



$$p \vee q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q \circ \neg r$$

$$p \vee q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q \circ q$$

$$p \vee q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q \circ r$$

$$\underline{q, \dots \circ q}$$

X

$$p, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q \circ q$$

$$p, \dots \circ q, p$$

X

$$p, q \vee r, \neg(r \wedge p) \rightarrow \neg q \circ q$$

$$\dots, q, \dots \circ q$$

X

$$p, r, \neg(r \wedge p) \rightarrow \neg q \circ q$$

$$p, r \circ q, \neg(r \wedge p)$$

$$p, r, r \wedge p \circ q$$

$$p, r \circ q$$

$$p, r, \neg q \circ q$$

$$p, r \circ q$$

only open branch: counterex.  
is  $v(p)=v(r)=1, v(q)=0$

$$p \vee q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q \vee r$$

Thursday, December 7, 2023 4:23 PM

$$\underline{p}, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q \vee r \quad q, p \rightarrow (q \vee r), \neg(r \wedge p) \rightarrow \neg q \vee r$$

$$\begin{array}{cc} p, \dots & \text{or}, p \\ \hline & \times \end{array} \quad \begin{array}{cc} p, q \vee r, & \neg(r \wedge p) \rightarrow \neg q \vee r \\ & \swarrow \quad \searrow \\ p, r, \dots & \text{or} \quad p, q, \neg(r \wedge p) \rightarrow \neg q \vee r \\ \hline & \times \end{array}$$

$$p, q, \neg q \vee r \quad \hline p, q \vee r, \neg(r \wedge p)$$

$$\begin{array}{c} p, q \vee r, q \\ \hline \times \end{array}$$

$$\begin{array}{c} p, q, r \wedge p \vee r \\ \hline p, q, r, p \vee r \quad \times \end{array}$$

1.  $P \rightarrow \neg(r \rightarrow q)$  given

2.  $\neg P$

3.  $\neg(r \rightarrow q)$   $E \rightarrow (1, 2)$

4.  $q$  ass

5.  $r$  ass

6.  $q$  (repeat 4)

7.  $r \rightarrow q$   $I \rightarrow (5, 6)$

8.  $\bot$   $E \neg (3, 7)$

9.  $\neg q$   $I \neg (4, 8)$

10.  $\neg r, \neg q, \neg(r \rightarrow q)$

11.  $\neg r$  (ass)

$\neg q \wedge r$

$P \rightarrow (\neg q \wedge r)$

1	$\neg(\neg p \vee \neg q)$	(given)
2	$\neg p$	(ass.)
3.	$\neg p \vee \neg q$	$I_{\vee}(2)$
4.	$\perp$	$E_{\neg}(1,3)$
5	$p$	$I_{\neg}(2,4)$
6	$\neg q$	(ass)
7	$\neg p \vee \neg q$	$I_{\vee}(6)$
8	$\perp$	$E_{\neg}(1,7)$
9	$q$	$I_{\neg}(6,8)$
10	$p \wedge q$	$I_{\wedge}(5,9)$

$$\cdot \forall x (p_x \vee \neg p_x) \wedge \forall x \forall y (s_{xy} \vee \neg s_{xy})$$

$$\cdot \exists x \exists y (p_x \wedge \neg p_y \wedge s_{xy})$$



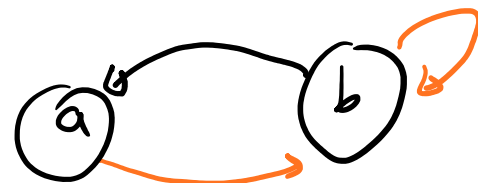
$$0 \quad \underline{\forall x ((\exists y R_{yx}) \rightarrow R_{xx})} \quad )(\text{exist})$$

$$\begin{array}{c} + \\ 0 \\ | \end{array} \quad (\exists y R_{ya}) \rightarrow \underline{R_{aa}}$$

$$\exists y R_{ya} \quad 0 \quad R_{aa}$$

$$\begin{array}{c} | \\ + \\ 0 \\ R_{ba} \quad 0 \quad R_{aa} \end{array}$$

exist.



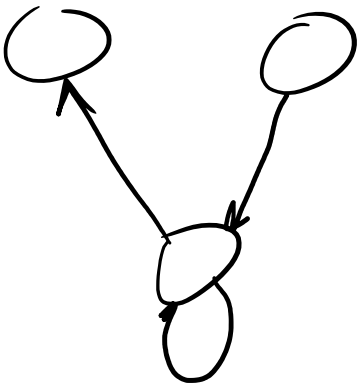
1.  $\forall x (P_x \rightarrow \exists y R_{xy})$
  2.  $\forall x (\exists y R_{yx} \rightarrow \neg P_x)$
  3.  $\exists x P_x$
- } given
4.  $P_c$  c exist (3)
  5.  $P_c \rightarrow \exists y R_{cy}$   $E\forall(1)$
  6.  $\exists y R_{cy}$   $E\rightarrow(4,5)$
  7.  $R_{cd}$  d exist const (6)
  8.  $\exists y R_{yd} \rightarrow \neg P_d$   $E\forall(2)$
  9.  $\exists y R_{yd}$   $I\exists(7) (c \rightarrow y)$
  10.  $\neg P_d$   $E\rightarrow(8,9)$
  11.  $\exists x (\neg P_x)$   $I\exists(10)$
  12.  $\neg \exists x (\neg P_x)$   $E\exists(7,11)$
  13.  $\neg \exists x (\neg P_x)$   $E\exists(4,12)$

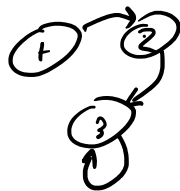


1.	$\forall x (A_x \rightarrow \neg B_x)$	} given
2	$\exists x (B_x \wedge C_x)$	
3	$B_c \wedge C_c$	c exist. const (2)
4	$A_c \rightarrow \neg B_c$	$E\forall (1)$
5	$C_c$	$E\wedge (3)$
6	$A_c$	assume.
7	$\neg B_c$	$E\rightarrow (4,6)$
8	$B_c$	$E\wedge (3)$
9	$\perp$	$E\neg (7,8)$
10	$\neg A_c$	$I\neg (6,9)$
11	$C_c \wedge \neg A_c$	$I\wedge (5,10)$
12	$\exists x (C_x \wedge \neg A_x)$	$I\exists (11)$
13	$\exists x (C_x \wedge \neg A_x)$	$E\exists (3,12)$

$X$	
$\neg X$	$I\neg$
$X$	
$\neg X$	
$\perp$	$E\neg (\cdot, \cdot)$

$$\left\{ \begin{array}{l} \underline{(1,1), (2,2), (3,3)} \\ \underline{(2,3), (3,1), (2,1)} \end{array} \right\}$$
$$(- \quad -)^*$$





$S$

$$R = \{ (1,2), (2,3), (3,3) \}$$

$$\subseteq S \times S$$

$$\subseteq S$$

$$S \times S \times S$$