$$\neg\langle\alpha\rangle\neg\varphi \quad =: [\alpha]\varphi$$

$\langle\alpha\rangle\varphi$: exists a way to execute α, such that φ is true

$[\alpha]\varphi$: for all executions of α, φ is true.

Can we derive $[\alpha]\varphi \vdash \langle\alpha\rangle\varphi$?

$1: [\alpha]\varphi$ ∘ $1: \langle\alpha\rangle\varphi$

|

$1:[\alpha]\varphi$, $1:[\alpha]\neg\varphi$ ∘

| $(!,!)$-rule ← WRONG! the preamble $1.\langle\alpha\rangle.2$ does not exist yet!

$1.\langle\alpha\rangle.2: \varphi$, $1:[\alpha]\neg\varphi$ ∘

| $(!,!)$-rule

$1.\langle\alpha\rangle.2:\varphi$, $1.\langle\alpha\rangle.2:\neg\varphi$ ∘

|

$1.\langle\alpha\rangle.2:\varphi$ ∘ $1.\langle\alpha\rangle.2:\neg\varphi$

X

$$\frac{1:\ \langle\alpha\rangle\psi,\ 1:[\alpha]\varphi \circ 1:\langle\alpha\rangle\varphi}{1.\langle\alpha\rangle.2:\psi,\ 1:[\alpha]\varphi \circ 1:\langle\alpha\rangle\varphi}\ \text{rule}(1,2)$$

$$\frac{1.\langle\alpha\rangle.2:\psi,\ 1:[\alpha]\varphi \circ 1:\langle\alpha\rangle\varphi}{1.\langle\alpha\rangle.2:\psi,\ 1.\langle\alpha\rangle.2:\varphi \circ 1:\langle\alpha\rangle\varphi}\ \text{rule}(1,1)$$

$$\frac{1.\langle\alpha\rangle.2:\psi,\ 1.\langle\alpha\rangle.2:\varphi,\ 1:[\alpha]\neg\varphi \circ}{\text{"},\ \text{"},\ 1.\langle\alpha\rangle.2:\neg\varphi \circ}\ \text{rule}(1,4)$$

$$\frac{}{1.\langle\alpha\rangle.2:\psi,\ 1.\langle\alpha\rangle.2:\varphi \circ 1.\langle\alpha\rangle.2:\varphi}\ \text{rule}(1,1)$$

$\times$

$\langle\alpha\rangle(p \to q) \models \langle\alpha\rangle q \vee \langle\alpha\rangle \neg p$

$1: \langle\alpha\rangle(p \to q) \circ 1: \langle\alpha\rangle q \vee \langle\alpha\rangle \neg p$

$1: \langle\alpha\rangle(p \to q) \circ 1: \langle\alpha\rangle q,\ 1: \langle\alpha\rangle \neg p$ rule 1, 2

$1.\langle\alpha\rangle.2: p \to q \circ$ " " rule 1, 3 Twice

$1.\langle\alpha\rangle.2: p \to q,\ 1: [\alpha] \neg q,\ 1: [\alpha] \neg\neg p \circ$ rule (1,1) Twice

$1.\langle\alpha\rangle.2: p \to q,\ 1.\langle\alpha\rangle.2: \neg q,\ 1.\langle\alpha\rangle.2: \neg\neg p \circ$

" × " $\circ$ $1.\langle\alpha\rangle.2: q$

" × × $\circ$ " , $1.\langle\alpha\rangle.2: \neg p$

$1.\langle\alpha\rangle.2: p \to q,\ 1.\langle\alpha\rangle.2: p \circ 1.\langle\alpha\rangle.2: q$

$1.\langle\alpha\rangle.2: p,\ 1.\langle\alpha\rangle.2: q \circ 1.\langle\alpha\rangle.2: q$ × | $1.\langle\alpha\rangle.2: p \circ 1.\langle\alpha\rangle.2: p,\ 1.\langle\alpha\rangle.2: q$ ×

Dangerous, not recommended

Tableau closed, inference valid.