#dn = np.random.randint(2, size=Ltxt)
```

```
520
```

In [9]:
```
# Create single pulse p(t) for display
ptlen = 1      # p(t) duration
Npt = round(FB*ptlen)
pn = np.zeros(Npt)
pn[round(Npt/2.0)] = 1
ttpt, pt = pam15(pn, FB, Fs, ptype, pparms)
ttpt = ttpt-round(Npt/2.0)/float(FB)
Pf = np.fft.fft(np.fft.fftshift(pt))/float(Fs)    # FT approximation
NPf = Pf.size
DPf = Fs/float(NPf)
ffPf = DPf*np.arange(NPf)-Fs/2.0
Pf = np.fft.fftshift(Pf)
```

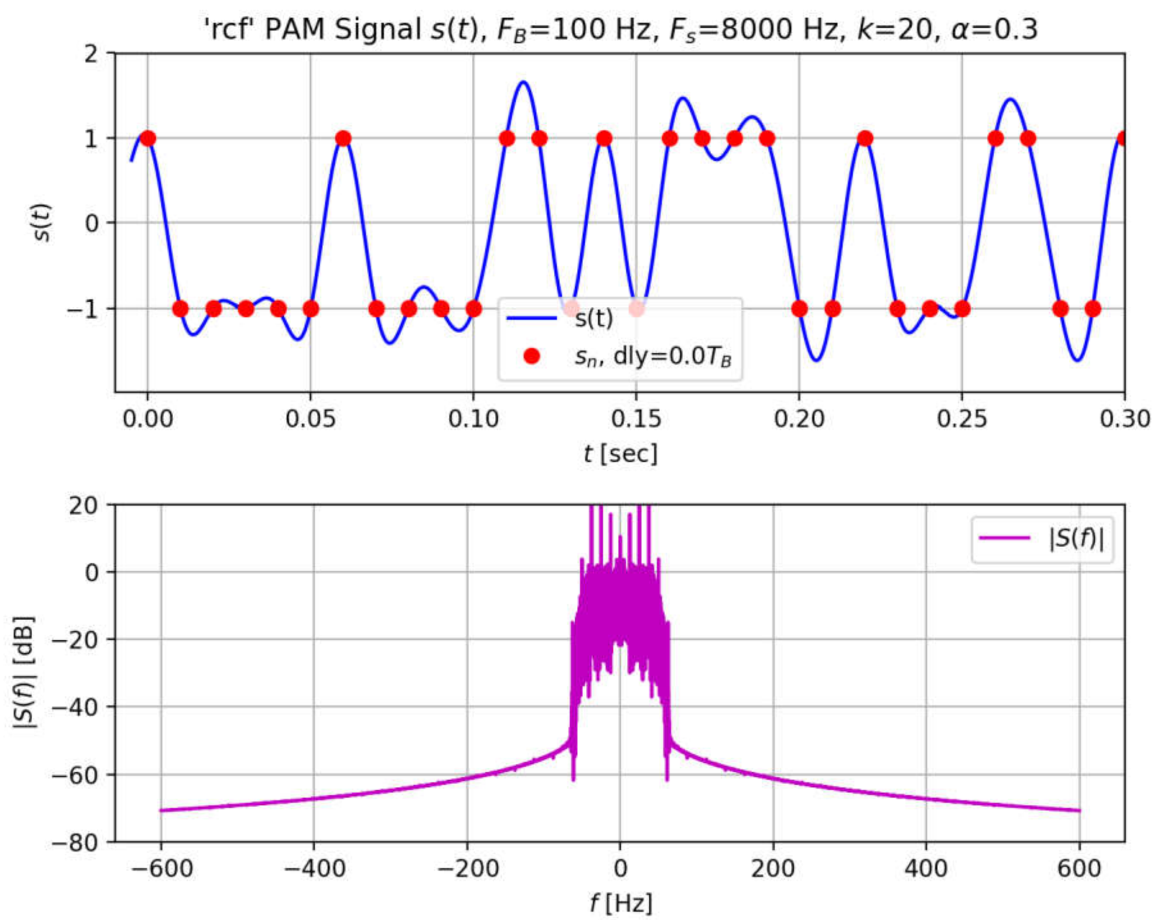
In [10]:
```
ttpt2 = 6/float(FB); ttpt1 = -ttpt2
ffPf2 = 6*FB; ffPf1 = -ffPf2
ixdttpt = np.where(np.logical_and(ttpt>=ttpt1, ttpt<=ttpt2))
ixdffPf = np.where(np.logical_and(ffPf>=ffPf1, ffPf<=ffPf2))
plt.figure(3, figsize=fsz)
plt.subplot(211)
plt.plot(ttpt[ixdttpt], pt[ixdttpt], '-b', label='$p(t)$')
strt3 = "PAM Pulse $p(t)$ '{}'".format(ptype)
strt3 = strt3 + ', $F_B$={} Hz'.format(FB)
strt3 = strt3 + ', $F_s$={} Hz'.format(Fs)
if ptype == 'sinc':
    strt3 = strt3 + ', $k$={}, $\\beta$={}'.format(*pparms)
if (ptype == 'rcf' or ptype == 'rrcf'):
    strt3 = strt3 + ', $k$={}, $\\alpha$={}'.format(*pparms)
plt.title(strt3)
plt.legend()
plt.grid()
plt.subplot(212)
plt.plot(ffPf[ixdffPf], 20*np.log10(np.abs(Pf[ixdffPf])), '-m', label='$|P(f)|$')
plt.ylim([-100, -20])
plt.legend()
plt.grid()
plt.tight_layout()
```



In [11]:
```
# Generate PAM signal s(t)
if polar:
    an = 2*dn-1
else:
    an = dn
tt, st = pam15(an, FB, Fs, ptype, pparms)
Sf = np.fft.fft(st)/float(Fs)
NSf = Sf.size
DSf = Fs/float(NSf)
ff = DSf*np.arange(NSf)-Fs/2.0
Sf = np.fft.fftshift(Sf)
```

In [12]:
```
# s(t) sampling times
dlys = 0.0    # sampling delay as fraction of TB
NSs = FB/float(Fs)*np.floor(st.size)
ixss = np.array(np.round(Fs/float(FB)*(0.5+np.arange(NSs)+dlys)),np.int64)
       # sampling times (n+dlys)*TB
ix = np.where(np.logical_and(ixss>=0, ixss<st.size))[0]
ixss = ixss[ix]      # trim ixss values to indexes in s(t)
sn = st[ixss]
```

In [13]:
```
ff2 = 6*FB; ff1 = -ff2
ixdff = np.where(np.logical_and(ff>=ff1, ff<=ff2))
plt.figure(7, figsize=fsz1)
plt.subplot(211)
plt.plot(tt, st, '-b', label='s(t)')
plt.plot(tt[ixss], st[ixss], 'or', label='$s_n$, dly={}$T_B$'.format(dlys))
plt.xlim([-0.01, 0.3])
strt7 = "'{}' PAM Signal $s(t)$".format(ptype)
strt7 = strt7 + ', $F_B$={} Hz'.format(FB)
strt7 = strt7 + ', $F_s$={} Hz'.format(Fs)
if ptype == 'sinc':
    strt7 = strt7 + ', $k$={}, $\\beta$={}'.format(*pparms)
if (ptype == 'rcf' or ptype == 'rrcf'):
    strt7 = strt7 + ', $k$={}, $\\alpha$={}'.format(*pparms)
plt.title(strt7)
plt.ylabel('$s(t)$')
plt.xlabel('$t$ [sec]')
plt.legend()
plt.grid()
plt.subplot(212)
plt.plot(ff[ixdff], 20*np.log10(np.abs(Sf[ixdff])), '-m', label='$|S(f)|$')
plt.ylim([-80, 20])
plt.ylabel('$|S(f)|$ [dB]')
plt.xlabel('$f$ [Hz]')
plt.legend()
plt.grid()
plt.tight_layout()
```



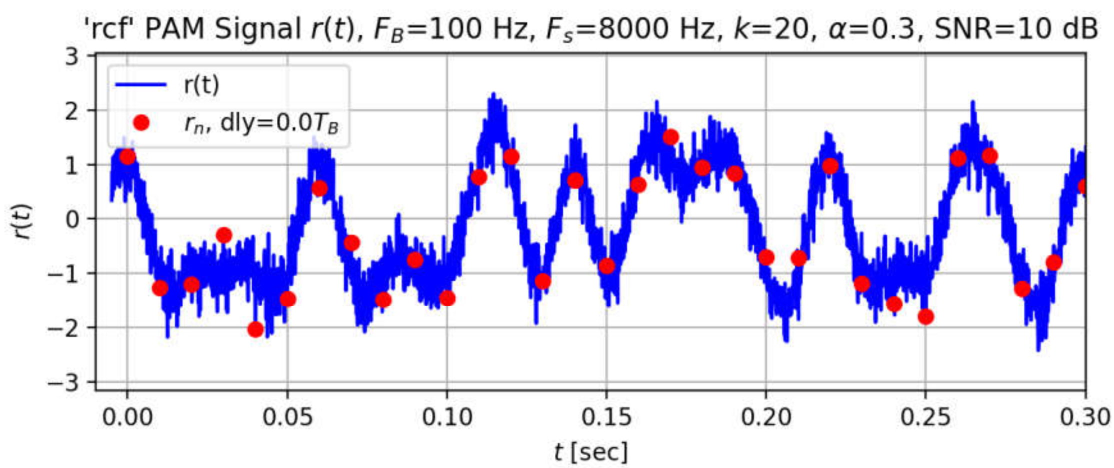
In [14]:
```
# Generate Gaussian noise
nt = np.random.randn(st.size)
P_nt = np.mean(np.power(nt, 2.0))  # randn noise power
SNR = 10**(SNRdB/10.0)
Ps = np.mean(np.power(st, 2.0))
An = np.sqrt(Ps/(SNR*P_nt))
P_Ant = np.mean(np.power(An*nt, 2.0))
print('Ps={:4.3f}, Pn={:5.4f}'.format(Ps, P_Ant))
```

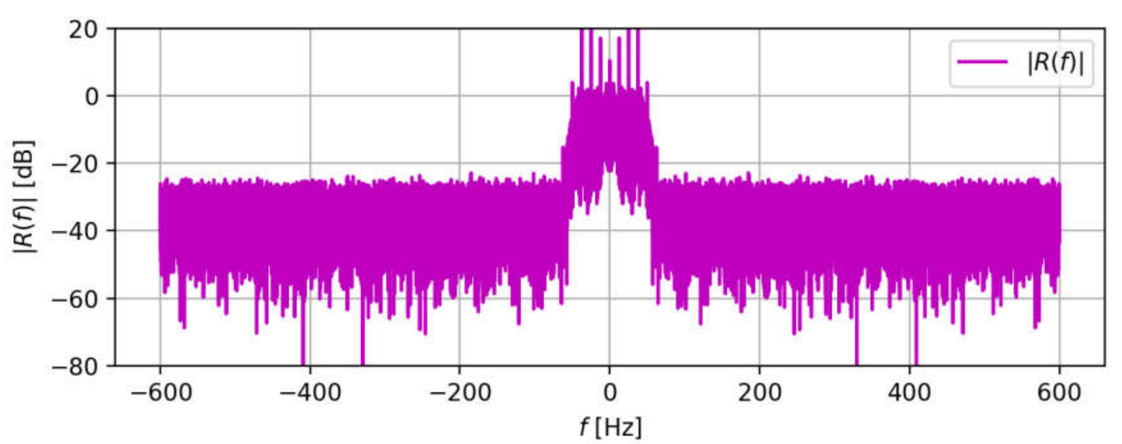
```
Ps=0.945, Pn=0.0945
```

In [15]:
```
# Received signal
rt = st + An*nt
Rf = np.fft.fft(rt)/float(Fs)
Rf = np.fft.fftshift(Rf)
```

In [16]:
```
# r(t) sampling times
dlyr = 0.0   # sampling delay as fraction of TB
NSr = FB/float(Fs)*np.floor(rt.size)
ixsr = np.array(np.round(Fs/float(FB)*(0.5+np.arange(NSr)+dlyr)),np.int64)
       # sampling times (n+dlyr)*TB
ix = np.where(np.logical_and(ixsr>=0, ixsr<rt.size))[0]
ixsr = ixsr[ix]     # trim ixss values to indexes in s(t)
rn = rt[ixsr]
```

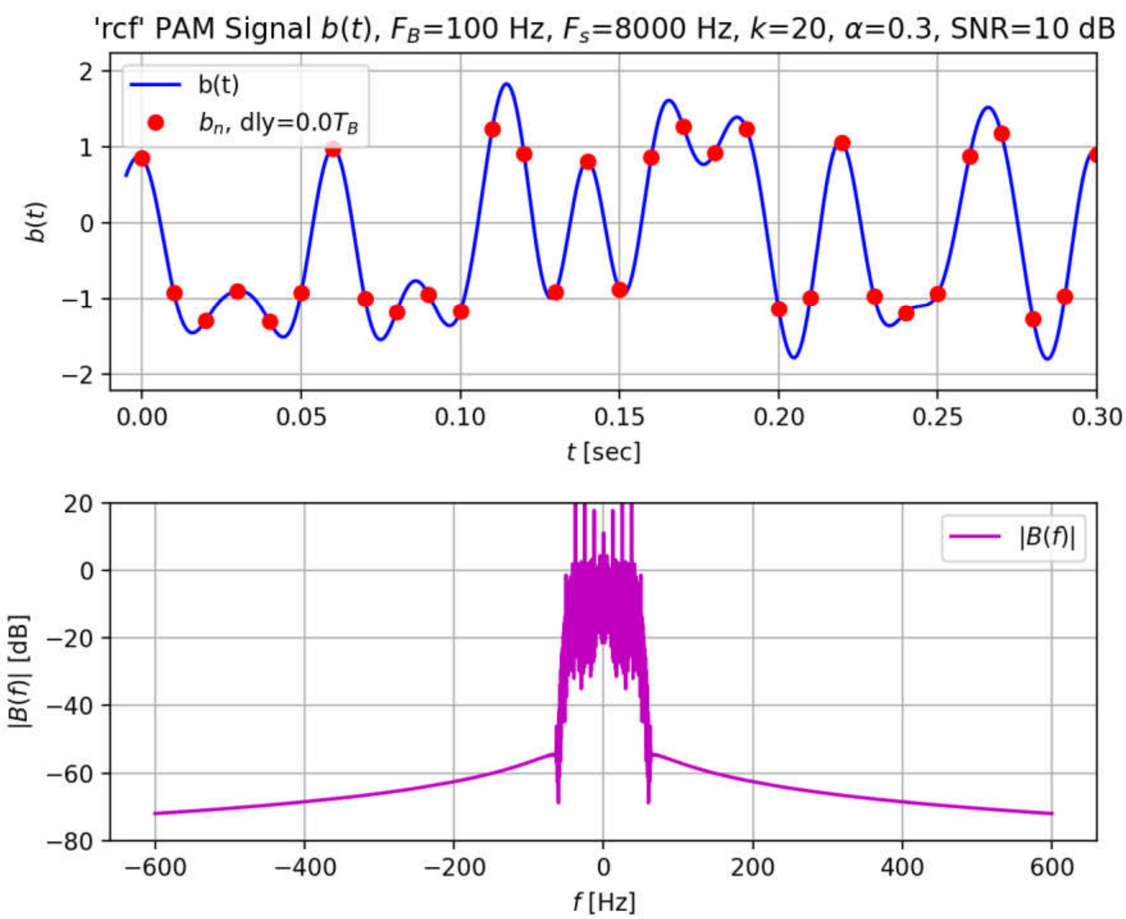
In [17]:
```
plt.figure(11, figsize=fsz1)
plt.subplot(211)
plt.plot(tt, rt, '-b', label='r(t)')
plt.plot(tt[ixsr], rt[ixsr], 'or', label='$r_n$, dly={}$T_B$'.format(dlyr))
plt.xlim([-0.01, 0.3])
strt11 = "'{}' PAM Signal $r(t)$".format(ptype)
strt11 = strt11 + ', $F_B$={} Hz'.format(FB)
strt11 = strt11 + ', $F_s$={} Hz'.format(Fs)
if ptype == 'sinc':
    strt11 = strt11 + ', $k$={}, $\\beta$={}'.format(*pparms)
if (ptype == 'rcf' or ptype == 'rrcf'):
    strt11 = strt11 + ', $k$={}, $\\alpha$={}'.format(*pparms)
strt11 = strt11 + ', SNR={} dB'.format(SNRdB)
plt.title(strt11)
plt.ylabel('$r(t)$')
plt.xlabel('$t$ [sec]')
plt.legend()
plt.grid()
plt.subplot(212)
plt.plot(ff[ixdff], 20*np.log10(np.abs(Rf[ixdff])), '-m', label='$|R(f)|$')
plt.ylim([-80, 20])
plt.ylabel('$|R(f)|$ [dB]')
plt.xlabel('$f$ [Hz]')
plt.legend()
plt.grid()
plt.tight_layout()
```





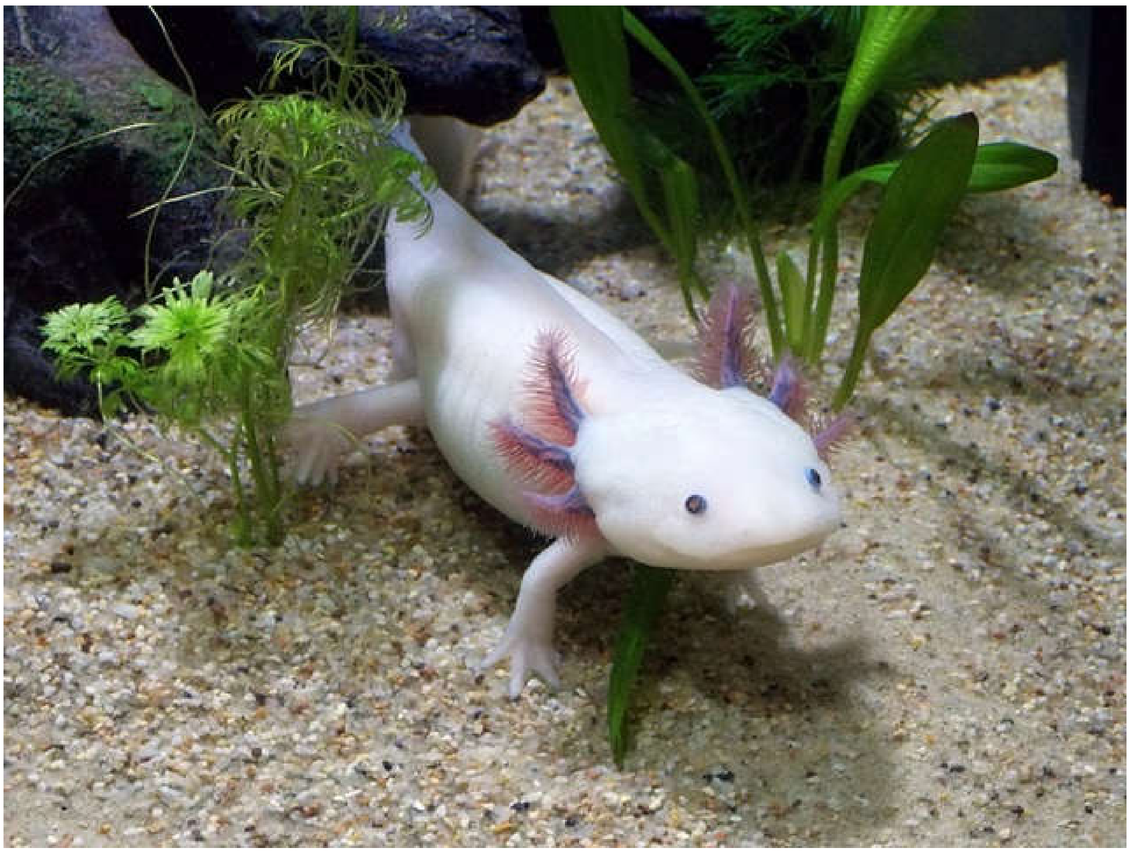
In [18]:
```
# Demodulated signal
dlyb = 0.0    # sampling delay as fraction of TB
bn, bt, ixn = pamrcvr15(tt, rt, [FB, dlyb], ptype, pparms)
Bf = np.fft.fft(bt)/float(Fs)
Bf = np.fft.fftshift(Bf)
```

In [19]:
```
plt.figure(15, figsize=fsz1)
plt.subplot(211)
plt.plot(tt, bt, '-b', label='b(t)')
plt.plot(tt[ixn], bt[ixn], 'or', label='$b_n$, dly={}$T_B$'.format(dlyb))
plt.xlim([-0.01, 0.3])
strt15 = "'{}' PAM Signal $b(t)$".format(ptype)
strt15 = strt15 + ', $F_B$={} Hz'.format(FB)
strt15 = strt15 + ', $F_s$={} Hz'.format(Fs)
if ptype == 'sinc':
    strt15 = strt15 + ', $k$={}, $\\beta$={}'.format(*pparms)
if (ptype == 'rcf' or ptype == 'rrcf'):
    strt15 = strt15 + ', $k$={}, $\\alpha$={}'.format(*pparms)
strt15 = strt15 + ', SNR={} dB'.format(SNRdB)
plt.title(strt15)
plt.ylabel('$b(t)$')
plt.xlabel('$t$ [sec]')
plt.legend()
plt.grid()
plt.subplot(212)
plt.plot(ff[ixdff], 20*np.log10(np.abs(Bf[ixdff])), '-m', label='$|B(f)|$')
plt.ylim([-80, 20])
plt.ylabel('$|B(f)|$ [dB]')
plt.xlabel('$f$ [Hz]')
plt.legend()
plt.grid()
plt.tight_layout()
```



In [21]:
```
# Recover received text
an_hat = bn
if polar:
    dn_hat = np.array((an_hat>=0), int)    # convert to integers
else:
#     dn_hat = np.array(np.round(rn), int)  # convert to integers
    dn_hat = np.array((an_hat>=0.5), int)    # convert to integers
txt_hat = bin2asc(dn_hat, bits)
print('Received txt:\n{}'.format(txt_hat))
```

```
Received txt:
AXOLOTL, also known as the Mexican Walking Fish,
this amphibious salamander is critically endangered,
and nearly went extinct in 2010.
Although the axolotl is colloquially known as a "walking fish",
it is not a fish, but an amphibian. The species was originally found
in several lakes, such as Lake Xochimilco underlying Mexico City.
Axolotls are unusual among amphibians in that they reach adulthood
without undergoing metamorphosis. Instead of developing lungs and
taking to the land, adults remain aquatic and gilled.
```



In [ ]: