

Practice Exam Questions Week 2, **Linear Algebra**, UCM

1. Consider the following matrix A and vector \mathbf{b} :

$$A = \begin{bmatrix} -1 & 2 & 3 & -4 & 8 \\ 3 & -6 & -4 & 7 & -9 \\ 2 & -4 & 1 & 1 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 7 \\ -6 \\ 7 \end{bmatrix}.$$

- (a) Are the columns of A linearly independent?
 - (b) Compute the solution set of the associated linear system of equations $A\mathbf{x} = \mathbf{b}$ and express it in parametric vector form.
2. True or false? If the given statement is true, briefly explain why. If it is false, give a counterexample.
- (a) If the columns of an augmented matrix are linearly independent, then the associated linear system of equations is inconsistent.
 - (b) Four different vectors in \mathbb{R}^3 always span \mathbb{R}^3 .
 - (c) The effect of adding a vector \mathbf{p} to a vector \mathbf{v} is to move the vector \mathbf{v} in a direction parallel to \mathbf{p} .
 - (d) If the augmented matrix of a linear system of equations has more rows than columns, then it cannot have infinitely many solutions.
 - (e) If A and B are matrices for which the product AB and the sum $A + B$ are both well defined, then the product BA is also well defined.
 - (f) If all the rows of an augmented matrix have a pivot, then the associated linear system of equations is inconsistent.
 - (g) If S and T are 2×2 matrices such that $ST = 0$, then also $TS = 0$.
 - (h) If the vectors $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^3$ are such that $\{\mathbf{x}, \mathbf{y}\}$ is linearly independent, $\{\mathbf{x}, \mathbf{z}\}$ is linearly independent, and $\{\mathbf{y}, \mathbf{z}\}$ is linearly independent, then also $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly independent.