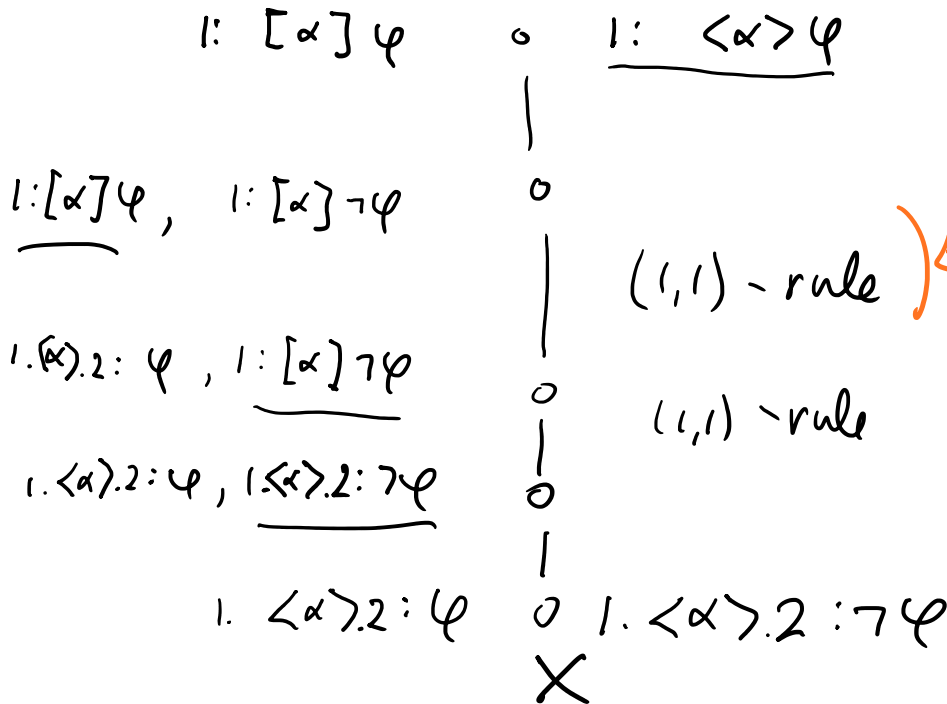$$\neg \langle \alpha \rangle \neg \varphi \quad =: \ [\alpha] \varphi$$

$\langle \alpha \rangle \varphi$: exists a way to execute α, such that
φ is true

$[\alpha] \varphi$: for all executions of α, φ is true.

Can we derive $[\alpha]\varphi \models \langle\alpha\rangle\varphi$?

1: $[\alpha]\varphi$ o   1: $\langle\alpha\rangle\varphi$

1: $[\alpha]\varphi$,   1: $[\alpha]\neg\varphi$ o

$(1,1)$~rule   **WRONG!** the preamble 1. $\langle\alpha\rangle$.2 does not exist yet!

1.$\langle\alpha\rangle$.2: φ , 1: $[\alpha]\neg\varphi$

$(1,1)$~rule

1.$\langle\alpha\rangle$.2: φ , 1$\langle\alpha\rangle$.2: $\neg\varphi$

1. $\langle\alpha\rangle$.2: φ o   1. $\langle\alpha\rangle$.2 : $\neg\varphi$

$\times$

$$1: \quad \underline{\langle \alpha \rangle \psi} \, , \, 1 : [\alpha] \varphi \qquad \circlearrowleft 1 : \langle \alpha \rangle \varphi \qquad \text{rule } (1,2)$$

$$1. \langle \alpha \rangle . 2 : \psi \, , \, \underline{1 : [\alpha] \varphi} \qquad \circ \qquad 1 : \langle \alpha \rangle \varphi \qquad \text{rule } (1,1)$$

$$1. \langle \alpha \rangle . 2 : \psi \, , \, 1. \langle \alpha \rangle . 2 : \varphi \qquad \circ \qquad \underline{1 : \langle \alpha \rangle \varphi} \qquad \text{rule } (1,4)$$

$$1. \langle \alpha \rangle . 2 : \psi \, , \, 1. \langle \alpha \rangle . 2 : \varphi \, , \, \underline{1 : [\alpha] \neg \varphi} \quad \circ \qquad \text{rule } (1,1)$$

$$'' \qquad , \qquad '' \qquad , \, \underline{1. \langle \alpha \rangle . 2 : \neg \varphi} \; \circ$$

$$1. \langle \alpha \rangle . 2 : \varphi \qquad , \, \underline{1. \langle \alpha \rangle . 2 : \varphi} \; \circlearrowleft \quad \underline{1. \langle \alpha \rangle . 2 : \varphi}$$

$$\times$$

$$\langle \alpha \rangle (p \rightarrow q) \models \langle \alpha \rangle q \vee \langle \alpha \rangle \neg p$$

$1: \langle \alpha \rangle (p \rightarrow q) \quad \circ \qquad 1: \underline{\langle \alpha \rangle q \vee \langle \alpha \rangle \neg p}$

$1: \underline{\langle \alpha \rangle (p \rightarrow q)} \quad \circ \quad 1: \underline{\langle \alpha \rangle q} , \quad 1: \underline{\langle \alpha \rangle \neg p} \qquad$ rule 1,2

$1. \neg \langle \alpha \rangle .2 : \underline{p \rightarrow q} \quad \circ \qquad \underline{\qquad '' \qquad} \qquad \underline{\qquad '' \qquad} \qquad$ rule 1,3 (Twice)

$1. \langle \alpha \rangle .2 : p \rightarrow q, \quad 1: \underline{[\alpha] \neg q} , \quad 1: \underline{[\alpha] \neg \neg p} \quad \circ \qquad$ rule (1,1) Twice

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$1. \langle \alpha \rangle .2 : p \rightarrow q, \quad 1. \underline{\langle \alpha \rangle .2 : \neg q} , \quad 1. \langle \alpha \rangle .2 : \neg \neg p \quad \circ$

$\qquad '' \qquad \times \qquad \underline{\qquad '' \qquad} \quad \circ \quad 1. \langle \alpha \rangle .2 : q$

$\qquad '' \qquad \times \qquad \times \qquad \circ \qquad '' \qquad , \quad 1. \underline{\langle \alpha \rangle .2 : \neg p}$

$1. \langle \alpha \rangle .2 : \underline{p \rightarrow q}, \quad 1. \langle \alpha \rangle .2 : p \quad \circ \quad 1. \langle \alpha \rangle .2 : q$

$1. \langle \alpha \rangle .2 : p, \quad 1. \langle \alpha \rangle .2 : q \quad \circ 1. \underline{\langle \alpha \rangle .2 : q} \quad \Big| \quad 1. \underline{\langle \alpha \rangle .2 : p} \quad \circ \quad \underline{1. \langle \alpha \rangle .2 : p , \quad 1. \langle \alpha \rangle .2 : q}$

$\qquad\qquad\qquad \times \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times$

Tableau closed, inference valid.

Dangerous, not recommended

rule 1,2 ... Twice

LECTURE 9 Page 4