Mathematics and Problem Solving

Lecture 1.0

Course Introduction

Hello!

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Overview

- The Course
  - Materials
  - Assessment
  - Content
- What is Mathematics? (Parts I, II, and III)

The Course

Course Materials

- Lectures
  - Online at https://mathsforcs.netlify.app
  - Videos, questions, exercises
  - Complete in your own time
- Attend practicals on Fridays

Practicals

- Software
  - Microsoft Teams
  - Replit https://repl.it
- Problem-based
  - Materials online in advance if you want them

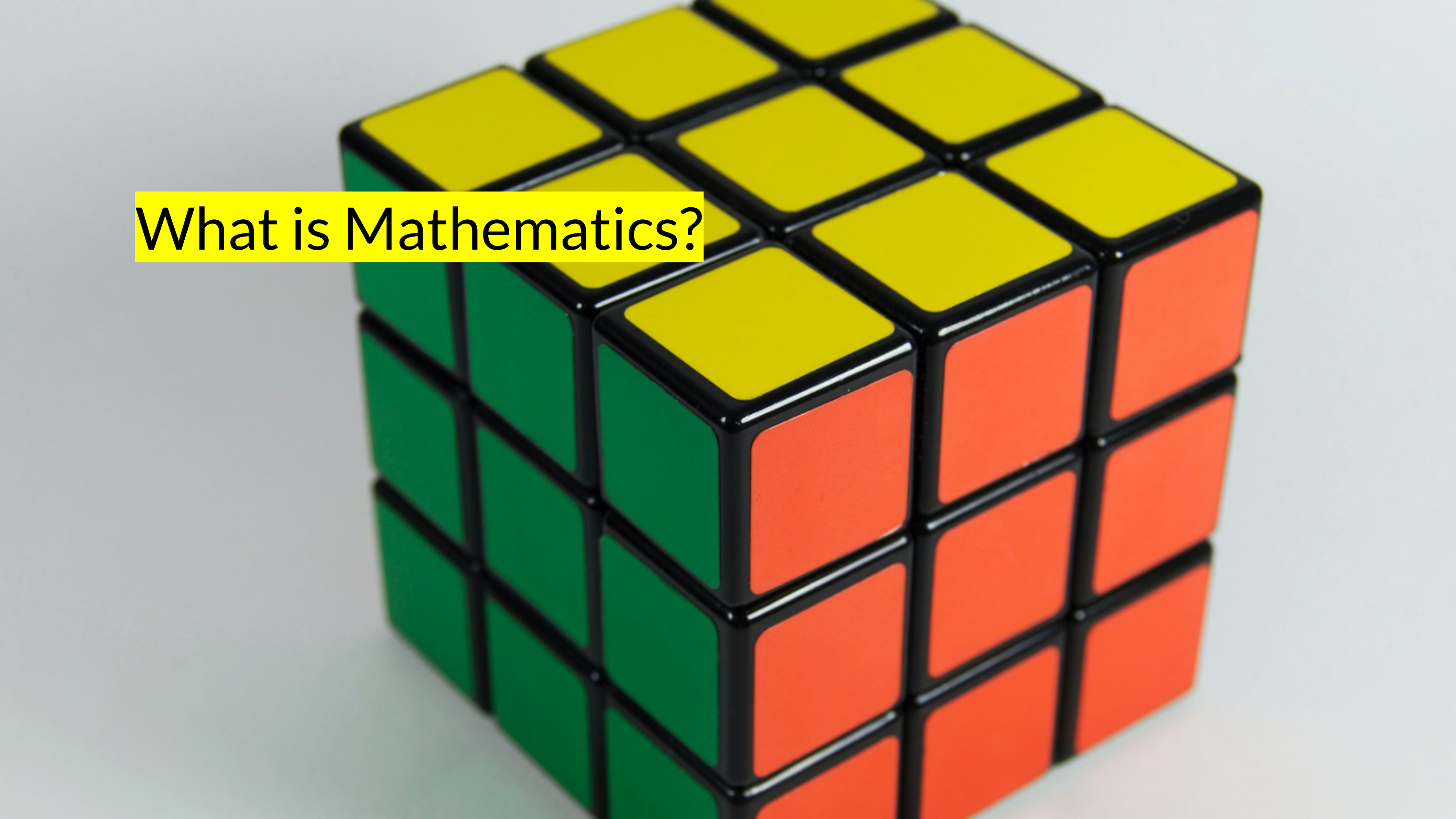
Assessment

- Coursework assignment
  - Multiple parts
  - Demonstrating knowledge and skills taught in the course
  - Apply knowledge to solve problems
- Prepare by completing questions from lectures and practicals

Content

Week	Topic	Lab Date
Week 1	Introduction (No preparation required for this week)	25/09/2020
Week 2	Numerical Systems	02/10/2020
Week 3	Modular Arithmetic	09/10/2020
Week 4	Asymptotic Complexity	16/10/2020
Week 5	Propositional Logic	23/10/2020
Week 6	Set Theory	30/10/2020
Week 7	Proof	06/11/2020
Week 8	Descriptive Statistics	13/11/2020
Week 9	(Reading week)	20/11/2020
Week 10	Probability	27/11/2020
Week 11	Inferential Statistics	04/12/2020
Week 12	TBD	11/12/2020

What is Mathematics?



Why was it invented?

- Problem solving
  - Area of fields
  - Inheritance
  - Taxes
  - Distribution of grain

Babylonians

- Algorithmic
- Base 60 number system (many different units)
- Zero as a placeholder

Indians

- Astronomy
- Calculating very big numbers
- Positional notation
- Oral transmission in verse
- Decimal number system

Greeks

- Aristocratic
- Geometry
- Proof from Axoims
- Euclid

Pythagoras

Islamic Empire

- Algebra
- Algorithms
- Still written in prose

Muhammad ibn Musa al-Khwarizmi

What is it?

- Solving problems
- Patterns
- Language to abstract these things

Mathematical Language

- Way of expressing problems and their solutions
  - Verse
  - Prose
  - Symbols
- To communicate knowledge

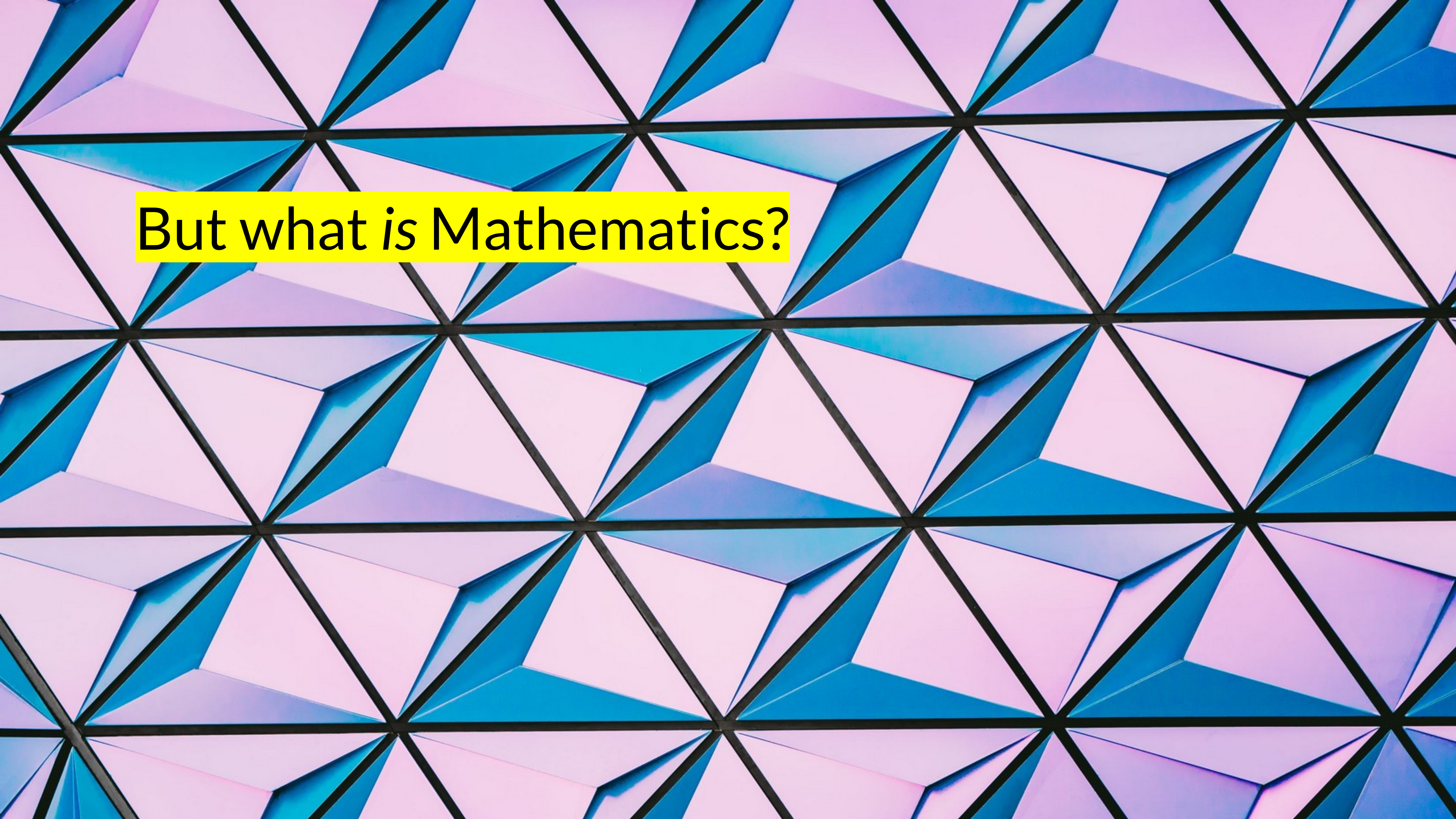
What sort of knowledge?

- Games
  - Drawing curves
  - Graphics/shaders
  - Path-finding
  - Physics engines
  - Simulations
- Software Engineering
  - Complexity
  - Networks
  - AI
  - Big data

Why Maths?

- Benefits of using maths for this
  - Precise
  - Unambiguous
  - Easy to interpret (once you know how)
  - Concise
- Perfect if you want to implement an algorithm

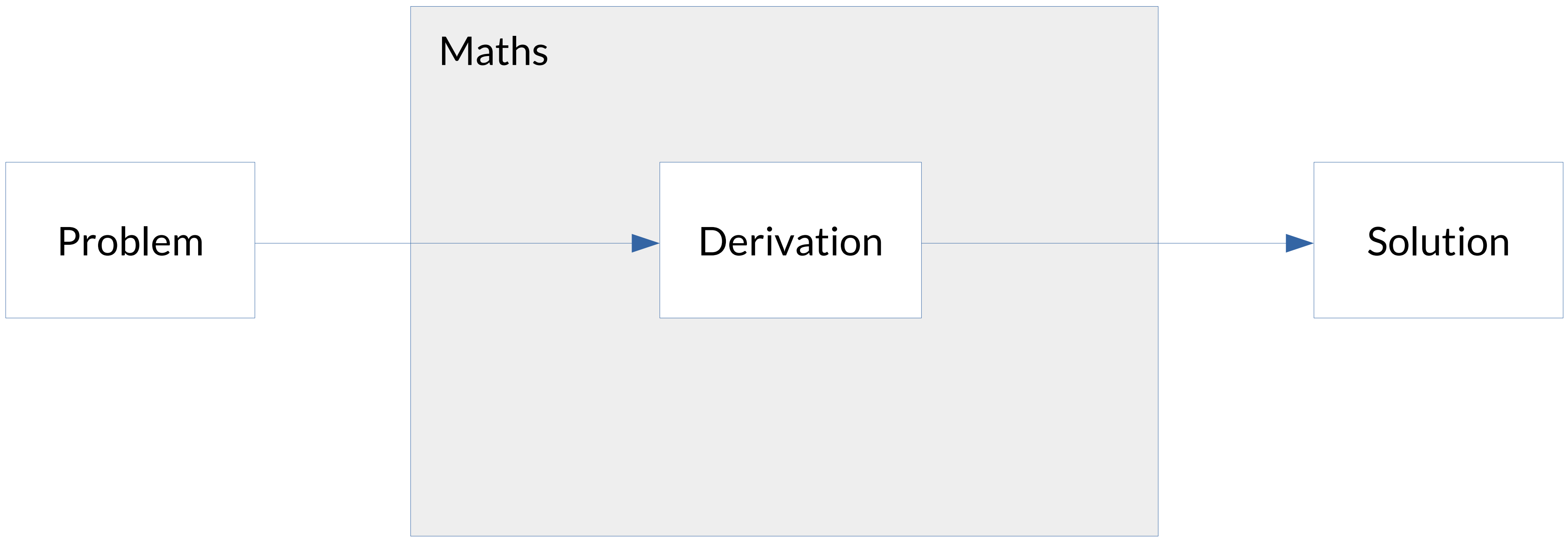
But what *is* Mathematics?



Maths as a Formal Language

- Formal Language
  - A
  - A→B
  - ∴ B
- Truth

Maths as Symbol Manipulation



Translation

- "My field should be $100m^2$, but the markers have been washed away. I know it's 10m long. How wide should it be?"
- $100 = 10w$

Derivation

- $100 = 10w$
- $10 = w$
- $w = 10$

Translation

- w = 10
- Means that the width of my field should be 10m




Abstraction

- Take only the things we think are essential
- Represent a complex system in terms of simple concepts
- Separate ideas from physical objects

Truth

- If our axioms are true, our conclusions are true
- All mathematical truth is relative to the truth of its axioms
- How we get to those axioms, however...

Meaning

- Our symbols are meaningful if there is some kind of isomorphism between our formal system and the real world
- Why does it need to be meaningful?
  - Formal systems are games
  - Some are useful, some not, some unexpectedly so
  - Usefulness, like a game, is not the point

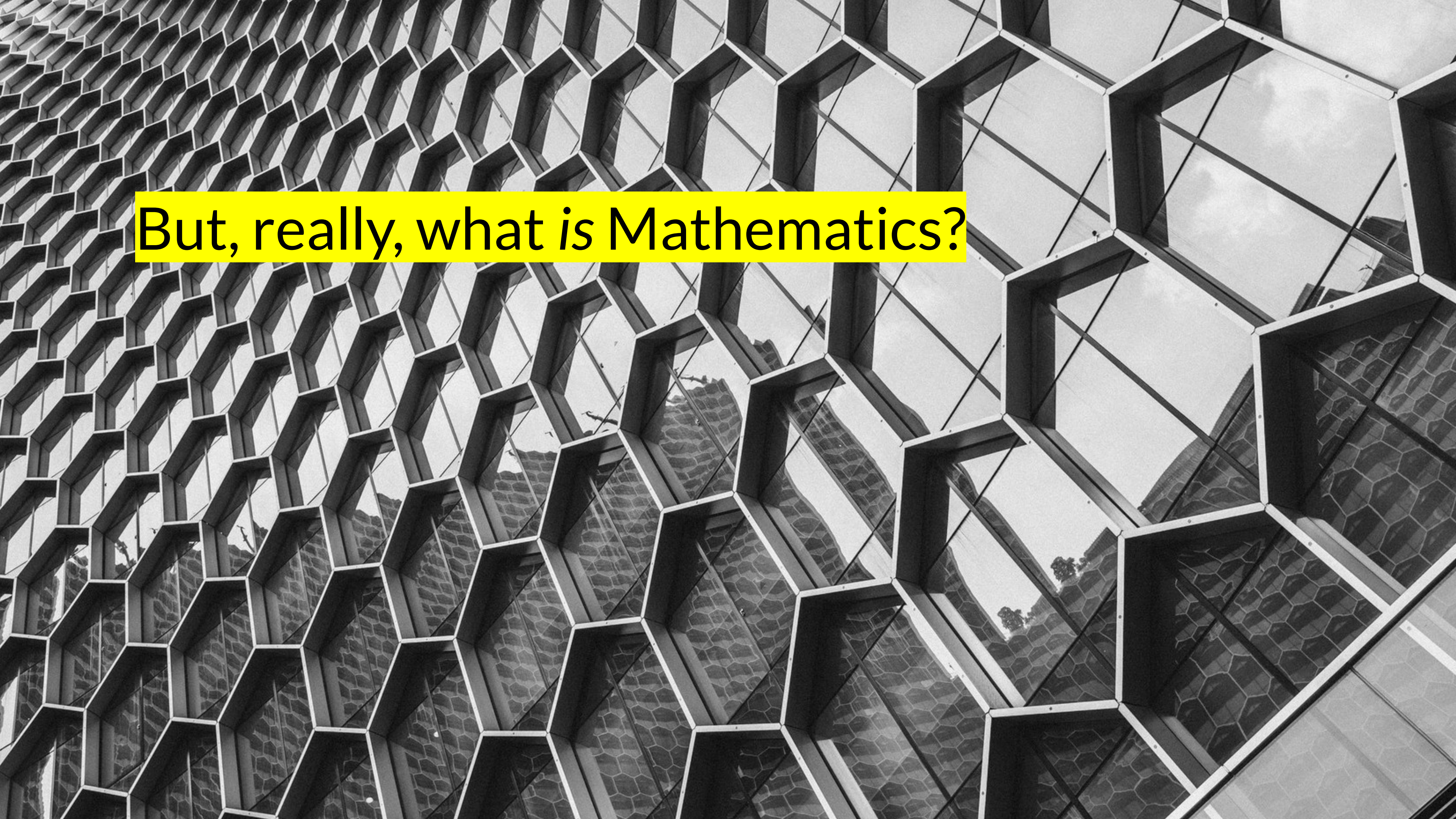
Hilbert’s Program

- Idealy we want out maths to be
  - Consistent
  - Complete

Gödel

- But any system complex enough to encode algebra is provably not complete
- There will always be things that are true that we can’t prove
  - We can’t know everything

But, really, what *is* Mathematics?



Numbers

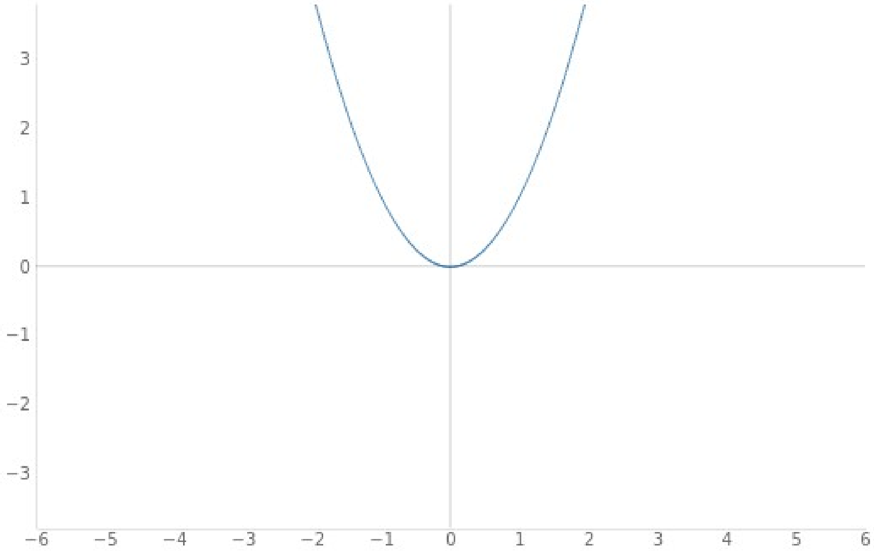
- Like 4
- How do we write them down?
  - Numerical Systems
- How do we add them up?
  - Algebra
- What if we change the rules?
  - Modular Arithmetic

Algebra (“reunion of broken parts”)

- Equations
  - [some stuff] = [some other stuff]
  - $y = ax^2 + bx + c$
- Do stuff to both sides
  - i.e. derive new equations that must also be true

Functions

- $f(x) = ax^2 + bx + c$
  - $f(1) = a + b + c$
  - $f(2) = 4a + 2b + c$
  - $f(3)$ ….



Important for algorithms

- How fast does a function grow?
  - Asymptotic Complexity

Logic

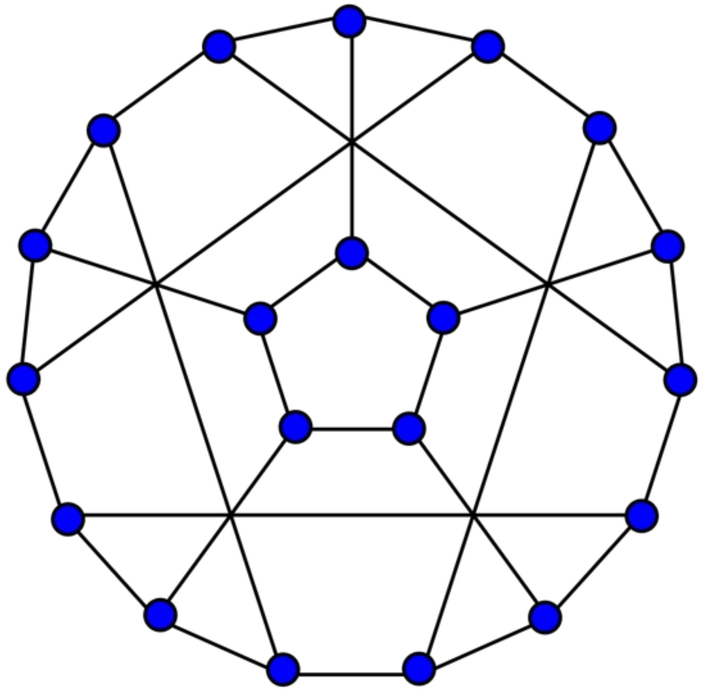
- Reasoning
  - If I am wearing socks I am not cold
  - I am wearing socks
  - I am not cold
- Propositional Logic
  - If A implies B, and A, then B
- Predicate Logic
- Second-order logic

Sets

- Like the **set** of natural numbers < 4
  - { 0, 1, 2, 3}
- Foundational in Mathematics
  - We may not be able to know everything, but we can know most stuff because of Set Theory

Graphs

- And extending the idea of a set, we could have a graph:
  - A set of nodes
  - A set of edges



Probability

- Using numbers to say how likely things are.
- The real world is messy
  - We never know things for sure
  - Things are just more or less likely
- We can model uncertainty using Probability

Statistics

- How do we make compelling arguments about this messy world?
  - Giving science a reliable foundation
  - By applying probability theory to our observations of the world using Statistics


Okay, that’s enough of that

The End

- See you on Friday
- No preparation required this week