Discrete Mathematics 2021/2022
Exam Questions (On-line version)

— *Do not turn this page before the official start of the exam!* —

First Name, Surname: ______________________________

Student ID: ______________________________

Program: Bachelor Data Science and Artificial Intelligence

Course code: KEN1130

Examiner: Dr. Steven Kelk, Dr. Marieke Musegaas and Dr. Stefan Maubach

Date/time: Friday 22nd October 2021, 13:00h-15:00h

Format: Closed book exam

Allowed aids: Pens, simple (non-programmable) calculator from the DKE-list of allowed calculators.

Instructions to students:

- The exam consists of 8 questions on 2 pages (excluding the 1 cover page(s)).
- Fill in your name and student ID number on each page, including the cover page.
- Ensure that you properly motivate your answers.
- Do not use red pens, and write in a readable way. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- Remember that $\mathbb{N} = \{1, 2, 3, \ldots\}$.
- **Good luck!**

The following table will be filled by the examiner:

Question:	1	2	3	4	5	6	7	8	Total
Points:	10	15	15	15	15	10	15	5	100
Score:									

Question 1 (10 points)
Fill in the truth table for the following logical proposition.

- $(p \Rightarrow (q \Rightarrow r)) \Leftrightarrow ((p \wedge q) \Rightarrow r)$

Question 2 (15 points)
Use induction to prove the following statement.

- For all integers $n \geq 1$,
$$\sum_{i=1}^{n} (-1)^i i^2 = \frac{(-1)^n n(n+1)}{2}$$

Question 3 (15 points)
Prove or disprove the following statements.

(a) *(7 points)* For all sets A, B, and C, $(B \cap C \subseteq A) \Rightarrow ((A \setminus B) \cap (A \setminus C) = \emptyset)$.

(b) *(8 points)* For all sets A, B, C and D, $(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D)$.

Question 4 (15 points)
This question is about *relations*.

(a) *(8 points)* Let $A = \{1, 2\}$. Draw a relation diagram on A that is reflexive and transitive, but not symmetric, or - if no such relation exists - explain why not. Note that the relation you draw does not have to have a 'real-world' or algebraic meaning!

(b) *(7 points)* Let $A = \mathbb{P}(\{a, b, c\})$. Let R be the relation on A defined as follows: XRY means "$|X| = |Y|$". This is an equivalence relation. (You do not need to prove this.) How many equivalence classes does R have? For each equivalence class, list explicitly which elements of A belong to the equivalence class.

Question 5 (15 points)
All the following questions are about *counting*. For all questions please give an exact number at the end (i.e. don't just leave your answer as a counting equation).

(a) *(5 points)* How many different functions are there from a set with 6 elements to a set with 3 elements?

(b) *(4 points)* How many different *invertible* functions are there from a set with 6 elements to a set with 3 elements?

(c) *(6 points)* A grandmother has 10 grandchildren and 20 *identical* chocolate bars. She wants every grandchild to have at least 1 chocolate bar, but for the rest she does not have any restrictions. In how many different ways can she distribute the chocolate bars over her grandchildren?

Question 6 (10 points)
Prove or disprove the following statements.

(a) (*5 points*) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x^2 y + 2x = x)$.

(b) (*5 points*) $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{N})((z^2 \geq x^2 + y^2) \wedge (z < 5))$.

Question 7 (15 points)
This is a question about *functions*.

(a) (*10 points*) Let $f : \mathbb{R} \setminus \{3\} \to \mathbb{R} \setminus \{1\}$ be the function defined as follows:

$$f(x) = \frac{x+2}{x-3}.$$

Prove that f is a bijection.

(b) (*5 points*) Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined as follows: $f(x) = -1 + x^2$. Let $g : \mathbb{N} \to \mathbb{R}$ be the function defined as follows: $g(x) = 4 - x^2$. Is $f \circ g$ a well-defined function? Is $g \circ f$ a well-defined function? Explain why or why not.

Question 8 (5 points)
Let $A = \{1\}$, $B = \{2, 3, 4\}$ and $C = \{3, 4, 5\}$. (Recall that $\mathbb{P}(.)$ denotes "powerset".)

(a) (*3 points*) Write down $((A \setminus B) \times (B \cap C)) \setminus \emptyset$.

(b) (*2 points*) Write down $\mathbb{P}(A \times A)$.