Practice Exam Questions Week 7, **Linear Algebra**

1. Let $V = \begin{bmatrix} 2 & -4 & 1 \\ -3 & -1 & 1 \\ 1 & 5 & 1 \end{bmatrix}$. Show that the columns of V are orthogonal to each other.

2. Consider the following matrix A and vectors $\mathbf{v}_1$ and $\mathbf{v}_2$:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

   a. Verify that $\mathbf{v}_1$ and $\mathbf{v}_2$ are eigenvectors of A.
   b. Orthogonally diagonalize the matrix A.

3. True or false? If the given statement is true, give a brief explanation. If it is false, give a counterexample.

   a. If U and V are 3×3 orthogonal matrices, then their product $W = UV$ is also a 3×3 orthogonal matrix.
   b. If the columns of a 3×3 matrix Q are orthogonal to each other, then $Q^T Q = I$.
   c. Every linearly independent set in $\mathbb{R}^n$ is an orthogonal set.
   d. If $A^T = A$ and if vectors $\mathbf{u}$ and $\mathbf{v}$ satisfy $A\mathbf{u} = 3\mathbf{u}$ and $A\mathbf{v} = 4\mathbf{v}$, then $\mathbf{u} \cdot \mathbf{v} = 0$.
   e. There are symmetric matrices that are not orthogonally diagonalizable.