Logic Lecture 1

Valid inference

suppose $\dfrac{A_1 \ldots A_c}{c}$ is valid

then

Modus tolems

$$\left[\begin{array}{l} \text{if } A, \text{ then } B \\ A. \\ \hline \text{then } B \end{array}\right. \qquad \begin{array}{l} \text{if } A, \text{ then } B \\ \neg B \\ \hline \text{so } \neg A \end{array}$$

there are infinitely many patterns of inferences.

Update and Consequences

Language of propositional logic

- Atomic statements
- Operators
- Language $\longrightarrow$ set of formulas:

(1) All the basic prop. are in $\mathcal{L}_P$

(2) If $\varphi \in \mathcal{L}_P$ and $\psi \in \mathcal{L}_P$, then

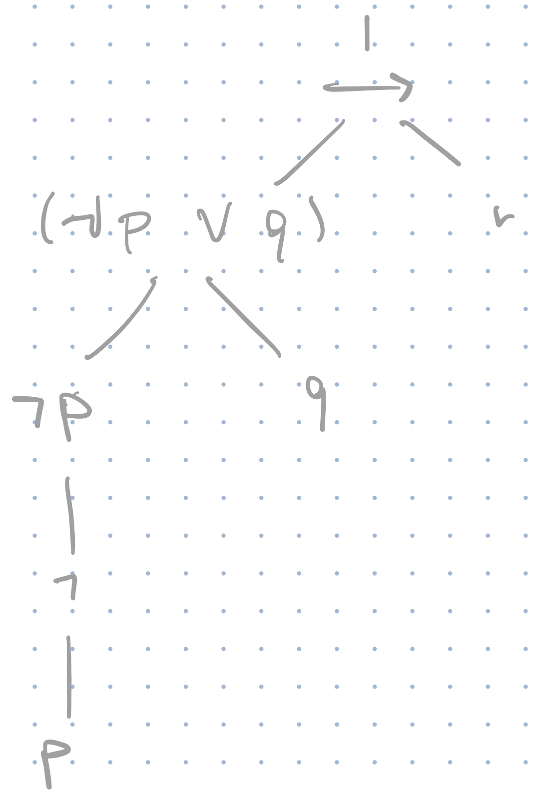
$$\neg\varphi \in \mathcal{L}_P,\ (\varphi \wedge \psi) \in \mathcal{L}_P,\ (\varphi \rightarrow \psi) \in \mathcal{L}_P$$
$$(\varphi \vee \psi) \in \mathcal{L}_P,\ (\varphi \leftrightarrow \psi) \in \mathcal{L}_P$$

(3) Nothing else in $\mathcal{L}_P$

Constructing formulas

- building a tree

$((\neg p \vee q) \rightarrow r)$ $\qquad\qquad$ $(\neg(p \vee q) \rightarrow r)$



Evaluating formulas

- Truth values

Semantics

Valuation $\longrightarrow$ Let $P = \{p, q, r, \ldots\}$ be a set of atomic propositions. A valuation V from P to $\{0,1\}$ assigns to each element of P a unique truth-value

$\hookrightarrow$ for P of n elements, there are 2^n valuations

$$\left.\begin{array}{lccccc} & (\neg & p) & \wedge & q \\ V: & 1 & 0 & 1 & 0 \end{array}\right] V \models (\neg p) \wedge q$$