Proof by induction

Example:

- $1 + 2 =$ 3
- $1 + 2 + 3 =$ 6
- $\underbrace{1 + 2 + \ldots + n}_{\sum_{k=1}^{n} k = \frac{1}{2} n(n+1)} =$ $\frac{1}{2} n (n+1)$

Proof by induction:

- Typically used to show $\forall n \geq \underline{N}, n \in \mathbb{N} : P(n)$
- Basic step : $P(N)$ holds (the first domino stone falls)
- Inductive step : if $P(n)$, then $P(n+1)$
  $n \geq N$
  (the n^{th} domino stone knocks the $(n+1)^{th}$ over)

Mathematical induction - example

$$\forall n \in \mathbb{N} : \sum_{k=1}^{n} k = \frac{1}{2} n(n+1)$$

- Basic step: P(1) holds

$$\sum_{k=1}^{1} k = 1 = \frac{1}{2} 1 (1+1)$$

- Inductive step: $P(n) \to P(n+1)$, $n \geq 1$

$$\sum_{k=1}^{n+1} k = \sum_{k=1}^{n} k + (n+1) = \frac{1}{2} n(n+1) + (n+1)$$

$$= (n+1)\left(\frac{1}{2} n + 1\right)$$

$$= \frac{1}{2}(n+1)(n+2) = \frac{1}{2}(n+1)(n+1+1)$$

Mathematical induction - example

$\forall n \in \mathbb{N} : 4^n - 1$ is divisible by 3

- Basic step: $P(1)$ is true

  for $n = 1$: $4^1 - 1 = 3$ is divisible by 3

- Inductive step: $\forall n \in \mathbb{Z}$ $P(n) \to P(n+1)$

  $4^{n+1} - 1 = \underbrace{4^n}_{= 3k+1} \cdot 4 - 1 = 4(3k+1) - 1 = 12k + 4 - 1$

  $= 12k + 3$

  is divisible by 3

$4^n - 1 = 3k$

$4^n = 3k + 1$