Practice Exam Questions Week 1, Linear Algebra,

1. Consider the following linear system of equations:

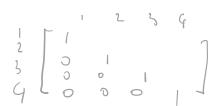
$$2x_1 - 3x_2 - 7x_3 + 13x_4 + x_5 = -3$$

$$x_1 + x_2 - x_3 + 4x_4 + 2x_5 = 4$$

$$-x_1 + x_2 + 3x_3 - 6x_4 - x_5 = 0$$

- (a) Determine the augmented matrix of this system and compute its reduced row echelon form.
- (b) Compute the whole solution set.
- 2. True or false? If the given statement is true, briefly explain why. If it is false, give a counterexample.
- (a) If a system of linear equations has a unique solution, then the number of basic variables is larger than the number of free variables.
- (b) If a linear system of four equations in four variables has a coefficient matrix with a pivot in each column, then the system has a unique solution.
- (c) A consistent system of linear equations with fewer equations than unknowns (also called underdetermined system) can never have an unique solution.
- 7 (d) Suppose a (3×5) coefficient matrix for a system has three pivot columns. Then the system is consistent.

 7 The the system is consistent.



(1)
$$\begin{bmatrix} 2 - 3 - 7 & 13 & 1 & -3 & 1 \\ -1 & 1 & -1 & 4 & 2 & 4 \\ -1 & 1 & +3 & -6 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & 4 & 2 & 4 \\ 2 - 3 & -7 & 13 & 1 & -3 \\ -1 & 1 & +3 & -6 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & 4 & 2 & 4 \\ 0 & -5 & -5 & 5 & -3 & -2/5 \\ 0 & 0 & 0 & -1/5 & -2/5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & 4 & 2 & 4 \\ 0 & 1 & -1 & -1 & 4 & 2 & 4 \\ 0 & 1 & 1 & -1 & 0 & 2 & 4 \\ 0 & 1 & 1 & -1 & 0 & 2 & 4 \\ 0 & 1 & 1 & -1 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 0 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 0 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 0 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 & 2 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 0 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 2 & 2 & 2 & 4 \\ 0 & 1 & 1 & -1 & 2 & 2 & 2 & 2 \\ 0 & 1 & 1 & -1 & 2 & 2 & 2 & 2 \\ 0 & 1 &$$

