Real functions

$f: D \to S$ on set D into set S is a rule that

Domain: $\mathbb{R}$

- convention: $\mathbb{R}$
- open interval: $(a,b) : \{x \in \mathbb{R} : a < x < b\}$
- closed interval: $[a,b] : \{x \in \mathbb{R} : a \leq x \leq b\}$

Co-domain

Always $\mathbb{R}$

Range → $\{f(x) | x \in D\}$ ← all existing images

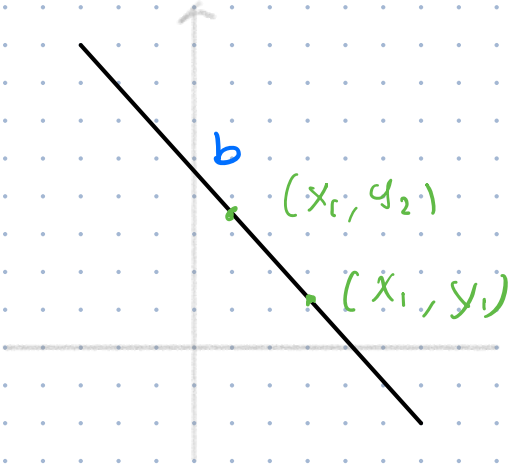
Important domains

- $\sqrt{x}$ → $[0, \infty)$
- $\frac{1}{x}$ → $\mathbb{R} - \{0\}$
- $\ln(x)$ → $(0, \infty)$

Linear functions

(a) $f(x) = \underbrace{a}_{\text{slope}} x + \underbrace{b}_{y\text{-intercept}}$

- $a < 0$ → ↘
- $a = 0$ → —



Determining the slope

$$a = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_1}{x_2 - x_1}$$

} we can plug into the equation

Parallel lines → same slope

Perpendicular lines → $a_1 \cdot a_2 = -1$

Exercises in class

$f(x) = x + 5$ $\quad$ $h(x) = x^2 + 2$

$g(x) = x^2 - 3$

$f(g(x)) = (x^2 - 3) + 5 = x^2 + 2$

(a) $y = -2x + 3$

$3 = (-2)(2) + 3$

$3 = -4 + 3$

(b) $y = \frac{x}{2} + 2$

$3 = 1 + 2$

(c) $y = 2x - 1$

$3 = 4 - 1$ ✓

$1 = 2 - 1$

Polynomial functions

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$

Degree → n (highest degree)

Root(s) → $P(r) = 0$

↳ number of (complex) roots = n

$$P(x) = (x - r)\,Q(x)$$

↑ root ↑ degree $n-1$

e.g. $x^2 - 3x + 2 = (x - r_1)(x + r_2) = (x - 2)(x - 1)$

Rational functions

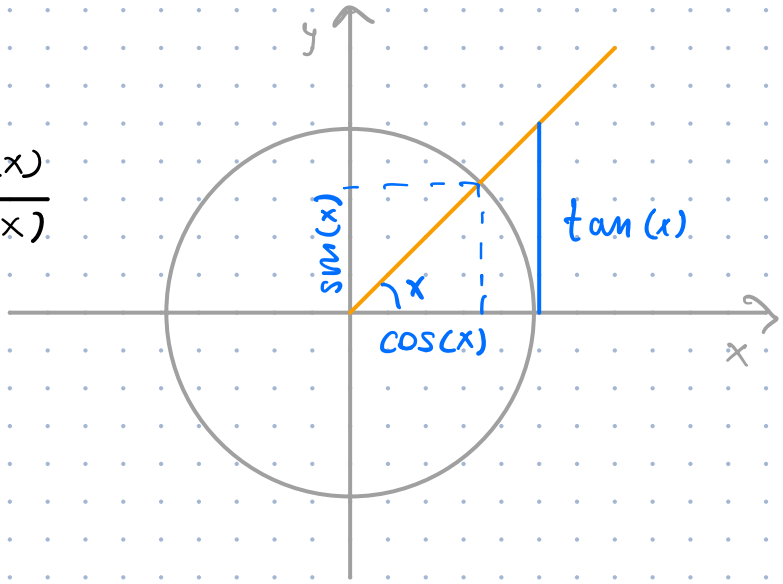
→ fraction of 2 polinomials

$$f(x) = \frac{P(x)}{Q(x)}$$

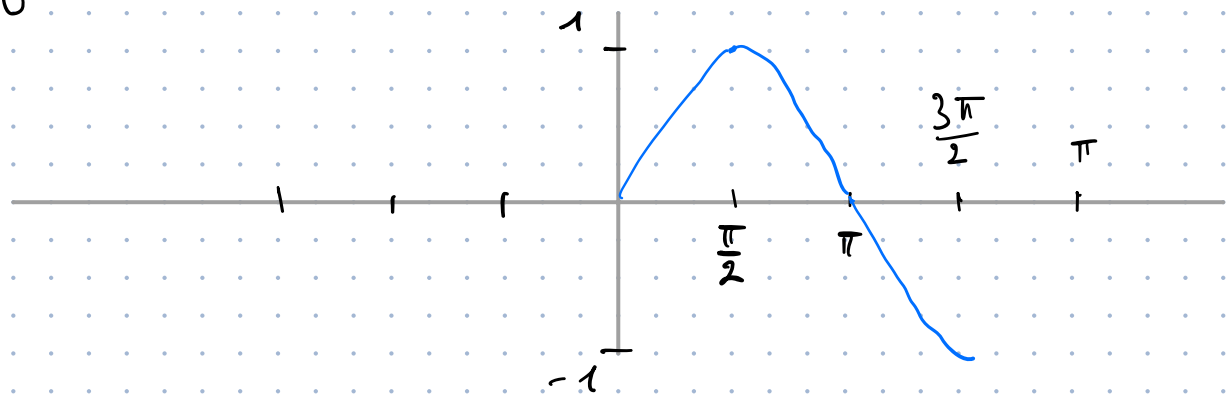
Domain = $\mathbb{R}$ - {roots of Q}

Trigonometric functions

- $\sin(x)$
- $\cos(x)$
- $\tan(x) = \frac{\sin(x)}{\cos(x)}$

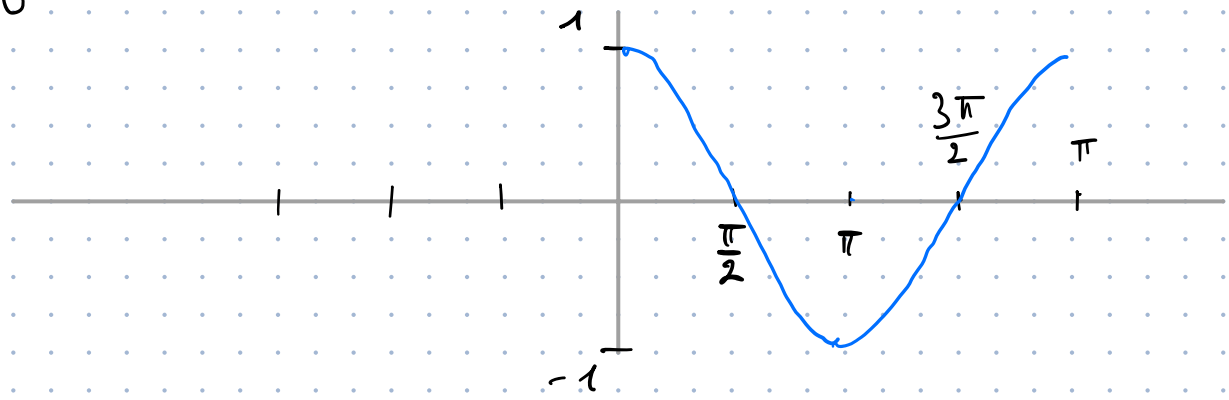


Graph of sin(x)



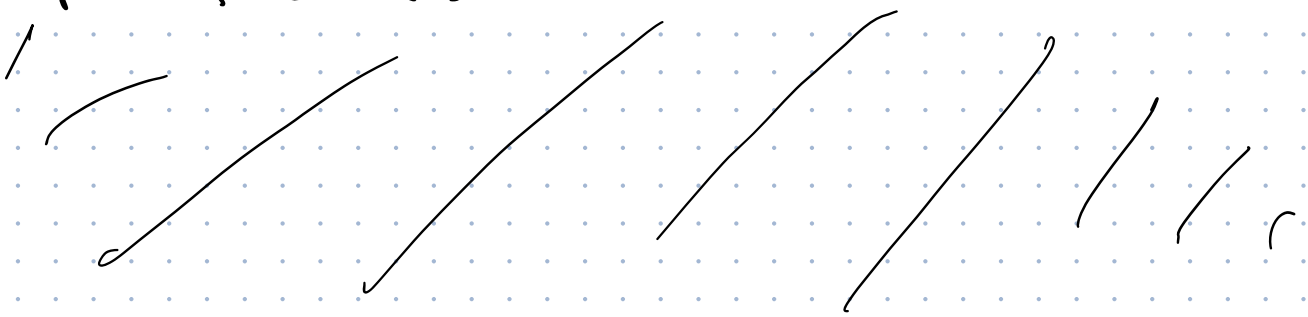
Period of 2π starting from 0

Graph of cos(x)



Period of 2π starting from 1

Graph of tan(x)



Even and odd functions

(1) even functions → $f(x) = f(-x)$
↳ mirror around y axis
e.g. $y = |x|$

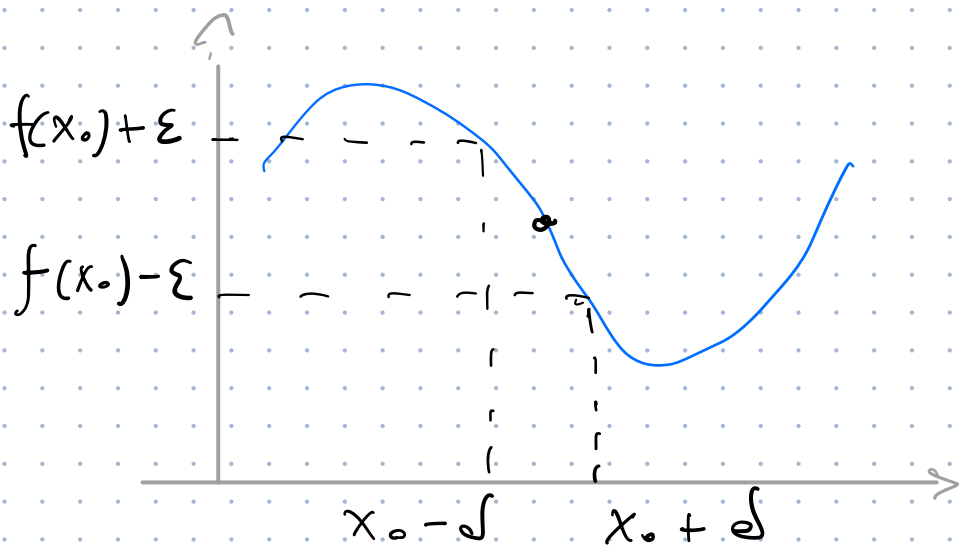
(2) odd functions → $f(-x) = -f(x)$
↳ mirror around origin
e.g. $\sin(x)$; $y = x^3$; $y = \sqrt[3]{x}$

Continuity

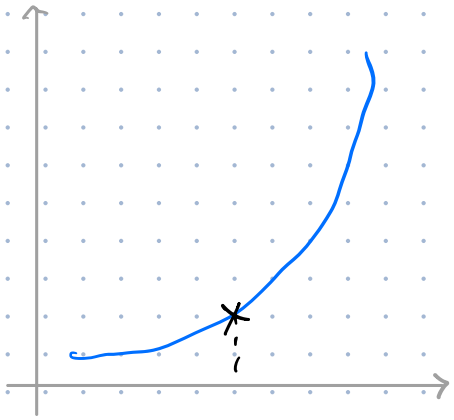
A function is continuous at an interior point x_0 of its domain if, for all points of domain

$$\forall \varepsilon > 0, \exists d > 0 : |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$$

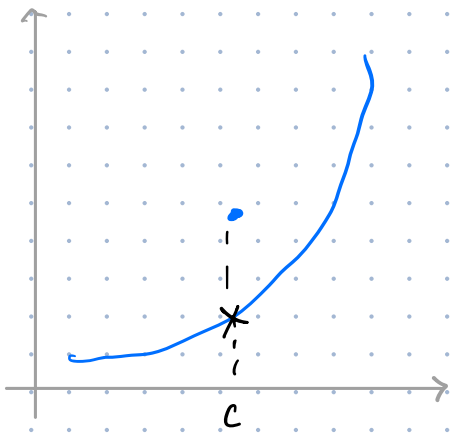
→ no jumps
→ if x approaches x_0, $f(x)$ approaches $f(x_0)$



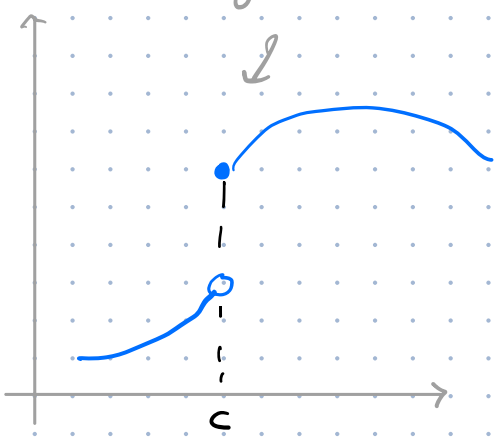
Discontinuous functions



Continuous



Removable disc.



Left / Right disc.

- every sum or product (or composition) of continuous functions results in a continuous function
- Continuity only makes sense on $\mathbb{R}$ (or $\mathbb{C}$)
- A function is continuous on an interval if it is continuous on all of the points of the interval
- In this course, a function is only discontinuous on its domain
  → no discontinuità di 2ª specie in punti che non appartengono al dominio!