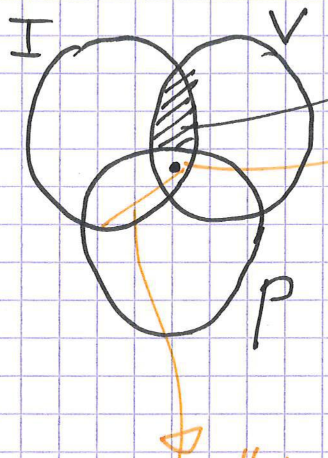
1.

a)



second sentence

first sentence

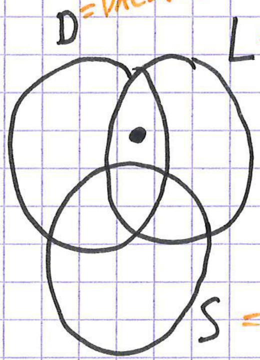
something here; so inference is valid (valid syllogism).

b)

Sentence 1:

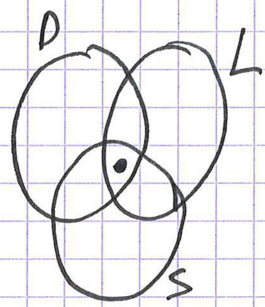
D = DACS student

L = logicians



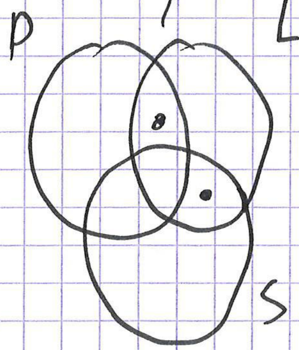
S = smart ppl

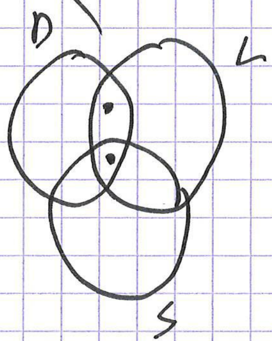
or



Already satisfies sentence 2.

sentence 2:





(already take into account)

Counterexample!

1 (i) a)

$$\frac{\exists x (Ix \land Vx) \qquad \forall x ((Ix \land Vx) \to Px)}{\exists x (Px \land Ix \land Vx)}$$

b)

$$\frac{\exists x (Dx \land Lx) \qquad \exists x (Lx \land Sx)}{\exists x (Dx \land Sx)}$$

$\exists x (Ix \land Vx), \ \forall x ((Ix \land Vx) \to Px) \circ \exists x (Px \land Ix \land Vx)$

$Ic \land Vc, \ \forall x ((Ix \land Vx) \to Px) \circ \exists x (Px \land Ix \land Vx)$

(twice)

$Ic \land Vc, \ (Ic \land Vc) \to Pc \circ Pc \land Ic \land Vc$

$Ic, Vc,$

$Ic \land Vc, Pc \circ Pc \land Ic \land Vc$ $\qquad$ $Ic \land Vc \circ Ic \land Vc, \ Pc \land Ic \land Vc$

×

$Ic \land Vc, Pc \circ Pc$ $\qquad$ $Ic \land Vc, Pc \circ Ic \land Vc$

× $\qquad$ ×

closed, so valid

1 (ii) b): $\underline{\exists x(Px \wedge Lx)}, \exists x(Lx \wedge Sx) \circ \exists x(Px \wedge Sx)$ (3)

$Pc \wedge Lc, \underline{\exists x(Lx \wedge Sx)} \circ \exists x(Px \wedge Sx)$

$Pc \wedge Lc, Ld \wedge Sd \circ \exists x(Px \wedge Sx)$

$Pc \underline{\wedge} Lc, Ld \underline{\wedge} Sd \circ Pc \wedge Sc, Pd \wedge Sd$

(twice)

$Pc, Lc, Ld, Sd \circ \underline{Pc \wedge Sc}, Pd \wedge Sd$

$\ldots \circ Pc \ldots$ | $Pc, Lc, Ld, Sd \circ Sc, \underline{Pd \wedge Sd}$

$Pc, Lc, Ld, Sd \circ Sc, Pd$ | $\ldots \circ \ldots Sd$

open

ex. 2

(i)

$\exists x (Rxx)$ true in (b), false in (a)

$\neg \exists x (Rxx)$

(ii)

$\forall x (Bx \vee \neg Bx)$, $\neg\neg \forall x (Bx \vee \neg Bx)$

(iii) $\neg \forall x (Bx \vee \neg Bx)$, $\neg\neg\neg \forall x (Bx \vee \neg Bx)$

ex 3

i) $\forall x (Px \to Qx) \circ \underline{\forall x (Px \vee Qx)}$

$+$

$\underline{\forall x (Px \to Qx)} \circ Pc \vee Qc$

$Pc \to Qc \circ \underline{Pc \vee Qc}$

$\underline{Pc \to Qc} \circ Pc, Qc$

$\circ Pc, Qc, Pc$ $Qc \circ Pc, Qc$

open, $\times$

invalid.

ex 3) (ii) $Pa, \forall x(Px \to Qx) \circ \exists x Qx$

$Pa, Pa \to Qa \circ \exists x Qx$

$Pa, Pa \to Qa \circ Qa$

$Pa \circ Qa, Pa$ $Pa, Qa \circ Qa$

$\times$ $\times$

closed, so valid.

(iii) $\top \models (\forall x Px) \leftrightarrow (\neg \exists x \neg Px)$

$\circ \forall x Px \leftrightarrow \neg \exists x \neg Px$

Left branch:

$\forall x Px \circ \neg \exists x \neg Px$

$\forall x Px, \exists x \neg Px \circ$

$\forall Px, \neg Pc \circ$

$Pc, \neg Pc \circ$

$Pc \circ Pc$

$\times$

Right branch:

$\neg \exists x \neg Px \circ \forall x Px$

$\circ \forall x Px, \exists x \neg Px$

$\circ Pc, \exists x \neg Px$

$\circ Pc, \neg Pc$

$Pc \circ Pc$

$\times$

closed, so valid

3 (iv) $\exists x \exists y (Rxy \vee Ryx) \circ \exists x \exists y Rxy$

$\exists y (Ray \vee Rya) \circ \exists x \exists y Rxy$

$Rab \vee Rba \circ \exists x \exists y Rxy$

$Rab \vee Rba \circ \exists y Ray, \exists y Rby$

$\ldots \circ Raa, Rab, \exists y Rby$

$Rab \vee Rba \circ Raa, Rab \quad Rba, Rbb$

$Rab \circ Raa, Rab, Rba, Rbb$ $\times$ | $Rba \circ Raa, Rab, Rba, Rbb$ $\times$

closed, hence valid.

4)

1	$\forall x (Ax \to Bx)$	given
2	$\exists x Ax$	ass.
3.	Ac	c exist. const.
4.	$Ac \to Bc$	$E_\forall (1)$
5	Bc	$E_\to (3,4)$
6	$\exists x Bx$	$I_\exists (5)$
7	$\exists x Bx$	$E_\exists (2, 3-6)$
8.	$(\exists x Ax) \to (\exists x Bx)$	$I_\to (2,7)$

Ex 4 (ii)

1. $Pa \to \forall x\,(Qx \to Qb)$ — given
2. Qa — given
3. $\neg Qb$ — given
4. Pa — assumption
5. $\forall x\,(Qx \to Qb)$ — $E_{\to}(1,4)$
6. $Qa \to Qb$ — $E_{\forall}(5)$
7. Qb — $E_{\to}(2,6)$
8. $\bot$ — $E_{\neg}(3,7)$
9. $\neg Pa$ — $I_{\neg}(4,8)$

4. (iii)

1. $\exists x\, Px$ — given
2. $\forall x\,(Px \to Qx)$ — given
3. $\forall x\, \neg Qx$ — ass.
4. Pc — c exist. constr (1)
5. $Pc \to Qc$ — $E_{\forall}(2)$
6. Qc — $E_{\to}(4,5)$
7. $\neg Qc$ — $E_{\forall}(3)$
8. $\bot$ — $E_{\neg}(6,7)$
9. $\bot$ — $E_{\exists}(1,4,8)$
10. $\neg \forall x \neg Qx$ — $I_{\neg}(3,9)$

4(iii) alternative attempt

1.	$\exists x\, Px$	
2	$\forall x\,(Px \to Qx)$	
3	Pc	c exist. (1)
4	$\forall x\, \neg Qx$	ass
5.	$\neg Qc$	$E_{\forall}(4)$
6.	$Pc \to Qc$	$E_{\forall}(2)$
7	Qc	$E_{\to}(3,6)$
8.	$\perp$	$E_{\neg}(5,7)$
9.	$\neg\forall x\, \neg Qx$	~~E~~ $I_{\neg}(4,8)$
10	$\neg\forall x\, \neg Qx$	$E_{\exists}(1,3,9)$