$\underline{p \to q} \circ$

$\circ\, p \qquad q \,\circ$

$\forall x\, Px \circ \underline{\neg \exists x \neg Px}$

$\forall x\, Px, \underline{\exists x \neg Px} \circ$

$\forall x\, Px, \underline{\neg Pc} \circ$

$\underline{\forall x\, Px} \circ Pc$

$\underline{Pc} \circ \underline{Pc}$

$\times$

So inference is valid

$\cancel{p \to q \circ}$

$\cancel{\circ\, p \qquad p, q \,\circ}$

$\forall x\, (Px \to Qx) \models \exists x\, (Px \wedge \neg Qx)$

$\forall x\,(P_x \to Q_x) \models \exists x\,(P_x \wedge \neg Q_x)$

$\forall x\,(P_x \to Q_x) \circ \exists x\,(P_x \wedge \neg Q_x)$

first time, introduce constant;

$P_c \to Q_c \circ \exists x\,(P_x \wedge \neg Q_x)$

already a constant, so no introducing new ones!

$P_c \to Q_c \circ P_c \wedge \neg Q_c$

$\circ P_c, P_c \wedge \neg Q_c$ $\qquad$ $Q_c \circ P_c \wedge \neg Q_c$

$\circ P_c, P_c$ $\qquad$ $\circ P_c, \neg Q_c$

open

counterexample:
$P_c$ does not hold
($Q_c$ doesn't matter)
$M = (D, I, g)$ $\qquad$ $D = \{c\}$

$$\underset{1}{\bullet} \longrightarrow \underset{2}{\bullet} \longrightarrow \underset{3}{\bullet} \rightarrow \underset{4}{\bullet} \rightarrow \bullet \cdots$$

$R \; x(x+1)$

$\forall y \exists x Rxy \models \exists x \forall y Rxy$

$\forall y \exists x Rxy \circ \exists x \forall y Rxy$ ) universal step

$\exists x Rxc \circ \exists x \forall y Rxy$ ) existential step

$\forall y \exists x Rxy, Rdc \circ \exists x \forall y Rxy$ ) universal step

" " $\circ$ $\forall y Rcy, \forall y Rdy$ ) existential

$\forall y \exists x Rxy, Rdc \circ Rce, \forall y Rdy, \exists x \forall y Rxy$ ) exist.

$\forall y \exists x Rxy, Rdc \circ Ree, Rdf, \exists x \forall y Rxy$ ) univ

$\exists x Rxc, \exists x Rxd, \exists x Rxe, \exists x Rxf, Rdc \circ Rce, Rdf, \exists x \forall y Rxy$

You need extended rules to solve this.

$\exists x \forall y Rxy \models \forall y \exists x Rxy$

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$\exists x \forall y Rxy \vDash \forall y \exists x Rxy$

$\underline{\exists x \forall y Rxy} \quad \circ \quad \forall y \exists x Rxy$ ) exist.

$\forall y Rcy \quad \circ \quad \underline{\forall y \exists x Rxy}$ ) exist.

$\underline{\forall y Rcy} \quad \circ \quad \exists x Rxd$ ) univ

$Rcc, Rcd \quad \circ \quad \underline{\exists x Rxd}$ ) univ

$Rcc, \underline{Rcd} \quad \circ \quad \underline{Rcd}, Rdd$

$\times$

So inference is valid.

$\exists x \forall y (P_x \to Q_y)$ o $\forall x \exists y (P_y \to Q_x)$

) exist .

$\forall y (P_c \to Q_y)$ o ''

) exist

$\forall y (P_c \to Q_y)$ o $\exists y (P_y \to Q_d)$

) unu

$P_c \to Q_c, P_c \to Q_d$ o ''

) um

'' , $P_c \to Q_d$ o $P_c \to Q_d, P_d \to Q_d$

X

$\forall x (Px \to \exists y Qy) \circ \exists x Qx \vee \neg \exists x Px$

$'' \circ \exists x Qx , \neg \exists x Px$

$\forall x (Px \to \exists y Qy) , \exists x Px \circ \exists x Qx$ (exist.)

$'' , Pc \circ ''$ ) univ.

$'' , Pc \circ Qc$ univ

$Pc \to \exists y Qy , Pc \circ Qc$

$Pc \circ Pc , Qc$

$\times$

$Pc , \exists y Qy \circ Qc$ (exist.

$\forall x (Px \to \exists y Qy) , Pc , Qd \circ Qc , \exists x Qx$

$'' , Pc , Qd \circ Qc , Qd$

$\times$

So: valid

$\forall x Px \to \forall x Qx \circ \underline{\forall x (Px \to Qx)}$ (exist)

$'' \circ Pc \to Qc$

$\forall x Qx \circ \underline{Pc \to Qc}$

$\underline{\forall x Qx}, Pc \circ Qc$ (univ)

$\underline{Qc}, Pc \circ \underline{Qc}$

X

$\circ \forall x Px, \underline{Pc \to Qc}$

$Pc \circ \underline{\forall x Px}, Qc$ (exist)

$Pc \circ Pd, Qc$

open.

Counterexample: $D = \{c, d\}$
$P = \{c\}$
$Q = \emptyset$

Thus: Not Valid