Maastricht University

Department of Data Science and Knowledge Engineering

Logic 2020/2021
Resit Exam Questions

— *Do not turn this page before the official start of the exam!* —

First Name, Surname: ______________________________

Student ID: ______________________________

Program: Bachelor Data Science and Knowledge Engineering

Course code: KEN1530

Examiners: dr. Otti D'Huys, dr. ir. ing. Nico Roos

Date/time: July 1^{st}, 2021, 9:30-11:30h

Format: Closed book exam.

Allowed aides: Pens.

Instructions to students:

General instructions:

- The exam consists of 10 questions on 12 pages (excluding the 1 cover page(s)).
- Answer every question on a separate piece of paper. Do not mix the answers on different exam questions.
- **Number each page of answers you submit in the top left corner.**
- Ensure that you properly motivate your answers.
- Do not use red pens, and write in a readable way. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- If you think a question is ambiguous, or even erroneous, explain this in detail in your answer.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.

For on-site exams:

- Fill in your name and student ID number on every page of answers you submit.
- Please make sure that communication devises and watches are not within reach.
- The use of pencils is not allowed.

For online proctored exams:

- The scan / photographs of your submitted answers must be readable.
- **Reserve space for your ID in the top right corner of every page of answers you submit.**

Success!

The following table will be filled by the examiner:

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	10	15	10	10	10	10	5	10	10	100
Score:											

Question 1 (10 points)

A B

- Alice likes running or Bob likes cycling. $a \vee b$
- C If Chris likes dancing, then Bob does not like cycling. $c \to \neg b$
- It is not the case that (Alice likes running and Chris likes dancing). $\neg(a \wedge c)$
- If Bob likes cycling and Alice does not like running, then Chris likes dancing. $(b \wedge \neg a) \to c$

Determine the answers of the following questions using a truth table. (Draw the full table!) Answer with: *Yes*, *No* or *Unknown*!

- Does Alice like running?
- Does Bob like cycling?
- Does Chris like dancing?

Use the atomic propositions a, b and c, use the truth-value 0 representing false and the truth-value 1 representing true, and enumerate the valuations of a, b and c as binary numbers from 0 to 7.

Solution:

	a	b	c	$a \vee b$	c	$\to$	$\neg$	b	$\neg$	$(a \wedge c)$	$(b$	$\wedge$	$\neg$	$a)$	$\to$	c
	~~0~~	~~0~~	~~0~~	0		1	1		1	0		0	1		1	
	~~0~~	~~0~~	~~1~~	0		1	1		1	0		0	1		1	
	~~0~~	~~1~~	~~0~~	1		1	0		1	0		1	1		0	
	~~0~~	~~1~~	~~1~~	1		0	0		1	0		1	1		1	
$\Rightarrow$	1	0	0	1		1	1		1	0		0	0		1	
	~~1~~	~~0~~	~~1~~	1		1	1		0	1		0	0		1	
$\Rightarrow$	1	1	0	1		1	0		1	0		0	0		1	
	~~1~~	~~1~~	~~1~~	1		0	0		0	1		0	0		1	

- a • Does Alice like running? **yes**
- b • Does Bob like cycling? **unknown** b can be true or false
- c • Does Chris like dancing? **no**

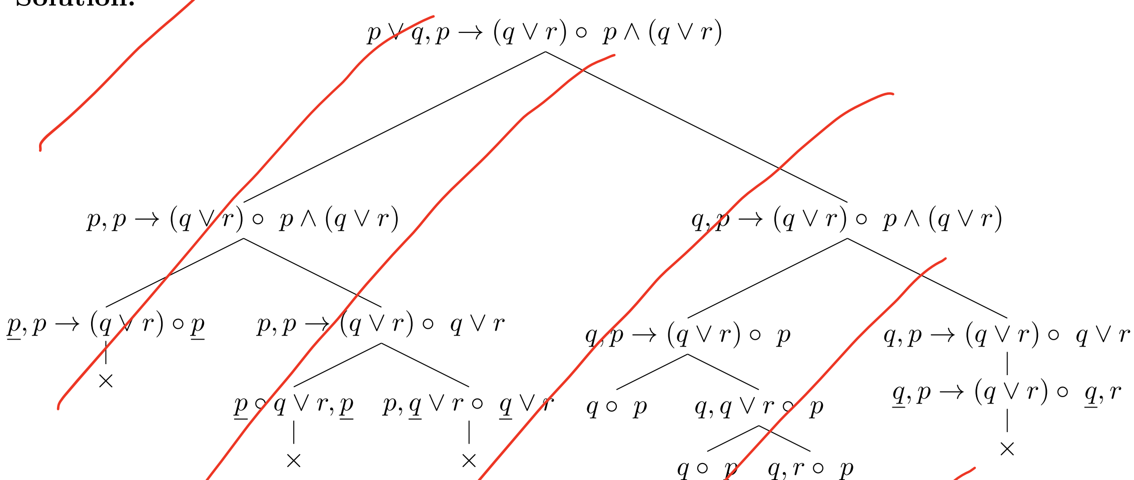
when are all of a, b and c true?

Question 2 (10 points)

Investigate the validity of the following inferences with the aid of a semantic tableau. If an inference is invalid, give **all** the counterexamples.

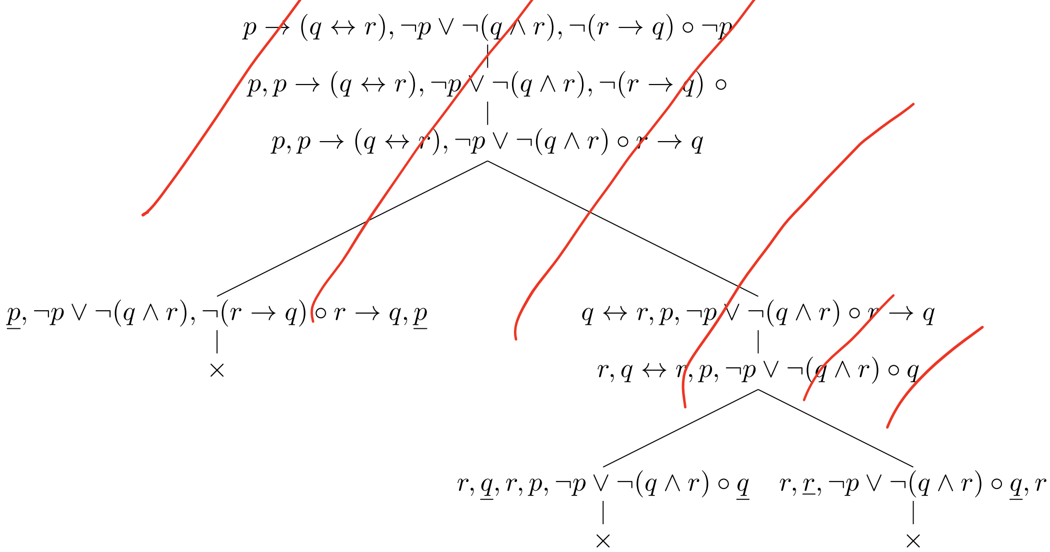
- $\{p \vee q, p \to (q \vee r), \neg(r \wedge p) \to \neg q\} \models p \wedge (q \wedge r)$
- $\{p \to (q \leftrightarrow r), \neg p \vee \neg(q \wedge r), \neg(r \to q)\} \models \neg p$

Solution:



Counter example 1: $V(p) = 0$, $V(q) = 1$, $V(r) = 0$.

Counter example 2: $V(p) = 0$, $V(q) = 1$, $V(r) = 1$.



Investigate the validity of the following inferences with the aid of a semantic tableau. If an inference is invalid, give **all** the counterexamples.

- $\{p \lor q, p \to (q \lor r), \neg(r \land p) \to \neg q\} \models p \land (q \land r)$
- $\{p \to (q \leftrightarrow r), \neg p \lor \neg(q \land r), \neg(r \to q)\} \models \neg p$

(2) $\{p \lor q,\ p \to (q \lor r),\ \neg(r \land p) \to \neg q\} \not\models p \land (q \land r)$

$p \lor q,\ p \to (q \lor r),\ \neg(r \land p) \to \neg q \ \circ\ p \land (q \land r)$

$p \lor q,\ p \to (q \lor r),\ \neg(r \land p) \to \neg q \ \circ\ p$

$//\ \circ\ q \land r$

$p, \ldots \circ p$

×

$q,\ p \to (q \lor r),\ \neg(r \land p) \to \neg q \ \circ\ p$

$q,\ p \to (q \lor r) \ \circ\ p,\ \neg(r \land p)$

$q,\ p \to (q \lor r),\ \neg q \ \circ\ p$

$r \land p,\ q,\ p \to (q \lor r) \ \circ\ p$

$q\ //\ \circ\ p, q$

×

$r, p,\ //\ \circ\ p$

×

* → $p \lor q,\ p \to (q \lor r),\ \neg(r \land p) \to \neg q \ \circ\ q \land r$

$p \land q,\ p \to (q \lor r),\ \neg(r \land p) \to \neg q \ \circ\ q$

$p \lor q,\ p \to (q \lor r),\ \neg(r \land p) \to \neg q \ \circ\ r$

$q, \ldots \circ q$

×

Question 3 (15 points)

Prove by means of natural deduction:

- From the premises $p \to \neg(r \to q)$ derive $p \to (\neg q \wedge r)$
- From the premise $\neg(\neg p \vee \neg q)$ derive $p \wedge q$

Solution:

1	$p \to \neg(r \to q)$	
2	$\quad p$	
3	$\quad \neg(r \to q)$	$\to$E, 1, 2
4	$\quad\quad \neg(\neg q \wedge r)$	
5	$\quad\quad\quad r$	
6	$\quad\quad\quad\quad \neg q$	
7	$\quad\quad\quad\quad \neg q \wedge r$	$\wedge$I, 5, 6
8	$\quad\quad\quad\quad \bot$	$\bot$I, 4, 7
9	$\quad\quad\quad q$	$\neg$E, 6–8
10	$\quad\quad r \to q$	$\to$I, 5, 9
11	$\quad\quad \bot$	$\bot$I, 3, 10
12	$\quad \neg q \wedge r$	$\neg$E, 4–11
13	$p \to (\neg q \wedge r)$	$\to$I, 2, 12

1	$\neg(\neg p \vee \neg q)$	
2	$\quad \neg p$	
3	$\quad \neg p \vee \neg q$	$\vee$I, 2
4	$\quad \bot$	$\bot$I, 1, 3
5	p	$\neg$E, 2–4
6	$\quad \neg q$	
7	$\quad \neg p \vee \neg q$	$\vee$I, 6
8	$\quad \bot$	$\bot$I, 1, 7
9	q	$\neg$E, 6–8
10	$p \wedge q$	$\wedge$I, 5, 9

Question 4 (10 points)

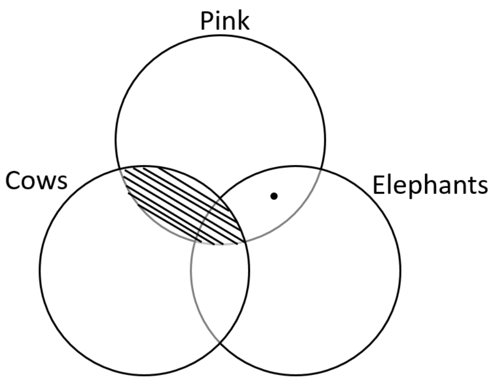
Consider the following two premises of a syllogism:

No cow is pink.
Some elephants are pink.
______
??

Complete the syllogism with a conclusion that makes it valid.

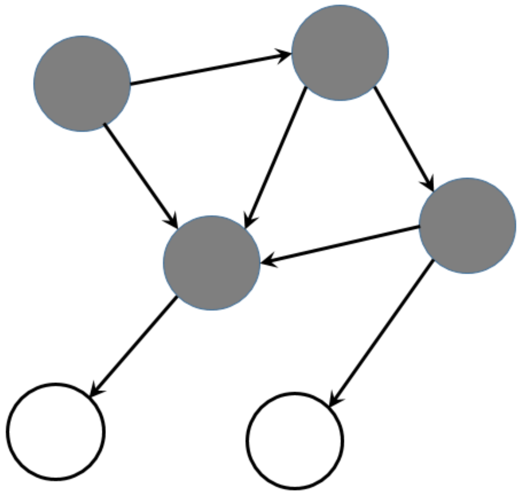
Demonstrate the validity of your syllogism using the method with Venn diagrams.

Solution: Some elephants are not cows.



Question 5 (10 points)

Consider the predicate logic model shown below. The model has a unary predicate P and a binary predicate S. Shaded objects have property P, a $\rightarrow$ from a to b meant that Sab is true in this model.



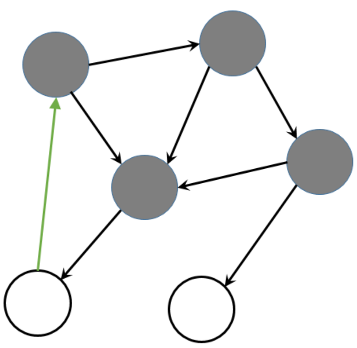
(a) State two closed formulas that are true in this model. Your formulas should include both predicates P and S.

(b) Add one pair to $I(S)$ (one arrow to the model representation), such that the statement $\forall x(Px \rightarrow \exists ySyx)$ becomes true.

Solution:

(a) Examples include $\forall x(\exists ySxy \rightarrow Px), \neg\exists x(Px \wedge Rxx), \forall x \forall y((Syx \wedge \neg Px) \rightarrow Py), \exists y \exists x(Sxy \wedge Py \wedge Px)$

(b) Any arrow arriving at the upper left object makes the formula true. An example is below



Question 6 (10 points)

Are the following formulas a tautology? Show by means of a semantic tableau.

- $\forall x \exists y\,(Ryx \to Rxx)$
- $\forall x\,((\exists y Ryx) \to Rxx)$

Solution:

$$\begin{array}{c}
\circ\underline{\forall x\exists y(Ryx \to Rxx)} \\
| \\
\circ^{+}\underline{\exists y(Rya \to Raa)} \\
| \\
\circ\underline{Raa \to Raa} \\
| \\
Raa \circ Raa \\
| \\
\times
\end{array}$$

This statement is a tautology

$$\begin{array}{c}
\circ\underline{\forall x((\exists y Ryx) \to Rxx)} \\
| \\
\circ^{+}\underline{(\exists y Rya) \to Raa} \\
| \\
\exists y Rya \circ Raa \\
| \\
Rba \odot^{+} Raa
\end{array}$$

This tableau has an open branch: we find a countermodel with two objects a and b, with b related to a, but a not related to itself.

Question 7 (10 points)

Prove by means of natural deduction:

- From the premises $\{\forall x(Px \to \exists yRxy), \forall x(\exists yRyx \to \neg Px), \exists xPx\}$ derive $\exists x \neg Px$
- From the premises $\{\forall x(Ax \to \neg Bx), \exists x(Bx \wedge Cx)\}$ derive $\exists x(Cx \wedge \neg Ax)$

Solution:

1	$\forall x(Px \to \exists yRxy)$	
2	$\forall x(\exists yRyx \to \neg Px)$	
3	$\exists xPx$	
4	$a \mid Pa$	
5	$\mid Pa \to \exists yRay$	$\forall$E, 1
6	$\mid \exists yRay$	$\to$E, 5, 4
7	$\mid b \mid Rab$	
8	$\mid \mid \exists yRyb$	$\exists$I, 7
9	$\mid \mid \exists yRyb \to \neg Pb$	$\forall$E, 2
10	$\mid \mid \neg Pb$	$\to$E, 9, 8
11	$\mid \mid \exists x \neg Px$	$\exists$I, 10
12	$\mid \exists x \neg Px$	$\exists$E, 6, 7–11
13	$\exists x \neg Px$	$\exists$E, 3, 4–12

Solution:

1	$\forall x(Ax \to \neg Bx)$	
2	$\exists x(Bx \land Cx)$	
3	$a \quad Ba \land Ca$	
4	$\quad Ca$	$\land$E, 3
5	$\quad Ba$	$\land$E, 3
6	$\quad\quad Aa$	
7	$\quad\quad Aa \to \neg Ba$	$\forall$E, 1
8	$\quad\quad \neg Ba$	$\to$E, 6, 7
9	$\quad\quad \bot$	$\bot$I, 5, 8
10	$\quad \neg Aa$	$\neg$I, 6–9
11	$\quad \neg Aa \land Ca$	$\land$I, 4, 10
12	$\quad \exists x(\neg Ax \land Cx)$	$\exists$I, 11
13	$\exists x(\neg Ax \land Cx)$	$\exists$E, 2, 3–12

Note: the topic of this question (epistemic logic) has been replaced in 2021/2022 by Hoare logic and tableau proofs for PDL. Please ignore this question.

Question 8 (5 points)

Translate the following sentence into a formula of epistemic logic, using appropriate translation keys.

- Bob does not know whether Chris knows whether Alice likes sailing.

Use the modal operators $\Box_A$, $\Box_B$ and $\Box_C$, and the atomic proposition a.

Solution:

- $\neg(\Box_B(\Box_C a \vee \Box_C \neg a) \vee \Box_B \neg(\Box_C a \vee \Box_C \neg a))$
  or
  $\neg\Box_B(\Box_C a \vee \Box_C \neg a) \wedge \neg\Box_B \neg(\Box_C a \vee \Box_C \neg a)$

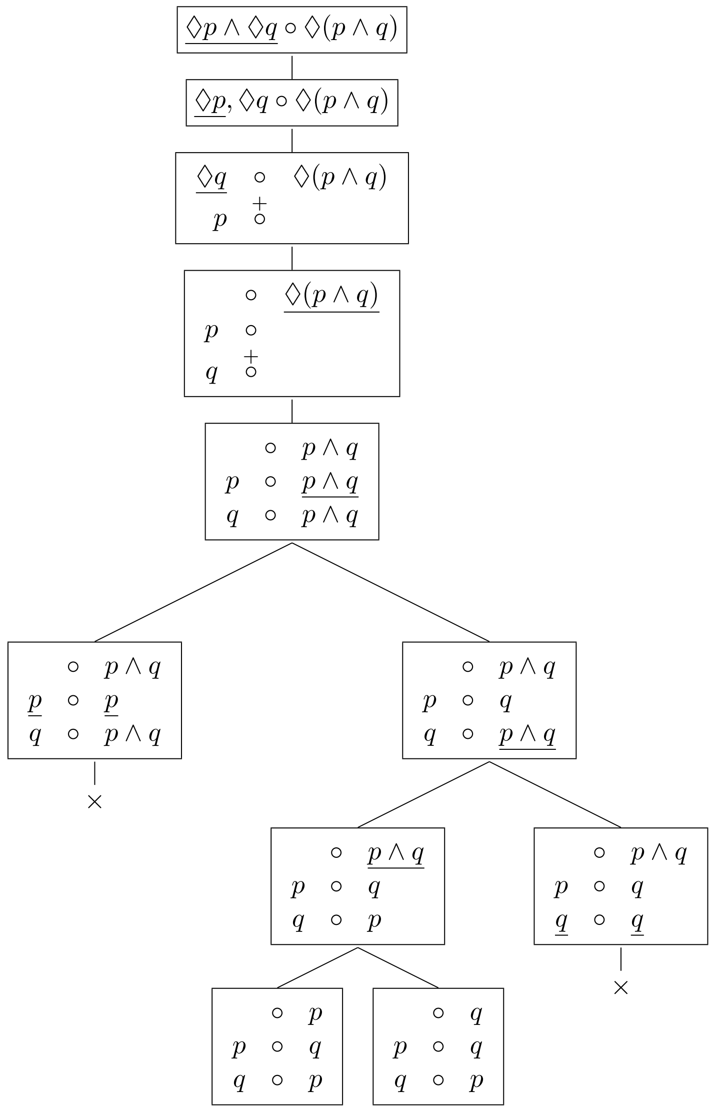
Note: the topic of this question (epistemic logic) has been replaced in 2021/2022 by Hoare logic and tableau proofs for PDL. Please ignore this question.

Question 9 (10 points)

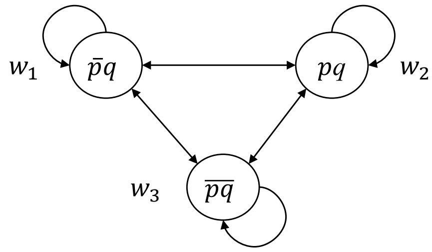
Investigate the validity of the following inferences with the aid of a semantic tableau. If an inference is invalid, give a counterexample.

- $\Diamond p \wedge \Diamond q \models \Diamond(p \wedge q)$
- $\neg\Diamond(p \vee q) \models \Box\neg p \wedge \Box\neg q$

Solution:

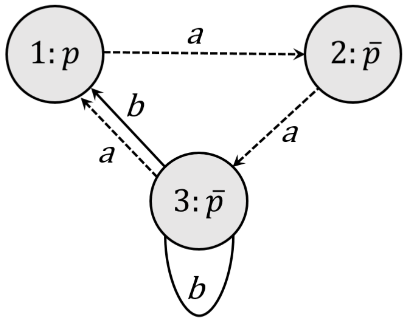


One of the three counter examples described by the open branches:


w_1
$\bar{p}q$
pq
w_2
w_3
$\overline{pq}$

Question 10 (10 points)

Consider the labelled transition system below, with 3 states and actions a and b.



(a) In which states are the following formula true?

- $\langle a^* \rangle p$.
- $[b]\neg p$

(b) Give all the elements of the relation defined by the action $(?\neg p; a \cup b)^*$.

Solution:

(a) In which states are the following formula true?

- $\langle a^* \rangle p$: true in all states.
- $[b]\neg p$: true in states 1 and 2.

(b) The elements of the relation defined by the action $(?\neg p; a \cup b)^*$ are $(1,1), (2,2), (3,3), (2,3), (3,1), (2,1)$