

$$\forall x \varphi(x) \models \exists x \varphi(x)$$

$$1. \forall x \varphi(x)$$

$$\exists x \varphi(x)$$

$$\forall x (P_x \rightarrow Q_x), \exists x P_x \models \exists z Q_z$$

$$1. \forall x (P_x \rightarrow Q_x)$$

(given)

$$2. \exists x P_x$$

(given)

3

$$P_c$$

(c, exist. constant(z))

4.

$$P_c \rightarrow Q_c$$

$E_{\forall}(1)$

5.

$$Q_c$$

$E_{\rightarrow}(3,4)$

6.

$$\exists z Q_z$$

$I_{\exists}(5)$

7

$$\exists z Q_z$$

$E_{\exists}(2,3,6)$

$$\forall x P_x, \forall x Q_x \vdash \forall x (P_x \wedge Q_x)$$

$$1. \forall x P_x \quad (\text{given})$$

$$2. \forall x Q_x \quad (\text{given})$$

$$3. \quad c, \text{ generic constant}$$

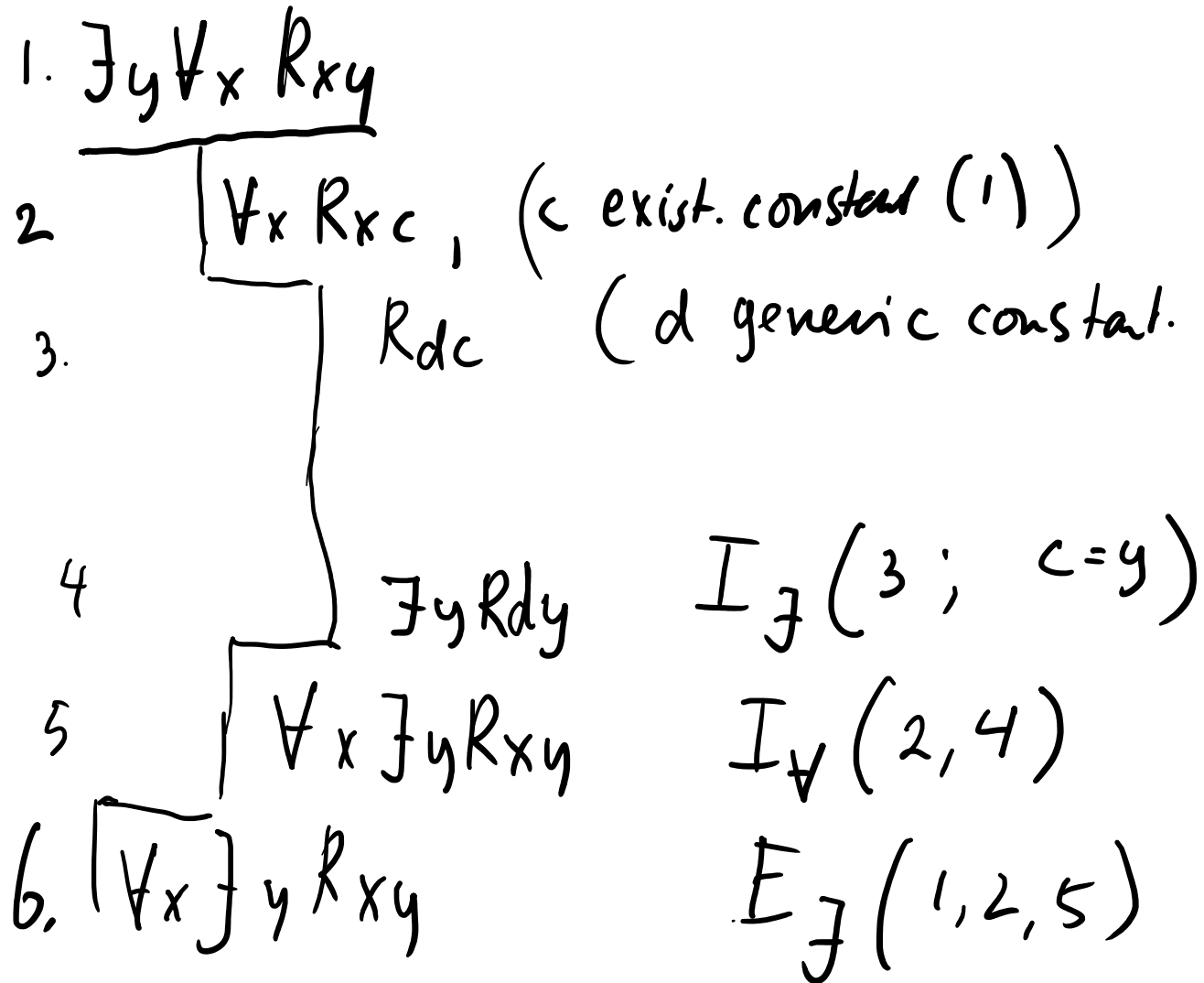
$$4. \quad P_c \quad E_{\forall}(1)$$

$$5. \quad Q_c \quad E_{\forall}(2)$$

$$6. \quad P_c \wedge Q_c \quad I_{\wedge}(4,5)$$

$$7. \quad \forall x (P_x \wedge Q_x) \quad I_{\forall}(3,6)$$

$$\exists y \forall x R_{xy} \models \forall x \exists y R_{xy}$$



Reconvene at 17.03

$$(1) \forall x \exists y Rxy \stackrel{?}{\models} \exists y \forall x Rxy \text{ (other way around)}$$

$$(2) \forall x \forall y Rxy \stackrel{?}{\models} \forall x Rxx$$

$$(3) \exists x (P_x \wedge R_x), \forall x (P_x \Rightarrow Q_x) \stackrel{?}{\models} \exists x (Q_x \wedge R_x)$$

$$\forall x \exists y Rxy \stackrel{?}{=} \exists y \forall x Rxy$$

1.	$\forall x \exists y Rxy$	(given)
2.	$c,$	(universal constant)
3.	$\exists y Rcy$	$E\forall(1, x=c)$
4.	Rcd	$E\exists(3), d \text{ exist. constant}$
5.	$\forall x Rxd$	$I\forall(2, 4)$
6.	$\exists y \forall x Rxy$	$I\exists(5)$
7.	$\exists y \forall x Rxy$	$E\exists(4, 6)$

This "proof" is
WRONG!

Why? line 2
line 4
line 5 get rid of
line 7 get rid of

(Remember what I said
with this example!)

$$\forall x \forall y Rxy \neq \forall x Rxx$$

1.	$\forall x \forall y Rxy$	(given)
2	c	(universal constant)
3	$\forall y Rcy$	$E_{\forall}(1, x=c)$
4	Rcc	$E_{\forall}(3, y=c)$
5.	$\forall x Rxx$	$I_{\forall}(2, 4)$

1.	$\forall x (P_x \rightarrow Q_x)$	(given)
2.	$\exists x (P_x \wedge R_x)$	(given)
3.	$P_c \wedge R_c$	$E_{\exists}(2, x=c), c \text{ exist. constant.}$
4.	$P_c \rightarrow Q_c$	$E_{\forall}(1, x=c)$
5.	P_c	$E_{\wedge}(3)$
6.	R_c	$E_{\wedge}(3)$
7.	Q_c	$E_{\rightarrow}(4, 5)$
8.	$Q_c \wedge R_c$	$I_{\wedge}(6, 7)$
9.	$\exists x (Q_x \wedge R_x)$	$I_{\exists}(8)$
10.	$\exists x (Q_x \wedge R_x)$	$E_{\exists}(2, 3, 9)$