

Practice Exam Questions Week 2, **Linear Algebra**, UCM

1. Consider the following matrix  $A$  and vector  $\mathbf{b}$ :

$$A = \begin{bmatrix} -1 & 2 & 3 & -4 & 8 \\ 3 & -6 & -4 & 7 & -9 \\ 2 & -4 & 1 & 1 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 7 \\ -6 \\ 7 \end{bmatrix}.$$

- (a) Are the columns of  $A$  linearly independent?
  - (b) Compute the solution set of the associated linear system of equations  $A\mathbf{x} = \mathbf{b}$  and express it in parametric vector form.
2. True or false? If the given statement is true, briefly explain why. If it is false, give a counterexample.
- (a) If the columns of an augmented matrix are linearly independent, then the associated linear system of equations is inconsistent.
  - (b) Four different vectors in  $\mathbb{R}^3$  always span  $\mathbb{R}^3$ .
  - (c) The effect of adding a vector  $\mathbf{p}$  to a vector  $\mathbf{v}$  is to move the vector  $\mathbf{v}$  in a direction parallel to  $\mathbf{p}$ .
  - (d) If the augmented matrix of a linear system of equations has more rows than columns, then it cannot have infinitely many solutions.
  - (e) If  $A$  and  $B$  are matrices for which the product  $AB$  and the sum  $A + B$  are both well defined, then the product  $BA$  is also well defined.
  - (f) If all the rows of an augmented matrix have a pivot, then the associated linear system of equations is inconsistent.
  - (g) If  $S$  and  $T$  are  $2 \times 2$  matrices such that  $ST = 0$ , then also  $TS = 0$ .
  - (h) If the vectors  $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^3$  are such that  $\{\mathbf{x}, \mathbf{y}\}$  is linearly independent,  $\{\mathbf{x}, \mathbf{z}\}$  is linearly independent, and  $\{\mathbf{y}, \mathbf{z}\}$  is linearly independent, then also  $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$  is linearly independent.