

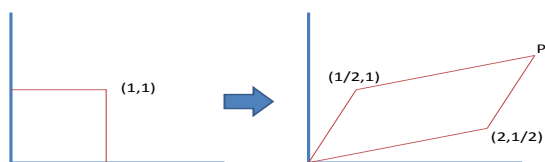
Practice Exam Questions Week 3, **Linear Algebra**

1. Consider the following matrix  $A$ :

$$A = \begin{bmatrix} 3 & -6 & 2 \\ 1 & -2 & 1 \\ -2 & 4 & -2 \end{bmatrix}.$$

- Are the columns of  $A$  linearly independent?
- Is  $A$  invertible? (Hint: use the answer from a.)

2. Consider the following transformation



- Construct the matrix which would give this transformation.
- Give the coordinates of point P.

3. Consider the following matrix  $A$ :

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix}.$$

- Compute the inverse of  $A$ .
- Let  $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$  and find a solution to the linear system of equations  $A\mathbf{x} = \mathbf{b}$ .
- Give an example of an alternative bottom row for  $A$  which would make it singular.

4. Compute the determinant of the following matrix:

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}.$$

5. True or false? If the given statement is true, briefly explain why. If it is false, give a counterexample.

- a. If  $S$  is a  $3 \times 3$  matrix such that  $S^2 = 0$ , then  $S^{-1}$  does not exist.

- b. If the equation  $A\mathbf{x} = \mathbf{b}$  is consistent, and the variable  $x_3$  is a free variable in the reduced echelon form of  $A$ , then there is a solution with  $x_3 = 4$ .
- c. If you take two vectors in  $\mathbb{R}^3$  they will never be linearly dependent.
- d. If  $F$  is  $(2 \times 2)$  with  $\det(F) = 0$  and  $g$  is a  $(2 \times 1)$  vector, then the matrix equation  $F\mathbf{x} = \mathbf{g}$  is always inconsistent.
- e. If  $G$  is a  $(3 \times 3)$  matrix for which  $G^2 = I$ , then  $\det(G) = 1$ .
- f. If  $\det(B) \neq 0$ , then  $B^T$  is invertible.
- g. If  $A$  is a  $(3 \times 4)$  matrix, then the transformation  $\mathbf{x} \mapsto A\mathbf{x}$  maps  $\mathbb{R}^3$  onto  $\mathbb{R}^4$ .
- h. An elementary row operation on  $A$  does not change the determinant of  $A$ .