Formula sheet Calculus

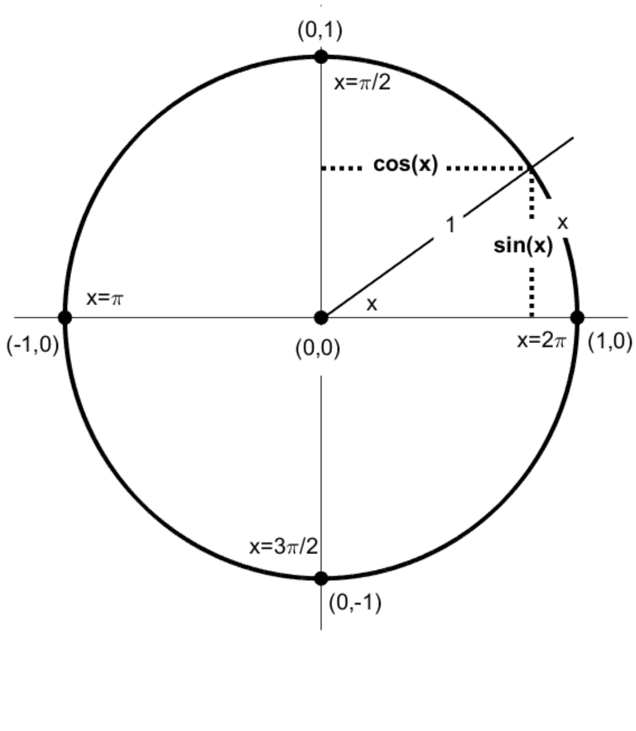
Derivatives and integrals

Table 1: Some important derivatives and indefinite integrals, $c \in \mathbb{R}$

$f(x)$	$f'(x)$	$\int f(x)dx$	remarks
x	1	$\frac{1}{2}x^2 + c$	
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\ln\lvert x\rvert + c$	
x^r	rx^{r-1}	$\frac{1}{r+1}x^{r+1} + c$	$r \neq -1$
e^x	e^x	$e^x + c$	
$\ln(x)$	$\frac{1}{x}$		$x > 0$
$\sin(x)$	$\cos(x)$	$-\cos(x) + c$	
$\cos(x)$	$-\sin(x)$	$\sin(x) + c$	
$\tan(x)$	$\frac{1}{\cos^2(x)}$		

Trigonometric identities

- $\sin^2(x) + \cos^2(x) = 1$
- $\sin(x + y) = \sin(x)\cos(y) + \sin(y)\cos(x)$
- $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
- $\sin(2x) = 2\sin(x)\cos(x)$
- $\cos(2x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$



Quadratic formula

If $Ax^2 + Bx + C = 0$, then $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Tangent line and tangent plane

- For a function $f(x)$ that is continuous and differentiable at $x = a$, the equation of the line tangent to the function at $(a, f(a)$ is given by $y = f'(a)(x - a) + f(a)$
- For a function $f(x, y)$ that is continuous at (x, y), and the partial derivatives exist, the equation of the plane tangent to the function at $(a, b, f(a, b))$ is given by $z = \frac{\partial f}{\partial a}(x-a) + \frac{\partial f}{\partial y}(y-b) + f(a, b)$

Convergence tests for series

nth term test for divergence

If a sequence $\{a_n\}$ does not converge to zero (i.e. if $\lim_{n\to\infty} a_n = L$, with $L \neq 0$, $\{a_n\}$ diverges or $\{a_n\}$ diverges to $\pm\infty$), then the series $\sum_{n=1}^{\infty} a_n$ diverges.

Convergence tests for positive series

- **Integral test**: Suppose that $a_n = f(n)$, where f is positive, continuous, and non-increasing on an interval $[N, \infty)$ for some positive integer N. Then $\int_N^\infty f(x)dx$ and $\sum_{n=1}^{\infty} a_n$ either both converge or both diverge to infinity.
- **Comparison test**: Let $\{a_n\}$ and $\{b_n\}$ be positive infinite sequences for which there exists a positive constant K such that, ultimately, $0 \leq a_n \leq Kb_n$.
  (a) If the series $\sum_{n=1}^{\infty} b_n$ converges, then so does the series $\sum_{n=1}^{\infty} a_n$
  (b) If the series $\sum_{n=1}^{\infty} a_n$ diverges to infinity, then so does the series $\sum_{n=1}^{\infty} b_n$
- **Limit comparison test**: Let $\{a_n\}$ and $\{b_n\}$ be positive infinite sequences and let $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, where L is a non-negative number or $+\infty$.
  (a) If $L < \infty$ and the series $\sum_{n=1}^{\infty} b_n$ converges, then so does the series $\sum_{n=1}^{\infty} a_n$
  (b) If $L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges to infinity, then so does the series $\sum_{n=1}^{\infty} a_n$
- **Ratio test**: Let $\{a_n\}$ be a positive infinite sequence, and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$, where ρ is a nonnegative number or $+\infty$.
  (a) If $\rho < 1$, the series $\sum_{n=1}^{\infty} a_n$ converges.
  (b) If $\rho > 1$, the sequence $\{a_n\}$ and the series $\sum_{n=1}^{\infty} a_n$ both diverge to infinity.
  (c) If $\rho = 1$, this test gives no information.

The alternating series test

Suppose $\{a_n\}$ is a sequence whose terms satisfy, for some positive integer N, the following three conditions:

(a) $a_n a_{n+1} < 0$ for $n > N$ (the sequence is alternating)

(b) $|a_{n+1}| < |a_n|$, for $n > N$ (the sequence decreases in absolute value)

(c) $a_n \to 0$ (the sequence converges to 0)

Then the series $\sum_{n=1}^{\infty} a_n$ converges.