Surname, First name

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Linear Algebra (KEN1410)
Exam

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

a b c d e f → b
a b c d e f → c
a b c d e f → a

Fill in your answer(s) to the multiple-choice questions as shown above (circles = one correct answer).

Program: Data Science and Artificial Intelligence
Course code: KEN1410
Examiners: dr. Marieke Musegaas and dr. Stefan Maubach
Date/time: Tuesday 29 March 2022, 09:00 - 11:00
Format: Closed book exam
Allowed aids: Pens, simple (non-programmable) calculator from the DKE-list of allowed calculators.
Instructions to students:

- The exam consists of 8 questions on 18 pages.
- Fill in your name and student ID number on the cover page.
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Before answering the questions, please first read all the exam questions, and then make a plan to spend the two hours.
- Only use black or dark blue pens, and write in a readable way. No Pencils. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

Exercise 1

Consider the following matrix A:

$$A = \begin{bmatrix} 2 & 0 & 2 & 4 \\ 1 & 2 & 1 & -2 \\ 2 & 0 & 1 & 2 \\ 1 & 0 & 1 & 2 \end{bmatrix}.$$

Define $T : \mathbb{R}^4 \to \mathbb{R}^4$ by $T(\mathbf{x}) = A\mathbf{x}$.

15p

1 Determine all $\mathbf{x} \in \mathbb{R}^4$ such that $T(\mathbf{x}) = \begin{bmatrix} 8 \\ 2 \\ 5 \\ 4 \end{bmatrix}$

$T(x) = \begin{bmatrix} 8 \\ 2 \\ 5 \\ 4 \end{bmatrix}$ $\quad T(x) = Ax$

$$\begin{bmatrix} 2 & 0 & 2 & 4 \\ 1 & 2 & 1 & -2 \\ 2 & 0 & 1 & 2 \\ 1 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 5 \\ 4 \end{bmatrix}$$

Exercise 2

Let $a, b, c, d \in \mathbb{R}$ and consider the following system of linear equations

$$\begin{aligned} x_1 + ax_2 + bx_3 &= 14 \\ cx_2 + dx_3 &= -40 \end{aligned}$$

This system of linear equations has a solution set that looks like

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix},$$

where $\lambda \in \mathbb{R}$.

15p **2** Compute a, b, c and d.

Exercise 3

5p **3** Provide, **IF POSSIBLE**, an example of a subset H of $\mathbb{R}^2$ that has the following three properties:

- the zero vector is in H,
- H is **NOT** closed under vector addition,
- H is closed under multiplication by scalars.

(Note: only provide your answer. An explanation is not required.)

Exercise 4
A matrix A has after a couple of row operations the following form

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 4 & -2 \\ 0 & -6 & 3 \\ 0 & 0 & 1 \end{bmatrix}.$$

5p **4a** Provide the dimensions of the following four vector spaces: Nul A, Col A, Row A and Nul A^T. (Note: only provide your answer. An explanation is not required.)

5p **4b** Provide, **IF POSSIBLE**, a basis of the following two vector spaces: Col A and Row A. If it is not possible to provide a basis for a vector space, explain why.

Exercise 5
Consider the following matrix A:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

5p **5a** Show that 3 is an eigenvalue of A (hint: find an eigenvector).

10p **5b** Show that 0 is an eigenvalue of A. And find two corresponding linearly independent eigenvectors.

5p **5c** Is the matrix A diagonalizable? Briefly explain.
(Note: you do not need to diagonalize A. You only need to state whether it is possible to diagonalize A.)

Exercise 6

Let $\mathbf{y} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$.

10p **6** Write $\mathbf{y}$ as the sum of a vector in $\text{Span}\{\mathbf{u}\}$ and a vector orthogonal to $\mathbf{u}$.

Exercise 7

10p **7** Prove or disprove the following statement.

Let $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^3$ be two vectors that have the same length ($||\mathbf{x}|| = ||\mathbf{y}||$) and define $\mathbf{u} = \mathbf{x} + \mathbf{y}$ and $\mathbf{v} = \mathbf{x} - \mathbf{y}$. Then, $\mathbf{u}$ and $\mathbf{v}$ are orthogonal to each other.

Exercise 8

True or False? For each of the statements below: state whether the statement is true or false. (Note: An explanation is not required. **Please read the multiple choice instructions on the cover page!**)

3p **8a** If two rows of a square matrix A are the same, then $\det A = 0$.

(a) True (b) False

3p **8b** If A is a 6×8 matrix, then it is possible that it has a 1-dimensional null space.

(a) True (b) False

3p **8c** Two orthogonal vectors are automatically also linearly independent.

(a) True (b) False

3p **8d** If λ is an eigenvalue of A, then it is also an eigenvalue of A^T.

(a) True (b) False

3p **8e** Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$. If $\mathbf{x} \cdot \mathbf{y} = 0$, then there does not exist a plane in $\mathbb{R}^3$ that contains both $\mathbf{x}$ and $\mathbf{y}$.

(a) True (b) False

Extra space

If you use these extra answer boxes, **please mention clearly in your main answer that part of your answer can be found here!**

9a

9b

9c

9d

9e