

## Exercises

1	2	3	4	5	6	7	8
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Surname, First name

## Linear Algebra (KEN1410)

Resit

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

a	<input checked="" type="radio"/>	c	d	e	f	→ b
a	b	<input checked="" type="radio"/>	d	e	f	→ c
<input checked="" type="radio"/>	b	c	<input checked="" type="radio"/>	e	f	→ a

Fill in your answer(s) to the multiple-choice questions as shown above (circles = one correct answer).

**Program:** Data Science and Artificial Intelligence

**Course code:** KEN1410

**Examiners:** dr. Marieke Musegaas and dr. Stefan Maubach

**Date/time:** 27-06-2022 9:00 - 11:00

**Format:** Closed book exam

**Allowed aids:** Pens, simple (non-programmable) calculator from the DKE-list of allowed calculators.

**Instructions to students:**

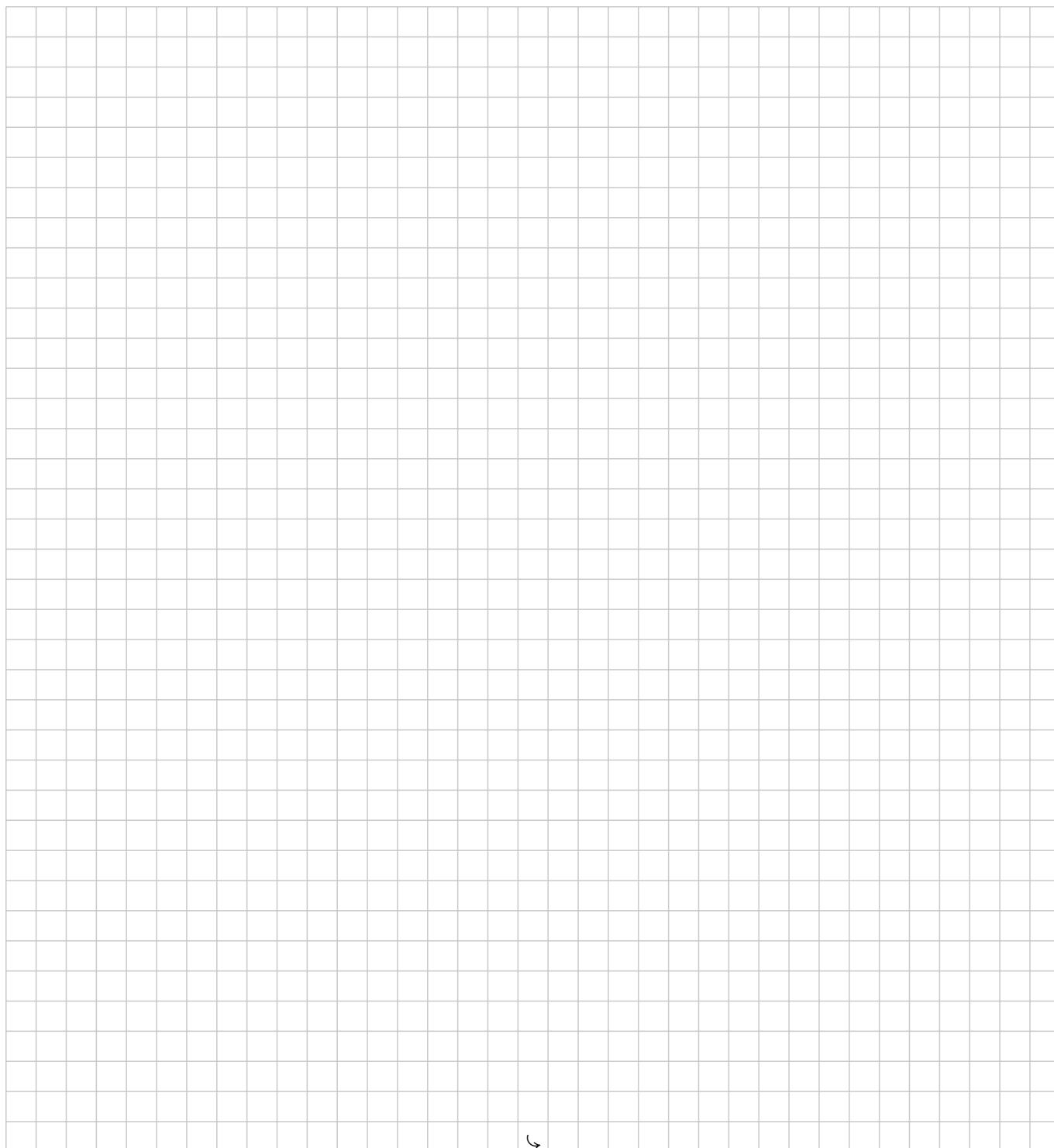
- The exam consists of 7 questions on 20 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

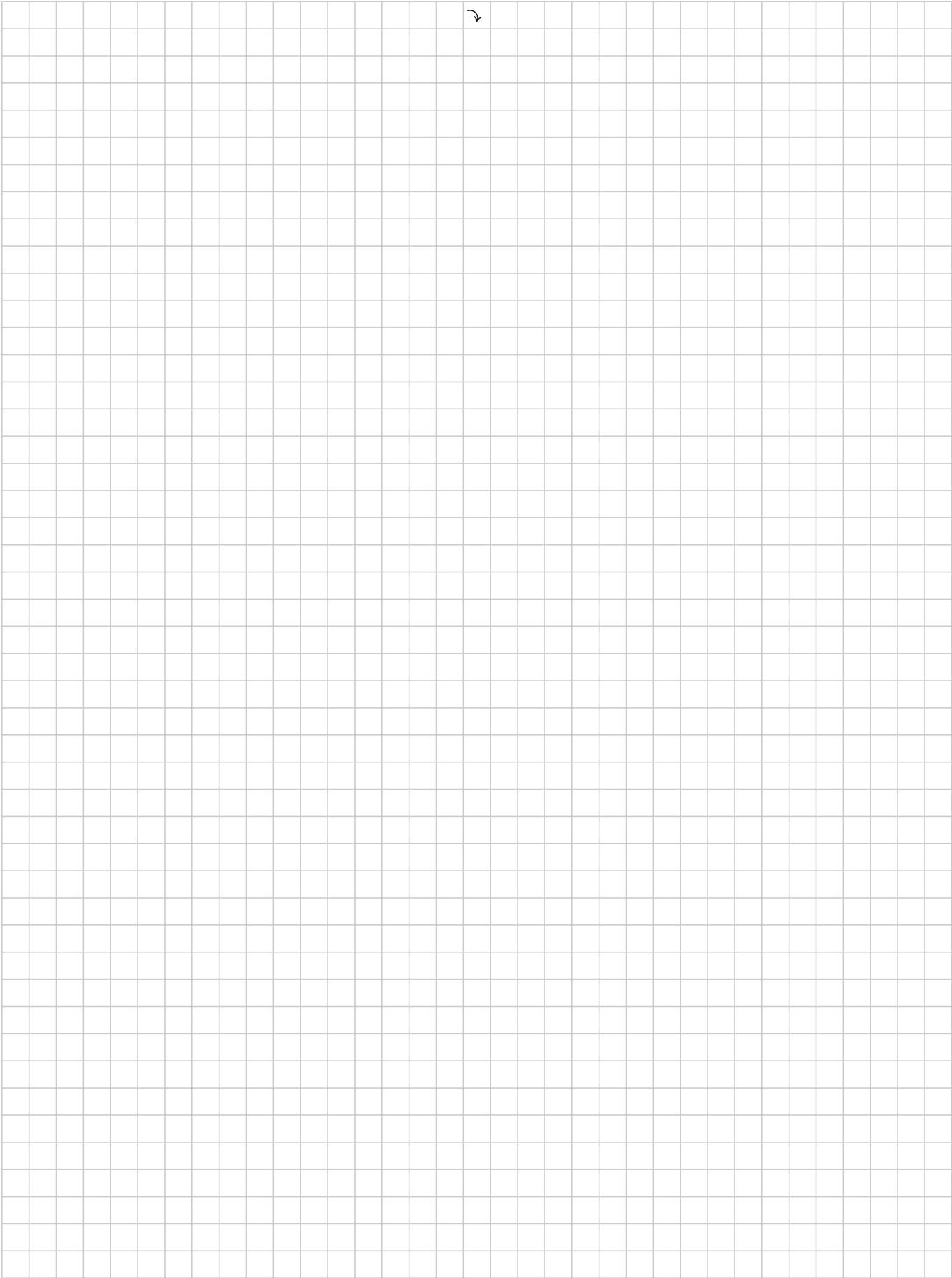
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**Exercise 1**

Let  $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$ ,  $\mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ ,  $\mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$ , and  $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ .

- 15p **1** Find two different ways to express  $\mathbf{b}$  as a linear combination of  $\mathbf{a}_1$ ,  $\mathbf{a}_2$  and  $\mathbf{a}_3$ .



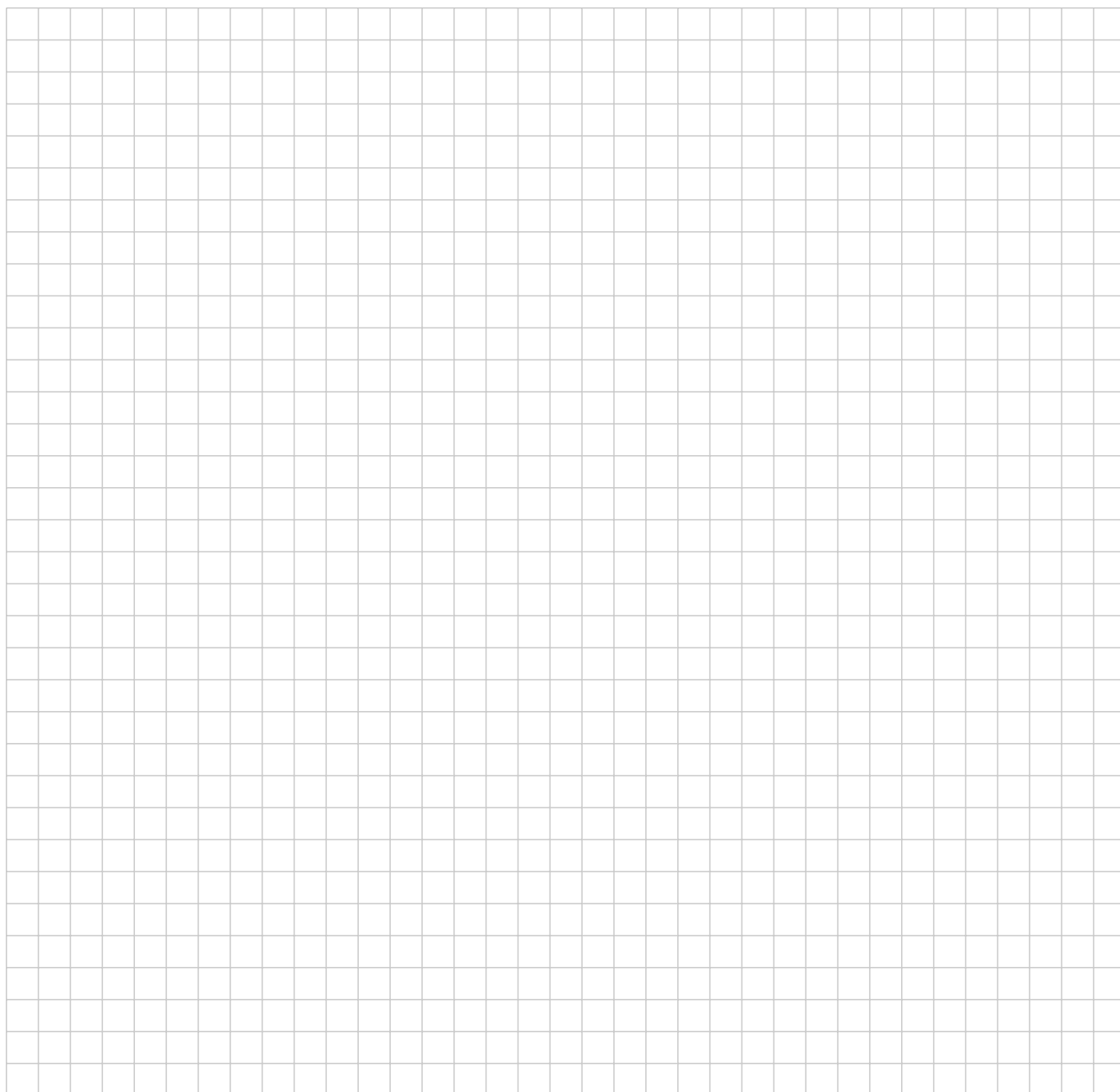


**Exercise 2**

Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^5$  be a mapping such that

$$T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -3 \\ 6 \\ 2 \\ 5 \end{bmatrix}.$$

10p **2** Can  $T$  be a linear transformation? Explain.



**Exercise 3**

Consider the following matrix  $A$  depending on a parameter  $p$ :

$$A = \begin{bmatrix} p & 0 & 0 \\ 1 & p & 2 \\ 0 & -1 & 2 \end{bmatrix}.$$

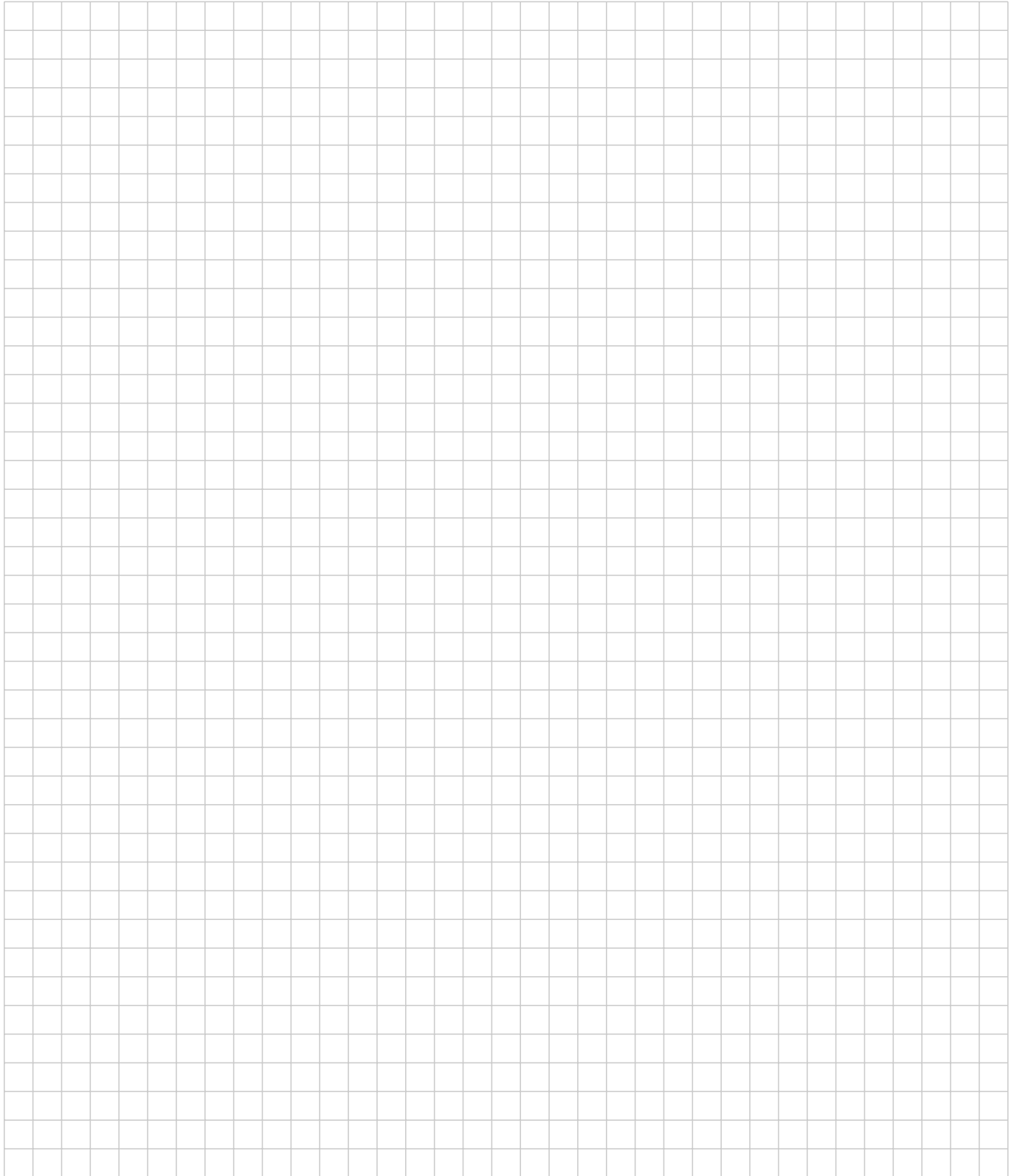
10p **3** For which value(s) of  $p$  is  $A$  not invertible?



**Exercise 4**

A  $3 \times 3$  matrix  $A$  has all its diagonal entries equal to 0. Moreover every row sum is equal to 3 and  $\det A = 6$ .

10p **4** Determine the eigenvalues of  $A$ .



**Exercise 5**

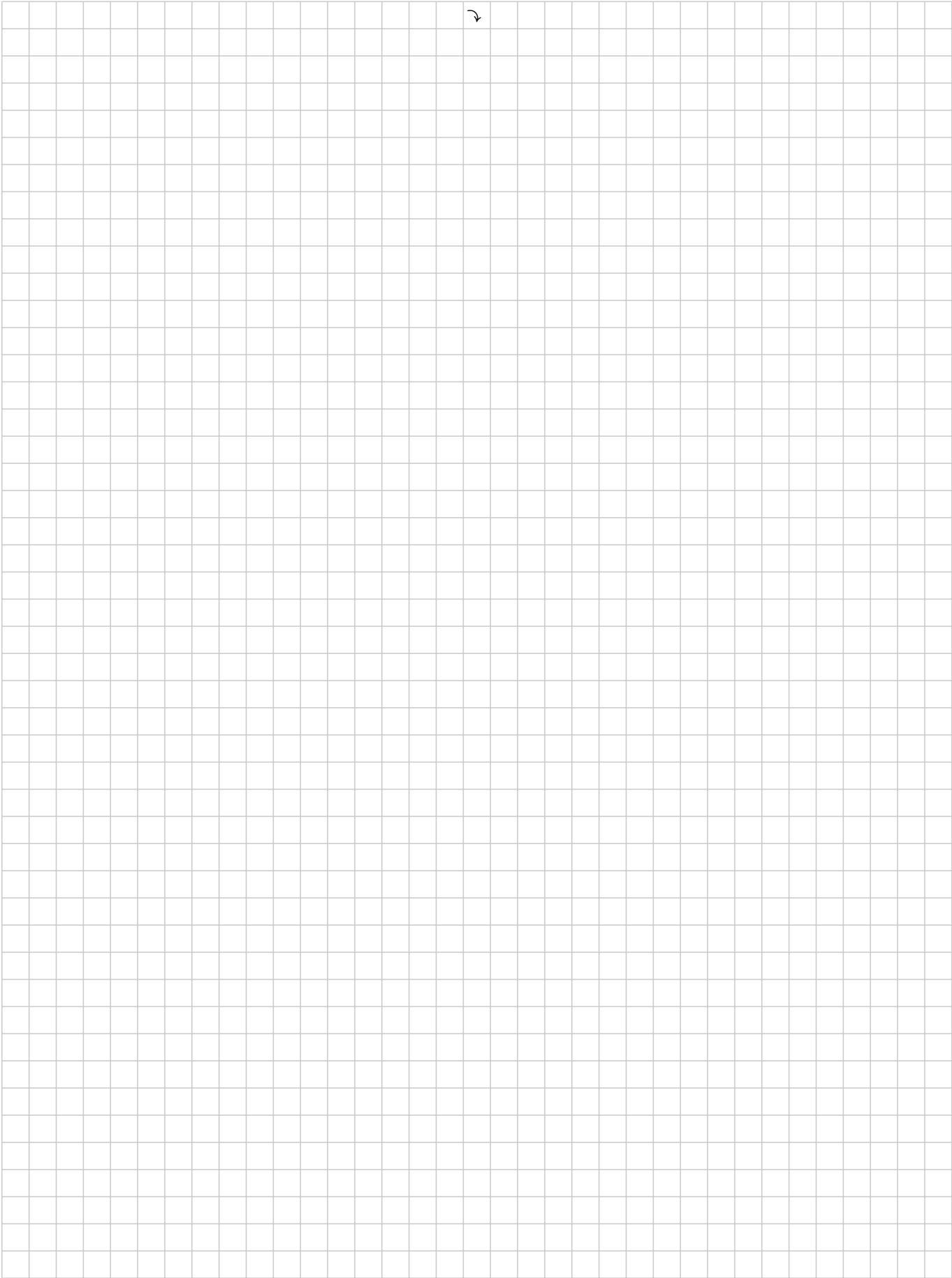
Consider the following matrix  $A$ :

$$A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}.$$

15p **5** Find two linearly independent vectors that are orthogonal to  $\text{Nul } A$ .



A large grid of graph paper for working out the solution. The grid is 30 columns wide and 30 rows high. A small arrow points to the bottom center of the grid.





**Exercise 6**

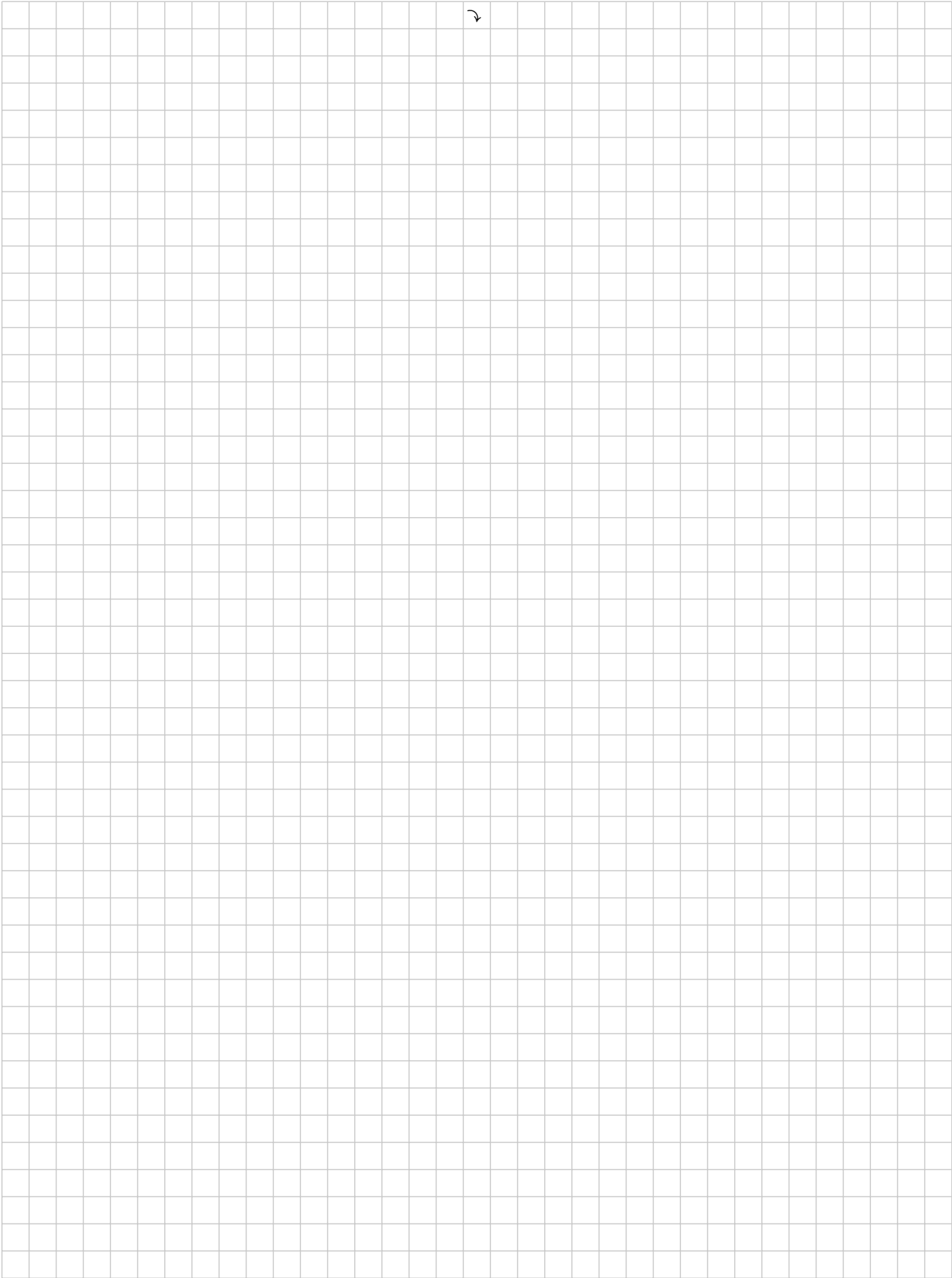
Consider the following matrix  $A$ :

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \end{bmatrix}.$$

15p

- 6 Determine a vector  $\mathbf{u}$  in Row  $A$  such that  $\mathbf{u} - \begin{bmatrix} 3 \\ 2 \\ 9 \end{bmatrix}$  is orthogonal to Row  $A$ .



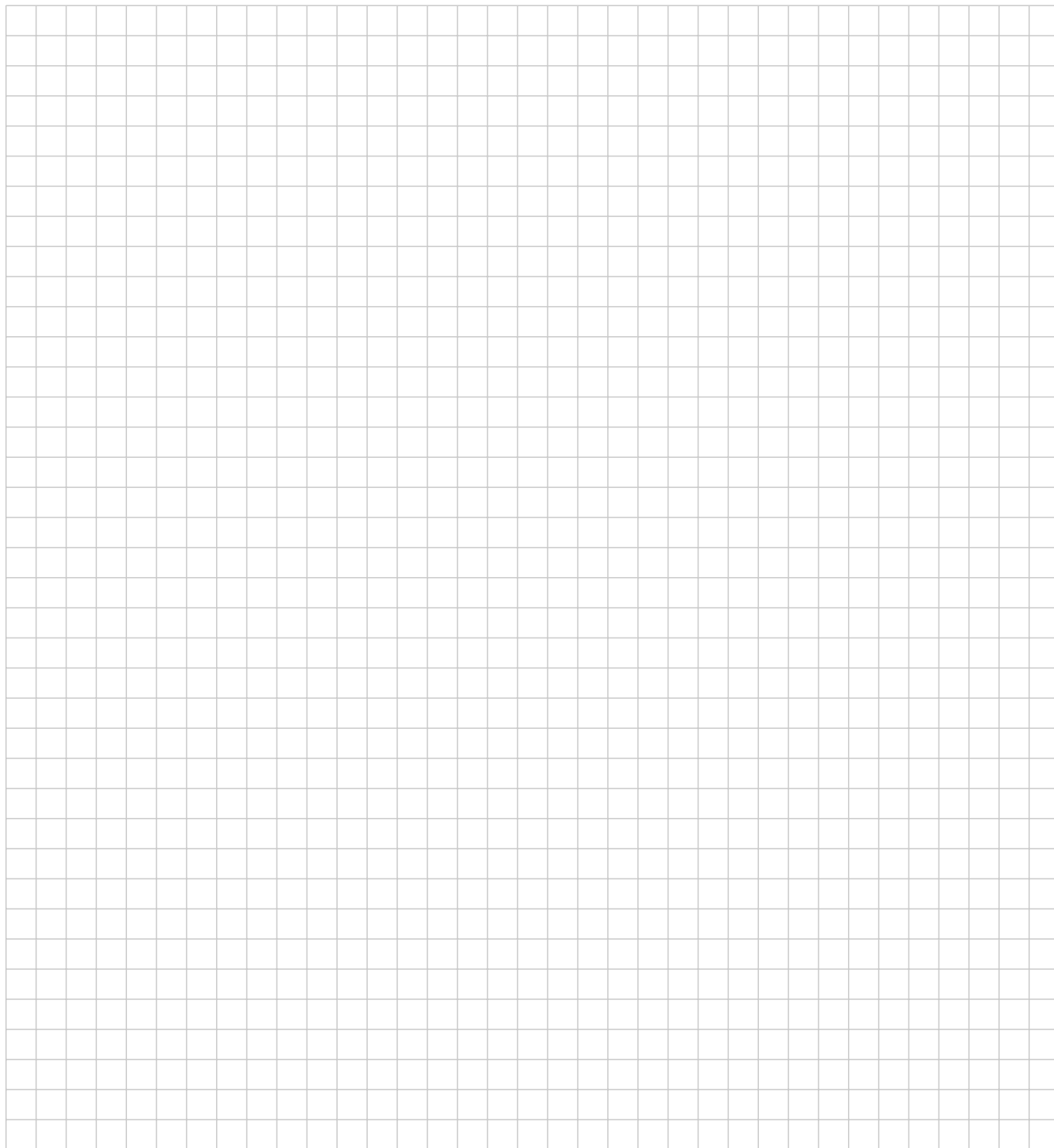


**Exercise 7**

For each of the questions below an explanation/derivation is not required; you only need to state the final answer.

5p

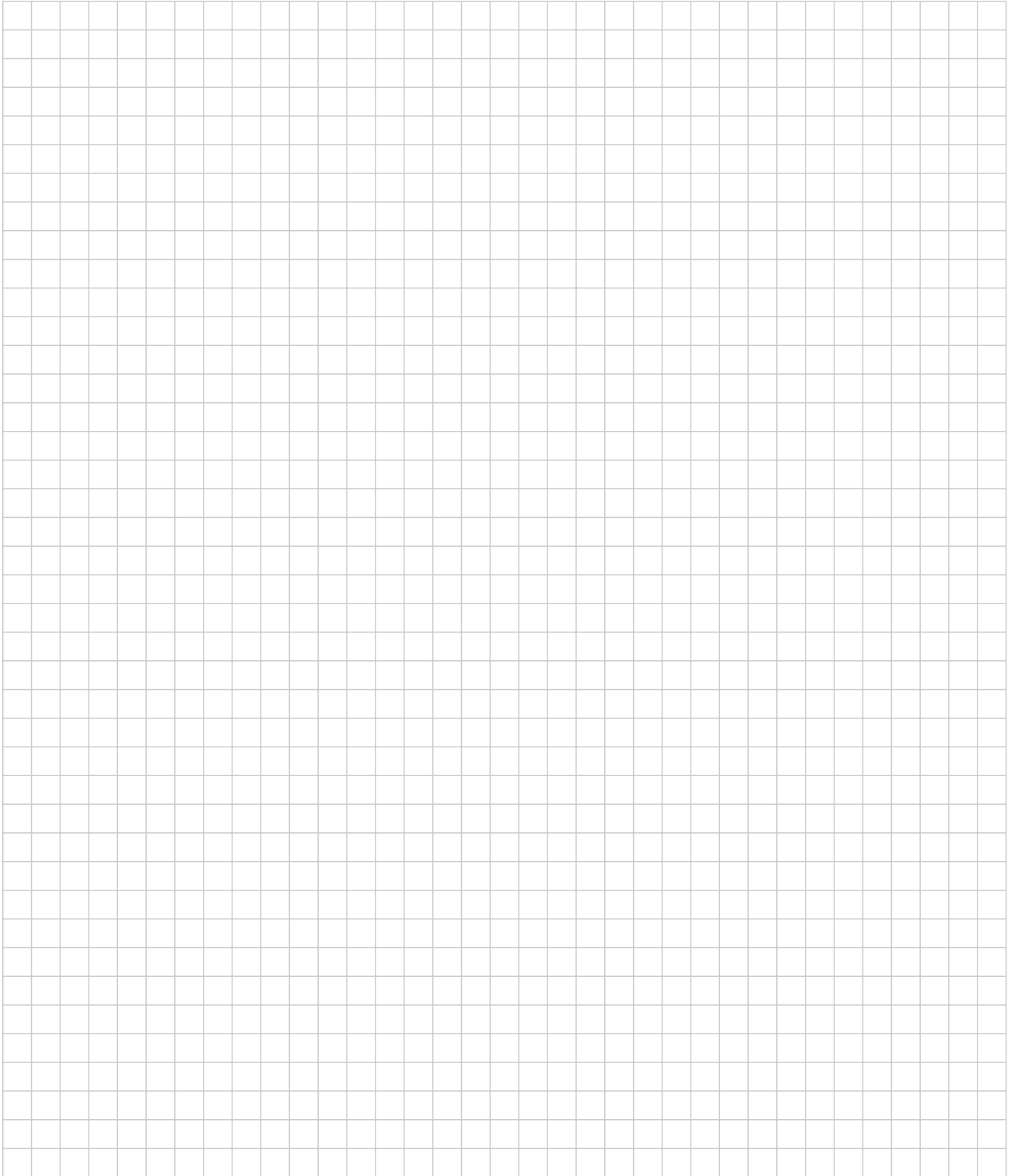
**7a** Determine two distinct vectors in  $\mathbb{R}^3$  with length 1 that are orthogonal to both  $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ .



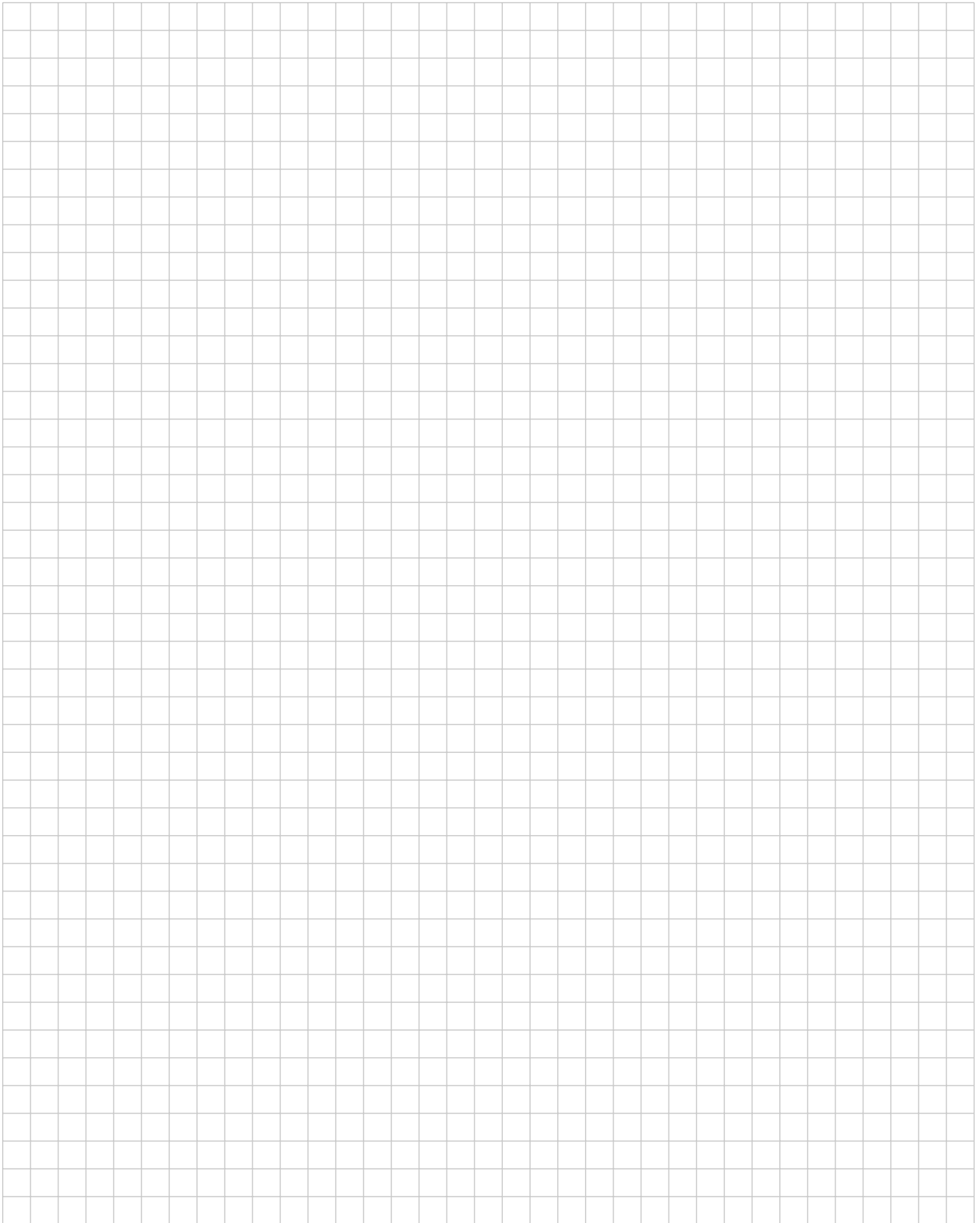
5p

**7b** Let  $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$  and  $P = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ .

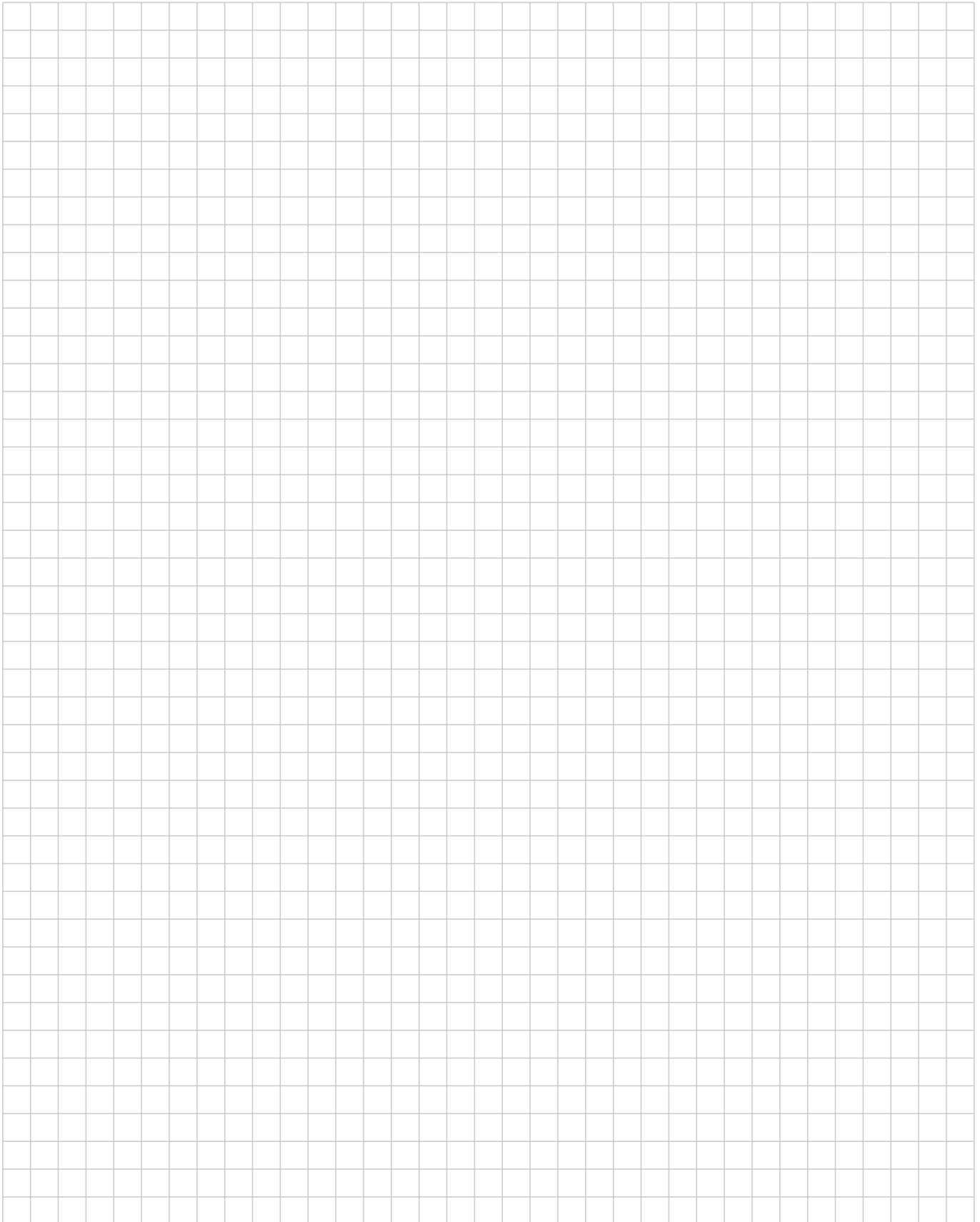
Determine a diagonal matrix  $D$  such that  $A = PDP^{-1}$ .



- 5p **7c** Determine vectors  $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$  such that  $\{\mathbf{u}, \mathbf{v}\}$  and  $\{\mathbf{v}, \mathbf{w}\}$  are linearly independent, but  $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$  is linearly dependent.



5p **7d** Provide a matrix  $A$  that is not invertible and such that  $\text{Nul } A = \{\mathbf{0}\}$ .

A large grid of graph paper, consisting of 20 columns and 30 rows of small squares, intended for the student to write their answer.

- 5p **7e** Provide an example of a subset  $H$  of  $\mathbb{R}^2$  that has the following three properties:
- the zero vector is in  $H$ ,
  - $H$  is closed under vector addition,
  - $H$  is **NOT** closed under multiplication by scalars.



### Extra space

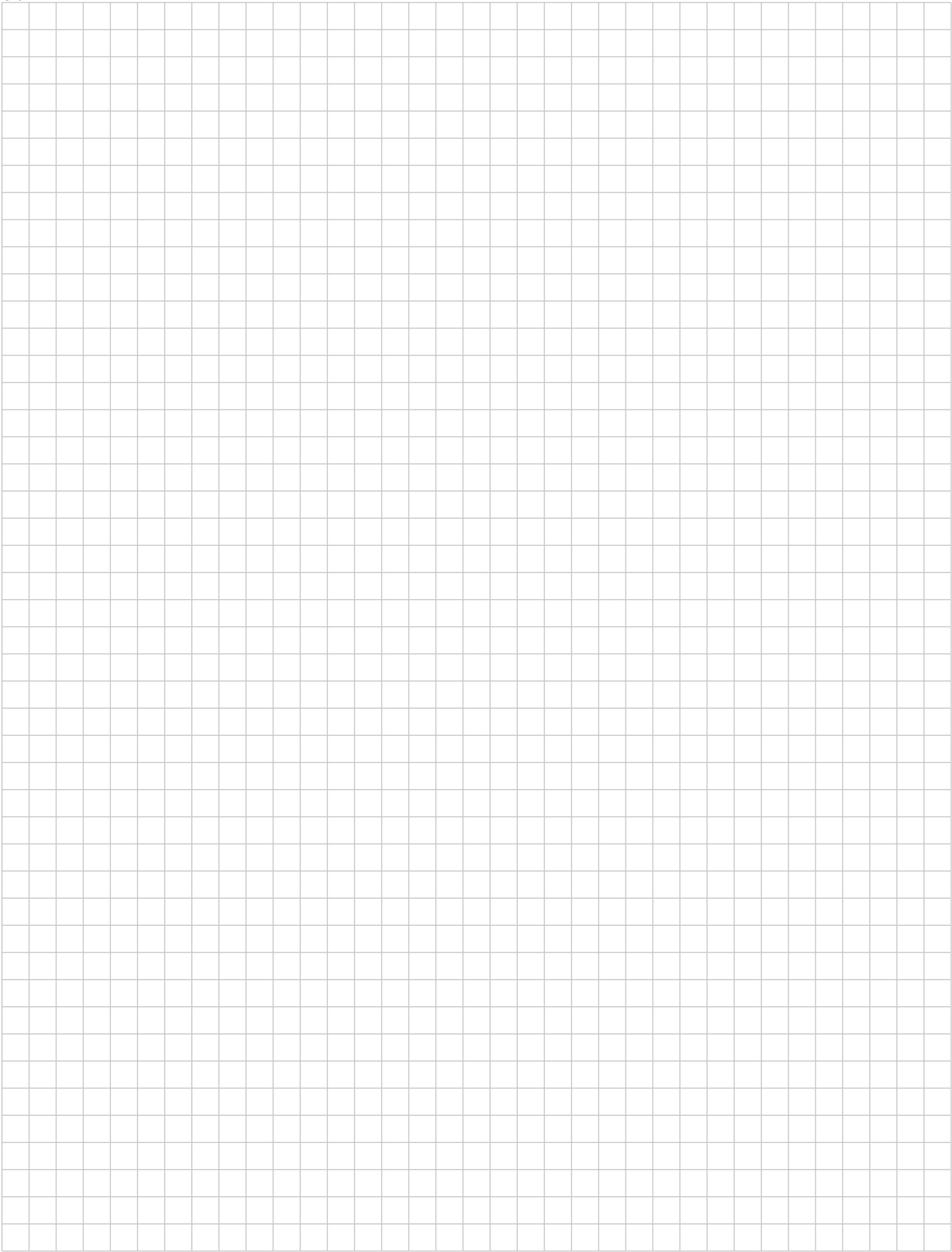
If you use these extra answer boxes, **please mention clearly in your main answer that part of your answer can be found here!**

**8a**





8b

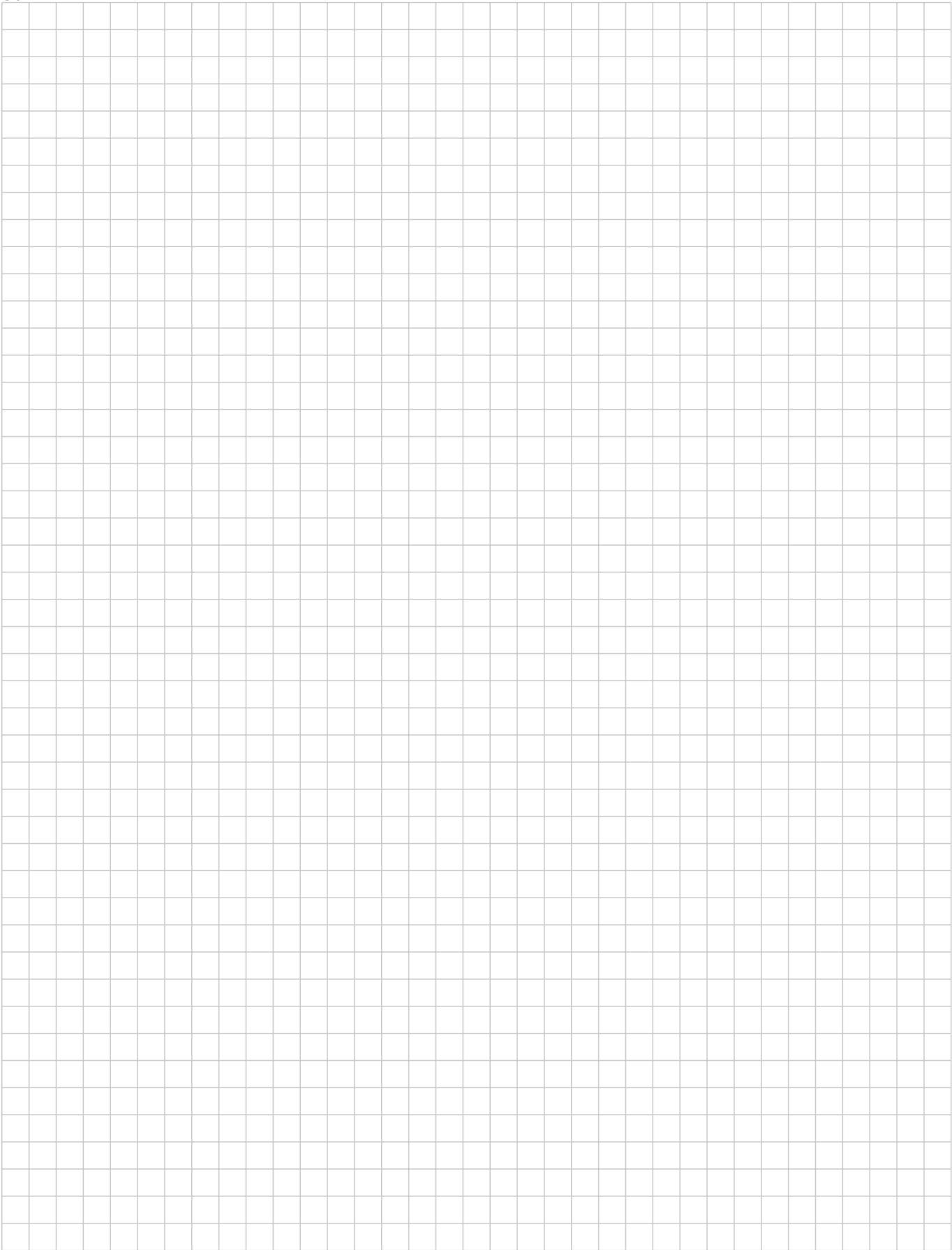


8c





8d



8e

