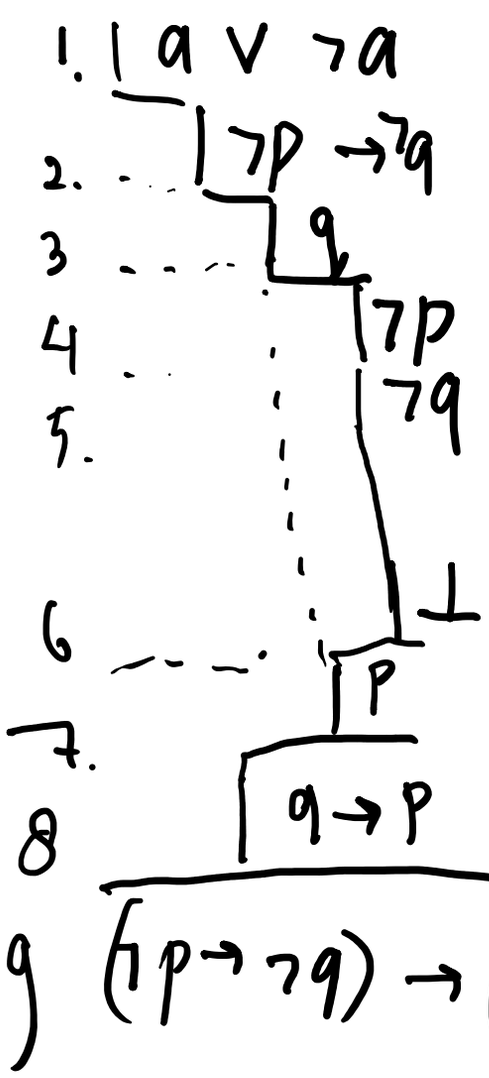
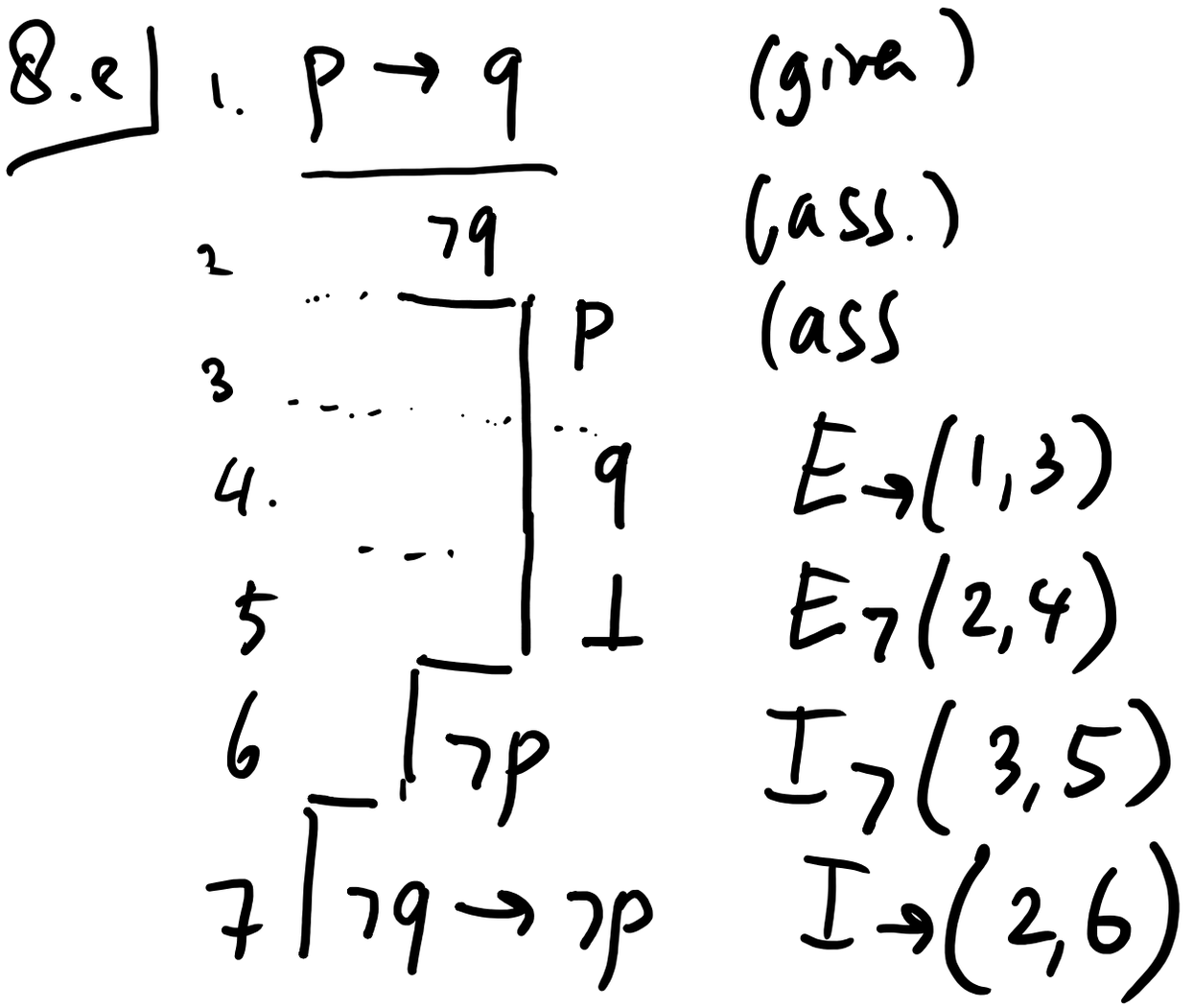
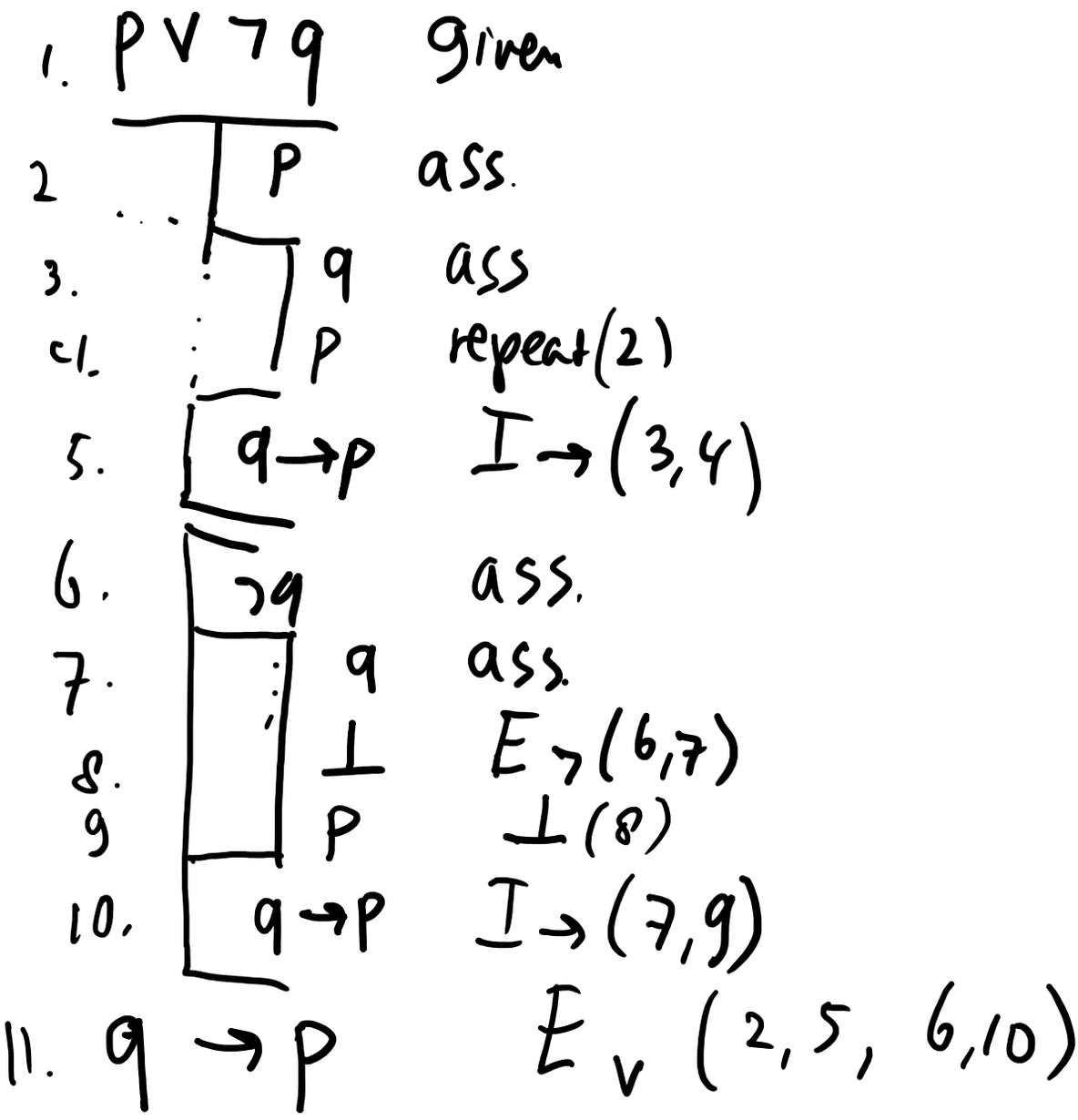
8ff $\quad a \vee \neg a \models (\neg p \to \neg q) \to (q \to p)$



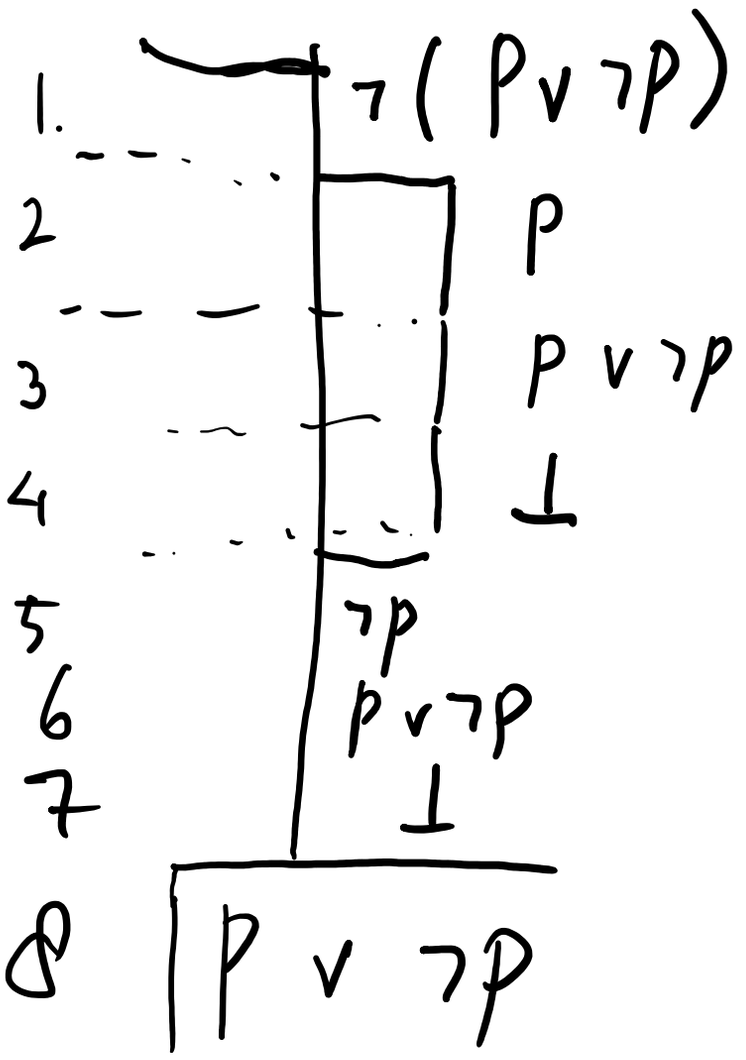
(given)
(assume)
(assume)
(assume)
$E\to(2,4)$
$E_\neg(3,5)$
$I_\neg(4,6)$
$I_\to(3,7)$
$I_\to(2,8)$


8.e
1. P → q
(give)
¬q
(ass.)
2
P
(ass
3
q
4.
E→(1,3)
⊥
5
E¬(2,4)
¬p
6
I¬(3,5)
q → ¬p
7
I→(2,6)

8d)


1. P ∨ ¬q given
2 P ass.
3. q ass
c1. P repeat(2)
5. q → p I→(3,4)
6. ¬q ass.
7. q ass.
8. ⊥ E¬(6,7)
9 P ⊥(8)
10. q → p I→(7,9)
11. q → p E∨(2,5, 6,10)

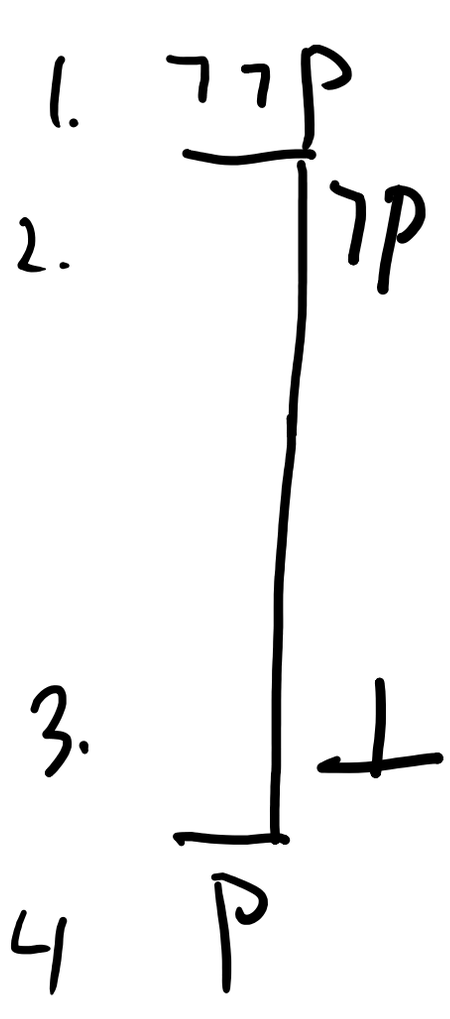
89)



8.b)

1. $P \to \neg q$ (given)
2. q (given)
3. P (ass.)
4. $\neg q$ $E\to(1,3)$
5. $\perp$ $E_\neg(2,4)$
6. $\neg P$ $I_\neg(3,5)$

8g)



$E_\neg(1,2)$

$I_\neg(2,3)$.

7c) $p \wedge q, \neg q \vdash r$

$p \wedge q \vdash r, q$

$p, q \vdash r, q$

×

So inference is valid

7b) $p \to q, \neg q \to r \vdash p \to r$

$p \to q, \neg q \to r, p \vdash r$

$p \to q, p, \neg r \vdash r$ | $p \to q, p \vdash \neg q, r$

$p \to q, p \vdash r$

$p, q \vdash r$ (open) | $p \vdash r, p$

Tableau open; Inference not valid!

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7g) $p, q \circ p \to q$

$p, q \circ q$

$\times$

6)

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

for matrix multiplication

$(p \to q) \wedge r \not\models p \to (q \wedge r)$

5g) φ contradiction

$\varphi \circ$

closed

$\circ \varphi$

closed

$\circ \neg \varphi$

closed

φ tautology

$\neg \varphi$ tautology

$\neg \varphi$ contradiction

$\leftrightarrow \varphi$ contradiction

5g) (cont'd)

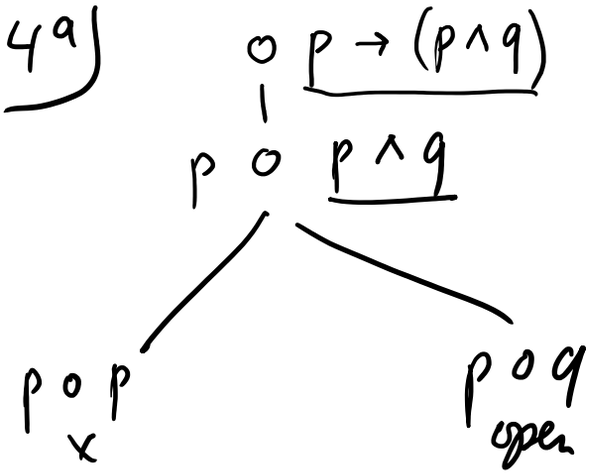
$$\frac{(p \to \neg q) \underline{\wedge} (q \to r) \wedge (r \to p) \circ}{p \to \neg q \;,\; (q \to r) \underline{\wedge} (r \to p) \circ}$$

$$\frac{p \to \neg q \;,\; \underline{q \to r} \;,\; r \to p \quad \circ}{\begin{array}{l|l} p \to \neg q \;,\; \underline{r \to p} \;,\; r \circ & \\ \hline \underline{p \to \neg q} \;,\; r \;,\; p \circ & p \to \neg q \;,\; r \underset{\times}{\circ} r \\ \hline r \;,\; p \;,\; \neg q \circ & r \;,\; p \underset{\times}{\circ} p \end{array} \qquad \begin{array}{l|l} & p \to \neg q \;,\; r \to p \circ q \end{array}}$$

$r \;,\; p \circ q$ open

open, so $\not\vdash \neg\varphi$ not valid, so φ is not a contradiction.

4g)



⇒ tableau open, $\not\vdash p \to (p \wedge q)$
not valid, so
not a tautology.

(a) Sentence (b) is true

(b) sentence (c) is false

(c) There are exactly two sentences correct

a = true → sentence 1 true
= false → sentence 1 false , b, c Similar

$a \leftrightarrow b$

$b \leftrightarrow \neg c$

$c \leftrightarrow ((a \wedge b \wedge \neg c) \vee (a \wedge \neg b \wedge c) \vee (\neg a \wedge b \wedge c))$

$c \leftrightarrow ((a \wedge b \wedge \neg c) \vee (a \wedge \neg b \wedge c) \vee (\neg a \wedge b \wedge c))$

a	b	c	
1	1	0	0
0	0	0	0
1	1	1	1
0	0	1	1

1
0
1
0

$b \leftrightarrow \neg c$