Practice Exam Questions - Tutorial 2

1. Use induction to prove the following statement

   (a) For all integers $n \geq 1$,
   $$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}.$$

   (b) For all integers $n \geq 1$, $7^n - 4^n$ is divisible by 3.

2. Use induction to prove the following statements.

   (a) For all integers $n \geq 1$,
   $$\sum_{i=1}^{n}(i \times (i!)) \quad = \quad (n+1)! - 1$$
   where as usual $n!$ refers to "n factorial" i.e. $n \times (n-1) \times ... \times 1$.

   (b) For all integers $n \geq 1$, $2^{3n} - 3^n$ is divisible by 5.

3. Use induction to prove the following statement.

   - For all integers $n \geq 1$,
   $$\sum_{i=1}^{n} i(i+2) = \frac{n(n+1)(2n+7)}{6}$$

4. Use induction to prove the following statement.

   - For all integers $n \geq 1$,
   $$\sum_{i=1}^{n}(2i-1)(2i) = \frac{n(n+1)(4n-1)}{3}$$

5. Let $A = \{\{7\}, 2, \{4, \{5, 6\}\}, 4\}$. Are the following statements true or false? Briefly motivate your answer.

   (a) $7 \in A$
   (b) $\{2, 4\} \subseteq A$
   (c) $\{5, 6\} \subseteq A$
   (d) $\{7\} \in A$
   (e) $\emptyset \subseteq A$
   (f) $\{4, \emptyset\} \subseteq A$
   (g) $|A| = 5$

6. Let $A = \{2, 3, 4, 5\}$, $B = \{4, 5, 6, 7\}$, $C = \{4, 5\}$. Which of the following statements are true? Briefly motivate your answer.

   (a) $(A \setminus C) \cup B = A \cup B$

(b) $(C \setminus B) \cup \{6, 7\} = \emptyset$

(c) $B \cap A = C$

(d) $A \cap (B \setminus C) \subseteq \emptyset$

(e) $\emptyset \in A \cap \{7, 8\}$

(f) $\{\{2\}\} \subseteq A$

7. Prove or disprove the following statement.

   - For all sets A, B and C, $(B \cap (A^c \cup C)^c = \emptyset) \Leftrightarrow (A \subseteq B^c \cup C)$.

8. Prove or disprove the following statement.

   - For all sets A, B and C, $(C \subseteq B \setminus A) \Leftrightarrow ((A \cap C = \emptyset) \wedge (B^c \subseteq C^c))$.

9. Prove or disprove the following statement.

   - For all sets A, B, and C, $(B \subseteq A^c \cup C) \Leftrightarrow ((A \cap B) \setminus (A \cap C) = \emptyset)$.

10. Prove or disprove the following statement.

    - For all sets A, B, and C, $(A \cup (C^c \setminus B)) = ((A \cup C^c) \setminus B)$.