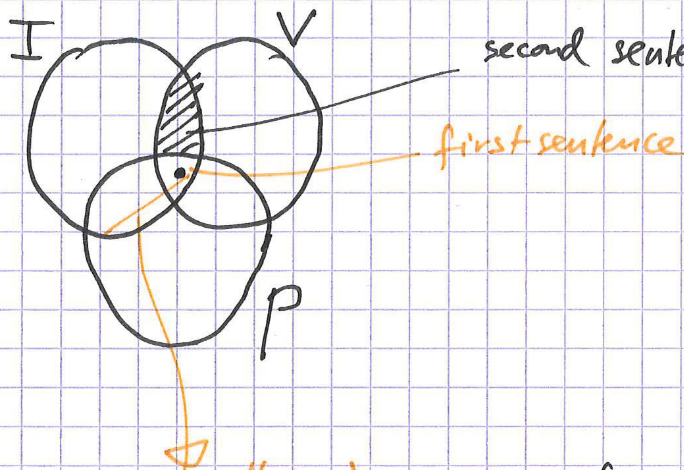
1.

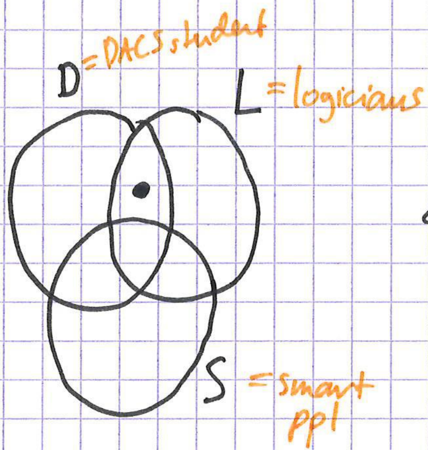
a)



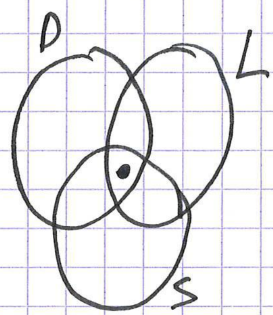
something here; so inference is valid (valid syllogism).

b)

Sentence 1:

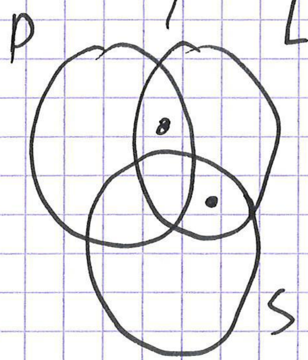


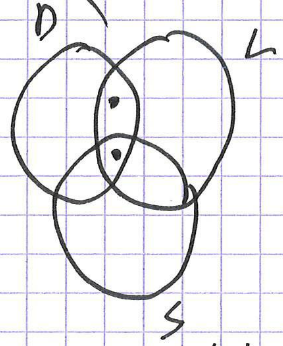
or



Already satisfies sentence 2.

sentence 2:





(already take into account)

counterexample!

1 (i)

a)

$$\begin{array}{l} \exists x\,(Ix \wedge Vx) \\ \forall x\,((Ix \wedge Vx) \to Px) \\ \hline \exists x\,(Px \wedge Ix \wedge Vx) \end{array}$$

b)

$$\begin{array}{l} \exists x\,(Dx \wedge Lx) \\ \exists x\,(Lx \wedge Sx) \\ \hline \exists x\,(Dx \wedge Sx) \end{array}$$

$\exists x\,(Ix \wedge Vx),\ \forall x\,((Ix \wedge Vx) \to Px) \circ \exists x\,(Px \wedge Ix \wedge Vx)$

$Ic \wedge Vc,\ \forall x\,((Ix \wedge Vx) \to Px) \circ \exists x\,(Px \wedge Ix \wedge Vx)$

(Twice)

$Ic \wedge Vc,\ (Ic \wedge Vc) \to Pc \circ Pc \wedge Ic \wedge Vc$

$Ic,\ Vc,$

$Ic \wedge Vc,\ Pc \circ Pc \wedge Ic \wedge Vc$ | $Ic \wedge Vc \circ Ic \wedge Vc,\ Pc \wedge Ic \wedge Vc$

$Ic \wedge Vc,\ Pc \circ Pc$ | $Ic \wedge Vc,\ Pc \circ Ic \wedge Vc$ | X

X | X

closed, so valid

1 (ci) b): $\underline{\exists x (Px \wedge Lx)}, \exists x (Lx \wedge Sx) \circ \exists x (Px \wedge Sx)$

$Pc \wedge Lc, \underline{\exists x (Lx \wedge Sx)} \circ \exists x (Px \wedge Sx)$

$Pc \wedge Lc, Ld \wedge Sd \circ \exists x (Px \wedge Sx)$

$Pc \,\underline{\wedge}\, Lc, Ld \,\underline{\wedge}\, Sd \circ Pc \wedge Sc, Pd \wedge Sd$

(twice)

$Pc, Lc, Ld, Sd \circ \underline{Pc \wedge Sc}, Pd \wedge Sd$

$\ldots \circ Pc \ldots$

$Pc, Lc, Ld, Sd \circ Sc, \underline{Pd \wedge Sd}$

$Pc, Lc, Ld, Sd \circ Sc, Pd$

open

$\ldots \circ Sd$

ex. 2

(i) $\exists x(Rxx)$ true in (b), false in (a)

$\neg\exists x(Rxx)$

(ii) $\forall x(Bx \vee \neg Bx)$, $\neg\neg\forall x(Bx \vee \neg Bx)$

(iii) $\neg\forall x(Bx \vee \neg Bx)$, $\neg\neg\neg\forall x(Bx \vee \neg Bx)$

ex 3

i) $\forall x(Px \to Qx) \circ \forall x(Px \vee Qx)$

$+$

$\forall x(Px \to Qx) \circ Pc \vee Qc$

$Pc \to Qc \circ Pc \vee Qc$

$Pc \to Qc \circ Pc, Qc$

$\circ Pc, Qc, Pc$	$Qc \circ Pc, Qc$
open, invalid.	$\times$

ex 3) (ii) $Pa, \forall x(Px \to Qx) \circ \exists x Qx$

$Pa, Pa \to Qa \circ \exists x Qx$

$Pa, Pa \to Qa \circ Qa$

$Pa \circ Qa, Pa$ $\times$ $\qquad$ $Pa, Qa \circ Qa$ $\times$

closed, so valid.

(iii) $\top \models (\forall x Px) \leftrightarrow (\neg \exists x \neg Px)$

$\circ \forall x Px \leftrightarrow \neg \exists x \neg Px$

$\forall x Px \circ \neg \exists x \neg Px$ $\qquad$ $\neg \exists x \neg Px \circ \forall x Px$

$\forall x Px, \exists x \neg Px \circ$ $\qquad$ $\circ \forall x Px, \exists x \neg Px$

$\forall Px, \neg Pc \circ$ $\qquad$ $\circ Pc, \exists x \neg Px$

$Pc, \neg Pc \circ$ $\qquad$ $\circ Pc, \neg Pc$

$Pc \circ Pc$ $\times$ $\qquad$ $Pc \circ Pc$ $\times$

closed, so valid

3 (iv) $\exists x \exists y (Rxy \vee Ryx) \circ \exists x \exists y Rxy$

$\exists y (Ray \vee Rya) \circ \exists x \exists y Rxy$

$Rab \vee Rba \circ \exists x \exists y Rxy$

$Rab \vee Rba \circ \exists y Ray, \exists y Rby$

. . . " . . $\circ Raa, Rab, \exists y Rby$

$Rab \vee Rba \circ Raa, Rab, Rba, Rbb$

$Rab \circ Raa, Rab, Rba, Rbb$ ×

$Rba \circ Raa, Rab, Rba, Rbb$ ×

closed, hence valid.

4)

1	$\forall x (Ax \to Bx)$	given
2	$\exists x Ax$	ass.
3.	Ac	c exist. const.
4.	$Ac \to Bc$	$E_\forall (1)$
5	Bc	$E_\to (3,4)$
6	$\exists x Bx$	$I_\exists (5)$
7	$\exists x Bx$	$E_\exists (2, 3-6)$
8.	$(\exists x Ax) \to (\exists x Bx)$	$I_\to (2,7)$

Ex 4 (ii)

1.	$Pa \to \forall x\,(Qx \to Qb)$	given
2.	Qa	given
3	$\neg Qb$	given
4	Pa	assumption
5.	$\forall x\,(Qx \to Qb)$	$E_{\to}(1,4)$
6	$Qa \to Qb$	$E_{\forall}(5)$
7	Qb	$E_{\to}(2,6)$
8	$\bot$	$E_{\neg}(3,7)$
9	$\neg Pa$	$I_{\neg}(4,8)$

4. (iii)

1.	$\exists x\, Px$	given
2	$\forall x(Px \to Qx)$	given
3	$\forall x \neg Qx$	ass.
4	Pc	c exist. constr (1)
5.	$Pc \to Qc$	$E_{\forall}(2)$
6	Qc	$E_{\to}(4,5)$
7	$\neg Qc$	$E_{\forall}(3)$
8	$\bot$	$E_{\neg}(6,7)$
9.	$\bot$	$E_{\exists}(1,4,8)$
10.	$\neg\forall x \neg Qx$	$I_{\neg}(3,9)$

4(iii) alternative attempt

1.	$\exists x\, P x$	
2	$\forall x\,(P x \to Q x)$	
3	$P c$	c exist. (1)
4	$\forall x\, \neg Q x$	ass
5.	$\neg Q c$	$E_\forall$ (4)
6.	$P c \to Q c$	$E_\forall$ (2)
7	$Q c$	$E_\to$ (3,6)
8.	$\perp$	$E_\neg$ (5,7)
9.	$\neg \forall x\, \neg Q x$	$\cancel{E}\, I_\neg$ (4,8)
10	$\neg \forall x\, \neg Q x$	$E_\exists$ (1,3,9)