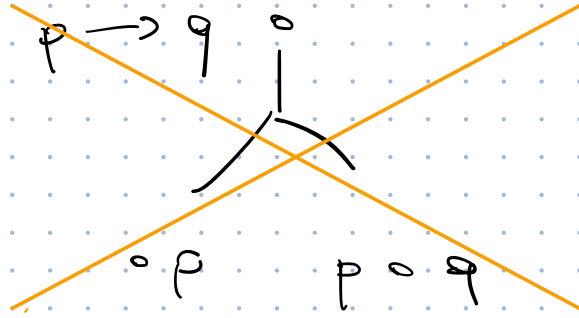
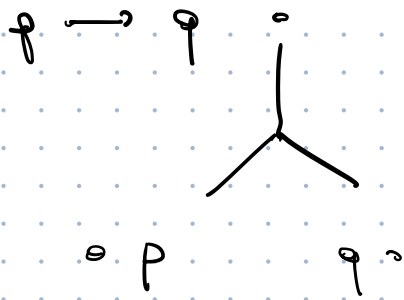


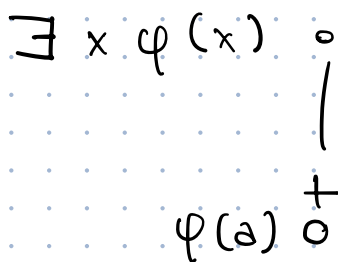
Tableau for predicate logic

Important observations



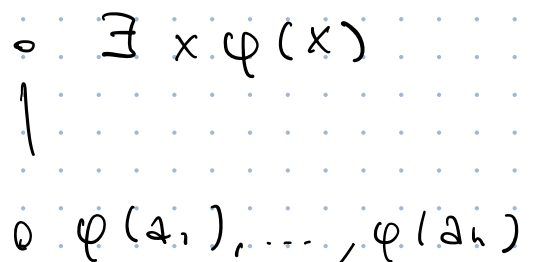
Extending tableaus and examples

Existential claim



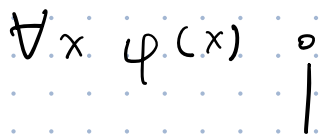
Introduce constant

Universal claim



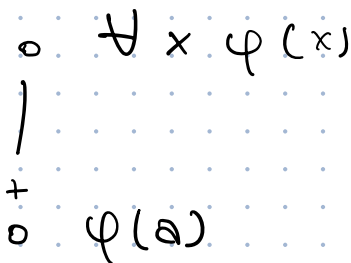
Substitute constants

Universal claim



$\phi(a_1), \dots, \phi(a_n)$

Existential claim



• If we have a universal claim with no names
↳ we add a new element → empty domains

- We deal with existential claims first
- Every time a new name is introduced, we need to reactivate all universal quantifiers

Suggested order

- (1) Logical connectives
- (2) Existential claims
- (3) Universal claims

Examples

$$(1) \quad \forall x P(x) \quad \circ \quad \neg \exists x \neg P(x)$$

existential



$$\forall x P_x, \quad \exists x \neg P_x \quad \circ$$

$$\forall x P_x, \quad \neg P_c \quad \circ$$

$$\forall x P_x \quad \circ \quad P_c$$

substitute c.

$$P_c \quad \circ \quad P_c$$

Closed \rightarrow valid
Open \rightarrow find counter.

↑ closed branch \rightarrow closed tableau
L \rightarrow valid inference

$$(2) \quad \forall x (P_x \rightarrow Q_x) \models \exists x (P_x \wedge \neg Q_x)$$

$$\forall x (P_x \rightarrow Q_x) \quad \circ \quad \exists x (P_x \wedge \neg Q_x)$$

Universal
on both sides

↓
introduce
constant

$$P_c \rightarrow Q_c$$

⊥

$$\exists x (P_x \wedge \neg Q_x)$$

We have constant \rightarrow subst.

$$P_c \rightarrow Q_c$$

∘

$$P_c \wedge \neg Q_c$$

false since P_c not hold

$$Q_c \quad \circ \quad P_c \wedge \neg Q_c$$

$$\circ \quad P_c, \quad P_c \wedge \neg Q_c$$

Open branch



$$\circ \quad P_c, \quad P_c$$

$$\circ \quad P_c, \quad \neg Q_c$$

P_c false, so $P_c \rightarrow Q_c$ is true
but $\exists c (P_c \wedge \neg Q_c)$ is false

Counterexample:

P_c does not hold
(Q_c does not matter)

$$M = (D, I, g) \quad D = \{c\}$$

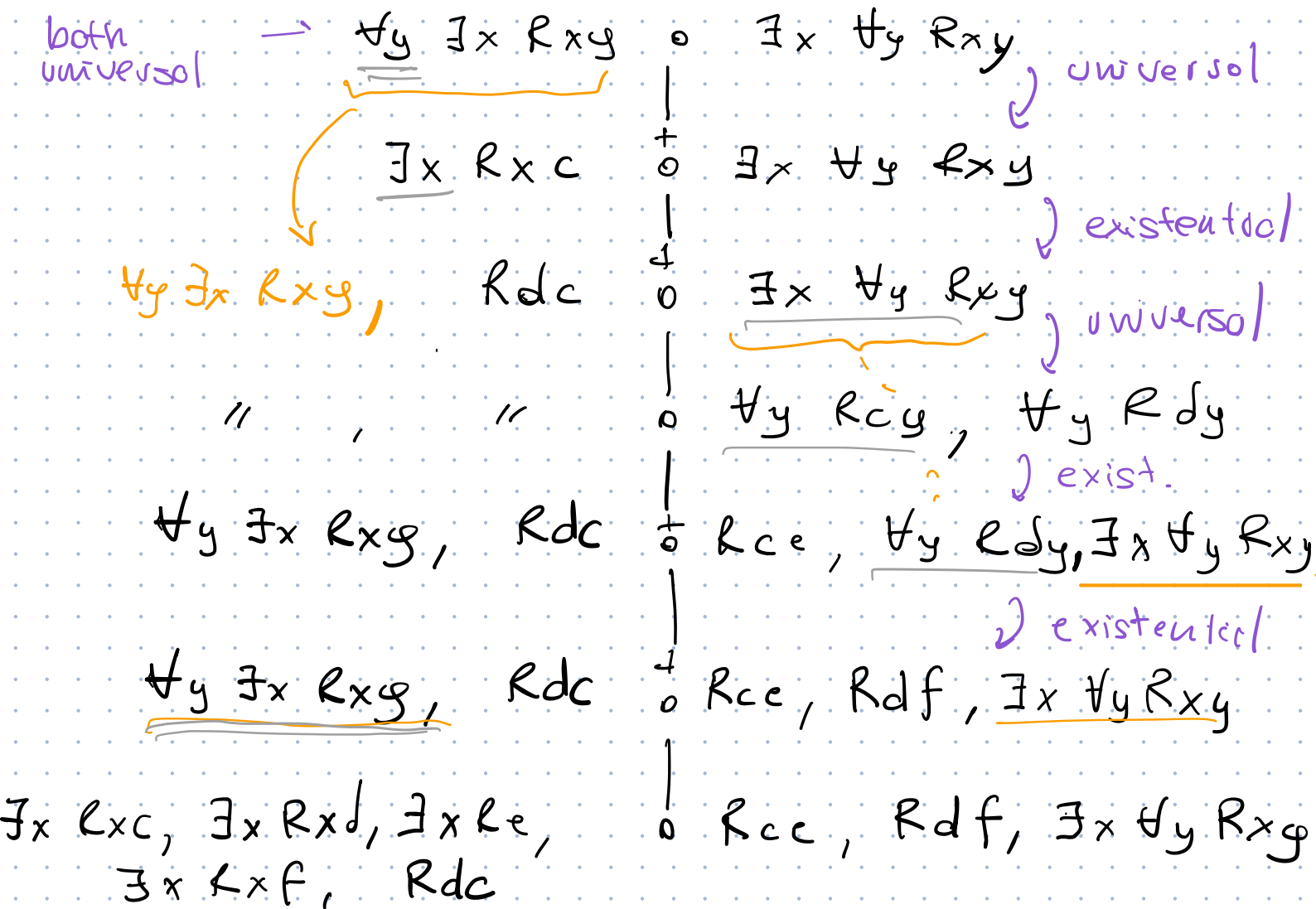
Infinite loops

Tableaux won't work with infinite models, only models with finite counterexamples

Tableaux are NOT COMPLETE for finding counterexamples in predicate logic

Exercises in class

$$(1) \forall y \exists x Rxy \models \exists x \forall y Rxy$$



↑ in this case, you need extended rules for existential claims, where on a branch you assume the newly introduced constant is one of the constants already present

↑ Not exam material

$$(2) \exists x \forall y Rxy \neq \forall y \exists x Rxy$$

$$\underline{\exists x \forall y Rxy} \circ \forall y \exists x Rxy$$

Exist - on both sides

$$\forall y Rcy \circ \forall y \exists x Rxy$$

$$\underline{\forall y Rcy} \circ \exists x Rxd$$

$$Rcc, Rcd \circ \exists x Rxd$$

$$Rcc, Rcd \circ Rcd, Rcd$$

x closed tableau

Valid inference

In exam we will NOT have cases with infinite counterexamples both exist-

$$(3) \exists x \forall y (Px \rightarrow Qy) \neq \forall x \exists y (Py \rightarrow Qx) \checkmark$$

$$\underline{\exists x \forall y (Px \rightarrow Qy)} \circ \forall x \exists y (Py \rightarrow Qx)$$

$$\forall y (Pc \rightarrow Qy) \circ \forall x \exists y (Py \rightarrow Qx)$$

$$\underline{\forall y (Pc \rightarrow Qy)} \circ \exists y (Py \rightarrow Qd)$$

$$Pc \rightarrow Qc, Pc \rightarrow Qd \circ \exists y (Py \rightarrow Qd)$$

$$Pc \rightarrow Qc, \underline{Pc \rightarrow Qd} \circ \underline{Pc \rightarrow Qd}, Pd \rightarrow Qd$$

Closed tableau \rightarrow valid inference

$$(4) \quad \forall x (P_x \rightarrow \exists y Q_y) \models \exists x Q_x \vee \neg \exists x P_x$$

$$\forall x (P_x \rightarrow \exists y Q_y) \models \exists x Q_x \vee \neg \exists x P_x$$

$$\forall x (P_x \rightarrow \exists y Q_y), \exists x P_x$$

$$\forall x (P_x \rightarrow \exists y Q_y), \quad Pc$$
$$P_c \rightarrow \exists y Ay, P_c$$

either P_C is false

PC


$$\vdash_x (P_x \rightarrow \exists_y Q_y), P_c, Q_d$$

substitute all constants

$$\forall x (Px \rightarrow \exists y Qy), Pc, \underline{Qd}$$


All branches are closed \rightarrow valid inference

(3) $\forall x P_x \rightarrow \forall x Q_x \models \forall x (P_x \rightarrow Q_x)$

$\forall x P_x \rightarrow \forall x Q_x \quad \circ \quad \forall x (P_x \rightarrow Q_x)$

it splits, so we want

\downarrow exist

$\forall x P_x \rightarrow \forall x Q_x \quad \circ \quad P_c \rightarrow Q_c$

$\forall x Q_x \quad \circ \quad P_c \rightarrow Q_c$

$\circ \quad \forall x P_x, P_c \rightarrow Q_c$

$\forall x Q_x, P_c \quad \circ \quad Q_c$

$P_c \quad \circ \quad \forall x P_x, Q_c$

$Q_c, P_c \quad \circ \quad Q_c$

$P_c \quad \circ \quad P_c, Q_c$

X

\downarrow open branch

Counterexample -

$D = \{c, d\}$

$P = \{c\}$

$Q = \emptyset$ Anything that doesn't contain c

$P_c \rightarrow Q_c$ not true