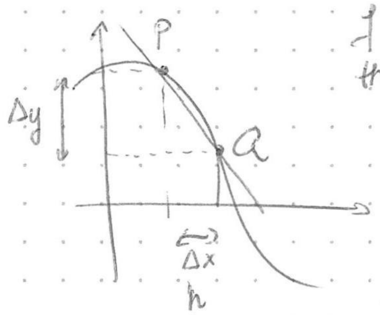
Calculus - lecture 3

DIFFERENTIATION

* tangent lines
* the derivative of a function - definition
* how to calculate the derivative
  - ↳ product rule
  - → chain rule
  - → trig. functions
  - → exp. / log

* higher order derivatives
* using derivatives for calculating limits - l'Hôpital rules

I. TANGENT LINES AND THEIR SLOPES



f continuous function

the line through P and Q becomes TANGENT as Q approaches P

* cfr. mean speed / average speed

The slope of the tangent line : $m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

$= \lim_{h \to 0} \frac{f(x_0+h) - f(x)}{h}$

↳ equation of the tangent line at (x_0, y_0)
$y = m(x - x_0) + y_0$

* can the tangent be vertical? yes, if $\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} = \pm\infty$

↳ for example $f(x) = \sqrt[3]{x}$

* does the tangent always exist: NO (only if $\lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$ exists)

↳ for example $f(x) = |x|$

- SLOPE OF A FUNCTION at x_0 = SLOPE of tangent line at x_0
  NORMAL TO a FUNCTION at x_0 = line $\perp$ to tangent, slope $\frac{-1}{m}$

II. THE DERIVATIVE

the derivative is defined as the slope of (the tangent of) a function

DEF: $f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$

at all points $x_0 \in D(f)$ for which this limit exists and is finite

* if the ~~lim~~ $f'(x_0)$ exists, we say that f is DIFFERENTIABLE at x_0
* the domain of f' may be smaller than the domain of f

$D(f') \subseteq D(f)$