PAUL'S ANSWERS TO Logic Bonus Assignment 1. 20/04/2023.

1 a) $\neg(\neg p \vee \neg(q \wedge p))$

p	q	$\neg$	($\neg p$	$\vee$	$\neg$	(q	$\wedge$	p))
1	1	1	0	0	0	1	1	1
1	0	0	0	1	1	0	0	1
0	1	0	1	1	1	1	0	0
0	0	0	1	1	1	0	0	0

Not a contradiction, not a tautology, but is satisfiable (when $p=1$, $q=1$)

Not a contradiction, not a tautology, but is satisfiable (when both p and q are true).

1 B) $(\neg a \vee \neg b) \leftrightarrow \neg(a \wedge b)$

a	b	($\neg a$	$\vee$	$\neg b$)	$\leftrightarrow$	$\neg$	(a	$\wedge$	b)
1	1	0	0	0	1	0	1	1	1
1	0	0	1	1	1	1	1	0	0
0	1	1	1	0	1	1	0	0	1
0	0	1	1	1	1	1	0	0	0

TAUTOLOGY, NOT A CONTRADICTION, satisfiable.

1 C) $\neg(a \rightarrow b) \rightarrow (a \rightarrow b)$

a	b	$\neg$	(a	$\rightarrow$	b)	$\rightarrow$	(a	$\rightarrow$	b)
1	1	0	1	1	1	1	1	1	1
1	0	1	1	0	0	0	1	0	0
0	1	0	0	1	1	1	0	1	1
0	0	0	0	1	0	1	0	1	0

Not a Tautology, Not a Contradiction, But is satisfiable.

1 D) $\neg b \to (a \to b)$

True when b = 1, regardless of a's value.
FALSE when b = 0, a = 1
True when b = 0, a = 0

So Not a TAUTOLOGY, not a CONTRADICTION, But Satisfiable.

2 A) $a \vDash (b \wedge c) \to (a \to b)$

a b c	a	$\vDash$	$(b \wedge c)$	$\to$	$(a \to b)$
1 1 1	1	1	1	1	1
1 1 0	1	1	0	1	1
1 0 1	1	1	0	1	0
1 0 0	1	1	0	1	0
0 1 1					
0 1 0					
0 0 1					
0 0 0					

VALID INFERENCE.

2B) $\{p \vee q, q \vee r\} \vDash (p \wedge r) \to \neg q$

p q r	$(p \vee q)$	$(q \vee r)$	$\vDash$	$(p \wedge r)$	$\to$	$\neg q$
1 1 1	1	1	0	1	0	0
1 1 0	1	1	1	0	1	0
1 0 1	1	1	1	1	1	1
1 0 0	1	0		0	1	1
0 1 1	1	1	1	0	1	0
0 1 0	1	1	1	0	1	0
0 0 1	0	1		0	1	1
0 0 0	0	0		0	1	1

NOT VALID (p = 1, q = 1, r = 1)

2c $\{p \to q, q \to r\} \models p \to r$

p	q	r	$p \to q$	$q \to r$	$\models$	$p \to r$
1	1	1	1	1	1	1
1	1	0	1	0		0
1	0	1	0	1		1
1	0	0	0	1		0
0	1	1	1	1	1	1
0	1	0	1	0		1
0	0	1	1	1	1	1
0	0	0	1	1	1	1

VALID INFERENCE :)

3.

Propositional logic:

$a \leftrightarrow b$

$b \leftrightarrow \neg c$

$* = c \leftrightarrow (a \wedge b \wedge \neg c) \vee (a \wedge \neg b \wedge c) \vee (\neg a \wedge b \wedge c) = *$

No configurations possible:

if a is true, b must be true, so c must not be true, but it is.

a is false, b must be false, so c must be true, but it is not

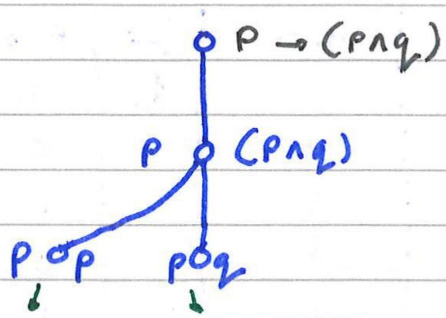
This problem is not satisfiable, it is a paradox.

a	b	c	$a \leftrightarrow b$	$b \leftrightarrow \neg c$	*
1	1	1	1	0	0
1	1	0	1	1	0
1	0	1	0	1	1
1	0	0	0	0	1

a	b	c	$a \leftrightarrow b$	$b \leftrightarrow \neg c$	*
0	1	1	0	0	1
0	1	0	0	1	1
0	0	1	1	1	0
0	0	0	1	0	1

4. Tableau method → TAUTOLOGIES → Logic in Action chp 8 slides
↳ slide 60 + 71

a)



closed

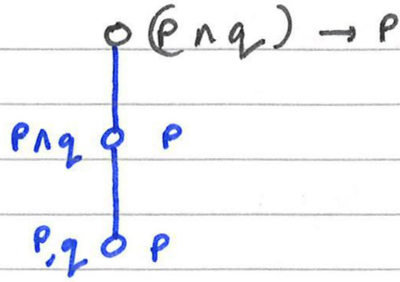
NOT CLOSED

↳ NOT A TAUTOLOGY

Counterexamples:

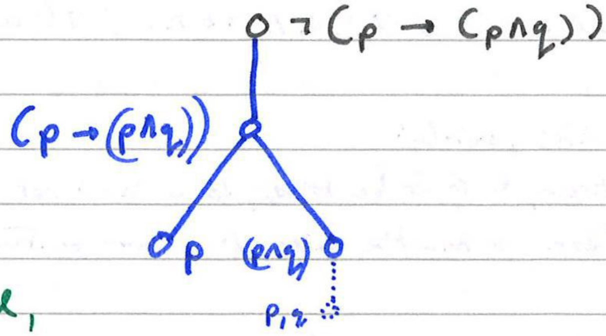
$p = 1$, $q = 0$

4b)



Closed, TAUTOLOGY

4c)



NOT closed,

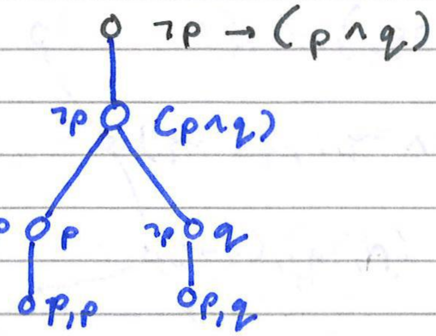
NOT A TAUTOLOGY

Counterexamples:

$p = 0$, $q = 0$ or 1.

$p = 1$, $q = 1$.

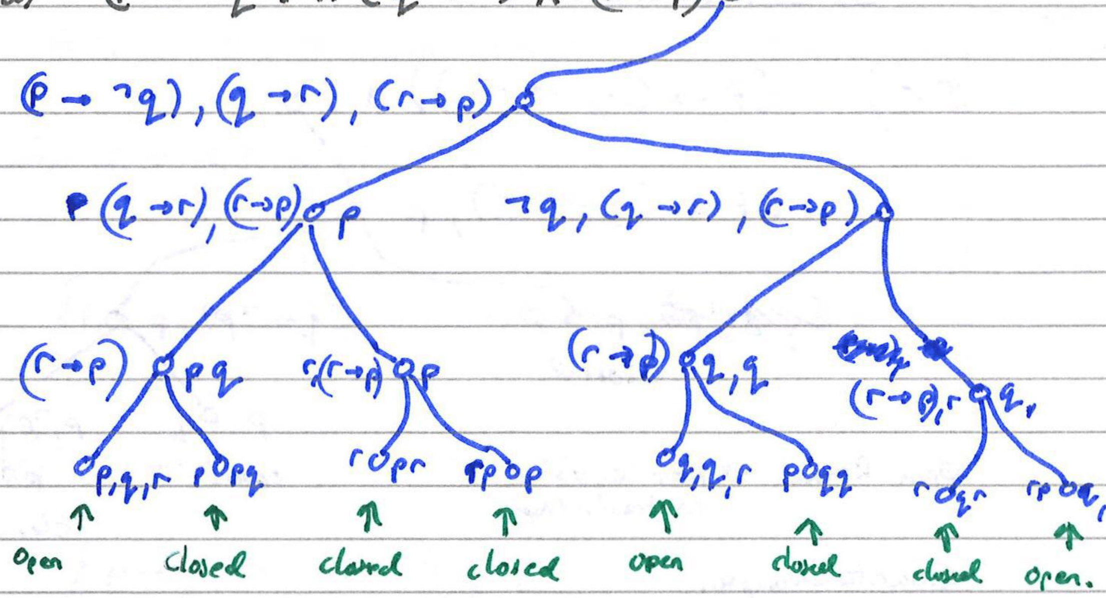
4 D



NOT CLOSED, NOT A TAUTOLOGY; Counterexample: $p=0$, $q=0$ or 1.

5 → Using the tableau method to ~~find~~ check for contradictions.

5a) $(p \to \neg q) \land (q \to r) \land (r \to p)$



3 open branches → Not a contradiction!

Counterexamples: $p=0$, $q=0$, $r=0$
$p=1$, $q=0$, $r=0$.
$p=1$, $q=0$, $r=1$.

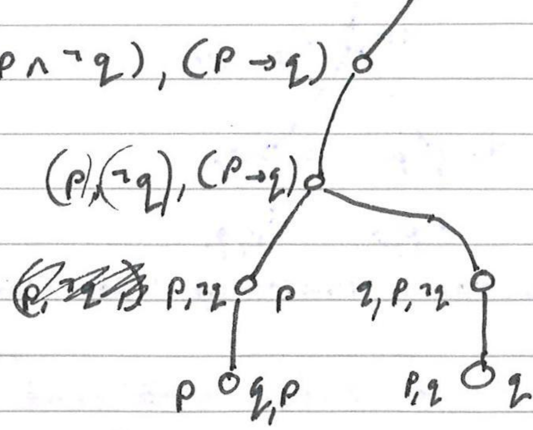
How did I find these? Look at the open branches!

$\neg p, \neg q, \neg r \Rightarrow p=0, q=0, r=0.$

$\neg q, \neg r \Rightarrow q=0, r=0, p=0$ or 1

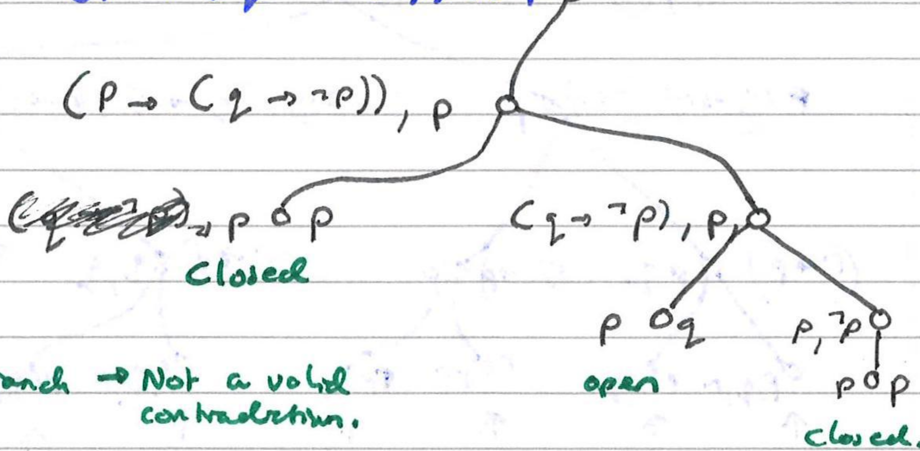
$r, p, \neg q \Rightarrow$

5B) $(p \land \neg q) \land (p \to q)$



All are closed
⇒ Contradiction.

5C) $(p \to (q \to \neg p)) \land p$



Open Branch → Not a valid contradiction.

Counterexample:

$p = 1, q = 0$

Remember, left of circle = positive / 1.
right of circle = negative / neutral / 0.

So $p \circ q \to p = 1, q = 0.$

↑
Counterexample.

5 What is wrong with the following formula?

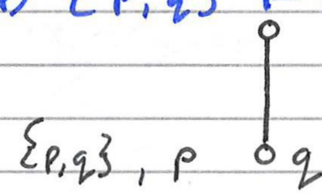
$(p \to q) \wedge (q \to p) \to (p \vee \neg q)$

→ There is nothing wrong with the formula.
When $p = 1$ and $q = 1$ → True.
$p = 1$ and $q = 0$ → True.
$p = 0$ and $q = 1$ → True
$p = 0$ and $q = 0$ → True.
IT IS A TAUTOLOGY.

Alternative answer:
The order in the formula is unclear. Is it
$((p \to q) \wedge (q \to p)) \to (p \vee \neg q)$
-- OR --
$(p \to q) \wedge ((q \to p) \to (p \vee \neg q))$
?

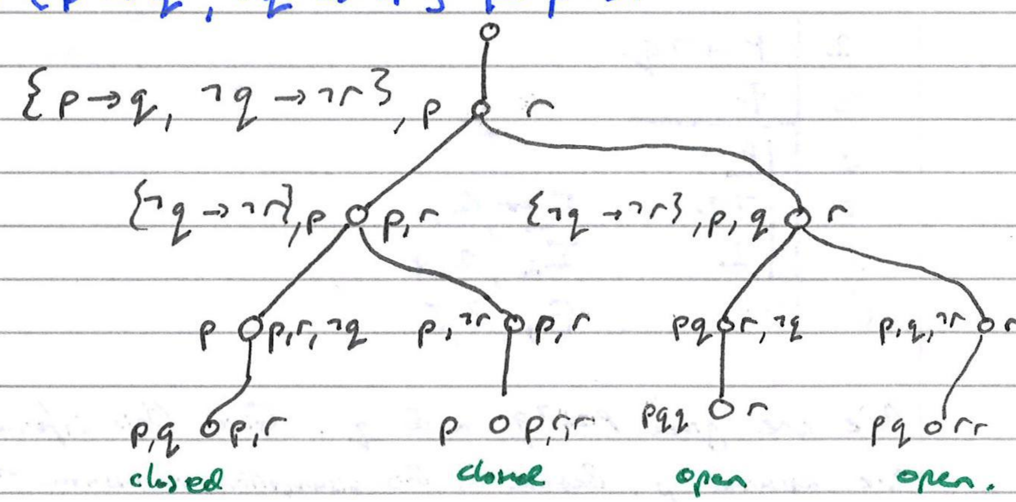
7. ~~A)~~ USE THE TABLEAU method to check if these inferences are Valid.

A) $\{p, q\} \models p \to q$



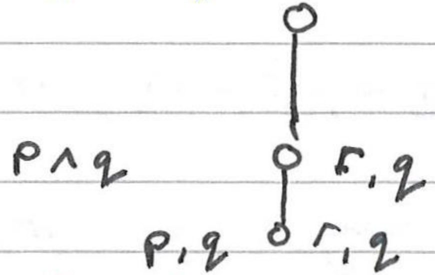
Closed, so valid inference.

B) $\{p \to q, \neg q \to \neg r\} \models p \to r$



Not a valid inference.

7. c) $\{p \wedge q, \neg q\} \models r$



Closed, so valid inference.

PAUL'S SOLUTIONS TO LOGIC BONUS 1 2023.

8 A) from $\neg\neg p$, derive p $\qquad \neg\neg p \models p$

1	$\neg\neg p$	given
2	$\quad \neg p$	assumption
3	$\quad \bot$	$E_\neg$ (1, 2)
4	p	$I_\neg$ (2,3)

8 B) from $p \to \neg q$ and q, derive $\neg p$

1	$p \to \neg q$	given
2	q	given
3	$\quad p$	Assumption
4	$\quad \neg q$	$E_\to$ (1, 3)
5	$\quad \bot$	$I_\bot$ (2, 4)
6	$\neg p$	$I_\neg$ (3,5)

Explanation: We are given $p \to \neg q$ and q. Given this information, We assume p. Based on this assumption, we assume $\neg q$, Since $p \to \neg q$ must hold. However, this contradicts line 2, where we assumed to be true. Thus, we derive $\neg p$ to be true.

8 C) Derive $p \vee \neg p$ $\qquad$ (so $\models p \vee \neg p$)

1	$\quad \neg p \vee p$	Assume Contrary.
2	$\quad\quad p$	Assume
3	$\quad\quad p \vee \neg p$	$E_\vee$ (2)
4	$\quad\quad \bot$	$I_\bot$ (1, 4)
5	$\quad \neg p$	~~Assume~~, $I_\neg$ (2,4)
6	$\quad \neg p \vee \neg\neg p$	$E_\vee$ (5)
7	$\quad \bot$	$I_\bot$ (1, 6)
8	$p \vee \neg p$	$I_\neg$ (1, 7)

from $p \vee \neg q$ derive $q \to p$

8 D)

1	$p \vee \neg q$	assume
2	p	assume (1)
3	$q \to p$	$I_\to$ (2)
4	$\neg q$	assume ($\vee$, 1)
5	q	assume
6	$\bot$	$\exists_\neg$ (4,5)
7	p	$I_\to$ (3,6)
8	$q \to p$	$I_\to$ (5,7)
9	$q \to p$	$E_\vee$ (1,2,3,4,8)

8 E) from $p \to q$ derive $\neg q \to \neg p$.

1.	$p \to q$	Given
2	$\neg q$	Assume
3	p	Assume
4	q	$E_\to$ (1,3)
5	$\bot$	$I_\neg$ (2,4)
6	$\neg p$	$I_\neg$ (3,5)
7	$\neg q \to \neg p$	$I_\to$ (2,6)

8 f)

1	$\neg p \to \neg q$	
2	q	
3	$\neg p$	
4	$\neg q$	mp (1,3)
5	$\bot$	$I_\bot$ (2,4)
6	p	$\exists_\neg$ (3-5.)
7	$q \to p$	$I_\to$ (2,-6)
8	$(\neg p \to \neg q) \to (q \to p)$	$I_\to$ (1-7)