PNMR - Prelab

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$$\frac{dM_{xy}}{dt} = \gamma \vec{M} \times \vec{B} = \gamma \begin{vmatrix} M_x \\ M_y \\ M_z \end{vmatrix} \times \begin{vmatrix} 0 \\ 0 \\ B_0 \end{vmatrix} = \gamma \begin{vmatrix} M_y B_0 \\ -M_x B_0 \\ 0 \end{vmatrix}$$

$$\frac{d\vec{M}_{xy}}{dt} = \begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ 0 \end{bmatrix} = \begin{bmatrix} \gamma M_y B_0 \\ -\gamma M_x B_0 \\ 0 \end{bmatrix}$$

solving:

$$\frac{dM_x}{dt} = \gamma M_y B_0 \qquad M_x = \int_0^t \gamma M_y B_0 \, dt$$

$$\frac{dM_y}{dt} = -\gamma M_x B_0 \qquad M_y = \int_0^t \gamma M_x B_0 \, dt$$

assume

$$M_x = A\sin(w_0 t) + B\cos(w_0 t) \quad \text{and} \quad M_y = C\sin(w_0 t) + D\cos(w_0 t)$$

$$\frac{dM_x}{dt} = A w_0 \cos(w_0 t) - B w_0 \sin(w_0 t) = \gamma B_0 M_y = \gamma B_0 (C\sin(w_0 t) + D\cos(w_0 t))$$

$$\frac{dM_y}{dt} = C w_0 \cos(w_0 t) - D w_0 \sin(w_0 t) = -\gamma B_0 M_x = \gamma B_0 (-A\sin(w_0 t) - B\cos(w_0 t))$$

$$\therefore \quad \gamma B_0 C = -B w_0 \quad , \quad \gamma B_0 D = A w_0$$

$$-\gamma B_0 A = -D w_0 \quad , \quad -\gamma B_0 B = C w_0$$

$$\Rightarrow \quad A = D \; , \quad w_0 = \gamma B_0 \qquad C = -B$$

thus:

$$M_x(t) = A\sin(w_0 t) + B\cos(w_0 t)$$

$$M_y(t) = -B\sin(w_0 t) + A\cos(w_0 t)$$

$$\vec{M}(t=0) = \hat{x} M_0 \rightarrow B = M_0 \; , A = 0$$

$$\therefore \quad M_x(t) = M_0 \cos(\gamma B_0 t)$$

$$M_y(t) = -M_0 \sin(\gamma B_0 t)$$

$$\therefore \quad \vec{M}(t) = M_0 \begin{bmatrix} \cos(\gamma B_0 t) \\ -\sin(\gamma B_0 t) \\ 0 \end{bmatrix} \qquad T = \frac{2\pi}{\gamma B_0} \qquad \omega = \frac{2\pi}{T} = \gamma B_0$$

precesses at $\omega_{precess} = \gamma B_0$

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now: $\vec{B}_n = \hat{z}(B_0 + \delta_n B)$

$$\frac{d\vec{M}}{dt} = \gamma \begin{vmatrix} M_x \\ M_y \\ 0 \end{vmatrix} \times \left(\begin{vmatrix} 0 \\ 0 \\ B_0 \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ \delta_n B \end{vmatrix} \right)$$

$$\frac{dM_x}{dt} = (\gamma B_0 + \gamma \delta_n B) M_y \qquad \frac{dM_y}{dt} = -(\gamma B_0 + \gamma \delta_n B) M_x$$

will yield
$$M_x = \cos\left([\gamma B_0 + \gamma \delta_n B] t \right)$$
$$M_y = -\sin\left([\gamma B_0 + \gamma \delta_n B] t \right)$$

$$\omega_{presess} \longrightarrow \omega_{precess} + \gamma \delta_n B$$

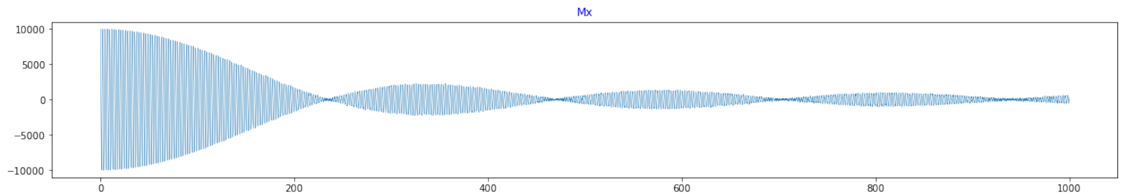
Plot $\vec{M}_{ensemble} = \sum_n \vec{M}_n(t) = \sum_n \begin{pmatrix} \cos[\gamma(B_0 + \delta_n B)t] \\ -\sin[\gamma(B_0 + \delta_n B)t] \\ 0 \end{pmatrix}$

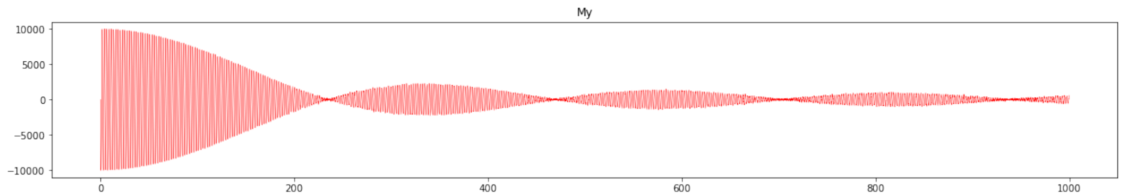
pnmr_prelab

November 3, 2022

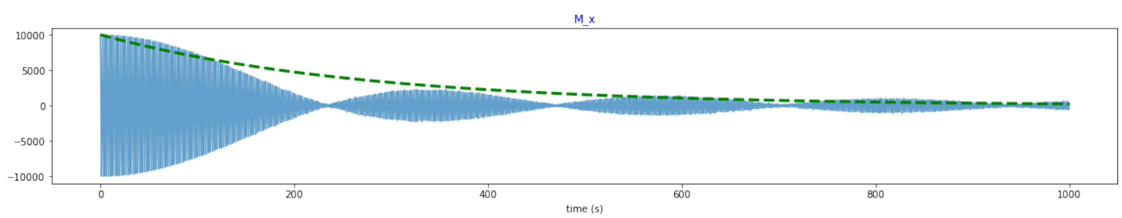
[56]:

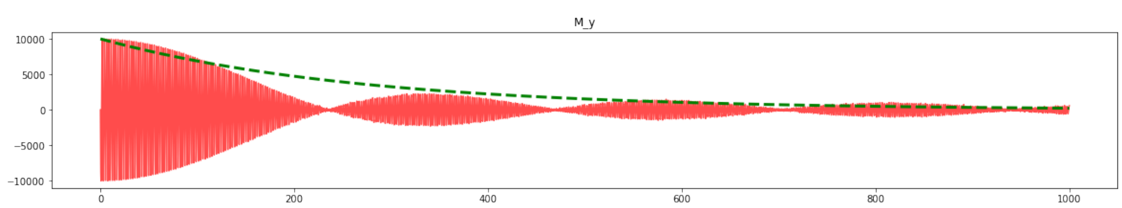
```python
import numpy as np
import matplotlib.pyplot as plt

N = 10000

B0 = 1
gamma = 2.675

smallB = B0/100

mx = []
my = []
ts = []

for t in np.arange(0, 1000, 0.1):

    deltas = np.random.rand(N)
    sumx = np.sum(np.cos(gamma*(B0+deltas*smallB)*t))
    sumy = -np.sum(np.sin(gamma*(B0+deltas*smallB)*t))

    mx.append(sumx)
    my.append(sumy)
    ts.append(t)

plt.rcParams["figure.figsize"] = (20,3)

plt.plot(ts, mx, linewidth=0.4 )
plt.title("Mx", color='b')
plt.show()

plt.clf()
plt.title("My")
plt.plot(ts,my, color='r', linewidth=0.4)
plt.show()
```



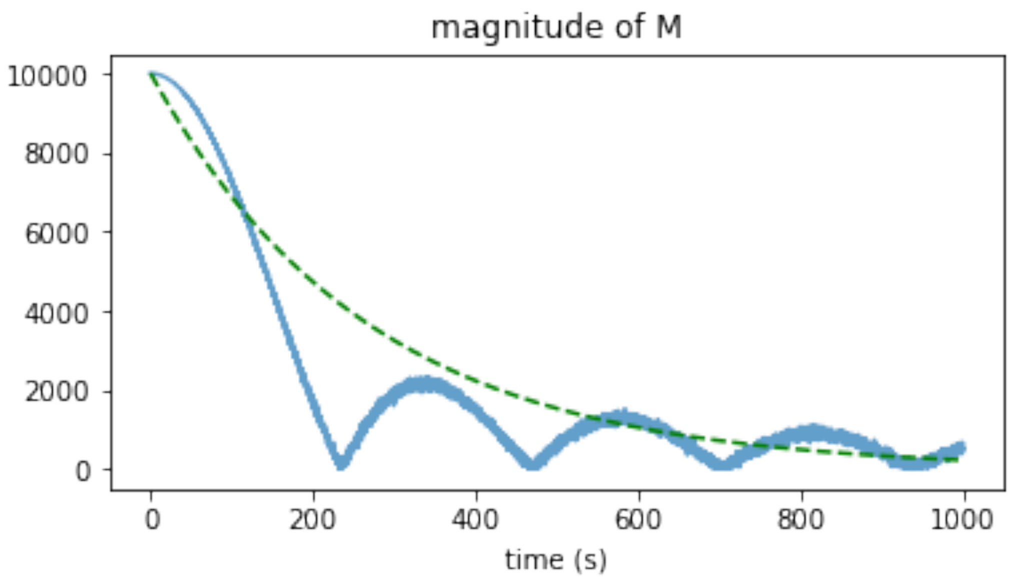


[58]:
```python
plt.plot(ts, mx, alpha=0.7)
plt.plot(ts, N*np.exp(-np.array(ts)/(gamma/smallB)), linewidth=3, linestyle='--', color='green')
plt.title("M_x", color='b')
plt.xlabel('time (s)')
plt.show()

plt.clf()
plt.title("M_y")
plt.plot(ts,my, color='r', alpha=0.7)
plt.plot(ts, N*np.exp(-np.array(ts)/(gamma/smallB)), linewidth=3, linestyle='--', color='green')
plt.xlabel('time (s)')
plt.show()
```





[61]:
```
M = np.sqrt(np.array(mx)**2 + np.array(my)**2)
plt.plot(ts, M, alpha=0.7)
plt.plot(ts, N*np.exp(-np.array(ts)/(gamma/smallB)), linestyle='--',␣
 ↪color='green')
plt.rcParams["figure.figsize"] = (4,3)
plt.title("magnitude of M")
plt.xlabel('time (s)')
plt.show()
```



We see that the M_x and M_y precesses and and their magnitudes decay and "pulse". The system acts as though $\vec{M}$ decays as an exponential

$$e^{-t/\tau}$$

of time constant

$$\tau = \gamma \cdot B$$

Plots are made above, with the units chosen arbitrarily