Calculus - lecture 7 : SERIES AND SEQUENCES

* Sequences (Thomas, 10.1 + Adams, 9.1)
* Infinite Series (Adams, 9.2)
* Convergence tests for positive series (Adams 9.3)
* Absolute and Conditional Convergence

I SEQUENCES

A sequence $\{a_n\}$ = list of numbers $a_1, a_2, \dots, a_n \dots$ in a given order

(a_1 → term, a_2 → index)

usually, sequences are infinite = have no last element

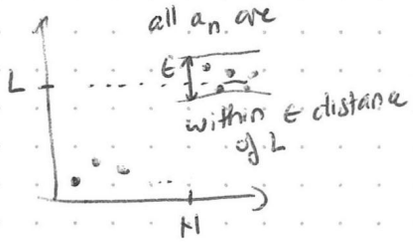
* a sequence can be seen as a function : $f: \mathbb{N} \to \mathbb{R} : n \to a_n = f(n)$

Examples
- $\sqrt{n}$, $\frac{1}{n}$, $(-1)^n$ (this is a formula for the general term)
- $a_1 = 1$, $a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$ (recursive formula)
  ↳ $1, 1, 2, 3, 5, \dots$ → FIBONNACI SEQUENCE
- $1, -\frac{x^2}{2}, \frac{x^4}{4!}, \dots$ (a pattern)

⇝ such sequences / series are powerful ways to approximate (transcendental) functions by polynomials, express irrational numbers, ...

* main question about sequences : does it converge ?

CONVERGENCE: a sequence $a_n \to L$ if $\forall \epsilon > 0 \ \exists N \in \mathbb{N}$, such that, if $n \geq N$, $|a_n - L| < \epsilon$



↳ L is the limit of a_n ($\lim_{n \to \infty} a_n = L$)

↳ if the limit does not exist, $\{a_n\}$ DIVERGES

↳ if the limit is infinity, a_n DIVERGES TO INFINITY

$a_n \to \infty$ if $\forall M \ \exists N$, such that, if $n \geq N$, $a_n > M$ (analogue for $-\infty$)

Examples : $\frac{1}{n} \to 0$, $\frac{n}{n+1} \to 1$, $(-1)^n$ diverges, $n \to +\infty$

→ if the sequence can be seen as a real function, i.e. if $f(x)$ ($x \in \mathbb{R}$) is defined for $x \geq n_0$, and $a_n = f(n)$ for $n \geq n_0$, then $\lim_{x \to \infty} f(x) = L \Rightarrow a_n \to L$

for example : $\frac{1}{n}$, $\sqrt{n}$, ...

opposite is not true : $\cos(2\pi n)$

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