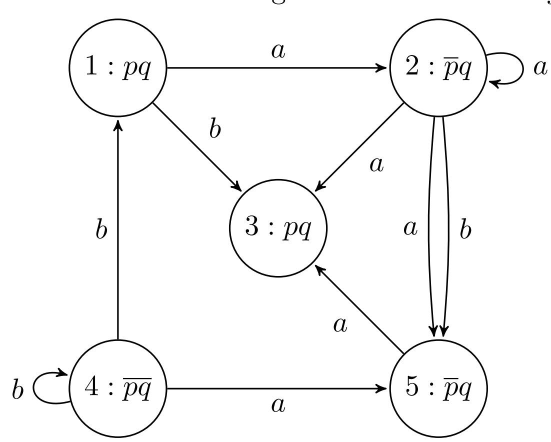
Logic 2022-2023 — Bonus Assignment 3

May 14, 2023

- This assignment must be made individually.
- Include your name and student number on the first sheet.
- Hand in your solution by uploading **a single PDF file** via Canvas.
- The deadline for uploading your solution is May 24th at 23:59.
- If graded as sufficient, you earn 0.33 bonus points to your final mark for the course.

1 The assignment

1. Consider the following labelled transition system.



(a) In which states are the following formulas true?

1. $\langle b \rangle p$ **Answer: 1, 4.**
2. $[b]p$ **Answer: 1, 3, 5.**
3. $\langle b* \rangle q$ **Answer: 1,2,3,4,5.**
4. $[a](q \leftrightarrow p)$ **Answer: 3.** and 5

5. $[b;a]\bot$ $[b;a]\top$ **Answer: 1 3 5.**

(b) Give a formula that is only true

1. In state 3 **Answer: (example)** $p \wedge q \wedge \langle b \rangle \bot$
2. In state 4 **Answer: (example)** $\langle b \rangle(\neg p \wedge \neg q)$

(c) Give all elements of the relations defined by the actions:

1. $(b; a)$ **Answer:** $\{(2,3),(4,5),(4,2)\}$
2. $q?; a; p?$ **Answer:** $\{(2,3),(5,3)\}$
3. $(a \cup b); b$ **Answer:** $\{(1,5),(2,5),(4,1),(4,4),(4,3)\}$

(d) Give a PDL expression that defines the relation $\{(4,5)\}$ in the graph. Hint: use a test action. **Answer:** $(\neg p \wedge \neg q)?; a$

2. Prove/disprove the following statements with a prefix tableau. In case a statement is not true, draw or describe a model that acts as a counterexample. Indicate which of the following rules you apply in your prefix tableaus using the following numbering scheme. For other rules you can use the rule names as used in tableau proofs for propositional logic.

$$11.\ \frac{x : [a]\varphi \circ}{x.\langle a\rangle.k : \varphi \circ} \qquad 12.\ \frac{x : \langle a\rangle\varphi \circ}{x.\langle a\rangle.k : \varphi \circ} \qquad 13.\ \frac{\circ\ x : [\alpha]\varphi}{x : \langle\alpha\rangle\neg\varphi \circ} \qquad 14.\ \frac{\circ\ x : \langle\alpha\rangle\varphi}{x : [\alpha]\neg\varphi \circ}$$

$$21.\ \frac{x : [\alpha;\beta]\varphi \circ}{x : [\alpha][\beta]\varphi \circ} \qquad 22.\ \frac{x : \langle\alpha;\beta\rangle\varphi \circ}{x : \langle\alpha\rangle\langle\beta\rangle\varphi \circ} \qquad 23.\ \frac{x : [\alpha+\beta]\varphi \circ}{x{:}[\alpha]\varphi\ ,\ x{:}[\beta]\varphi \circ} \qquad 24.\ \frac{x{:}\langle\alpha+\beta\rangle\varphi\circ}{x{:}\langle\alpha\rangle\varphi \circ \parallel x{:}\langle\beta\rangle\varphi \circ}$$

$$31.\ \frac{x : [\psi?]\varphi \circ}{x{:}\neg\psi \circ \parallel x{:}\varphi \circ} \qquad 32.\ \frac{x : \langle\psi?\rangle\varphi \circ}{x{:}\psi\ ,\ ,x{:}\varphi \circ} \qquad 33.\ \frac{x : [\alpha^*]\varphi \circ}{x{:}\varphi\ ,\ x{:}[\alpha][\alpha^*]\varphi \circ} \qquad 34.\ \frac{x : \langle\alpha^*\rangle\varphi \circ}{x{:}\varphi \circ \parallel x : \langle\alpha\rangle\langle\alpha^*\rangle\varphi \circ}$$

(a) $\models (\langle p? \rangle q) \rightarrow (p \wedge q)$

(b) $\langle a; b \rangle p \wedge [a]q \models \langle a; q? \rangle p$

(c) $\langle q?; a \rangle p \models \langle a \rangle p$

(d) $[a^*]p \wedge [a^*]q \models [a](p \vee q)$

Answers at end

3. The following Java code defines a function that computes the n-th Fibonacci number (for any $n \geq 0$). Prove that the code is correct by formulating a loop invariant for the while loop and explaining how this loop invariant implies that `b` equals the `n`-th Fibonacci number after the while loop terminates. To formulate the loop invariant you may use expressions of the form "`x=fib(e)`" (variable `x` equals the `e`-th Fibonacci number).

```
public static int fibonacci(int n) {

  if (n <= 0) return 0;

  int a = 0;
  int b = 1;

  int i = 1;

  while (i < n) {
    int c = a + b;
    a = b;
    b = c;
    i = i + 1;
  }

  return b;
}
```

Answer: the loop invariant is the condition

$$\texttt{a} = \texttt{fib(i-1)} \wedge \texttt{b} = \texttt{fib(i)} \wedge \texttt{i} \leq \texttt{n}.$$

We have that:

(a) The loop invariant is true before the while loop starts.

(b) If the loop invariant is true before an iteration of the while loop, it is also true after an iteration of the while loop.

(c) When the while loop ends we have $\texttt{i} \geq \texttt{n}$. Together with the loop invariant this implies that $\texttt{i} = \texttt{n}$ and hence $\texttt{b} = \texttt{fib(n)}$.

2a. $\vDash (\langle p? \rangle q) \to (p \wedge q)$

Statement is true:

	$0: (\langle p? \rangle q) \to (p \wedge q)$
	$\to R$
$0: \langle p? \rangle q$	$0: p \wedge q$
$\underline{32}$	
$0:p \quad 0:q$	$0: p \wedge q$
	$\wedge R$
$0:p \quad 0:q$	$0:p \quad 0:q$
	X

② b $\langle a;b\rangle p \wedge [a]q \models \langle a;q?\rangle p$

Statement is false:

$0: \langle a;b\rangle p \wedge [a]q$ $\quad$ $0: \langle a;q?\rangle p$

$\wedge L$

$\underline{0\ \langle a;b\rangle p}$ $\quad$ $0: [a]q$ $\quad$ $0: \langle a;q?\rangle p$

22

$\underline{0: \langle a\rangle\langle b\rangle p}$ $\quad$ $0: [a]q$ $\quad$ $0: \langle a;q?\rangle p$

12

$\underline{0.\langle a\rangle.1: \langle b\rangle p}$ $\quad$ $0: [a]q$ $\quad$ $0: \langle a;q?\rangle p$

12

$0.\langle a\rangle.1.\langle b\rangle.2: p$ $\quad$ $\underline{0: [a]q}$ $\quad$ $0: \langle a;q?\rangle p$

11

$0.\langle a\rangle.1.\langle b\rangle.2: p$ $\quad$ $0.\langle a\rangle.1: q$ $\quad$ $\underline{0: \langle a;q?\rangle p}$

14

$0\langle a\rangle 1\langle b\rangle 2: p$ $\quad$ $0.\langle a\rangle.1: q$ $\quad$ $\underline{0: [a;q?]\neg p}$

21

" $\quad$ " $\quad$ $\underline{0: [a][q?]\neg p}$

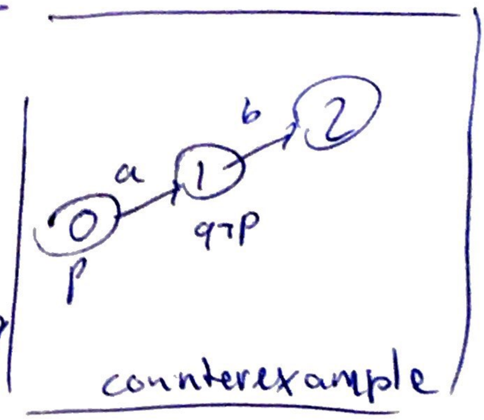
12

" $\quad$ " $\quad$ $\underline{0: \langle a\rangle.1: [q?]\neg p}$

31

$0\langle a\rangle.1\langle b\rangle 2: p$ $\quad$ $0: \langle a\rangle.1: q$ $\quad$ $0.\langle a\rangle.1: \neg p$

open



$0.\langle a\rangle.1.\langle b\rangle.2: p$ $\quad$ $0: \langle a\rangle.1: q$ $\quad$ $0\langle a\rangle.1: \neg q$

$\neg L$

$0: \langle a\rangle 1: q$ $\quad$ $0.\langle a\rangle.1: \neg q$

closed

② c $\langle q?;a\rangle p \vDash \langle a\rangle p$

Statement is true:

$0:\langle q?;a\rangle p$ $\qquad$ $\dfrac{0:\langle a\rangle p}{14}$

$\dfrac{0:\langle q?;a\rangle p}{22}$ $\qquad$ $0:[a]\neg p$

$\dfrac{0.\langle q?\rangle\langle a\rangle p}{32}$ $\qquad$ $0:[a]\neg p$

$0:q$ $\qquad$ $\dfrac{0:\langle a\rangle p}{12}$ $\qquad$ $0:[a]\neg p$

$0:q$ $\qquad$ $0.\langle a\rangle.1:p$ $\qquad$ $\dfrac{0:[a]\neg p}{11}$

$0:q$ $\qquad$ $0.\langle a\rangle.1:p$ $\qquad$ $\underline{0.\langle a\rangle.1:\neg p}$

$0:q$ $\qquad$ $0.\underline{\langle a\rangle}.1:p$ $\qquad$ $0:\langle \underline{a}\rangle.1:p$

X

Closed

② d $\quad [a^*]p \wedge [a^*]q \vDash [a](p \vee q)$

Statement is true

$0 : [a^*]p \wedge [a^*]q \qquad 0 : [a](p \vee q)$

$0 : [a^*]p \qquad 0 : [a^*]q \qquad 0 : [a](p \vee q)$

$\underset{33}{0 : [a^*]p} \qquad 0 : [a^*]q \qquad 0 : [a]p \qquad 0 : [a]q$

forget, we don't need it

$0 : p \qquad 0 : [a][a^*]p \qquad 0 : [a^*]q \qquad \underset{13}{0 : [a]p} \qquad 0 : [a]q$

forget

$0 : p \qquad 0 : [a][a^*]p \qquad \underset{12}{0 : \langle a \rangle \neg p} \qquad 0 : [a]q$

$\underset{11}{0 : [a][a^*]p} \qquad 0.\langle a \rangle.1 : \neg p \qquad ..$

$\underset{33}{0.\langle a \rangle.1 : [a^*]p} \qquad 0.\langle a \rangle.1 : \neg p \qquad ..$

$0.\langle a \rangle.1 : p \qquad 0.\langle a \rangle.1 : [a][a^*]p \qquad 0.\langle a \rangle.1 : \neg p \qquad ..$

$0.\langle a \rangle.1 : p \qquad 0.\langle a \rangle.1 : p$

×

Closed