$\underline{p \to q} \circ$

$\circ\, p \qquad q \circ$

~~$\underline{p \to q} \circ$~~

~~$\circ\, p \qquad p, q \circ$~~

$\forall x\, Px \circ \underline{\neg \exists x \neg Px}$

$\forall x\, Px, \underline{\exists x \neg Px} \circ$

$\forall x\, Px, \underline{\neg Pc} \circ$

$\underline{\forall x\, Px} \circ Pc$

$\underline{Pc} \circ \underline{Pc}$

$\times$

So inference is valid

$\forall x\,(Px \to Qx) \models \exists x\,(Px \land \neg Qx)$

$\forall x \, (Px \to Qx) \models \exists x \, (Px \land \neg Qx)$

$\forall x \, (Px \to Qx) \circ \exists x \, (Px \land \neg Qx)$

$Pc \to Qc \circ \exists x \, (Px \land \neg Qx)$

first time, introduce constant;

$Pc \to Qc \circ Pc \land \neg Qc$

already a constant, so no introducing new ones!

$\circ Pc, Pc \land \neg Qc$ $\qquad$ $Qc \circ Pc \land \neg Qc$

$\circ Pc, Pc$ $\qquad$ $\circ Pc, \neg Qc$

open

counterexample:
Pc does not hold
(Qc doesn't matter)
$M = (D, I, g)$ $\qquad$ $D = \{c\}$

$$\underset{1}{\bullet} \to \underset{2}{\bullet} \to \underset{3}{\bullet} \to \underset{4}{\bullet} \to \bullet \;\cdots\; -$$

$R \quad x(x+1)$

$\forall y \exists x Rxy \models \exists x \forall y Rxy$

$\exists x \forall y Rxy \models \forall y \exists x Rxy$

$\forall y \exists x Rxy \circ \exists x \forall y Rxy$) universal step

$\exists x Rxc \circ \exists x \forall y Rxy$) existential step

$\forall y \exists x Rxy, Rdc \circ \exists x \forall y Rxy$) universal step

" " $\circ$ $\forall y Rcy, \forall y Rdy$) existential

$\forall y \exists x Rxy, Rdc \circ Rce, \forall y Rdy, \exists x \forall y Rxy$) exist.

$\forall y \exists x Rxy, Rdc \circ Ree, Rdf, \exists x \forall y Rxy$) univ

$\exists x Rxc, \exists x Rxd, \exists x Rxe, \exists x Rxf, Rdc \circ Rce, Rdf, \exists x \forall y Rxy$

You need extended rules to solve this.

$\exists x \forall y Rxy \models \forall y \exists x Rxy$

So inference is valid.

$\exists x \forall y Rxy \;\circ\; \forall y \exists x Rxy$) exist.

$\forall y Rcy \;\overset{+}{\circ}\; \forall y \exists x Rxy$) exist.

$\forall y Rcy \;\overset{+}{\circ}\; \exists x Rxd$) univ

$Rcc, Rcd \;\circ\; \exists x Rxd$) univ

$Rcc, Rcd \;\circ\; Rcd, Rdd$

X

$\exists x \forall y (P_x \to Q_y)$	o	$\forall x \exists y (P_y \to Q_x)$
		) exist.
$\forall y (P_c \to Q_y)$	o	"
		) exist
$\forall y (P_c \to Q_y)$	o	$\exists y (P_y \to Q_d)$
		) un
$P_c \to Q_c,\ P_c \to Q_d$	o	"
		) un
" $,\ P_c \to Q_d$	o	$P_c \to Q_d,\ P_d \to Q_d$
	X	

$\forall x(Px \to \exists y Qy) \circ \exists x Qx \vee \neg \exists x Px$

$" \circ \exists x Qx , \neg \exists x Px$

$\forall x(Px \to \exists y Qy) , \exists x Px \circ \exists x Qx$

(exist.)

$" , Pc \circ "$) univ.

$" , Pc \circ Qc$ univ

$Pc \to \exists y Qy , Pc \circ Qc$

$Pc \circ Pc , Qc$ ×

$Pc , \exists y Qy \circ Qc$ (exist.)

$\forall x(Px \to \exists y Qy) , Pc , Qd \circ Qc , \exists x Qx$

$" , Pc , Qd \circ Qc , Qd$ ×

So: valid

$\forall x Px \to \forall x Qx \circ \underline{\forall x (Px \to Qx)}$ (exist)

$'' \circ Pc \to Qc$

$\forall x Qx \circ \underline{Pc \to Qc}$

$\underline{\forall x Qx}, Pc \circ Qc$

(univ)

$\underline{Qc}, Pc \circ \underline{Qc}$

X

$\circ \forall x Px, \underline{Pc \to Qc}$

$Pc \circ \underline{\forall x Px}, Qc$

(exist)

$Pc \circ Pd, Qc$

open.

Counterexample: $D = \{c, d\}$

$P = \{c\}$

$Q = \emptyset$

Thus: Not Valid