Probability Theory Basics

Experiment → action whose outcome is det. by chance

Sample Space → Set of possible outcomes
$S = \{1, 2, 3 \dots\}$

Events → subset of sample space
$E \subseteq S$, $E = \{1, 2\}$
↳ All possible events : P^* of S

If we have 2 events (or more), we can compute their complement, intersections...

No intersections between events → **Mutually Exclusive**

Combinatorics

- Ordered sampling

- Unordered sampling without replacement
  $\binom{n}{k}$ with k items, n positions

- Unordered sampling with replacement
  $\binom{n+k-1}{k}$

Random Variables

X is a rand. var. for sample space S if it assigns a real number to each element of S

ex: $X : S \rightarrow \mathbb{R}$

$S = \{1, 2, 3, 4\}$

X: square $\forall s \in S : X(s) = s^2$

$y : \begin{cases} 2s & \text{if odd} \\ \frac{s}{2} & \text{if even} \end{cases} \rightarrow y$

ex 2: Rolling a die until 6 comes up

$S = \{6, N6, NN6, NNN6, \ldots\}$

X: number of rolls required

$X(6) = 1, \quad X(N6) = 2 \ldots$

Discrete vs. Continuous RV

X is discrete if S is finite or countable, otherwise X is continuous

Discrete: mapping to a finite/countable set

Probability distribution

$f(x) = P(X = x)$ for a discrete RV

Cumulative Distribution

$F(x) = P(X \leq x) = \sum_{y \leq x} f(y)$

Exercise

20 laptops shipped, 3 defective.

2 are ordered by a school

X = # defective laptops (3)

$$P(X = x) = \frac{\binom{17}{2-x}\binom{3}{x}}{\binom{20}{2}}$$

Continuous Random Variables