

## Practice Exam Questions - Tutorial 1

1. Fill in the truth table for the following logical proposition.

- $(p \vee (\neg q \Leftrightarrow r)) \wedge (\neg p \Rightarrow r)$

2. Fill in the truth table for the following logical proposition.

- $((r \vee \neg q) \Rightarrow (q \Leftrightarrow p)) \wedge (r \vee p)$

3. Fill in the truth table for the following logical proposition.

- $((q \wedge \neg p) \vee \neg r) \Leftrightarrow (r \Rightarrow \neg q)$

4. Fill in the truth table for the following logical proposition.

- $((p \wedge \neg q) \Rightarrow \neg r) \Leftrightarrow (p \Rightarrow (q \vee r)).$

5. Consider the table below.

<b>1</b>		<b>8</b>	<b>11</b>	
	<b>2</b>			
<b>3</b>		<b>4</b>	<b>5</b>	<b>10</b>
	<b>6</b>		<b>9</b>	<b>13</b>
<b>14</b>	<b>7</b>	<b>12</b>		

For the benefit of those viewing in black and white: numbers 1,4,5,6,14 are *red*, numbers 3,8,9,13 are *green*, and numbers 2,7,10,11,12 are *blue*. For each of the statements listed, decide whether it is true or false, and then justify your answer as rigorously as you can. Note that, when I am talking about “numbers” and “colours”, I mean *those in the table* – I don’t write this explicitly simply to save space.

- All the numbers in the table are even if and only if all the numbers in the table are odd.
- For every blue number  $x$ ,  $x$  is even or  $x$  is prime.
- There is a colour  $c$  such that for every prime number  $y$ ,  $y$  does not have colour  $c$ .
- For every column  $d$ , if  $d$  contains three or more numbers, then  $d$  contains at least three colours.
- For every red number  $x$  and for every green number  $y$ , there is a blue number  $z$  such that  $x < z < y$ .

- (f) There is an even number  $x$  and there is an odd number  $y$  such that  $x$  and  $y$  are both the same colour.
- (g) For every even number  $x$ , there is a prime number  $y$  such that  $y$  is in the same row as  $x$ .
- (h) For every column  $d$ , if  $d$  contains four or more numbers, then  $d$  contains at least four colours.
- (i) For every red number  $x$  and for every green number  $y$ , if  $x \leq y$  then there is a blue number  $z$  such that  $x < z < y$ .
- (j) There is an odd number  $x$ , such that for every even number  $y$ , the column that  $y$  is in is strictly<sup>1</sup> to the left of the column that  $x$  is in.

6. Prove or disprove the following statements.

- (a)  $(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})(\frac{x+y}{3} \notin \mathbb{N})$
- (b)  $(\exists x \in \mathbb{N})(\forall y \in \mathbb{Z})(3x + 4y \neq 20)$
- (c)  $\neg((\exists x \in \mathbb{N})(\exists y \in \mathbb{R})(2x - y \leq 10))$
- (d)  $(\exists x \in \mathbb{N})(\forall y \in \mathbb{Z})(\exists z \in \mathbb{N})(x + y + z \geq -32)$

7. Prove or disprove the following statements.

- (a)  $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})((x = y + 10) \vee (y \neq 0))$
- (b)  $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{N})(y \geq \frac{1}{2}x^2 - 20)$
- (c)  $\neg((\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x^2 - x + y \text{ is odd}))$
- (d)  $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(\exists z \in \mathbb{N})(y < x < z)$ .

8. Prove or disprove the following statements.

- (a)  $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{N})((x > 0) \wedge (x + y^2 \text{ is even})),$
- (b)  $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{N})(\forall z \in \mathbb{N})(x - y < z - 10).$

9. Prove or disprove the following statements.

- (a)  $\neg((\forall x \in \mathbb{Z})(\exists y \in \mathbb{N})(-y < x < y))$
- (b)  $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{N})(\forall z \in \mathbb{N})(x + y \neq z).$

10. Prove or disprove the following statement.

$$(\forall x \in \mathbb{R})(x \in \mathbb{N} \Leftrightarrow (2x \in \mathbb{N} \wedge 3x \in \mathbb{N})).$$

11. Prove or disprove the following statement.

$$(\forall m \in \mathbb{Z})(\forall n \in \mathbb{Z})((mn \text{ is even}) \Leftrightarrow ((m \text{ is even}) \vee (n \text{ is even}))).$$

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<sup>1</sup>I use *strictly* here in the same sense as strict inequality i.e. 1 is strictly less than 3, but 3 is not strictly less than 3.