

Exercises

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Surname, First name

Linear Algebra (KEN1410)

Exam

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

a	<input checked="" type="radio"/>	c	d	e	f	→ b
a	b	<input checked="" type="radio"/>	d	e	f	→ c
<input checked="" type="radio"/>	b	c	<input checked="" type="radio"/>	e	f	→ a

Fill in your answer(s) to the multiple-choice questions as shown above (circles = one correct answer).

Program: Data Science and Artificial Intelligence

Course code: KEN1410

Examiners: Dr. Marieke Musegaas and Dr. Stefan Maubach

Date/time: Monday 3 April 2023, 09:00 - 11:00

Format: Closed book exam

Allowed aids: Pens, simple (non-programmable) calculator from the DACS-list of allowed calculators.

Instructions to students:

- The exam consists of 7 questions on 20 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

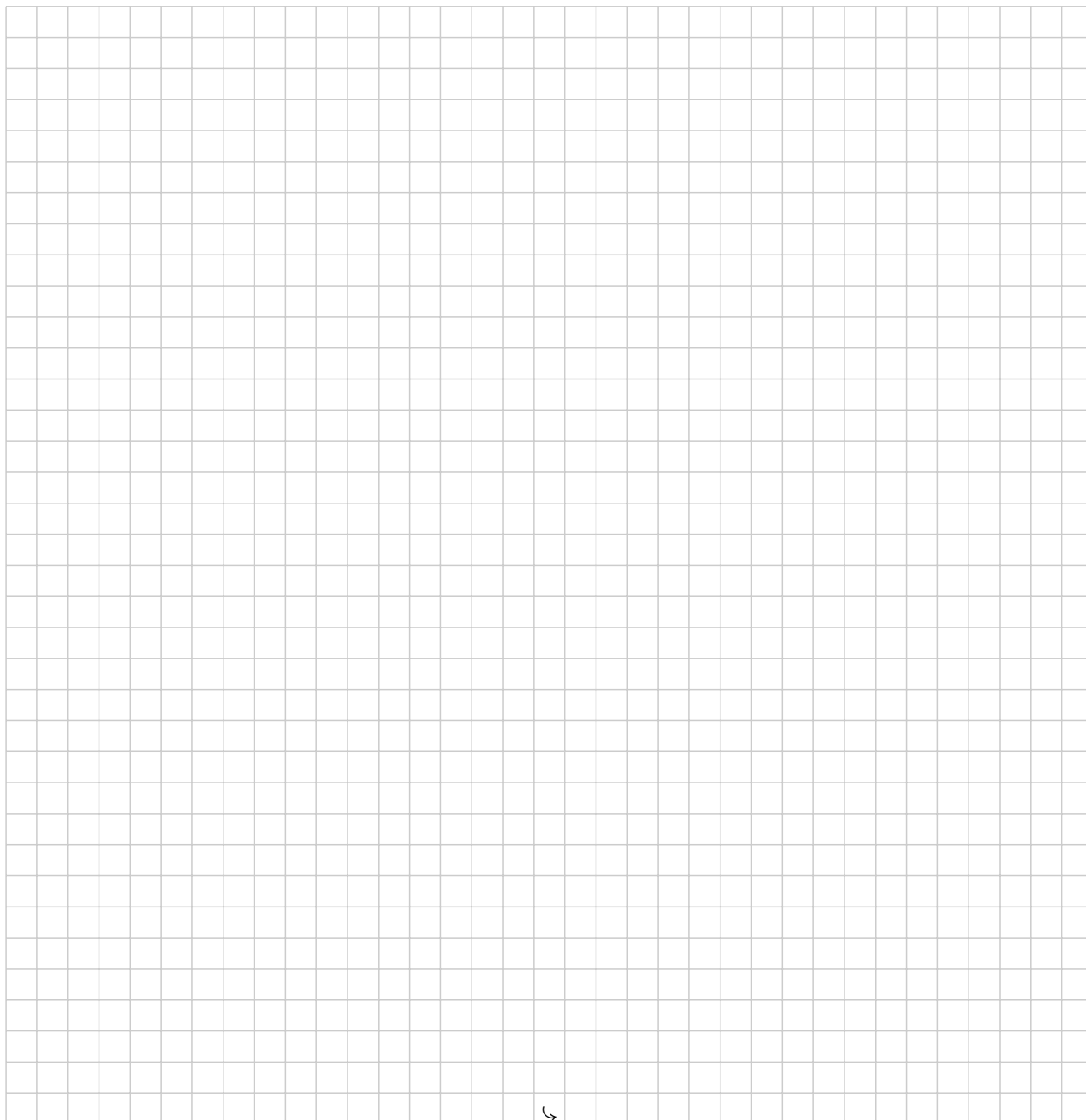
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Question 1

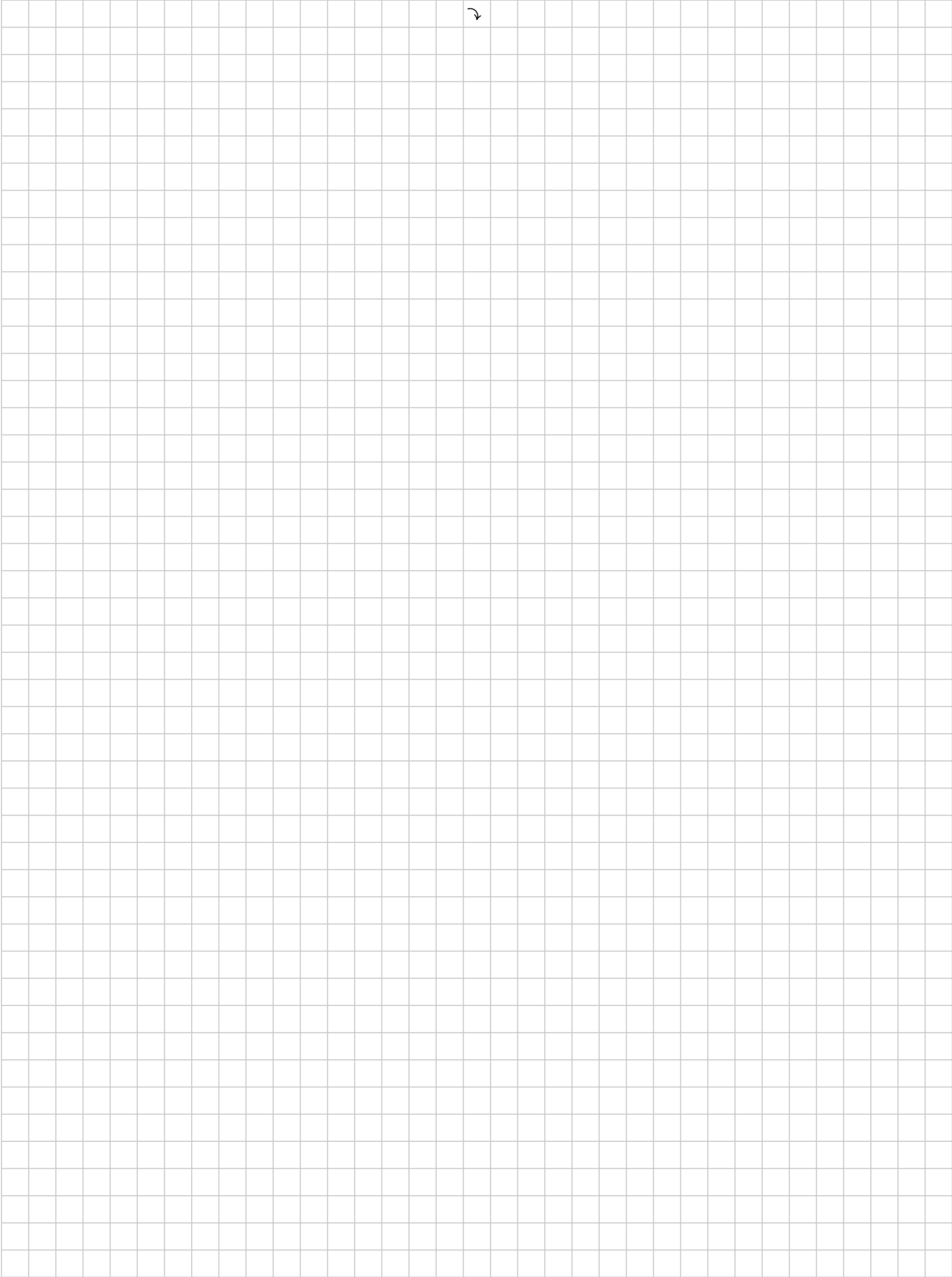
10p **1** Consider the following matrix A depending on a parameter p :

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & p & 3 \\ 5 & 3 & p \end{bmatrix}.$$

Determine for which value(s) of p the matrix A is **not** invertible.



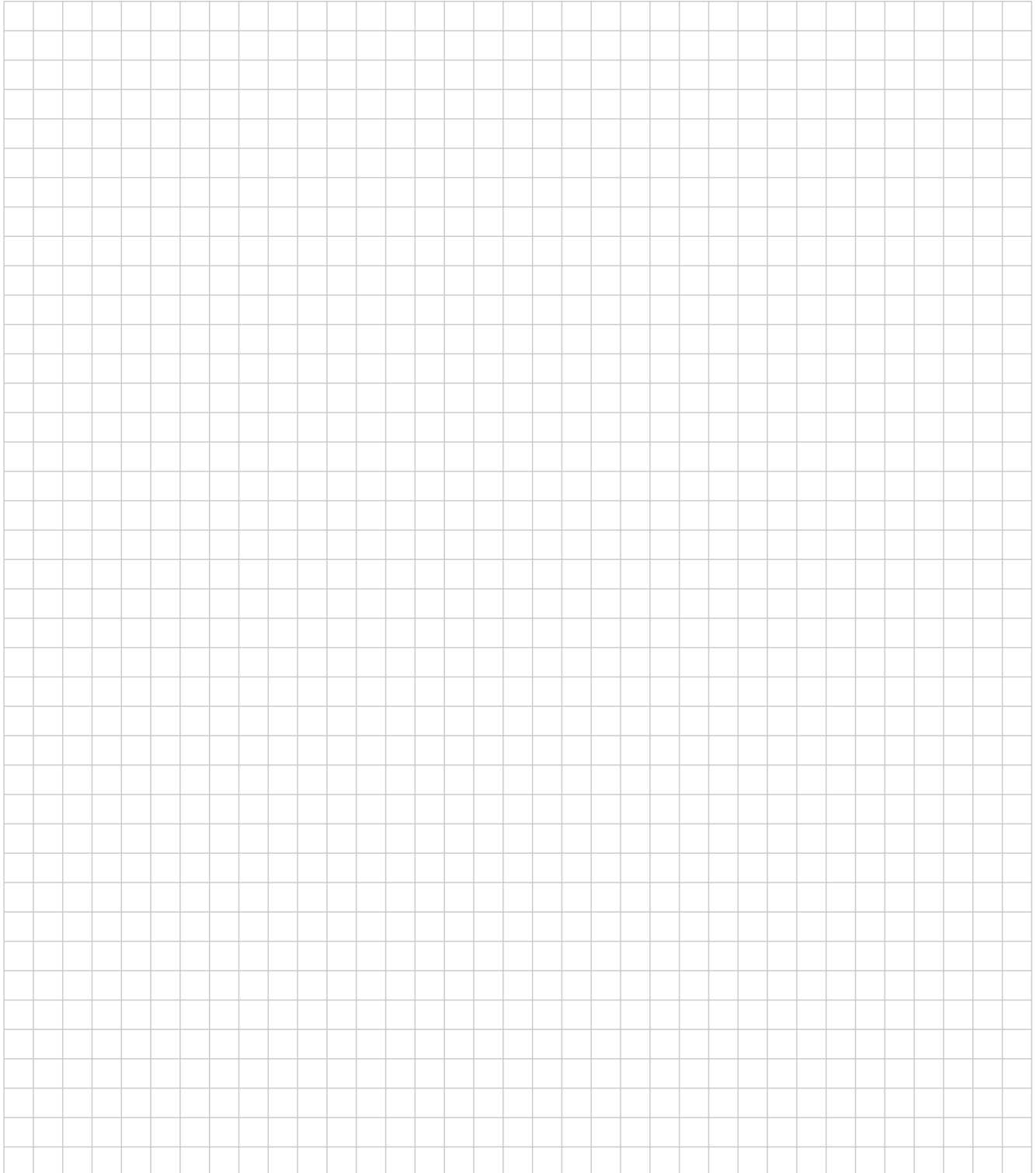
A large grid of graph paper for working out the solution. At the bottom center of the grid, there is a small icon of a hand pointing to the right.



Question 2

5p **2** Is the following statement true? If yes, provide a proof. If no, provide a counterexample.

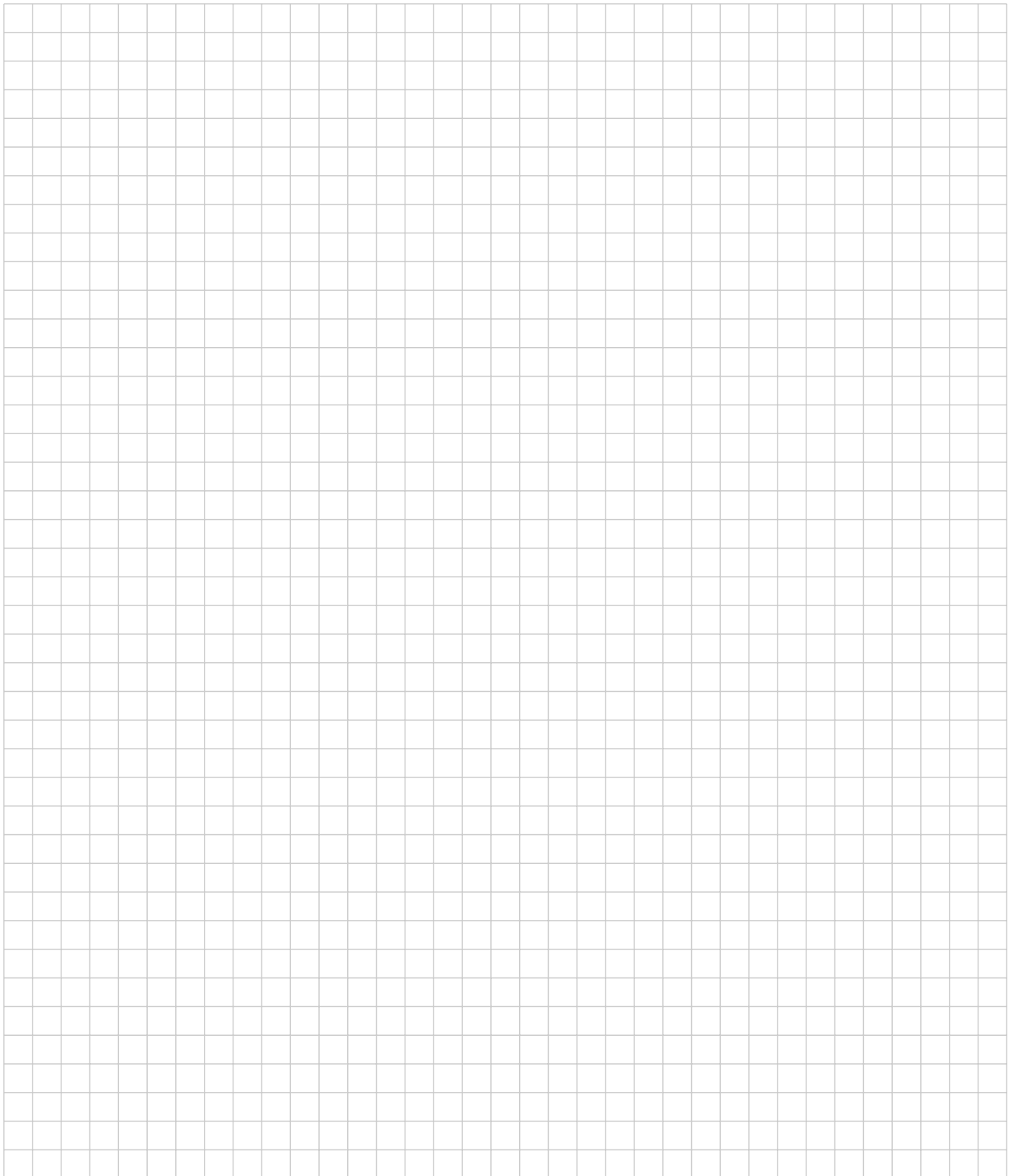
"If an $n \times n$ matrix A is symmetric, then A is invertible."



Question 3

5p

3 Let $\mathbf{u} = \begin{bmatrix} \sqrt{2} \\ \sqrt{3} \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} \sqrt{3} \\ \sqrt{2} \end{bmatrix}$. Determine whether $\{\mathbf{u}, \mathbf{v}\}$ is a basis for \mathbb{R}^2 .



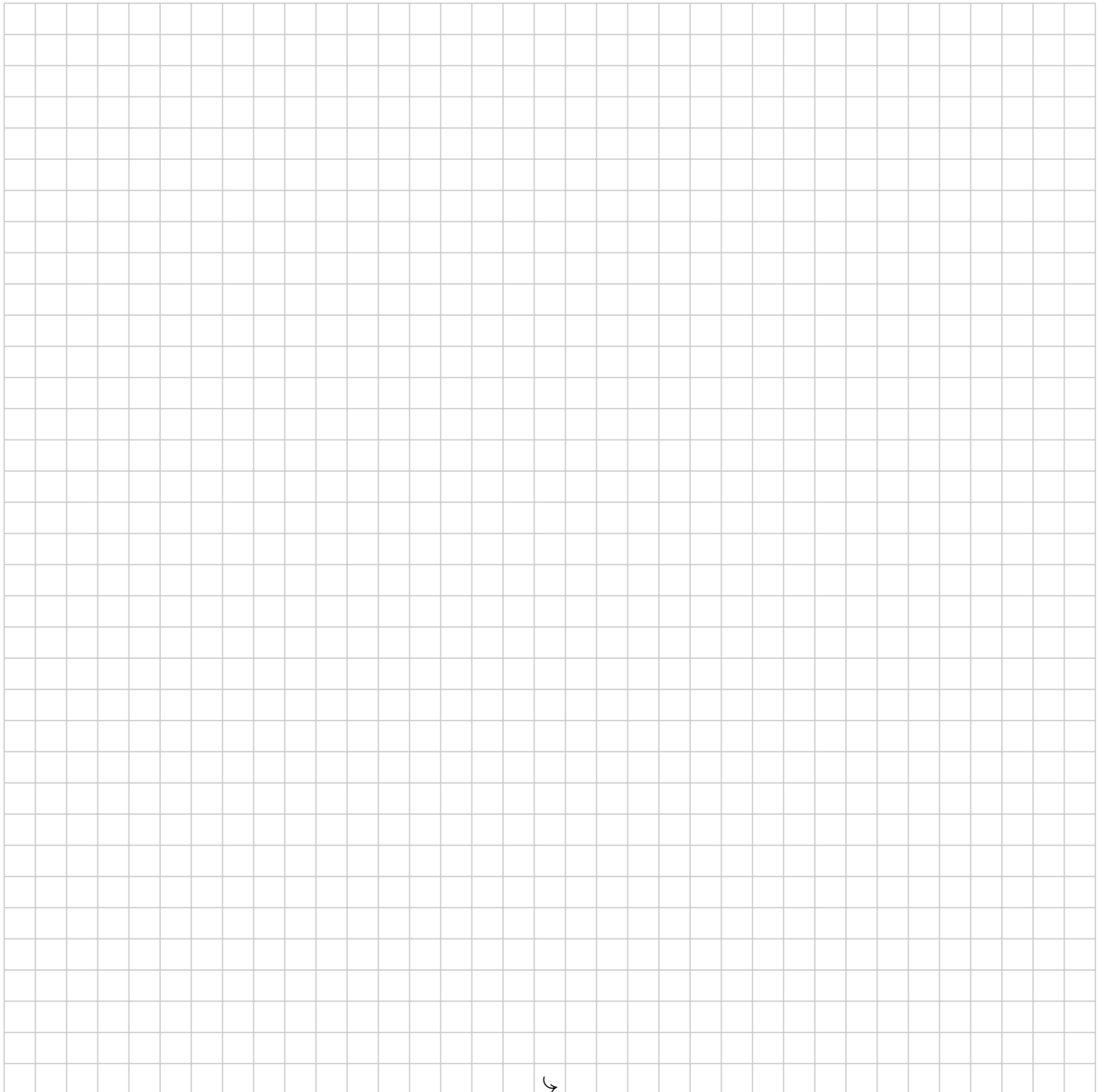
Question 4

10p **4** Consider the following matrix A :

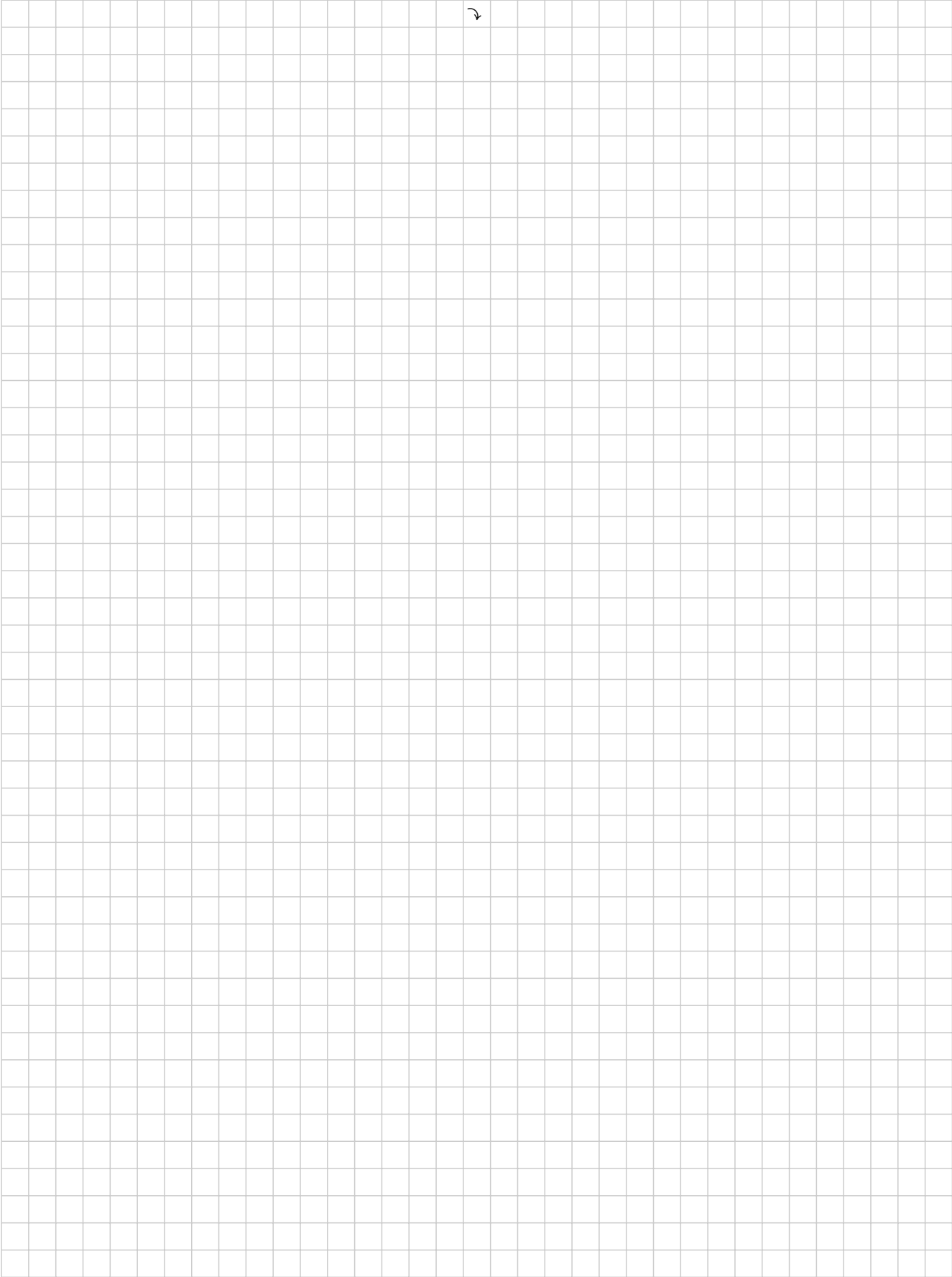
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}.$$

Is the matrix A diagonalizable?

(Note: you do not need to diagonalize A . You only need to state if it is possible to diagonalize A .)



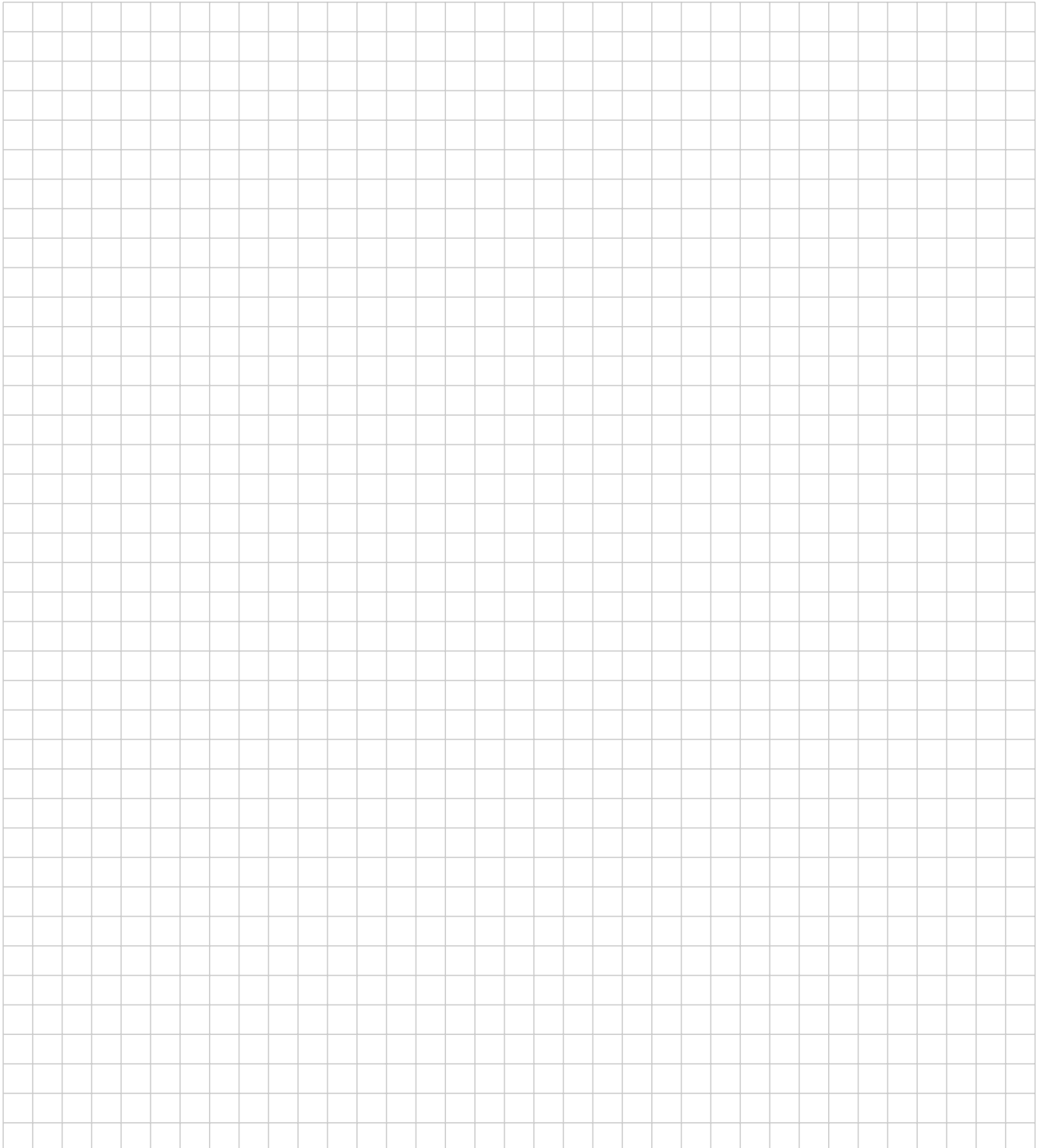
A large grid of graph paper for working out the solution. At the bottom center of the grid, there is a small icon of a hand pointing to the right.



Question 5

5p

5 Determine two distinct vectors in $\text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 3 \end{bmatrix} \right\}$ with length 1.



Question 6


Consider the following matrix A :

$$A = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}.$$

It is given that the matrix A has eigenvalues 1 and -2.

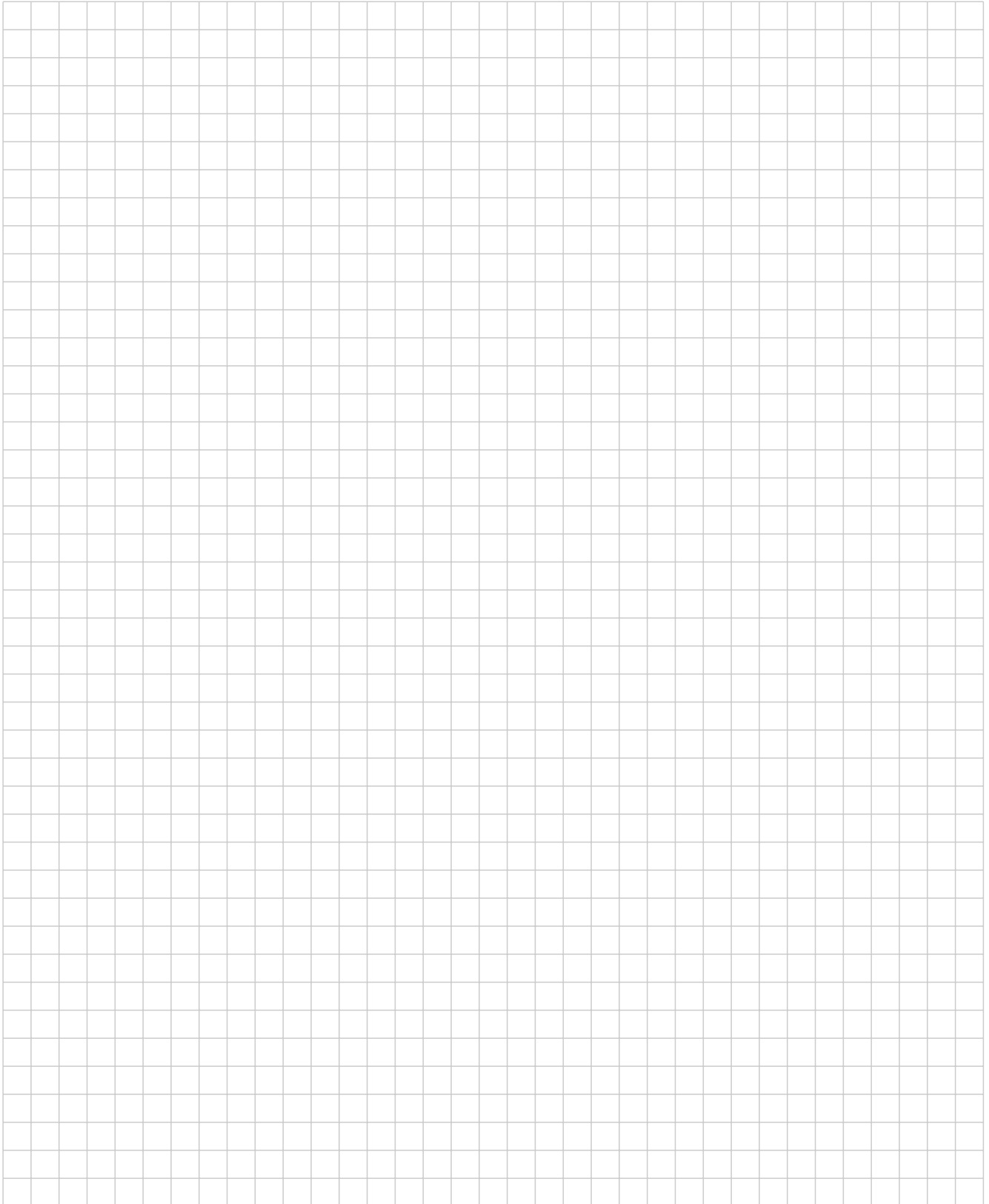
5p

6a Show that $\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right\}$ forms a basis for the eigenspace corresponding to the eigenvalue 1.

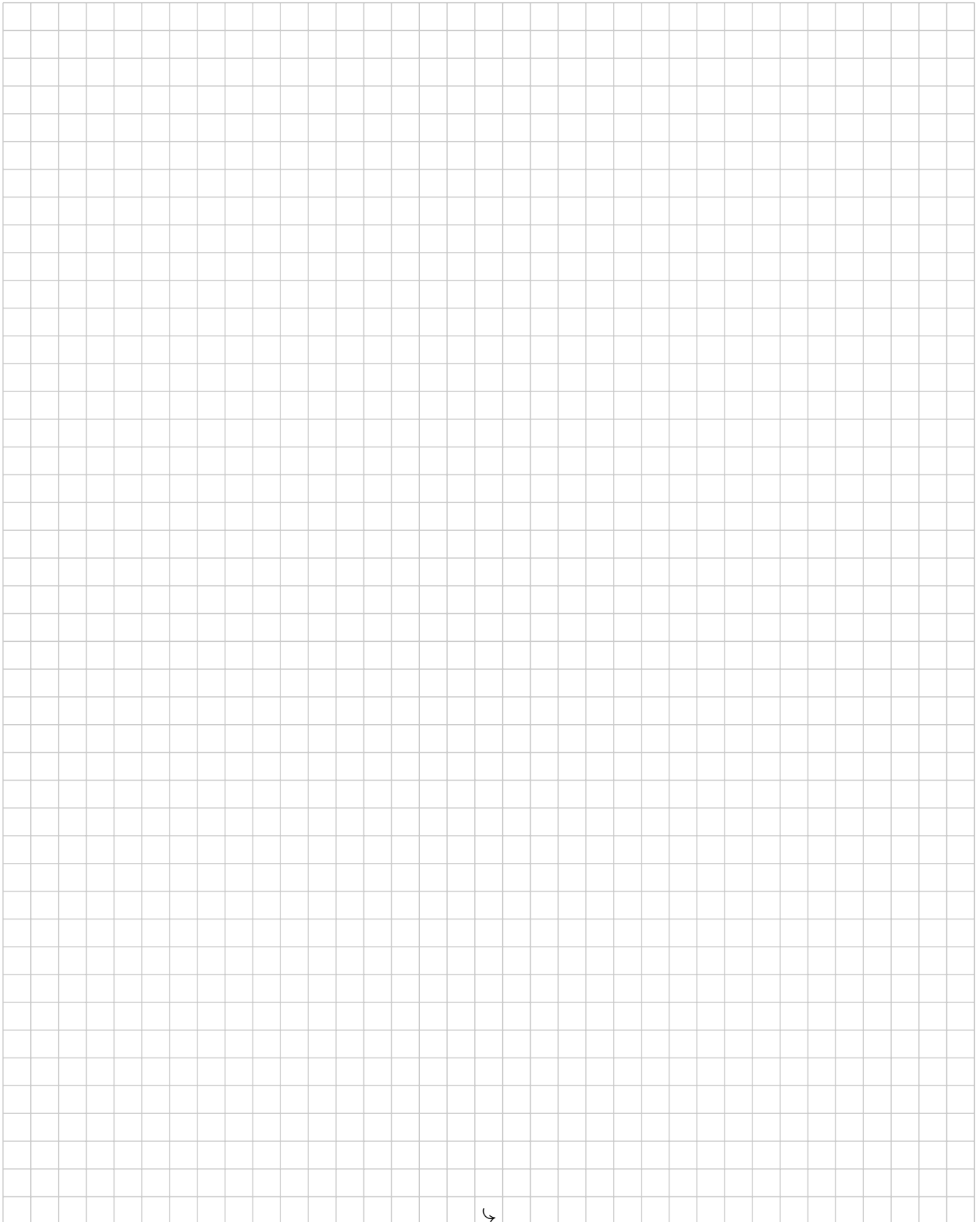


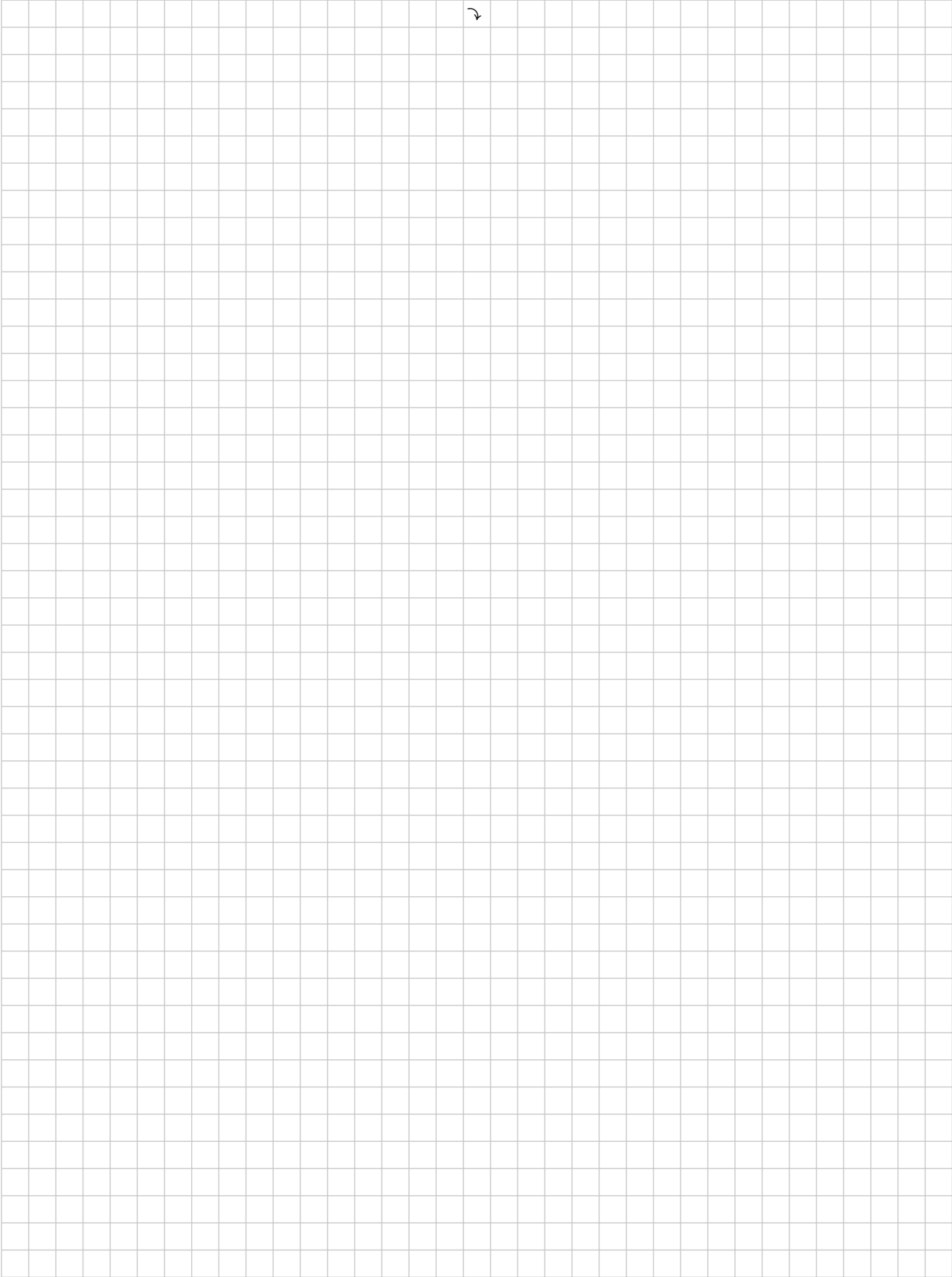
5p

6b Show that $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ forms a basis for the eigenspace corresponding to the eigenvalue -2.



- 10p **6c** Orthogonally diagonalize matrix A , i.e. find an orthogonal matrix P and a diagonal matrix D such that $A = PDP^T$.





Question 7

Answer the following multiple-choice questions.

An explanation is not required.

For every multiple-choice question, there is only **one correct answer**.

Please read the multiple choice instructions on the cover page!

5p **7a** Consider the following matrices A , B and C :

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}.$$

Which of the following operations **can not** be performed?

- ☐ a AB
- ☐ b AC
- ☐ c $2A + 3C$
- ☐ d $3B + 5C$
- ☐ e All of the above operations can be performed.

5p **7b** Find a value k such that the system of linear equations $A\mathbf{x} = \mathbf{b}$ corresponding to the following augmented matrix has infinitely many solutions.

$$\left[\begin{array}{cccc} 2 & 2 & -4 & 3 \\ 1 & 3 & -2 & 4 \\ -4 & k & 8 & -6 \end{array} \right].$$

- ☐ a $k = -12$
- ☐ b $k = -4$
- ☐ c $k = 4$
- ☐ d $k = 12$
- ☐ e None of these k values will give infinitely many solutions.

- 5p **7c** The following augmented matrix is almost in reduced echelon form. What is the solution of the corresponding system of linear equations $Ax = b$?

$$\left[\begin{array}{ccccc} 0 & 0 & 1 & 0 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right].$$

(a) $x = \begin{bmatrix} 2 \\ 5 \\ 0 \\ 7 \end{bmatrix}$

(b) $x = \begin{bmatrix} 5 \\ 2 \\ 7 \\ 0 \end{bmatrix}$

(c) $x = \begin{bmatrix} 7 \\ 2 \\ 5 \\ 0 \end{bmatrix}$

(d) The system of linear equations does not have a solution.

(e) The system of linear equations has infinitely many solutions.

5p

7d Let $u = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 0 \\ 5 \\ -1 \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ 2 \\ a \\ b \end{bmatrix}$. If w is a linear combination of u and v , then we must have

(a) $a = 3$ and $b = 1$

(b) $a = 2$ and $b = 0$

(c) $a = 2$ and $b = -1$

(d) $a = 1$ and $b = 3$

(e) $a = 0$ and $b = 2$

(f) $a = -1$ and $b = 2$

(g) None of the above

5p

7e Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation such that $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

Is T injective (one-to-one)? Is T surjective (onto)?

- ☐ (a) T is both injective and surjective
- ☐ (b) T is injective, but not surjective
- ☐ (c) T is surjective, but not injective
- ☐ (d) T is neither injective nor surjective

5p

7f Let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Which one of the following subsets of \mathbb{R}^3 is a subspace of \mathbb{R}^3 ?

- ☐ (a) $\{\mathbf{x} \in \mathbb{R}^3 \mid \mathbf{x} + \mathbf{u} = \mathbf{0}\}$
- ☐ (b) $\{\mathbf{x} \in \mathbb{R}^3 \mid \mathbf{x} \cdot \mathbf{u} = 0\}$
- ☐ (c) $\{\mathbf{x} \in \mathbb{R}^3 \mid \mathbf{x} \cdot \mathbf{x} = 1\}$
- ☐ (d) $\{\mathbf{x} \in \mathbb{R}^3 \mid \mathbf{x} \cdot \mathbf{u} = 1\}$
- ☐ (e) None of the above.

5p

7g Recall that \mathbb{P}_3 denotes the set of polynomials of degree at most 3. In other words, \mathbb{P}_3 consists of all polynomials of the form

$$\mathbf{p}(t) = a_0 + a_1t + a_2t^2 + \dots + a_nt^n,$$

where the coefficients a_0, \dots, a_n and the variable t are real numbers.

Let H be the subspace of \mathbb{P}_3 made up of only those polynomials $\mathbf{p}(t)$ such that $\mathbf{p}(0) = 0$, i.e.

$$H = \{\mathbf{p} \in \mathbb{P}_3 \mid \mathbf{p}(0) = 0\}.$$

Then,

- ☐ (a) $\{t, t^2, t^3\}$ is a basis for H .
- ☐ (b) $\{1, t, t^2, t^3\}$ is a basis for H .
- ☐ (c) None of the above.

5p **7h** Consider the following matrix A :

$$A = \begin{bmatrix} 1 & 1 & -2 & -1 \\ -1 & 4 & -3 & 1 \\ 0 & 7 & 1 & -8 \end{bmatrix}.$$

A vector in $\text{Nul}(A)$ is

- (a) $\begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}$
- (b) $\begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}$
- (c) $\begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}$
- (d) $\begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}$
- (e) None of the above.

5p 7i Consider the following matrix A :

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix}.$$

A vector in $\text{Col}(A)$ is

- a $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
- b $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$
- c $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$
- d $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$
- e None of the above.

Extra space

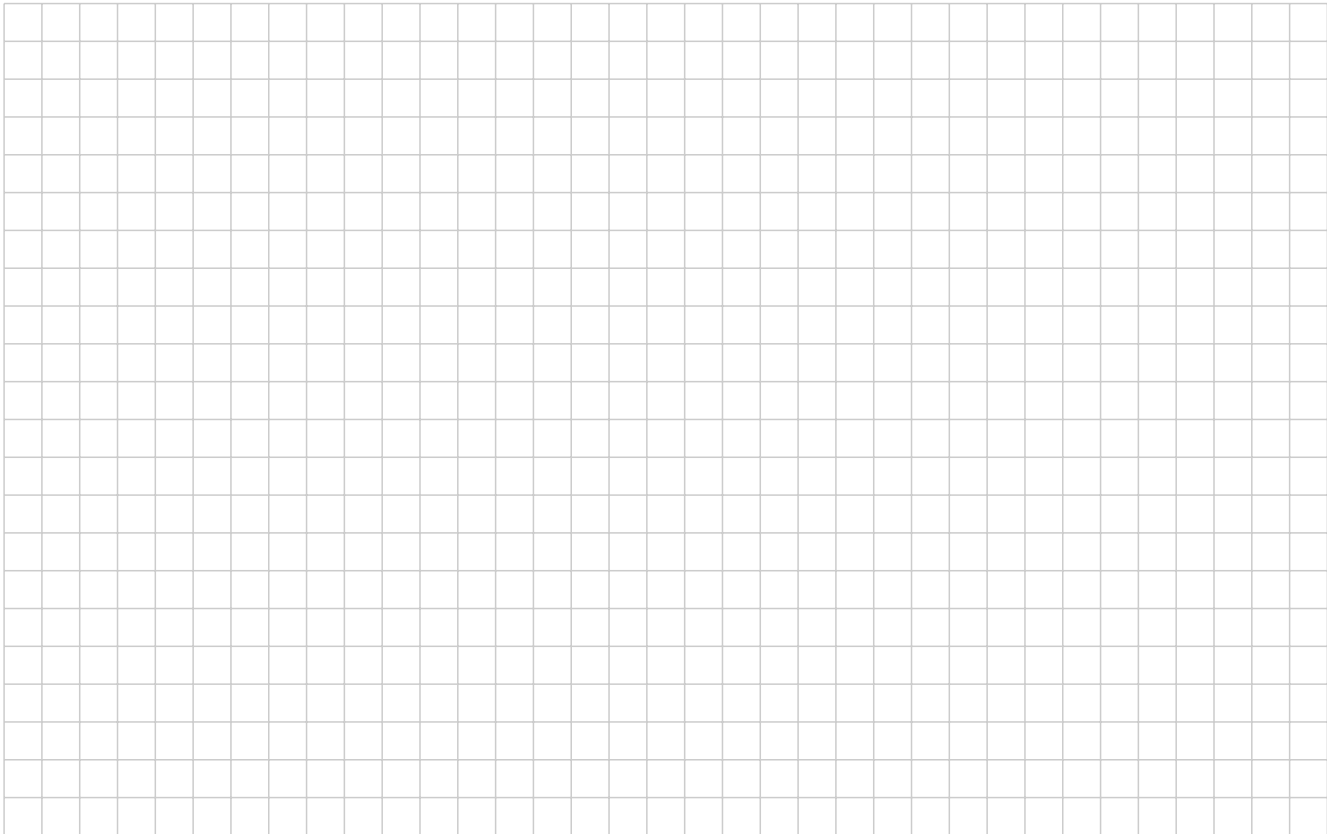
If you use these extra answer boxes, **please mention clearly in your main answer that part of your answer can be found here!**

8a

8b



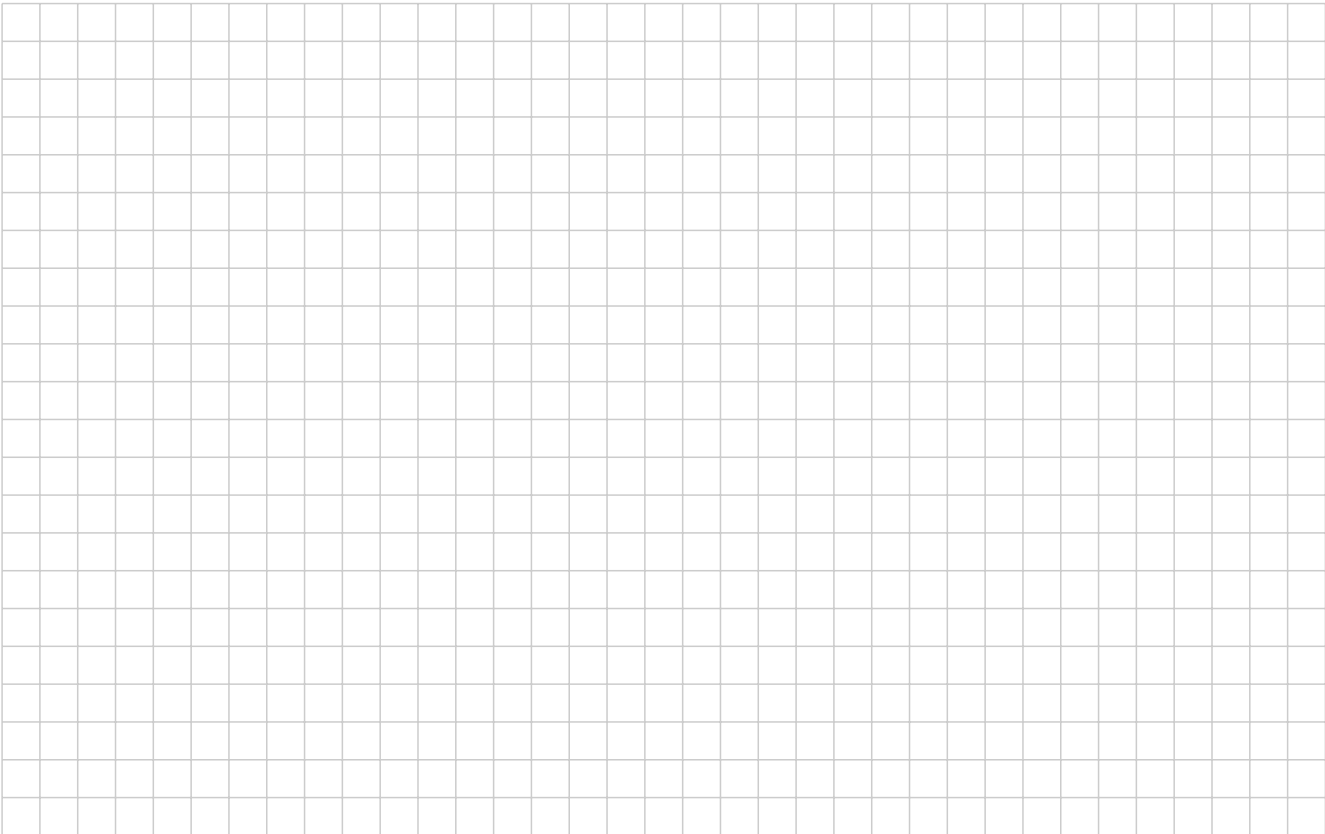
8c



8d



8e



8f

