Class notes

Exercises

(a) is $(q \wedge (p \to q)) \to p$ valid? (tautology)

$\circ \; (q \wedge (p \to q)) \to p$

remove $\to$, wgh? (we need to have 2 formulas)

$q \wedge (p \to q) \circ \; p$

remove $\wedge$ on LHS

$q, \; p \to q \circ \; p$

remove $\to$ on LHS

$q \circ \; p, p$ $\qquad\qquad$ $q, q \circ \; p$

If we have the same statement, we can ignore it

$q \circ \; p$ $\qquad\qquad$ $q \circ \; p$

Tableau is open $\to$ formula is not valid

↳ it would be false if

$V(q) = 1$, $V(p) = 0$, then $V((q \wedge (p \to q)) \to p) = 0$

$\to$ this is a counterexample

$\to$ if the question asks if a statement is a tautology, it is enough to expand on a single branch that leads to a counter example

Terminology of tableaus

- sequent ⟶ each node of the tree
- Closed branch ⟶ on the end sequent there is the same formula on LHS & RHS
- Closed tableau ⟶ all the branches are closed
- Open branch ⟶
- Open tableau ⟶ at least 1 open branch

General tableau method

$$\frac{\varphi_1, \varphi_2, \ldots \varphi_n}{\psi} \text{ valid ?}$$

$\iff (\varphi_1, \varphi_2, \varphi_n) \to \psi$ valid

$$\begin{array}{lcl} & \circ & (\varphi_1 \wedge \varphi_2 \wedge \ldots \varphi_n) \to \psi \quad (\to, \text{ right}) \\ & | & \\ \varphi_1 \wedge \varphi_2 \wedge \varphi_n & \circ & \psi \\ & | & (\wedge, \text{ left}, n-1 \text{ times}) \\ \varphi_1, \varphi_2 \ldots \varphi_n & \circ & \psi \end{array}$$

If tableau closed ⟶ valid
If tableau open ⟶ find counter ex.

Tableau use cases

(1) Check if a formula is valid
(2) Check if formula is satisfiable or contradiction
(3) Check if a set of formulas is satisfiable
(4) Check if 2 formulas are equivalent

(1) $\dfrac{\varphi \to \psi, \quad \psi}{\varphi}$ valid ?

$\varphi \to \psi, \ \psi \circ \varphi$

($\to$, left)

$\psi \circ \varphi, \psi$ $\qquad\qquad$ $\psi, \ \psi \circ \varphi$

$\psi \circ \varphi$ $\qquad\qquad\qquad$ ψ

open branch:

$V(\psi) = 1, \ V(\varphi) = 0$

$\longrightarrow$ complete example notes

(4) Check if φ and ψ are equivalent

$\longrightarrow \varphi \models \psi \ \wedge \ \psi \models \varphi$

2 ways:

(1) $\varphi \circ \psi$ closed $\wedge$ $\psi \circ \varphi$ closed

(2) $\varphi \leftrightarrow \psi$ is tautology

$\circ \ \varphi \leftrightarrow \psi$ is closed

$\varphi \circ \psi$ $\qquad$ $\psi \circ \varphi$

Notes on tableaus

- Tableau builds a model with specific requirem..
- Tableau method is complete for proving validity and for finding counterexamples
- Tablaus can generate all counterexamples
- Tableaus can be used with quantifiers too
  - ↳ problems with checking completeness

Exercises in class

De Morgan law

(1) Is $\neg A \wedge \neg B \models \neg(A \vee B)$ valid?

$\neg A \wedge \neg B \circ \neg(A \vee B)$ ← is it closed?

($\wedge$, left)

$\neg A, \neg B \circ \neg(A \vee B)$

why and didn't split?

($\neg$, right)

$\neg A, \neg B, A \vee B \circ$

Don't do them at once!

($\neg$, left)

$\neg B, A \vee B \circ A$

order of these 2 steps doesn't matter

We CAN do it (But only with the NOT operator)

$A \vee B \circ A, B$

the $\vee$ splits

why $\vee$ split?

$A \circ A, B$ X

$B \circ A, B$ X

Tableau is closed ⟶ valid inference

$\neg(A \vee B) \models \neg A \vee \neg B$

(2) $\neg(A \vee B) \models \neg A \wedge \neg B$

$\neg(A \vee B) \circ \neg A \wedge \neg B$

($\neg$, left and right)

$\circ \neg A \wedge \neg B, A, B$

$\circ \neg A, A, B$

$\circ \neg B, A, B$

$A \circ A, B$

$B \circ A, B$

Why did the OR not split, but the AND did