Discrete Mathematics 2021/2022 (resit)
Exam Questions

— Do not turn this page before the official start of the exam! —

First Name, Surname: ______________________________

Student ID: ______________________________

Program: Bachelor Data Science and Artificial Intelligence

Course code: KEN1130

Examiner: Dr. Steven Kelk, Dr. Marieke Musegaas and Dr. Stefan Maubach

Date/time: Wednesday 26th January 2022, 9:00h-11:00h

Format: Closed book exam

Allowed aids: Pens, simple (non-programmable) calculator from the DKE-list of allowed calculators.

Instructions to students:

- The exam consists of 8 questions on 16 pages (excluding the 1 cover page(s)).
- Fill in your name and student ID number on each page, including the cover page.
- Answer every question at the reserved space below the questions. If you run out of space, continue on the back side, and if needed, use the extra blank page.
- Ensure that you properly motivate your answers.
- Do not use red pens, and write in a readable way. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- Remember that $\mathbb{N} = \{1, 2, 3, \ldots\}$.
- **Good luck!**

The following table will be filled by the examiner:

Question:	1	2	3	4	5	6	7	8	Total
Points:	10	15	15	15	15	10	15	5	100
Score:									

Question 1 (10 points)

Fill in the truth table for the following logical proposition.

- $(p \Rightarrow q) \Rightarrow (((q \Rightarrow r) \land \neg r) \Rightarrow \neg p)$

Question 2 (15 points)

Use induction to prove the following statement.

- For all integers $n \geq 1$,

$$\sum_{i=1}^{n} i^3 = \frac{1}{4}n^2(n+1)^2$$

Question 3 (15 points)

Prove or disprove the following statements.

(a) *(7 points)* For all sets A, B, and C, if $A \not\subseteq B$ and $B \not\subseteq C$ then $A \not\subseteq C$.

(b) *(8 points)* For all sets A, B and C, if $A \subseteq B$ then $A \cap (B \cap C)^c = \emptyset$.

Question 4 (15 points)

Let $A = \mathbb{P}(\{a, b, c\})$. Let R be the relation on A defined as follows:

$$XRY \text{ means "} X \backslash Y \neq \emptyset \text{ and } Y \backslash X \neq \emptyset \text{".}$$

Is R reflexive? Symmetric? Transitive? Anti-symmetric? For each of these properties, prove or disprove that it has that property.

Question 5 (15 points)

All the following questions are about *counting*. For all questions please give an exact number at the end (i.e. don't just leave your answer as a counting equation).

(a) (*3 points)* When you decide to order pizza, you must first choose the type of crust: thin crust or deep dish. Next, you choose one topping: cheese, pepperoni, or sausage. How many possibilities are there for your pizza?

(b) (*6 points)* How many bit sequences of length eleven are there which start with one of the two bit sequences 101 or 010?

(c) (*6 points)* A student possesses six distinct books on economics, and six distinct books on mathematics. The books are arranged in a sequence on a book shelf. How many options are there, if the books on mathematics should be on the left?

Question 6 (10 points)
Prove or disprove the following statements.

Let $X = \{-2, -1, 0, 1, 2\}$ and $Y = \{-2, -1, 0, 1, 2\}$.

(a) (*2 points*) $(\forall x \in X)(\exists y \in Y)(x + y = 0)$.

(b) (*2 points*) $(\exists x \in X)(\forall y \in Y)(x + y = y)$.

(c) (*6 points*) $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{N})((z^2 \geq x^2 + y^2) \wedge (z \geq 5))$.

Question 7 (15 points)
This is a question about *functions*.

(a) (*10 points*) Let $f : \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus \{1\}$ be the function defined as follows:

$$f(x) = \frac{x+1}{x}.$$

Prove that f is a bijection.

(b) (*5 points*) Construct sets A, B and functions $f : A \to B$, $g : B \to A$ such that

- f is not an injective function,
- but, g and $f \circ g$ are both injective functions.

Question 8 (5 points)
This is a question about *set theory*.

(a) (*2 points*) Is $\{\{a, d, e\}, \{b, c\}, \{d, f\}\}$ a partition of $\{a, b, c, d, e, f\}$? Why/Why not?

(b) (*3 points*) Let $A = \{1, 2\}$, $B = \{2, 3\}$ and $C = \{3, 4\}$. Write down $(A \times B) \cap (A \times C)$.