Practice Exam Questions Week 5, Linear Algebra,

1. Consider the following matrix A and vectors \mathbf{v}_1 and \mathbf{v}_2 :

$$A = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & 1 \\ -1 & 0 & 1 & 1 \\ 3 & -1 & -2 & -1 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

- a. Show that \mathbf{v}_1 and \mathbf{v}_2 are both eigenvectors of A. What are the corresponding eigenvalues?
- b. Show that 0 is an eigenvalue of A.
- c. Compute a basis for the eigenspace of A for the eigenvalue 0.
- d. Compute a basis for Nul A.
- e. Is the matrix A diagonalizable? If it is, determine a matrix M such that $M^{-1}AM$ is a diagonal matrix. If it is not, explain why not.
- f. Compute the matrix A^9 .
- 2. True or false? If the given statement is true, give a brief explanation. If it is false, give a counterexample.
 - a. If A = QR and Q is invertible, then A is similar to B = RQ;
 - b. An elementary row operation on A does not change the determinant of A.
 - c. If λ is an eigenvalue of A, then it is also an eigenvalue of A^{T}
 - d. Each eigenvalue of A is also an eigenvalue of A^2 .
 - e. If M is a (2×2) matrix such that dim Nul A equals 1, then M has one eigenvalue equal to 0.
 - f. Let B be an $(n \times n)$ matrix. Let \mathbf{e}_1 be the first column of the identity matrix I_n . If \mathbf{e}_1 is an eigenvector of B with eigenvalue 1, then the first column of B is \mathbf{e}_1 .
 - g. Any invertible matrix can also be diagonalized.
 - h. If A and B are 2×2 matrices which can both be diagonalized, then their sum C = A + B can also be diagonalized.
 - i. If K and L are 3×3 matrices and \mathbf{v} is an eigenvector of K and also of L, then \mathbf{v} is an eigenvector of the matrix product KL.
 - j. If **x** is an eigenvector of an invertible matrix P, then it is also an eigenvector of P^{-1}
- 3. Consider the following matrix Q:

$$Q = \left[\begin{array}{rrr} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{array} \right]$$

Find the characteristic polynomial and the eigenvalues of Q.