Modeling the Chaotic Rhythms of a Dripping Faucet with the Noisy 1-D Logistic Map

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1 Abstract

As the drip rate of water is varied for a leaky faucet, various chaotic behaviour is exhibited. We examine the chaotic behaviour and attempt to model specific instances of the chaos using the 1-D logistic map. Specifically, we note that at certain valve levels (ie 317), the patterns of the chaotic behaviour can be modeled by the 1-D logistic map equation with an added noise term

$$x_{n+1} = p \cdot x_n \cdot (1 - x_n) + \zeta \cdot \sigma$$

with

$$p = 3.6$$
$$\zeta = 0.02$$
$$\sigma \sim N(0, 1)$$

where $N(0, 1)$is random noise sampled from a standard normal with mean 0 and standard deviation of 1.

2 Anlaysis

The experiment is setup as follows: drip rates from a leaky faucet is measured via a phototransistor, which counts the successive time between water droplets passing between it. A valve controls the openness of the faucet, with 0 being fully closed and 340 begin fully open. The level of the valve is varied and 1000 drip rates are recorded at each level. We analyze the trend of data collected at each level, and specifically take note of the chaotic behaviours that occur when the level of the valve is at 317.

For a valve level of 317 the drip rate vs drip number is plotted in Figure 1-A. The behaviour is chaotic and no simple trend can be observed for the data. We also plot the drip rates of the successive drops T_{n+1} vs T_n. in Figure 1-C. We note that within this chaotic system, a level of determinism between the drip rates of each successive drops can be seen, which roughly follows a parabolic pattern.

We compare the experimental results with the 1-D logistic map equation with an added noise term

$$x_{n+1} = p \cdot x_n \cdot (1 - x_n) + \zeta \cdot \sigma \tag{1}$$

as suggested by Robert Shaw in his book "The Dripping Faucet as a Model Chaotic System" (1984). The parameters for the model are

$$p = 3.6$$

$$\zeta = 0.02$$

$$\sigma \sim N(0, 1)$$

where $N(0, 1)$ the standard normal with mean 0 and standard deviation of 1.

Using this model, 2000 points $\{x_1, x_2, ..., x_{2000}\}$ are generated from equation (1), with an initial random value of x_1 chosen between 0 and 1, and only the last 1000 points are retained to give the system time to reach chaos. To fit the model to our observed data at valve level = 317, we take the negative values of the data-points $\{-x_{1000}, -x_{1001}, ..., -x_{2000}\}$. We then plot the values of each x in Figure 1-B and note that chaos is observed for the sequence generated. Plotting subsequent points x_{n+1} vs x_n (Figure 1-D) shows a nearly identical quadratic trend to what is seen from the experimental data.

3 Conclusion

From this analysis, we see that the chaotic behaviour of a dripping faucet at a valve level 317 can described by the model of the 1-D logistic map with noise.

Figure 1: Chaotic behaviour of drips at valve level = 317 (blue) compared to negative 1-D logistic map with noise (green)