

Logic 2020/2021
Resit Exam Questions

— Do not turn this page before the official start of the exam! —

First Name, Surname: _____

Student ID: _____

Program: Bachelor Data Science and Knowledge Engineering

Course code: KEN1530

Examiners: dr. Otti D’Huys, dr. ir. ing. Nico Roos

Date/time: July 1st, 2021, 9:30-11:30h

Format: Closed book exam.

Allowed aides: Pens.

Instructions to students:

General instructions:

- The exam consists of 10 questions on 12 pages (excluding the 1 cover page(s)).
- Answer every question on a separate piece of paper. Do not mix the answers on different exam questions.
- **Number each page of answers you submit in the top left corner.**
- Ensure that you properly motivate your answers.
- Do not use red pens, and write in a readable way. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- If you think a question is ambiguous, or even erroneous, explain this in detail in your answer.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.

For on-site exams:

- Fill in your name and student ID number on every page of answers you submit.
- Please make sure that communication devices and watches are not within reach.
- The use of pencils is not allowed.

For online proctored exams:

- The scan / photographs of your submitted answers must be readable.
- **Reserve space for your ID in the top right corner of every page of answers you submit.**

Success!

The following table will be filled by the examiner:

[illegible]

Question 1 (10 points)

- Alice likes running or Bob likes cycling.

- If Chris likes dancing, then Bob does not like cycling.

- It is not the case that (Alice likes running and Chris likes dancing).

- If Bob likes cycling and Alice does not like running, then Chris likes dancing.

$$a \vee b$$

$$c \rightarrow \neg b$$

$$\neg(a \wedge c)$$

$$(b \wedge \neg a) \rightarrow c$$

Determine the answers of the following questions using a truth table. (Draw the full table!)

Answer with: *Yes*, *No* or *Unknown*!

- Does Alice like running?
- Does Bob like cycling?
- Does Chris like dancing?

Use the atomic propositions a , b and c , use the truth-value 0 representing false and the truth-value 1 representing true, and enumerate the valuations of a , b and c as binary numbers from 0 to 7.

Solution:

a	b	c	$a \vee b$	$c \rightarrow \neg b$	$\neg(a \wedge c)$	$(b \wedge \neg a) \rightarrow c$
0	0	0	0	1	1	1
0	0	1	0	1	1	1
0	1	0	1	1	0	0
0	1	1	1	0	0	1
1	0	0	1	1	1	0
1	0	1	1	1	0	1
1	1	0	1	1	0	0
1	1	1	1	0	0	1

- Does Alice like running? **yes**

- Does Bob like cycling? **unknown** *b can be true or false*

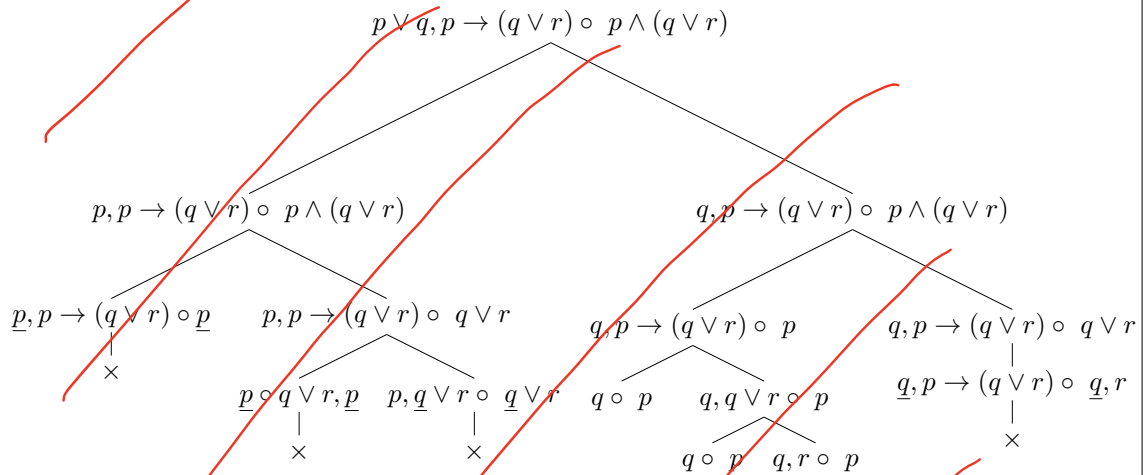
- Does Chris like dancing? **no**

↑ when are all of a , b and c true?

Question 2 (10 points)

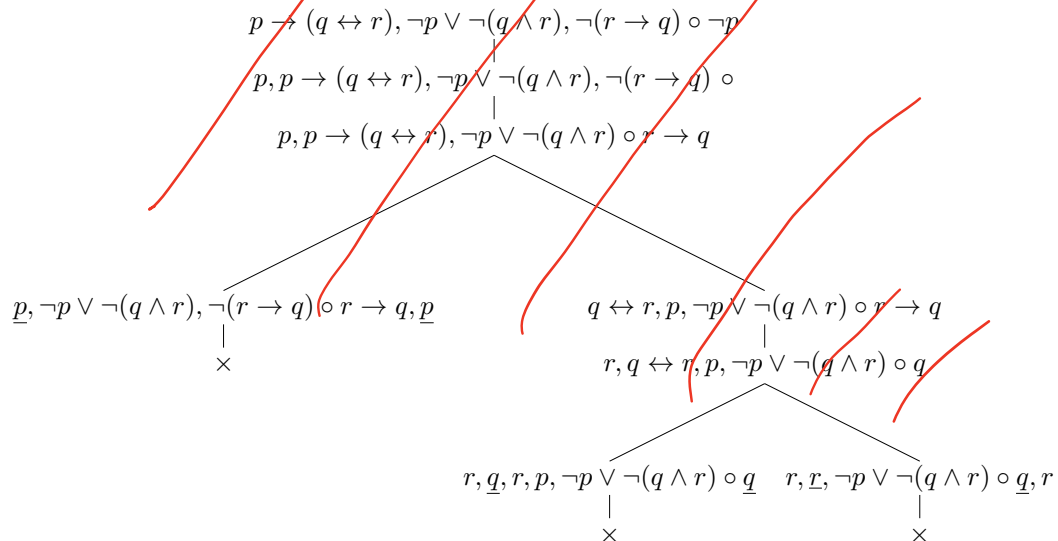
Investigate the validity of the following inferences with the aid of a semantic tableau. If an inference is invalid, give **all** the counterexamples.

- $\{p \vee q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q\} \models p \wedge (q \wedge r)$
- $\{p \rightarrow (q \leftrightarrow r), \neg p \vee \neg(q \wedge r), \neg(r \rightarrow q)\} \models \neg p$

Solution:

Counter example 1: $V(p) = 0, V(q) = 1, V(r) = 0$.

Counter example 2: $V(p) = 0, V(q) = 1, V(r) = 1$.



Investigate the validity of the following inferences with the aid of a semantic tableau. If an inference is invalid, give **all** the counterexamples.

- $\{p \vee q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q\} \models p \wedge (q \wedge r)$
- $\{p \rightarrow (q \leftrightarrow r), \neg p \vee \neg(q \wedge r), \neg(r \rightarrow q)\} \models \neg p$

(2) $\{p \vee q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q\} \models p \wedge (q \wedge r)$

$$p \vee q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q \quad \circ \quad \underline{p \wedge (q \wedge r)}$$

$$\underline{p \vee q}, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q \quad \circ \quad p \quad \boxed{// \quad \circ \quad q \wedge r}$$

$$p, \dots \quad \circ \quad p \quad \swarrow \quad q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q \quad \circ \quad p$$

X

$$q, p \rightarrow (q \vee r) \quad \circ \quad p, \neg(r \wedge p)$$

$$q, p \rightarrow (q \vee r), \neg q \quad \circ \quad p$$

$$\underline{r \wedge p}, q, p \rightarrow (q \vee r) \quad \circ \quad p$$

$$q \quad // \quad \circ \quad p, q$$

X

$$r, p, // \quad \circ \quad p$$

X

* $\rightarrow p \vee q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q \quad \circ \quad q \wedge r$

$$p \wedge q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q \quad \circ \quad q$$

$$q, \dots \quad \circ \quad q$$

X

$$p \vee q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q$$

$$\circ \quad r$$

Question 3 (15 points)

Prove by means of natural deduction:

- From the premises $p \rightarrow \neg(r \rightarrow q)$ derive $p \rightarrow (\neg q \wedge r)$
- From the premise $\neg(\neg p \vee \neg q)$ derive $p \wedge q$

Solution:

1	$p \rightarrow \neg(r \rightarrow q)$	
2	p	
3	$\neg(r \rightarrow q)$	$\rightarrow E, 1, 2$
4	$\neg(\neg q \wedge r)$	
5	r	
6	$\neg q$	
7	$\neg q \wedge r$	$\wedge I, 5, 6$
8	\perp	$\perp I, 4, 7$
9	q	$\neg E, 6-8$
10	$r \rightarrow q$	$\rightarrow I, 5, 9$
11	\perp	$\perp I, 3, 10$
12	$\neg q \wedge r$	$\neg E, 4-11$
13	$p \rightarrow (\neg q \wedge r)$	$\rightarrow I, 2, 12$

1	$\neg(\neg p \vee \neg q)$	
2	$\neg p$	
3	$\neg p \vee \neg q$	$\vee I, 2$
4	\perp	$\perp I, 1, 3$
5	p	$\neg E, 2-4$
6	$\neg q$	
7	$\neg p \vee \neg q$	$\vee I, 6$
8	\perp	$\perp I, 1, 7$
9	q	$\neg E, 6-8$
10	$p \wedge q$	$\wedge I, 5, 9$

Question 4 (10 points)

Consider the following two premises of a syllogism:

No cow is pink.

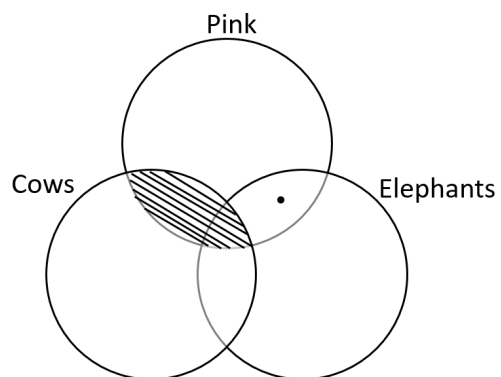
Some elephants are pink.

??

Complete the syllogism with a conclusion that makes it valid.

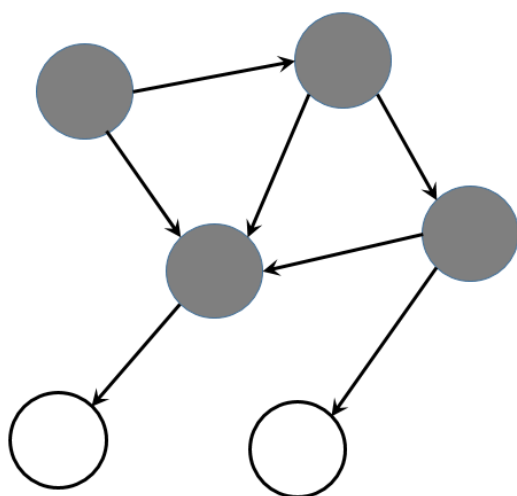
Demonstrate the validity of your syllogism using the method with Venn diagrams.

Solution: Some elephants are not cows.



Question 5 (10 points)

Consider the predicate logic model shown below. The model has a unary predicate P and a binary predicate S . Shaded objects have property P , a \rightarrow from a to b meant that Sab is true in this model.

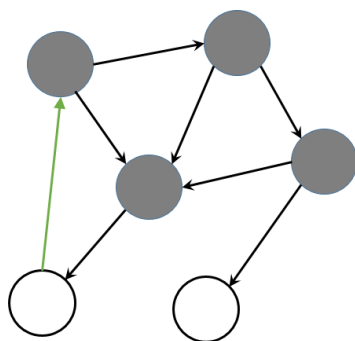


- State two closed formulas that are true in this model. Your formulas should include both predicates P and S .
- Add one pair to $I(S)$ (one arrow to the model representation), such that the statement $\forall x(Px \rightarrow \exists ySxy)$ becomes true.

Solution:

- Examples include $\forall x(\exists ySxy \rightarrow Px)$, $\neg\exists x(Px \wedge Rx)$, $\forall x\forall y((Sxy \wedge \neg Px) \rightarrow Py)$, $\exists y\exists x(Sxy \wedge Py \wedge Px)$

- Any arrow arriving at the upper left object makes the formula true. An example is below



Question 6 (10 points)

Are the following formulas a tautology? Show by means of a semantic tableau.

- $\forall x \exists y (Ryx \rightarrow Rxx)$
- $\forall x ((\exists y Ryx) \rightarrow Rxx)$

Solution:

$$\begin{array}{c}
 \circ \forall x \exists y (Ryx \rightarrow Rxx) \\
 | \\
 \circ^+ \exists y (Rya \rightarrow Raa) \\
 | \\
 \circ \underline{Raa \rightarrow Raa} \\
 | \\
 Raa \circ Raa \\
 | \\
 \times
 \end{array}$$

This statement is a tautology

$$\begin{array}{c}
 \circ \forall x ((\exists y Ryx) \rightarrow Rxx) \\
 | \\
 \circ^+ (\exists y Ryx) \rightarrow Raa \\
 | \\
 \exists y Ryx \circ Raa \\
 | \\
 Rba \odot^+ Raa
 \end{array}$$

This tableau has an open branch: we find a countermodel with two objects a and b , with b related to a , but a not related to itself.

Question 7 (10 points)

Prove by means of natural deduction:

- From the premises $\{\forall x(Px \rightarrow \exists yRxy), \forall x(\exists yRyx \rightarrow \neg Px), \exists xPx\}$ derive $\exists x\neg Px$
- From the premises $\{\forall x(Ax \rightarrow \neg Bx), \exists x(Bx \wedge Cx)\}$ derive $\exists x(Cx \wedge \neg Ax)$

Solution:

1		$\forall x(Px \rightarrow \exists yRxy)$	
2		$\forall x(\exists yRyx \rightarrow \neg Px)$	
3		$\exists xPx$	
4		a Pa	
5		$Pa \rightarrow \exists yRay$	$\forall E, 1$
6		$\exists yRay$	$\rightarrow E, 5, 4$
7		b Rab	
8		$\exists yRyb$	$\exists I, 7$
9		$\exists yRyb \rightarrow \neg Pb$	$\forall E, 2$
10		$\neg Pb$	$\rightarrow E, 9, 8$
11		$\exists x\neg Px$	$\exists I, 10$
12		$\exists x\neg Px$	$\exists E, 6, 7-11$
13		$\exists x\neg Px$	$\exists E, 3, 4-12$

Solution:

1	$\forall x(Ax \rightarrow \neg Bx)$	
2	$\exists x(Bx \wedge Cx)$	
3	a	$Ba \wedge Ca$
4	Ca	$\wedge E, 3$
5	Ba	$\wedge E, 3$
6	Aa	
7	$Aa \rightarrow \neg Ba$	$\forall E, 1$
8	$\neg Ba$	$\rightarrow E, 6, 7$
9	\perp	$\perp I, 5, 8$
10	$\neg Aa$	$\neg I, 6-9$
11	$\neg Aa \wedge Ca$	$\wedge I, 4, 10$
12	$\exists x(\neg Ax \wedge Cx)$	$\exists I, 11$
13	$\exists x(\neg Ax \wedge Cx)$	$\exists E, 2, 3-12$

Note: the topic of this question (epistemic logic) has been replaced in 2021/2022 by Hoare logic and tableau proofs for PDL. Please ignore this question.

Question 8 (5 points)

Translate the following sentence into a formula of epistemic logic, using appropriate translation keys.

- Bob does not know whether Chris knows whether Alice likes sailing.

Use the modal operators \Box_A , \Box_B and \Box_C , and the atomic proposition a .

Solution:

- $\neg(\Box_B(\Box_C a \vee \Box_C \neg a) \vee \Box_B \neg(\Box_C a \vee \Box_C \neg a))$
or
 $\neg\Box_B(\Box_C a \vee \Box_C \neg a) \wedge \neg\Box_B \neg(\Box_C a \vee \Box_C \neg a)$

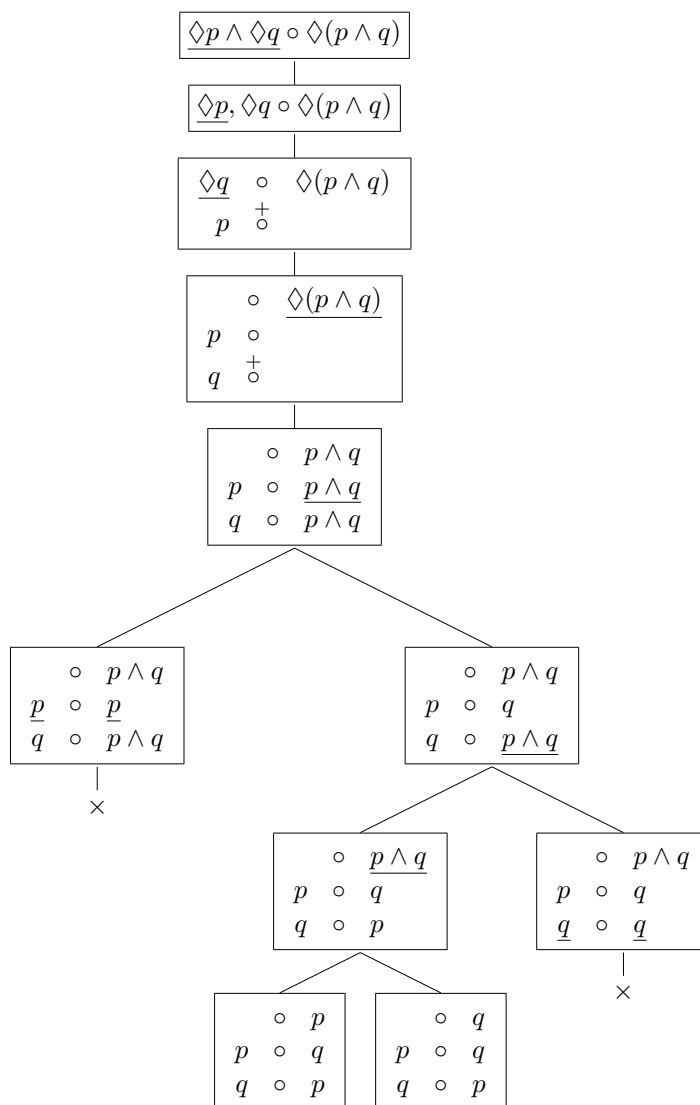
Note: the topic of this question (epistemic logic) has been replaced in 2021/2022 by Hoare logic and tableau proofs for PDL. Please ignore this question.

Question 9 (10 points)

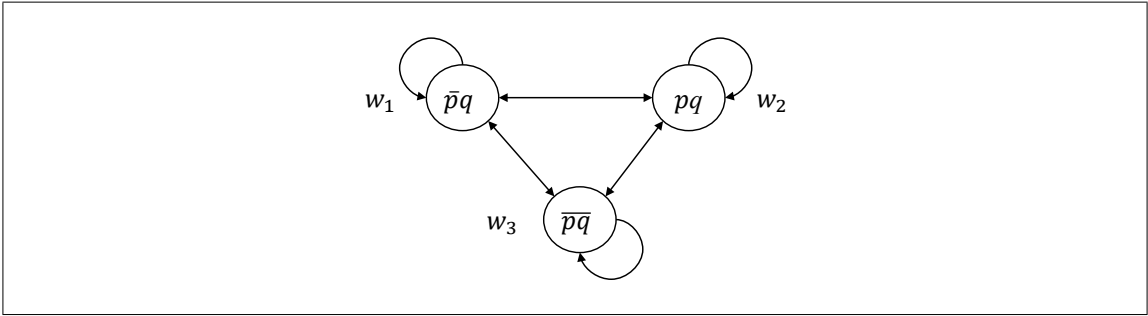
Investigate the validity of the following inferences with the aid of a semantic tableau. If an inference is invalid, give a counterexample.

- $\Diamond p \wedge \Diamond q \models \Diamond(p \wedge q)$
- $\neg \Diamond(p \vee q) \models \Box \neg p \wedge \Box \neg q$

Solution:

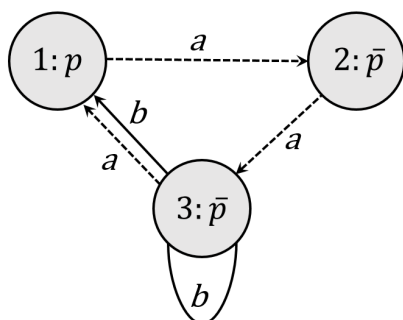


One of the three counter examples described by the open branches:



Question 10 (10 points)

Consider the labelled transition system below, with 3 states and actions a and b .



(a) In which states are the following formula true?

- $\langle a^* \rangle p$.
- $[b] \neg p$

(b) Give all the elements of the relation defined by the action $(? \neg p; a \cup b)^*$.

Solution:

(a) In which states are the following formula true?

- $\langle a^* \rangle p$: true in all states.
- $[b] \neg p$: true in states 1 and 2.

(b) The elements of the relation defined by the action $(? \neg p; a \cup b)^*$ are

$(1, 1), (2, 2), (3, 3), (2, 3), (3, 1), (2, 1)$