PAUL'S ANSWERS TO LOGIC BONUS ASSIGNMENT 1. 20/04/2023.

1 a) $\neg(\neg p \vee \neg(q \wedge p))$

p	q	$\neg$	$(\neg p$	$\vee$	$\neg$	$(q$	$\wedge$	$p))$
1	1	1	0	0	0	1	1	1
1	0	0	0	1	1	0	0	1
0	1	0	1	1	1	1	0	0
0	0	0	1	1	1	0	0	0

Not a contradiction, not a tautology, but is satisfiable (when $p=1$, $q=1$)

Not a contradiction, not a tautology, but is satisfiable (when both p and q are true).

1 B) $(\neg a \vee \neg b) \leftrightarrow \neg(a \wedge b)$

a	b	$(\neg a$	$\vee$	$\neg b)$	$\leftrightarrow$	$\neg$	$(a$	$\wedge$	$b)$
1	1	0	0	0	1	0	1	1	1
1	0	0	1	1	1	1	1	0	0
0	1	1	1	0	1	1	0	0	1
0	0	1	1	1	1	1	0	0	0

TAUTOLOGY, NOT A CONTRADICTION, satisfiable.

1 C) $\neg(a \rightarrow b) \rightarrow (a \rightarrow b)$

a	b	$\neg$	$(a$	$\rightarrow$	$b)$	$\rightarrow$	$(a$	$\rightarrow$	$b)$
1	1	0	1	1	1	1	1	1	1
1	0	1	1	0	0	0	1	0	0
0	1	0	0	1	1	1	0	1	1
0	0	0	0	1	0	1	0	1	0

Not a Tautology
Not a Contradiction,
But is satisfiable.

1 D) $\neg b \rightarrow (a \rightarrow b)$

True when b = 1, regardless of a's value.
FALSE when b = 0, a = 1
True when b = 0, a = 0

So Not a TAUTOLOGY, not a CONTRADICTION, But Satisfiable.

2 A) $a \models (b \wedge c) \rightarrow (a \rightarrow b)$

a b c	a	$\models$	$(b \wedge c)$	$\rightarrow$	$(a \rightarrow b)$
1 1 1	1	1	1	1	1
1 1 0	1	1	0	1	1
1 0 1	1	1	0	1	0
1 0 0	1	1	0	1	0
0 1 1					
0 1 0					
0 0 1					
0 0 0					

VALID INFERENCE.

2 B) $\{p \vee q, q \vee r\} \models (p \wedge r) \rightarrow \neg q$

p q r	$(p \vee q)$	$(q \vee r)$	$\models$	$(p \wedge r)$	$\rightarrow$	$\neg q$
1 1 1	1	1	0	1	0	0
1 1 0	1	1	1	0	1	0
1 0 1	1	1	1	1	1	1
1 0 0	1	0	[illegible]	0	1	1
0 1 1	1	1	1	0	1	0
0 1 0	1	1	1	0	1	0
0 0 1	0	1		0	1	1
0 0 0	0	0		0	1	1

NOT VALID (p=1, q=1, r=1)

2c $\{p \to q,\ q \to r\} \models p \to r$

p	q	r	$p \to q$	$q \to r$	$\models$	$p \to r$
1	1	1	1	1	1	1
1	1	0	1	0		0
1	0	1	0	1		1
1	0	0	0	1		0
0	1	1	1	1	1	1
0	1	0	1	0		1
0	0	1	1	1	1	1
0	0	0	1	1	1	1

VALID INFERENCE :)

3.

Propositional logic:

$a \leftrightarrow b$

$b \leftrightarrow \neg c$

$* = c \leftrightarrow (a \wedge b \wedge \neg c) \vee (a \wedge \neg b \wedge c) \vee (\neg a \wedge b \wedge c) = *$

No configurations possible:

if a is true, b must be true, so c must not be true, but it is.

a is false, b must be false, so c must be true, but it is not

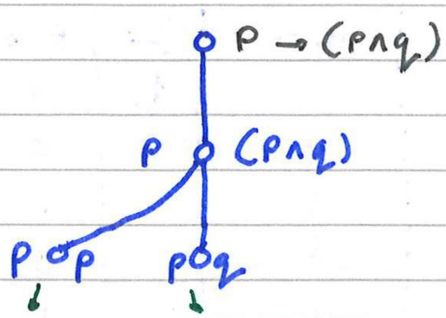
This problem is not satisfiable, it is a paradox.

abc	$a \leftrightarrow b$	$b \leftrightarrow \neg c$	*
111	1	0	0
110	1	1	0
101	0	1	1
100	0	0	1

abc	$a \leftrightarrow b$	$b \leftrightarrow \neg c$	*
011	0	0	1
010	0	1	1
001	1	1	0
000	1	0	1

4. Tableau method → TAUTOLOGIES → Logic in Action chp 8 slides
↳ slide 60 + 71

a)



closed

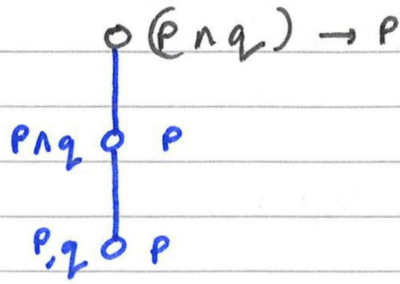
NOT CLOSED

↳ NOT A TAUTOLOGY

Counterexamples:

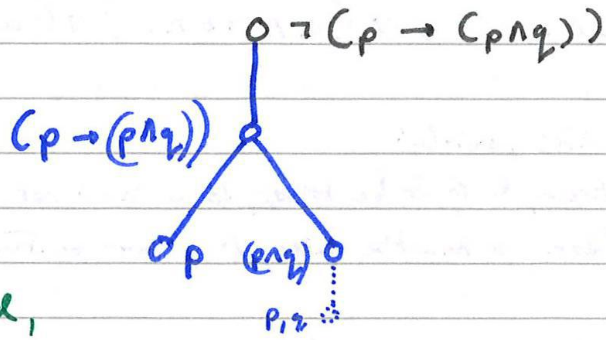
$p = 1$, $q = 0$

4b)



Closed, TAUTOLOGY

4c)



NOT closed,

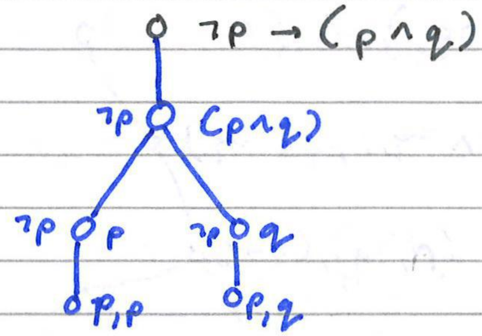
NOT A TAUTOLOGY

Counterexamples:

$p = 0$, $q = 0$ or 1.

$p = 1$, $q = 1$.

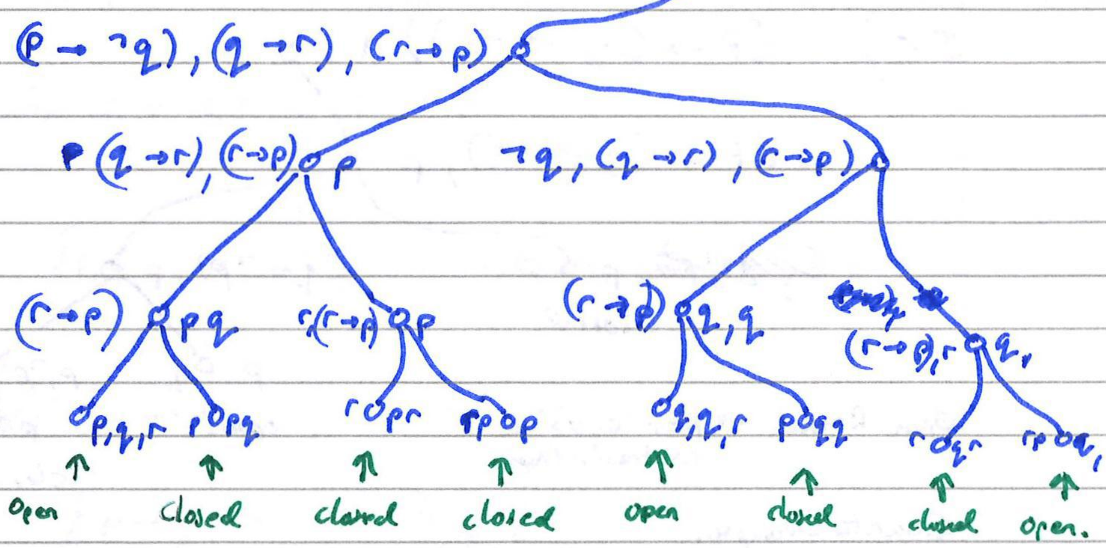
4 D



NOT CLOSED, NOT A TAUTOLOGY; Counterexample: $P=0$, $q=0$ or 1.

5 → Using the tableau method to ~~find~~ check for contradictions.

5a) $(p \to \neg q) \wedge (q \to r) \wedge (r \to p)$ ○



3 open branches → Not a contradiction!

Counterexamples: $p=0$, $q=0$, $r=0$

$p=1$, $q=0$, $r=0$.

$p=1$, $q=0$, $r=1$.

→ How did I find these? Look at the open branches!

○ p, q, r ⇒ $p=0$, $q=0$, $r=0$.

○ q, q, r ⇒ $q=0$, $r=0$, $p=0$ or 1

r p ○ q ⇒

5B) $(p \wedge \neg q) \wedge (p \to q)$ ○

$(p \wedge \neg q), (p \to q)$ ○

$(p), (\neg q), (p \to q)$ ○

~~$p, \neg q$~~ $p, \neg q$ ○ p $\quad$ $q, p, \neg q$ ○

p ○ q, p $\quad$ p, q ○ q

All are closed
⇒ Contradiction.

5C) $(p \to (q \to \neg p)) \wedge p$ ○

$(p \to (q \to \neg p)), p$ ○

~~$(q \to \neg p)$~~ $\neg p$ ○ p $\quad$ $(q \to \neg p), p,$ ○

Closed

p ○ q $\quad$ $p, \neg p$ ○

open $\quad$ p ○ p

closed.

Open Branch → Not a valid contradiction.

Counterexample:
$p = 1, \ q = 0$

Remember, left of circle = positive / 1.
right of circle = negative / neutral / 0.

So p ○ q → $p = 1, \ q = 0$.

↑

Counterexample.

6 What is wrong with the following formula?

$(p \rightarrow q) \wedge (q \rightarrow p) \rightarrow (p \vee \neg q)$

→ There is nothing wrong with the formula.

When $p = 1$ and $q = 1$ → True.

$p = 1$ and $q = 0$ → True.

$p = 0$ and $q = 1$ → True

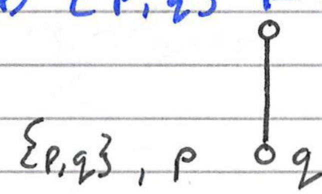
$p = 0$ and $q = 0$ → True.

IT IS A TAUTOLOGY.

Alternative answer:
The order in the formula is unclear. Is it
$((p \rightarrow q) \wedge (q \rightarrow p)) \rightarrow (p \vee \neg q)$
- - OR - -
$(p \rightarrow q) \wedge ((q \rightarrow p) \rightarrow (p \vee \neg q))$
?

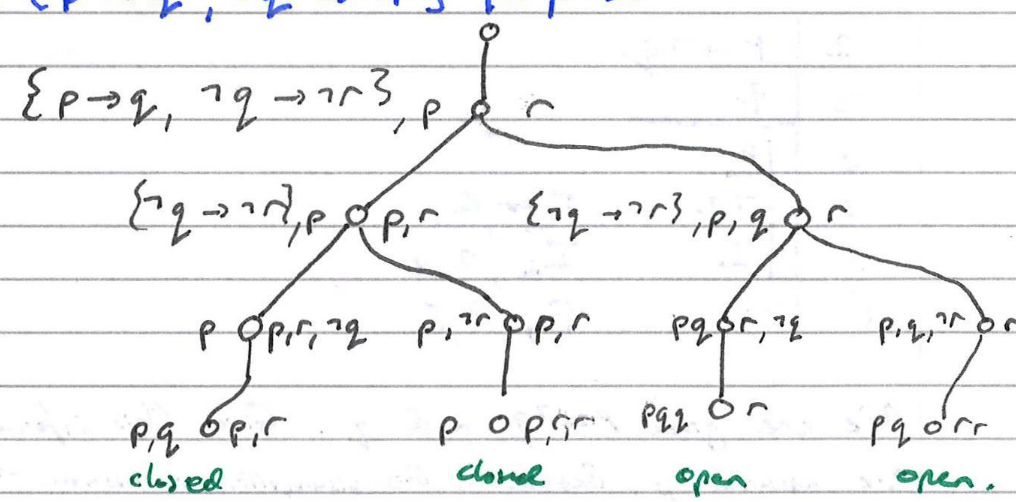
7. ~~A)~~ Use THE TABLEAU method to check if these inferences are Valid.

A) $\{p, q\} \models p \rightarrow q$



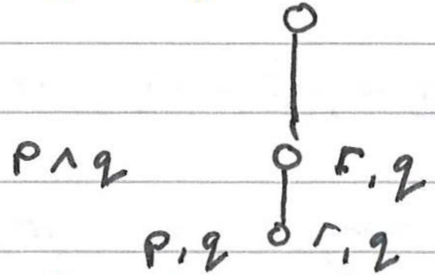
Closed, so valid inference.

B) $\{p \rightarrow q, \neg q \rightarrow \neg r\} \models p \rightarrow r$



Not a valid inference.

7. c) $\{p \wedge q, \neg q\} \models r$



Closed, so valid inference.

PAUL'S SOLUTIONS TO LOGIC BONUS 1 2023.

8 A) from $\neg\neg p$, derive p $\quad \neg\neg p \vDash p$

1	$\neg\neg p$	given
2	$\neg p$	assumption
3	$\bot$	$E_\neg$ (1, 2)
4	p	$I_\neg$ (2,3)

8B) from $p \to \neg q$ and q, derive $\neg p$

1	$p \to \neg q$	given
2	q	given
3	p	Assumption
4	$\neg q$	$E_\to$ (1, 3)
5	$\bot$	$I_\bot$ (2, 4)
6	$\neg p$	$I_\neg$ (3,5)

Explanation: We are given $p \to \neg q$ and q. Given this information, We assume p. Based on this assumption, we assume $\neg q$, Since $p \to \neg q$ must hold, However, this contradicts line 2, where we assumed to be true. Thus, we derive $\neg p$ to be true.

8 C) Derive $p \vee \neg p$ $\quad$ (so $\vDash p \vee \neg p$)

1	$\neg p \vee p$	Assume contrary.
2	p	Assume
3	$p \vee \neg p$	$E_\vee$ (2)
4	$\bot$	$I_\bot$ (1, 4)
5	$\neg p$	~~Assume~~ $I_\neg$ (2,4)
6	$\neg p \vee \neg\neg p$	$E_\vee$ (5)
7	$\bot$	$I_\bot$ (1, 6)
8	$p \vee \neg p$	$I_\neg$ (1, 7)

from $P \vee \neg q$ derive $q \to p$

8 D)

1	$P \vee \neg q$	assume
2	P	assume (1)
3	$q \to p$	$I_{\to}$ (2)
4	$\neg q$	assume ($\vee$, 1)
5	q	assume
6	$\bot$	$\exists_{\neg}$ (4,5)
7	p	$I_{\to}$ (3, 6)
8	$q \to p$	$I_{\to}$ (5, 7)
9	$q \to p$	$E_{\vee}$ (1, 2, 3, 4, 8)

8 E) from $p \to q$ derive $\neg q \to \neg p$.

1.	$p \to q$	Given
2	$\neg q$	Assume
3	P	Assume
4	q	$E_{\to}$ (1, 3)
5	$\bot$	$I_{\neg}$ (2,4)
6	$\neg p$	$I_{\neg}$ (3,5)
7	$\neg q \to \neg p$	$I_{\to}$ (2, 6)

8 f)

1	$\neg p \to \neg q$	
2	q	
3	$\neg p$	
4	$\neg q$	mp (1, 3)
5	$\bot$	$I_{\bot}$ (2,4)
6	p	$\exists_{\neg}$ (3-5)
7	$q \to p$	$I_{\to}$ (2-6)
8	$(\neg p \to \neg q) \to (q \to p)$	$I_{\to}$ (1-7)