

Deduce  $\varphi \vee \psi$  from  $\psi \vee \varphi$

1.	$\psi \vee \varphi$	(given)	
2.	$\psi$	(assumption)	←
3.	$\psi \vee \psi$	$I_{\vee}(2)$	×
4.	$\varphi$	(assumption)	←
5.	$\varphi \vee \varphi$	$I_{\vee}(4)$	×
6.	$\varphi \vee \psi$	$E_{\vee}(1, 2, 3, 4, 5)$	×

1.	$\varphi$	(assumption)
2.	$\varphi$	(assumption)
3.	$\varphi$	(repeat of 1)
4.	$\varphi \rightarrow \varphi$	$I \rightarrow (2, 3)$
5.	$\varphi \rightarrow (\varphi \rightarrow \varphi)$	$I \rightarrow (1, 4)$

(Intermezzo:

$$\neg \varphi = "\varphi \rightarrow \perp"$$

$$\perp = "p \wedge \neg p"$$

but please forget  
that  $\neg$  ;)

Prove  $(p \wedge Q) \rightarrow (p \vee Q)$

1.  $p \wedge Q$  (assume)

2.  $Q$   $E \wedge (1)$

3.  $p \vee Q$   $I \vee (2)$

4.  $(p \wedge Q) \rightarrow (p \vee Q)$   $I \rightarrow (1, 3)$

prove  $\neg(\neg P \wedge \neg Q)$  from  $(P \vee Q)$

1.	$P \vee Q$	(given)
2.	$\neg P \wedge \neg Q$	(assumption)
3.	$\neg P$	$E_{\wedge}(2)$
4.	$\neg Q$	$E_{\wedge}(2)$
5.	$P$	(assumption)
6.	$\perp$	$E_{\neg}(5, 3)$
7.	$Q$	(assumption)
8.	$\perp$	$E_{\neg}(7, 4)$
9.	$\perp$	$E_{\vee}(1, 5, 6, 7, 8)$
10.	$\neg(\neg P \wedge \neg Q)$	$I_{\neg}(2, 9)$

Forgotten rule:

$$\frac{\perp}{\varphi} \quad "\perp"$$

Give proofs using natural deduction:

① deduce  $(R \rightarrow P) \rightarrow (R \rightarrow Q)$  from  $P \rightarrow Q$

②  $((P \vee Q) \wedge R) \rightarrow ((P \vee R) \wedge (Q \vee R))$

③  $P \vee \neg P$  (Hurd.)

④  $((P \rightarrow R) \wedge (S \rightarrow R)) \rightarrow ((P \vee S) \rightarrow R)$

⑤  $(\neg Q \wedge \neg P) \rightarrow (P \rightarrow Q)$

(5):

1.	$\neg Q \wedge \neg P$	(assumption)
2	$P$	(assumption)
3.	$\neg P$	$E \wedge(1)$
4	$\perp$	$E \neg(2, 3)$
5	$Q$	$\perp(4)$
6	$P \rightarrow Q$	$I \rightarrow(2, 6)$
7	$(\neg Q \wedge \neg P) \rightarrow (P \rightarrow Q)$	

1.	$(P \rightarrow R) \wedge (S \rightarrow R)$	(ass.)	
2.	$P \vee S$	(ass.)	
3.	$P \rightarrow R$	$E \wedge(1)$	} could have done after line 1, would've been better
4.	$S \rightarrow R$	$E \wedge(1)$	
5.	$P$	(ass.)	
6.	$R$	$E \rightarrow(3,5)$	
7.	$S$	(ass.)	
8.	$R$	$E \rightarrow(4,7)$	
9.	$R$	$E \wedge(2,5,6,7,8)$	
10.	$(P \vee S) \rightarrow R$	$I \rightarrow(2,9)$	
11.	$((P \rightarrow R) \wedge (S \rightarrow R)) \rightarrow ((P \vee S) \rightarrow R)$	$I \rightarrow(1,10)$	

①  $(P \rightarrow Q) \text{ imples } ((R \rightarrow P) \rightarrow (R \rightarrow Q))$

1.	$P \rightarrow Q$	(given)
2.	$R \rightarrow P$	(ass)
3.	$R$	(ass)
4.	$P$	$E \rightarrow (2, 3)$
5.	$Q$	$E \rightarrow (1, 4)$
6.	$R \rightarrow Q$	$I \rightarrow (3, 5)$

7.  $(R \rightarrow P) \rightarrow (R \rightarrow Q) \quad I \rightarrow (2, 6)$



1.	$(P \vee Q) \wedge R$	(ass)	
2.	$P \vee Q$	$E \wedge (1)$	← line 2 not needed!
3.	$R$	$E \wedge (1)$	
4.	$Q \vee R$	$I \vee (3)$	
5.	$P \vee R$	$I \vee (3)$	
6.	$(P \vee R) \wedge (Q \vee R)$	$I \wedge (4, 5)$	
7.	$((P \vee Q) \wedge R) \rightarrow ((P \vee R) \wedge (Q \vee R))$	$I \rightarrow (1, 6)$	

1.	$\neg (p \vee \neg p)$	
2.	$p$	(ass.
3.	$p \vee \neg p$	$\vee(2)$
4.	$\perp$	$E \neg(1,3)$
5.	$\neg p$	$I \neg(2,4)$
6.	$p \vee \neg p$	$\vee(5)$
7.	$\perp$	$E \neg(1,6)$
8.	$p \vee \neg p$	$I \neg(1,7)$