Integration techniques

Substitution

$$\int \frac{d}{dx}\left[f(g(x))\right] dx = \int \left[f'(g(x)) \cdot g'(x)\right] dx$$

$$f(g(x)) + C \qquad f'(u) \qquad du$$

$$\begin{cases} u = g(x) \\ du = g'(x)\,dx \end{cases}$$

Examples

- $\int \sin(3x)\,dx = \int \sin(u)\,\frac{1}{3}\,du$

  $u = 3x$ $\quad = \frac{1}{3}\int \sin u\,du$

  $du = 3\,dx$

  $= -\frac{1}{3}\cos(3x) + C$

- $\int \frac{dx}{x+1} = \int \frac{du}{u} = \ln|u| + C = \ln|x+1| + C$

  $u = x+1$

  $du = dx$

- $\int \tan(x)\,dx = \int \frac{\sin x}{\cos x}\,dx = \int \frac{-du}{u}$

  $\begin{cases} u = \cos x \\ du = -\operatorname{sen} x\,dx \end{cases}$

Integration by parts

$$\int \frac{d}{dx}(u \cdot v)\,dx = \int u' \cdot v\,dx + \int v' \cdot u\,dx$$

$$v \cdot u = \int v\,du + \int u\,dv \longrightarrow \int u \cdot v = u \cdot v - \int v\,du$$

Examples

$$\int x e^x\,dx = x e^x - \int e^x\,dx = x e^x - e^x + c$$

$$u = x \longrightarrow du = dx$$

$$dv = e^x\,dx \qquad v = e^x$$