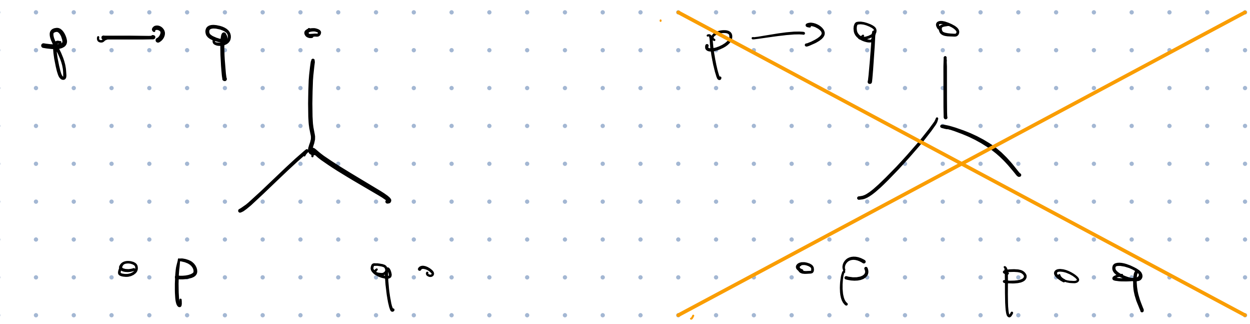
Tableau for predicate logic

Important observations



Extending tableaus and examples

LHS universal statement

Existential claim

$\exists x \varphi(x)$ ∘

|

$\varphi(a)$ ∘ +

Introduce constant

Universal claim

∘ $\exists x \varphi(x)$

|

∘ $\varphi(a_1), \dots, \varphi(a_n)$

Substitute constants

Universal claim

$\forall x \varphi(x)$ ∘

|

$\varphi(a_1), \dots \varphi(a_n)$ ∘

Existential claim

∘ $\forall x \varphi(x)$

|

+ ∘ $\varphi(a)$

- If we have a universal claim with no names,
  ↳ we add a new element → empty domains

- We deal with existential claims first
- Every time a new name is introduced, we need to reactivate all universal quantifiers

Suggested order

(1) Logical connectives
(2) Existential claims
(3) Universal claims

Examples

Closed $\to$ valid

Open $\to$ find counter.

(1) $\forall x\, P(x) \circ \neg \exists x\, \neg P(x)$

existential

$\forall x\, Px,\ \exists x\, \neg Px \circ$

$\forall x\, Px,\ \neg Pc \circ$

$\forall x\, Px \circ Pc$

substitute c.

$Pc \circ Pc$

closed branch $\to$ closed tableau

$\to$ valid inference

(2) $\forall x\,(Px \to Qx) \vDash \exists x\,(Px \wedge \neg Qx)$

$\forall x\,(Px \to Qx) \circ \exists x\,(Px \wedge \neg Qx)$

Universal on both sides $\to$ introduce constant

$Pc \to Qc \circ \exists x\,(Px \wedge \neg Qx)$

We have constant $\to$ subst.

$Pc \to Qc \circ Pc \wedge \neg Qc$

false since Pc not hold

$\circ Pc,\ Pc \wedge \neg Qc$ $\qquad$ $Qc \circ Pc \wedge \neg Qc$

Open branch

$\circ Pc,\ Pc$ $\qquad$ $\circ Pc,\ \neg Qc$

Pc false, so $Pc \to Qc$ is true

but $\exists c\,(Pc \wedge \neg Qc)$ is false

Counterexample:

Pc does not hold

(Qc does not matter)

$M = (D, I, g)$ $\quad D = \{c\}$

Infinite loops

Tableaus won't work with infinite models, only models with finite counterexamples

Tableaus are NOT COMPLETE for finding counterexamples in predicate logic

Exercises in class

(1) $\forall y\, \exists x\, Rxy \models \exists x\, \forall y\, Rxy$

both universal → $\forall y\, \exists x\, Rxy$	$\exists x\, \forall y\, Rxy$	universal
$\exists x\, Rxc$	$\exists x\, \forall y\, Rxy$	existential
$\forall y\, \exists x\, Rxy$, Rdc	$\exists x\, \forall y\, Rxy$	universal
″ , ″	$\forall y\, Rcy$, $\forall y\, Rdy$	exist.
$\forall y\, \exists x\, Rxy$, Rdc	Rce, $\forall y\, Rdy$, $\exists x\, \forall y\, Rxy$	existential
$\forall y\, \exists x\, Rxy$, Rdc	Rce, Rdf, $\exists x\, \forall y\, Rxy$	
$\exists x\, Rxc$, $\exists x\, Rxd$, $\exists x\, Rxe$, $\exists x\, Rxf$, Rdc	Rce, Rdf, $\exists x\, \forall y\, Rxy$	

⌊ in this case, you need extended rules for existential claims, where on a branch you assume the newly introduced constant is one of the constants already present

⌊ Not exam material

(2) $\exists x \forall y Rxy \models \forall y \exists x Rxy$

$\exists x \forall y Rxy \circ \forall y \exists x Rxy$ Exist - on both sides

exist

$\forall y Rcy \circ \forall y \exists x Rxy$

exist

$\forall y Rcy \circ \exists x Rxd$

universal

$Rcc, Rcd \circ \exists x Rxd$

uni

$Rcc, Rcd \circ Rcd, Rdd$

X closed tableau

Valid inference

In exam we will NOT have cases with infinite counterexamples

both exist.

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(3) $\exists x \forall y (Px \to Qy) \models \forall x \exists y (Py \to Qx)$

$\exists x \forall y (Px \to Qy) \circ \forall x \exists y (Py \to Qx)$

exist

$\forall y (Pc \to Qy) \circ \forall x \exists y (Py \to Qx)$

exist

$\forall y (Pc \to Qy) \circ \exists y (Py \to Qd)$

univers

$Pc \to Qc, Pc \to Qd \circ \exists y (Py \to Qd)$

univers

$Pc \to Gc, Pc \to Qd \circ Pc \to Qd, Pd \to Qd$

X

Closed tableau $\to$ valid inference

(4) $\forall x\,(Px \to \exists y\, Qy) \models \exists x\, Qx \lor \neg \exists x\, Px$

$\forall x\,(Px \to \exists y\, Qy) \circ \exists x\, Qx \lor \neg \exists x\, Px$

$\quad // \quad \circ \quad \exists x\, Qx,\ \neg \exists x\, Px$

$\forall x\,(Px \to \exists y\, Qy),\ \exists x\, Px \circ \exists x\, Qx$ *

exist.

$\quad // \quad ,\ Pc \circ \exists x\, Qx$

uni

$\forall x\,(Px \to \exists y\, Qy),\ Pc \circ Qc$

uni

$Pc \to \exists y\, Qy,\ Pc \circ Qc$

Rule for $\to$

either Pc is false

$Pc \circ Pc,\ Qc$

X

$Pc,\ \exists y\, Qy \circ Qc$

exist *

$\forall x\,(Px \to \exists y\, Qy),\ Pc,\ Qd \circ Qc,\ \exists x\, Qx$

substitute all constants

$\forall x\,(Px \to \exists y\, Qy),\ Pc,\ Qd \circ Qc,\ Qd$

X

All branches are closed $\to$ valid inference

(5) $\forall x\, Px \to \forall x\, Qx \models \forall x\, (Px \to Qx)$

$\forall x\, Px \boxed{\to} \forall x\, Qx \circ \forall x\, (Px \to Qx)$

it splits, so we wait

exist

$\forall x\, Px \to \forall x\, Qx \overset{+}{\circ} Pc \to Qc$

$\forall x\, Qx \circ Pc \to Qc$ | $\circ\ \forall x\, Px,\ Pc \to Qc$

$\forall x\, Qc,\ Pc \circ Qc$ | $Pc \circ \forall x\, Px,\ Qc$

own | exist

$Qc,\ Pc \circ Qc$ | $Pc \overset{+}{\circ} Pd,\ Qc$

$\times$ | $\downarrow$ open branch

Counterexample:

$D = \{c, d\}$

$P = \{c\}$

$Q = \emptyset$ Anything that doesn't contain c

$Pc \to Qc$ not true