Thursday, December 7, 2023 4:02 PM

$p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \ \circ\ p \wedge (q \wedge r)$

$p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \ \circ\ p$ $\qquad$ $p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \ \circ\ q \wedge r$

$p, \dots \circ p$ $\qquad$ $q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \ \circ\ p$

X

$q,\ p \to (q \vee r)\ \circ\ p,\ \neg(r \wedge p)$ $\qquad$ $q,\ p \to (q \vee r),\ \neg q\ \circ\ p$

$r \wedge p,\ q,\ p \to (q \vee r)\ \circ\ p$ $\qquad$ q " $\circ\ p, q$

X

$r, p,$ "" $\circ\ p$

X

$$p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \circ q \wedge r$$

$$p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \circ q \qquad\qquad p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \circ r$$

$$q, \ldots \circ q \quad \times \qquad\qquad p,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \circ q$$

$$p, \ldots \circ q, p \quad \times \qquad\qquad p,\ q \vee r,\ \neg(r \wedge p) \to \neg q \circ q$$

$$\ldots, q, \ldots \circ q \quad \times \qquad\qquad p,\ r,\ \neg(r \wedge p) \to \neg q \circ q$$

$$p, r \circ q, \neg(r \wedge p) \qquad\qquad p, r, \neg q \circ q$$

$$p, r, r \wedge p \circ q \qquad\qquad p, r \circ q$$

$$p, r \circ q$$

only open branch: countex. is $V(p) = V(r) = 1$, $V(q) = 0$

$p \vee q, p \to (q \vee r), \neg(r \wedge p) \to \neg q$ or r

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$p, p \to (q \vee r), \neg(r \wedge p) \to \neg q$ or r

$q, p \to (q \vee r), \neg(r \wedge p) \to \neg q$ or r

$p, \dots$ or, p

×

$p, q \vee r, \neg(r \wedge p) \to \neg q$ or r

$p, r, \dots$ or

×

$p, q, \neg(r \wedge p) \to \neg q$ or r

$p, q, \neg q$ or r

p, q or r, $\neg(r \wedge p)$

p, q or r, q

×

$p, q, r \wedge p$ or r

p, q, r, p or r ×

$\neg r, \neg q, \neg(r \to q)$

1. $p \to \neg(r \to q)$ given
2. p
3. $\neg(r \to q)$ $E\to(1,2)$
4. q ass
5. r ass
6. q (repeat 4)
7. $r \to q$ $I\to(5,6)$
8. $\bot$ $E\neg(3,7)$
9. $\neg q$ $I\neg(4,8)$

10 $\neg r$ (ass)

11

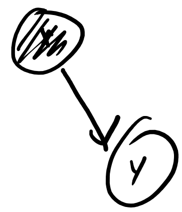
⋮

$\neg q \wedge r$

$p \to (\neg q \wedge r)$

1	$\neg(\neg p \lor \neg q)$	(given)
2	$\neg p$	(ass.)
3.	$\neg p \lor \neg q$	$I_\lor(2)$
4.	$\bot$	$E_\neg(1,3)$
5	p	$I_\neg(2,4)$
6	$\neg q$	(ass)
7	$\neg p \lor \neg q$	$I_\lor(6)$
8	$\bot$	$E_\neg(1,7)$
9	q	$I_\neg(6,8)$
10	$p \land q$	$I_\land(5,9)$

- $\forall x (Px \lor \neg Px) \land \forall x \forall y (Sxy \lor \neg Sxy)$
- $\exists x \exists y (Px \land \neg Py \land Sxy)$



0 $\underline{\forall x}((\exists y R_{yx}) \to R_{xx}))$ (exist)

|

+ $(\exists y R_{ya}) \underline{\to} R_{aa}$

0

|

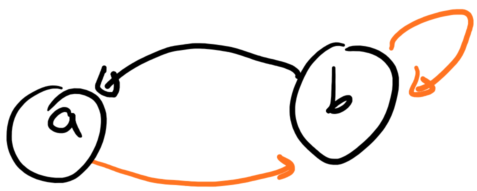
$\exists y R_{ya}$ 0 R_{aa}

|

$\perp$

R_{ba} 0 R_{aa}

exist.



$$\left.\begin{array}{ll} 1. & \forall x\,(Px \to \exists y\, Rxy) \\ 2. & \forall x\,(\exists y\, Ryx \to \neg Px) \\ 3. & \exists x\, Px \end{array}\right\}\ \text{given}$$

4	Pc	c exist (3)
5.	$Pc \to \exists y\, Rcy$	$E_\forall$ (1)
6.	$\exists y\, Rcy$	$E_\to$ (4,5)
7.	Rcd	d exist const (6)
8.	$\exists y\, Ryd \to \neg Pd$	$E_\forall$ (2)
9	$\exists y\, Ryd$	$I_\exists$ (7) ($c \to y$)
10	$\neg Pd$	$E_\to$ (8,9)
11	$\exists x(\neg Px)$	$I_\exists$ (10)
12	$\exists x(\neg Px)$	$E_\exists$ (7,11)
13	$\exists x(\neg Px)$	$E_\exists$ (4,12)

1.	$\forall x (Ax \to \neg Bx)$	given
2	$\exists x (Bx \land Cx)$	given
3	$Bc \land Cc$	c exist. const (2)
4	$Ac \to \neg Bc$	$E\forall$ (1)
5	Cc	$E\land$ (3)
6	Ac	assume.
7	$\neg Bc$	$E\to$ (4,6)
8	Bc	$E\land$ (3)
9	$\bot$	$E\neg$ (7,8)
10	$\neg Ac$	$I\neg$ (6,9)
11	$Cc \land \neg Ac$	$I\land$ (5,10)
12	$\exists x (Cx \land \neg Ax)$	$I\exists$ (11)
13	$\exists x (Cx \land \neg Ax)$	$E\exists$ (3,12)

X

$\bot$

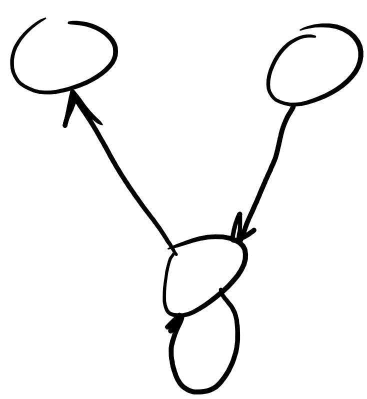
$\neg X$ $\quad I\neg$

X

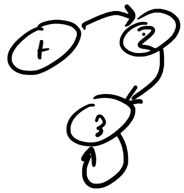
$\neg X$

$\bot$ $\quad E\neg(\cdot , \cdot)$

$$\{(1,1),(2,2),(3,3),(2,3),(3,1),(2,1)\}$$



$$(\_ \quad \_)^*$$



$R = \{(1,2), (2,3), (3,3)$

$\subseteq S \times S$

S

$\subseteq S$

$S \times S \times S$