Chain Rules for functions of one and multiple variables

Observe the use of $d(.)$ or $\partial(.)$ in the equations below!

1 Chain Rules for a function of one variable f(x)

- The derivative of a composite function of one variable $f(g(x))$ is:

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$$

- The derivative of a composite function $f(t) = f(g(x,y))$ of one variable t, where a function of 2 variables replaces t ($t = g(x,y)$), is given by the same chain rule, where $g'(x)$ is replaced by partial derivatives:

$$\frac{\partial}{\partial x}f(g(x,y)) = f'(g(x,y)) \cdot \frac{\partial g(x,y)}{\partial x}$$

$$\frac{\partial}{\partial y}f(g(x,y)) = f'(g(x,y)) \cdot \frac{\partial g(x,y)}{\partial y}$$

2 Chain Rules for a function of two variables f(x,y)

- The derivative of $z = f(x(t), y(t))$, a composite function of two variables x, y that depend on **one** variable t ($x(t), y(t)$), is given by the chain rule:

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

- The derivative of $z = f(x(s,t), y(s,t))$, a composite function of two variables x, y that depend on **two** variables s, t is given by the same chain rule, but with partial derivatives for each s, t:

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$