

- $(p \Rightarrow q) \Rightarrow (((q \Rightarrow r) \wedge \neg r) \Rightarrow \neg p)$

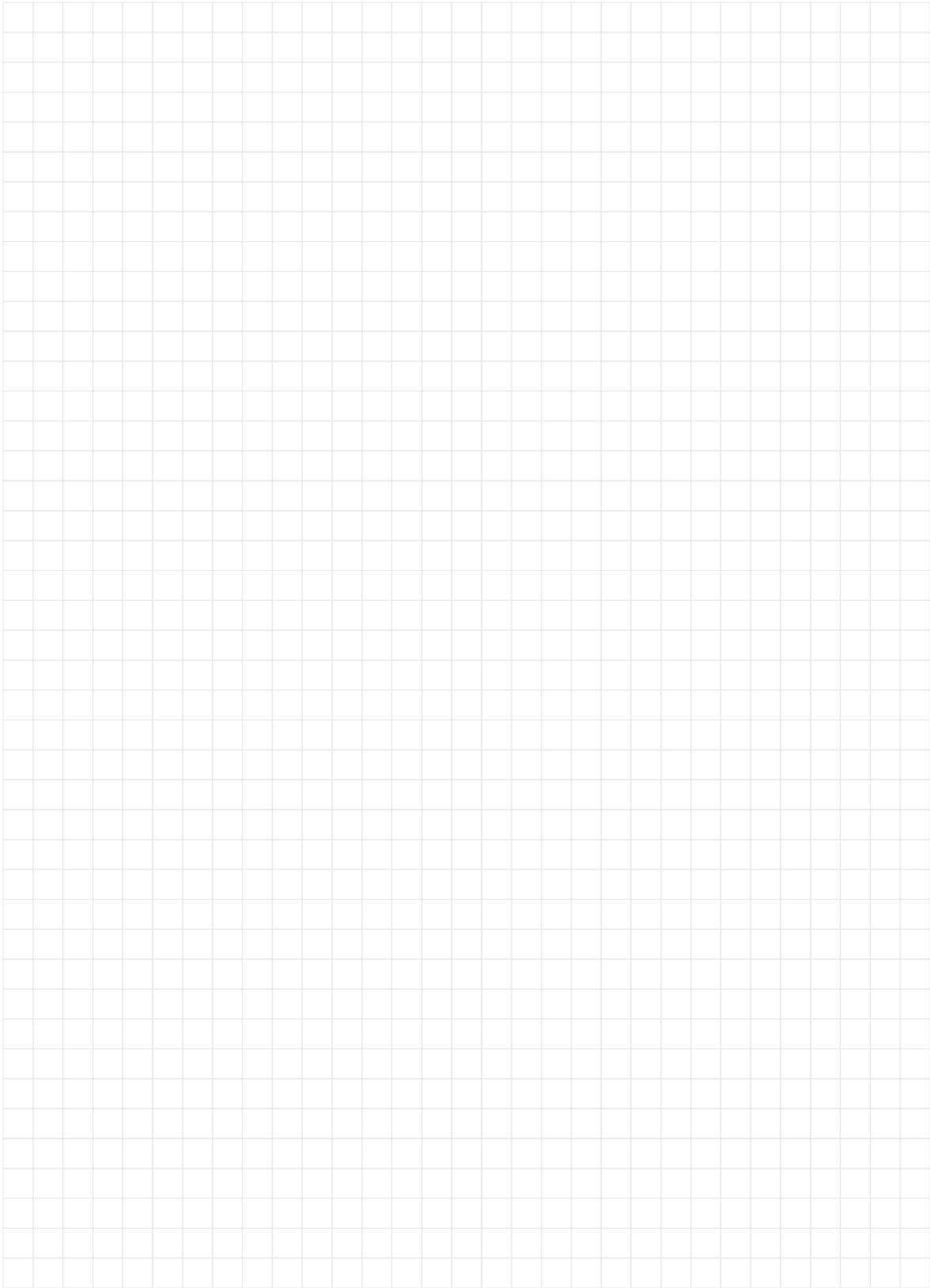


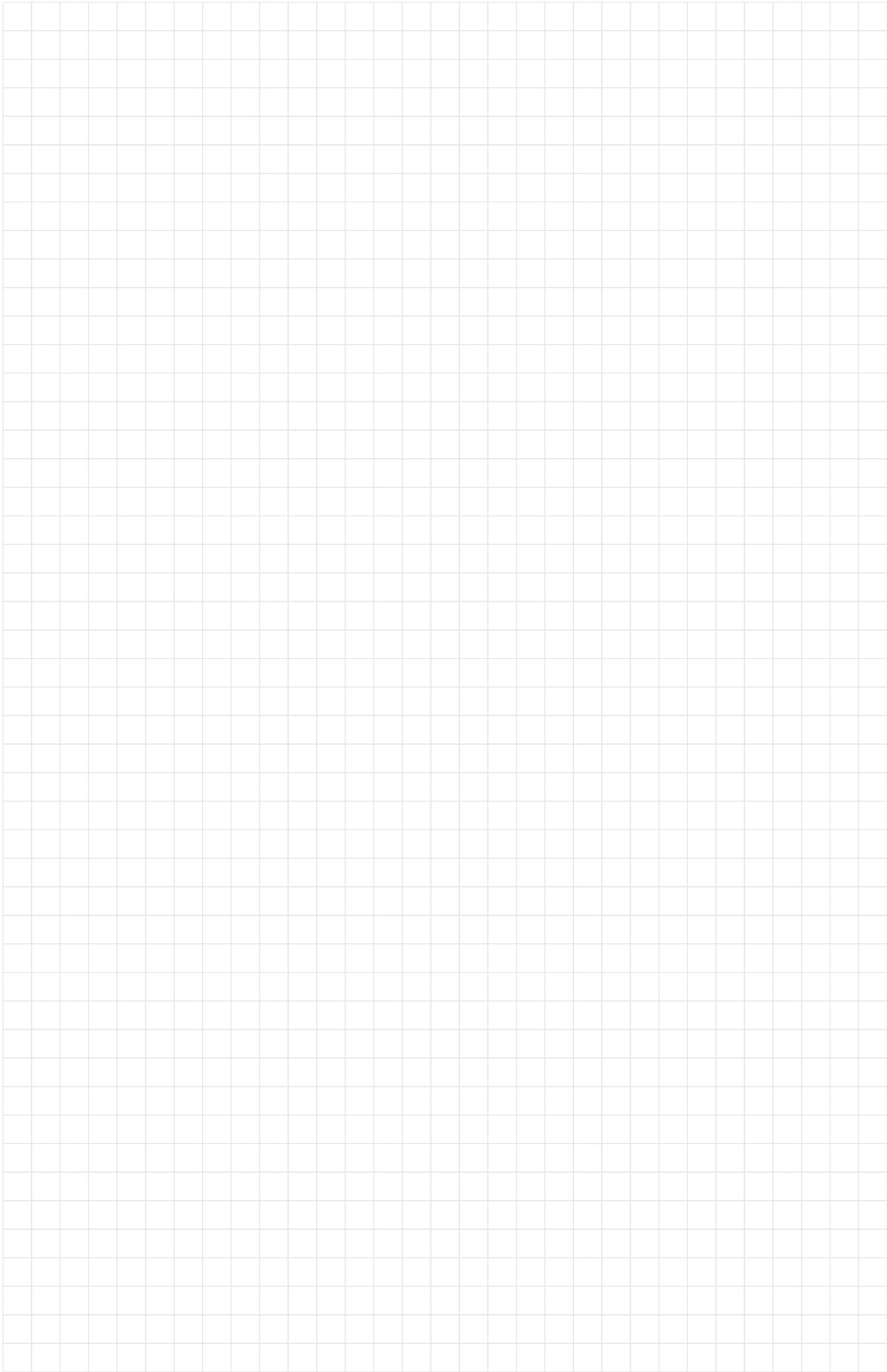
Question 2 (15 points)

Use induction to prove the following statement.

- For all integers $n \geq 1$,

$$\sum_{i=1}^n i^3 = \frac{1}{4}n^2(n+1)^2$$





Question 3 (15 points)

Prove or disprove the following statements.

- (a) (*7 points*) For all sets A , B , and C , if $A \not\subseteq B$ and $B \not\subseteq C$ then $A \not\subseteq C$.
- (b) (*8 points*) For all sets A , B and C , if $A \subseteq B$ then $A \cap (B \cap C)^c = \emptyset$.





Question 4 (15 points)

Let $A = \mathbb{P}(\{a, b, c\})$. Let R be the relation on A defined as follows:

XRY means “ $X \setminus Y \neq \emptyset$ and $Y \setminus X \neq \emptyset$ ”.

Is R reflexive? Symmetric? Transitive? Anti-symmetric? For each of these properties, prove or disprove that it has that property.





Question 5 (15 points)

All the following questions are about *counting*. For all questions please give an exact number at the end (i.e. don't just leave your answer as a counting equation).

- (a) (*3 points*) When you decide to order pizza, you must first choose the type of crust: thin crust or deep dish. Next, you choose one topping: cheese, pepperoni, or sausage. How many possibilities are there for your pizza?
- (b) (*6 points*) How many bit sequences of length eleven are there which start with one of the two bit sequences 101 or 010?
- (c) (*6 points*) A student possesses six distinct books on economics, and six distinct books on mathematics. The books are arranged in a sequence on a book shelf. How many options are there, if the books on mathematics should be on the left?

A large grid of graph paper, consisting of 20 columns and 30 rows of small squares, intended for the student to write their answers to the questions.



Question 6 (10 points)

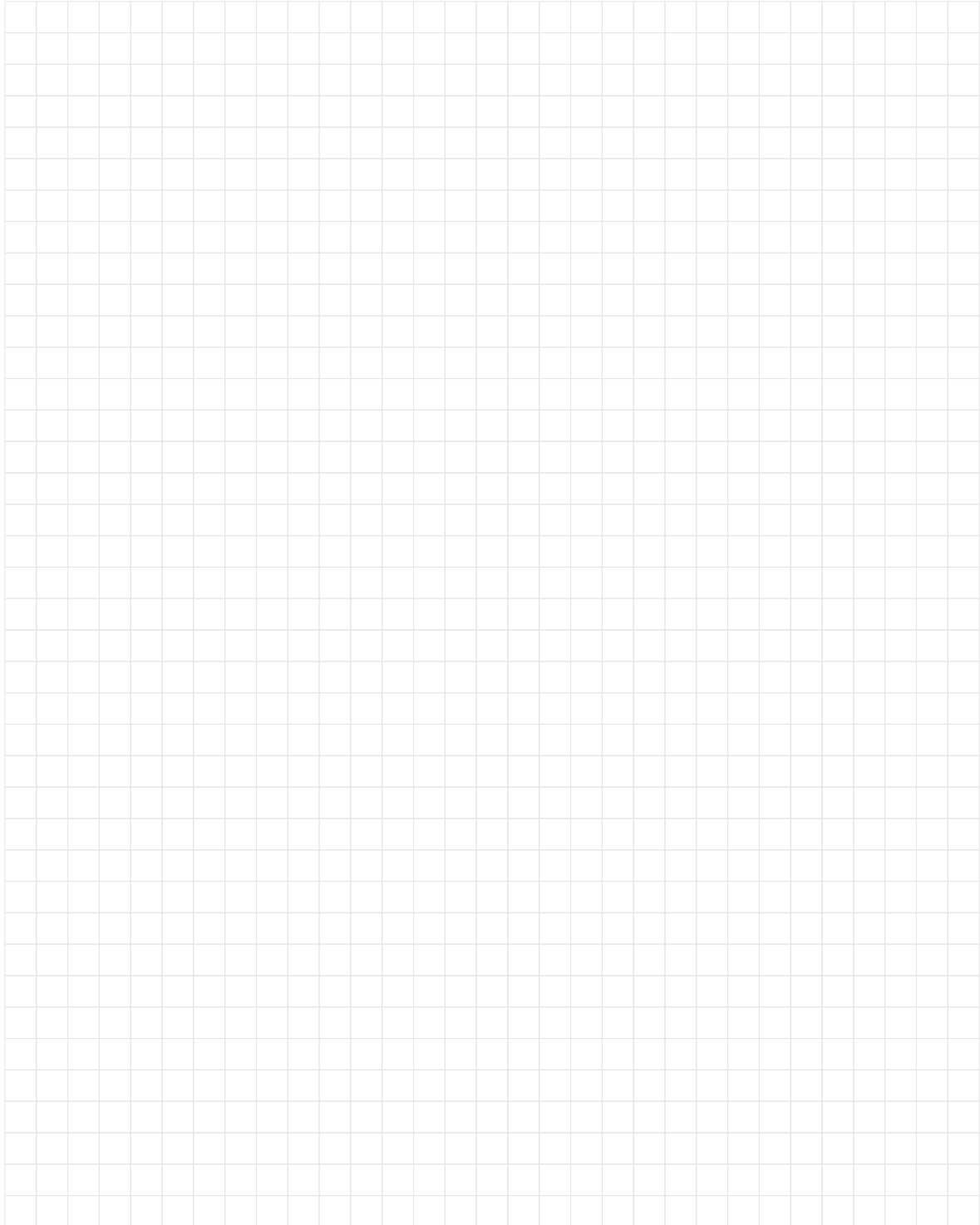
Prove or disprove the following statements.

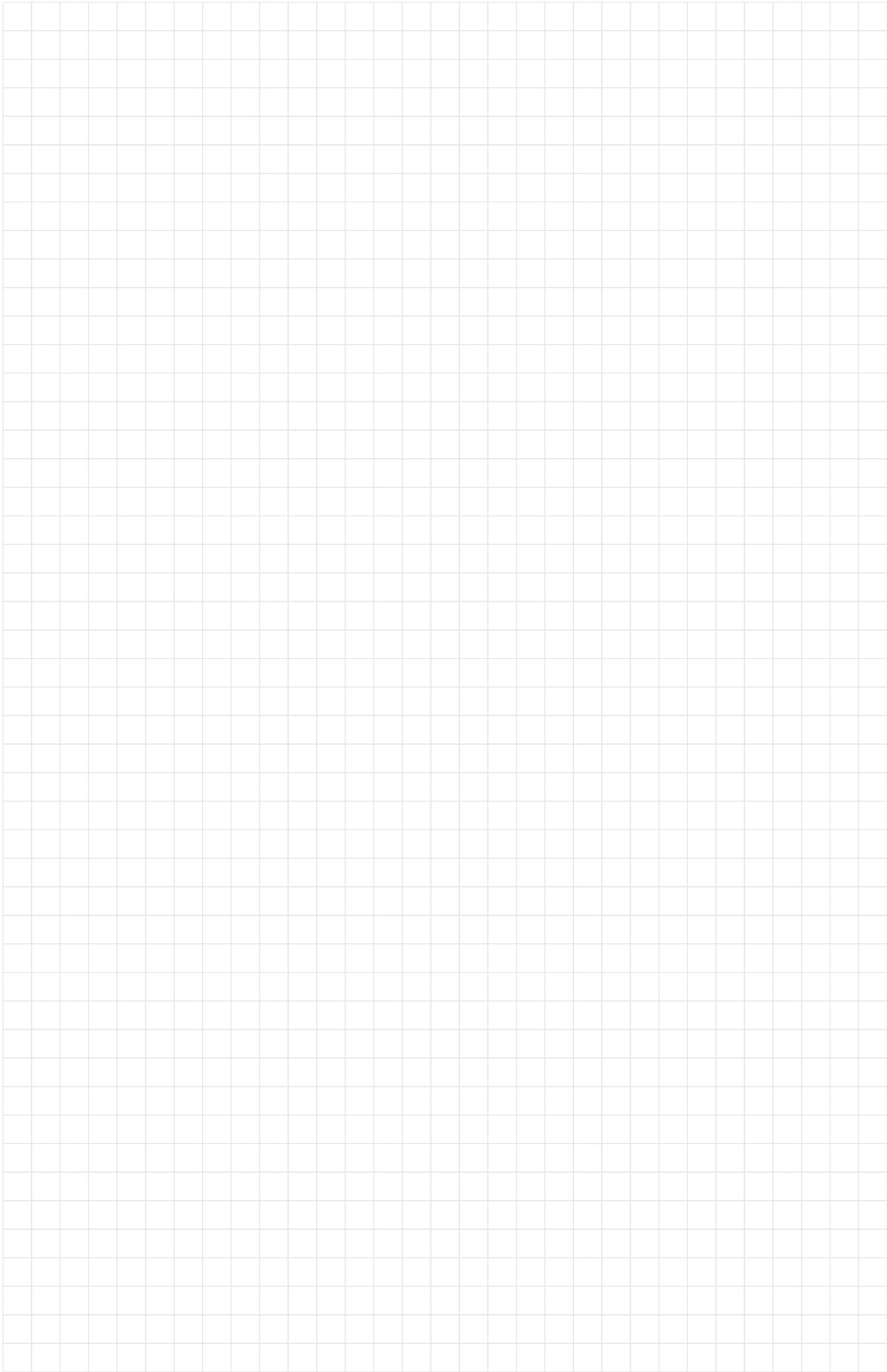
Let $X = \{-2, -1, 0, 1, 2\}$ and $Y = \{-2, -1, 0, 1, 2\}$.

(a) (2 points) $(\forall x \in X)(\exists y \in Y)(x + y = 0)$.

(b) (2 points) $(\exists x \in X)(\forall y \in Y)(x + y = y)$.

(c) (6 points) $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{N})((z^2 \geq x^2 + y^2) \wedge (z \geq 5))$.





Question 7 (15 points)

This is a question about *functions*.

- (a) (10 points) Let $f : \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} \setminus \{1\}$ be the function defined as follows:

$$f(x) = \frac{x+1}{x}.$$

Prove that f is a bijection.

- (b) (5 points) Construct sets A , B and functions $f : A \rightarrow B$, $g : B \rightarrow A$ such that

- f is not an injective function,
- but, g and $f \circ g$ are both injective functions.





- (a) (2 points) Is $\{\{a, d, e\}, \{b, c\}, \{d, f\}\}$ a partition of $\{a, b, c, d, e, f\}$? Why/Why not?
- (b) (3 points) Let $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{3, 4\}$. Write down $(A \times B) \cap (A \times C)$.

