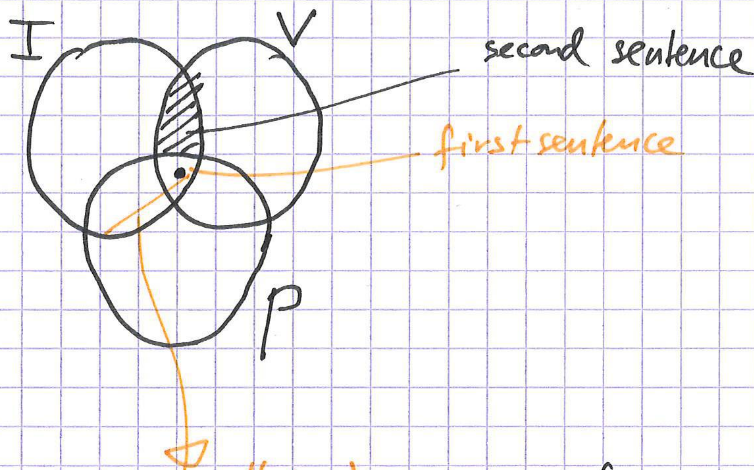
1.

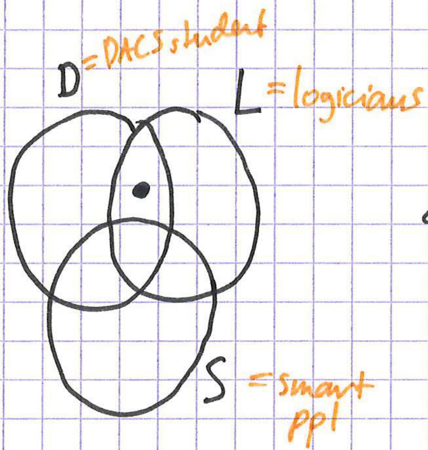
a)



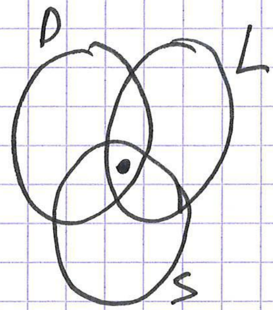
something here; so inference is valid (valid syllogism).

b)

Sentence 1:

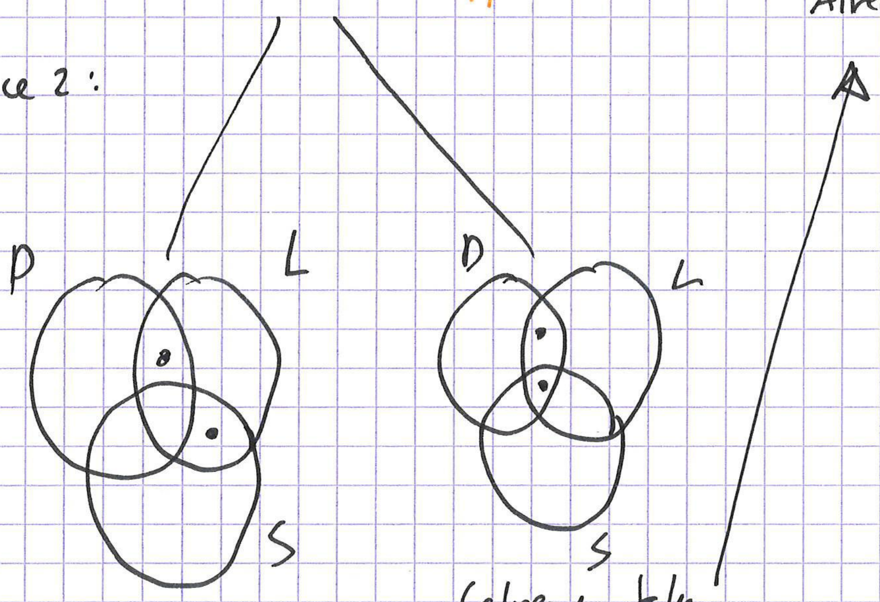


or



Already satisfies sentence 2.

sentence 2:



(already take into account)

counterexample!

1 (i) a)

$$\begin{array}{l} \exists x(Ix \land Vx) \\ \underline{\forall x((Ix \land Vx) \to Px)} \\ \exists x(Px \land Ix \land Vx) \end{array}$$

b)

$$\begin{array}{l} \exists x(Dx \land Lx) \\ \underline{\exists x(Lx \land Sx)} \\ \exists x(Dx \land Sx) \end{array}$$

$\underline{\exists x(Ix \land Vx)}, \forall x((Ix \land Vx) \to Px) \circ \exists x(Px \land Ix \land Vx)$

$Ic \land Vc, \forall x((Ix \land Vx) \to Px) \circ \exists x(Px \land Ix \land Vx)$

(twice)

$\underline{Ic \land Vc}, (Ic \land Vc) \to Pc \circ \underline{Pc \land Ic \land Vc}$

$Ic, Vc,$

$Ic \land Vc, Pc \circ Pc \land Ic \land Vc$ | $\underline{Ic \land Vc} \circ \underline{Ic \land Vc}, Pc \land Ic \land Vc$

X

$Ic \land Vc, \underline{Pc} \circ \underline{Pc}$ | $\underline{Ic \land Vc}, Pc \circ \underline{Ic \land Vc}$

X | X

closed, so valid

1 (ii) b): $\exists x(Px \wedge Lx), \exists x(Lx \wedge Sx) \circ \exists x(Px \wedge Sx)$

$Pc \wedge Lc, \exists x(Lx \wedge Sx) \circ \exists x(Px \wedge Sx)$

$Pc \wedge Lc, Ld \wedge Sd \circ \exists x(Px \wedge Sx)$

$Pc \wedge Lc, Ld \wedge Sd \circ Pc \wedge Sc, Pd \wedge Sd$

(twice)

$Pc, Lc, Ld, Sd \circ Pc \wedge Sc, Pd \wedge Sd$

$\ldots \circ Pc \ldots$ $\qquad$ $Pc, Lc, Ld, Sd \circ Sc, Pd \wedge Sd$

$Pc, Lc, Ld, Sd \circ Sc, Pd$ $\qquad$ $\ldots \circ Sd$

open

ex. 2

(i) $\exists x (Rxx)$ true in (b), false in (a)

$\neg \exists x (Rxx)$

(ii) $\forall x (Bx \vee \neg Bx)$, $\neg\neg \forall x (Bx \vee \neg Bx)$

(iii) $\neg \forall x (Bx \vee \neg Bx)$, $\neg\neg\neg \forall x (Bx \vee \neg Bx)$

ex 3

i) $\forall x (Px \rightarrow Qx) \circ \underline{\forall x (Px \vee Qx)}$

$+$

$\underline{\forall x (Px \rightarrow Qx)} \circ Pc \vee Qc$

$Pc \rightarrow Qc \circ \underline{Pc \vee Qc}$

$\underline{Pc \rightarrow Qc} \circ Pc , Qc$

$\circ Pc , Qc , Pc$ — open, invalid.

$Qc \circ Pc , Qc$ — $\times$

ex 3) (ii) $Pa, \underline{\forall x(Px \to Qx)} \circ \exists x Qx$

$Pa, Pa \to Qa \circ \underline{\exists x Qx}$

$Pa, \underline{Pa \to Qa} \circ Qa$

$\underline{Pa} \circ Qa, \underline{Pa}$ ✗ $\qquad\qquad$ $Pa, \underline{Qa} \circ \underline{Qa}$ ✗

closed, so valid.

(iii) $\top \models (\forall x Px) \leftrightarrow (\neg \exists x \neg Px)$

$\circ\ \forall x Px \underline{\leftrightarrow} \neg \exists x \neg Px$

Left branch:

$\forall x Px \circ \underline{\neg \exists x \neg Px}$

$\forall x Px, \underline{\exists x \neg Px} \circ$

$\underline{\forall Px}, \neg Pc \circ$

$Pc, \underline{\neg Pc} \circ$

$\underline{Pc} \circ \underline{Pc}$ ✗

Right branch:

$\underline{\neg \exists x \neg Px} \circ \forall x Px$

$\circ\ \underline{\forall x Px}, \exists x \neg Px$

$\circ\ Pc, \underline{\exists x \neg Px}$

$\circ\ Pc, \underline{\neg Pc}$

$Pc \circ Pc$ ✗

closed, so valid

3 (iv) $\exists x \exists y (Rxy \vee Ryx) \circ \exists x \exists y Rxy$

$\exists y (Ray \vee Rya) \circ \exists x \exists y Rxy$

$Rab \vee Rba \circ \exists x \exists y Rxy$

$Rab \vee Rba \circ \exists y Ray, \exists y Rby$

$\dots " \dots \circ Raa, Rab, \exists y Rby$

$Rab \vee Rba \circ Raa, Rab \quad Rba, Rbb$

$Rab \circ Raa, Rab, Rba, Rbb$ ×

$Rba \circ Raa, Rab, Rba, Rbb$ ×

closed, hence valid.

4)

1	$\forall x (Ax \to Bx)$	given
2	$\exists x Ax$	ass.
3.	Ac	← exist. const.
4.	$Ac \to Bc$	$E_{\forall}(1)$
5	Bc	$E_{\to}(3,4)$
6	$\exists x Bx$	$I_{\exists}(5)$
7	$\exists x Bx$	$E_{\exists}(2, 3-6)$
8.	$(\exists x Ax) \to (\exists x Bx)$	$I_{\to}(2,7)$

Ex 4 (ii)

1.	$Pa \to \forall x (Qx \to Qb)$	given
2.	Qa	given
3	$\neg Qb$	given
4	Pa	assumption
5.	$\forall x (Qx \to Qb)$	$E_{\to}(1,4)$
6	$Qa \to Qb$	$E_{\forall}(5)$
7	Qb	$E_{\to}(2,6)$
8	$\perp$	$E_{\neg}(3,7)$
9	$\neg Pa$	$I_{\neg}(4,8)$

4. (iii)

1.	$\exists x Px$	given
2	$\forall x (Px \to Qx)$	given
3	$\forall x \neg Qx$	ass.
4	Pc	c exist. constr (1)
5.	$Pc \to Qc$	$E_{\forall}(2)$
6	Qc	$E_{\to}(4,5)$
7	$\neg Qc$	$E_{\forall}(3)$
8	$\perp$	$E_{\neg}(6,7)$
9.	$\perp$	$E_{\exists}(1,4,8)$
10.	$\neg \forall x \neg Qx$	$I_{\neg}(3,9)$

4(iii) alternative attempt

1.	$\exists x\, Px$	
2	$\forall x\,(Px \to Qx)$	
3	Pc	c exist. (1)
4	$\forall x\, \neg Qx$	ass
5.	$\neg Qc$	$E_\forall(4)$
6.	$Pc \to Qc$	$E_\forall(2)$
7	Qc	$E_\to(3,6)$
8.	$\perp$	$E_\neg(5,7)$
9.	$\neg\forall x\,\neg Qx$	$\cancel{E}\, I_\neg(4,8)$
10	$\neg\forall x\,\neg Qx$	$E_\exists(1,3,9)$