$p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \ \circ\ p \wedge (q \wedge r)$

$p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \ \circ\ p$ $\qquad$ $p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \ \circ\ q \wedge r$

$p, \dots \circ p$ $\qquad$ $q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \ \circ\ p$

X

$q,\ p \to (q \vee r) \ \circ\ p,\ \neg(r \wedge p)$ $\qquad$ $q,\ p \to (q \vee r),\ \neg q \ \circ\ p$

$r \wedge p,\ q,\ p \to (q \vee r) \ \circ\ p$ $\qquad$ $q \quad " \quad \circ\ p, q$

X

$r, p, \quad "" \quad \circ\ p$

X

$p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \circ q \wedge r$

$p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \circ q$

$p \vee q,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \circ r$

$q, \dots \circ q$

×

$p,\ p \to (q \vee r),\ \neg(r \wedge p) \to \neg q \circ q$

$p, \dots \circ q, p$

×

$p,\ q \vee r,\ \neg(r \wedge p) \to \neg q \circ q$

$\dots, q, \dots \circ q$

×

$p,\ r,\ \neg(r \wedge p) \to \neg q \circ q$

$p, r \circ q, \neg(r \wedge p)$

$p, r, r \wedge p \circ q$

$p, r \circ q$

$p, r, \neg q \circ q$

$p, r \circ q$

only open branch: countex. is $V(p)=V(r)=1$, $V(q)=0$

$p \lor q,\ p \to (q \lor r),\ \neg(r \land p) \to \neg q \circ r$

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$p,\ p \to (q \lor r),\ \neg(r \land p) \to \neg q \circ r$ $\qquad$ $q,\ p \to (q \lor r),\ \neg(r \land p) \to \neg q \circ r$

$p, \dots \circ r, p$ $\qquad$ $p,\ q \lor r,\ \neg(r \land p) \to \neg q \circ r$

X

$p, r, \dots \circ r$ $\qquad$ $p,\ q,\ \neg(r \land p) \to \neg q \circ r$

X

$p,\ q,\ \neg q \circ r$ $\qquad$ $p,\ q \circ r,\ \neg(r \land p)$

$p,\ q \circ r,\ q$ $\qquad$ $p,\ q,\ r \land p \circ r$

X

$p,\ q,\ r,\ p \circ r$ $\quad$ X

1. $p \to \neg(r \to q)$ given

2. p

3. $\neg(r \to q)$ $E\to(1,2)$

4 q ass

5 r ass

6 q (repeat 4)

7 $r \to q$ $I\to(5,6)$

8 $\bot$ $E\neg(3,7)$

9 $\neg q$ $I\neg(4,8)$

$\neg r, \neg q, \neg(r \to q)$

10 $\neg r$ (ass)

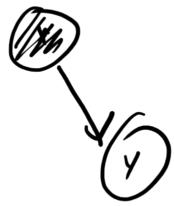
11

$\neg q \wedge r$

$p \to (\neg q \wedge r)$

1	$\neg(\neg p \vee \neg q)$	(given)
2	$\neg p$	(ass.)
3.	$\neg p \vee \neg q$	$I_\vee(2)$
4.	$\perp$	$E_\neg(1,3)$
5	p	$I_\neg(2,4)$
6	$\neg q$	(ass)
7	$\neg p \vee \neg q$	$I_\vee(6)$
8	$\perp$	$E_\neg(1,7)$
9	q	$I_\neg(6,8)$
10	$p \wedge q$	$I_\wedge(5,9)$

- $\forall x(Px \lor \neg Px) \land \forall x \forall y(Sxy \lor \neg Sxy)$
- $\exists x \exists y(Px \land \neg Py \land Sxy)$



0 $\underline{\forall x}((\exists y R_{yx}) \to R_{xx})$) (exist)

| +

0 $(\exists y R_{ya}) \underline{\to} R_{aa}$

|

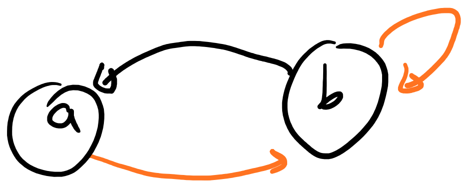
$\exists y R_{ya}$ 0 R_{aa}

|

+

R_{ba} 0 R_{aa}

exist.



1. $\forall x (Px \to \exists y Rxy)$
2. $\forall x (\exists y Ryx \to \neg Px)$
3. $\exists x Px$

given

4 Pc c exist (3)
5. $Pc \to \exists y Rcy$ $E\forall$ (1)
6. $\exists y Rcy$ $E\to$ (4,5)
7. Rcd d exist const (6)
8. $\exists y Ryd \to \neg Pd$ $E\forall$(2)
9 $\exists y Ryd$ $I\exists$ (7) (c→y)
10 $\neg Pd$ $E\to$ (8,9)
11 $\exists x (\neg Px)$ $I\exists$ (10)
12 $\exists x (\neg Px)$ $E\exists$ (7,11)
13 $\exists x (\neg Px)$ $E\exists$ (4,12)

1.	$\forall x\,(Ax \to \neg Bx)$	} given
2	$\exists x\,(Bx \land Cx)$	
3	$Bc \land Cc$	c exist. const (2)
4	$Ac \to \neg Bc$	$E\forall$ (1)
5	Cc	$E\land$ (3)
6	Ac	assume.
7	$\neg Bc$	$E\to$ (4,6)
8	Bc	$E\land$ (3)
9	$\bot$	$E\neg$ (7,8)
10	$\neg Ac$	$I\neg$ (6,9)
11	$Cc \land \neg Ac$	$I\land$ (5,10)
12	$\exists x(Cx \land \neg Ax)$	$I\exists$ (11)
13	$\exists x\,(Cx \land \neg Ax)$	$E\exists$ (3,12)

X

$\bot$

$\neg X$ $\quad I\neg$

X

$\neg X$

$\bot$ $\quad E\neg(\cdot\,,\,)$

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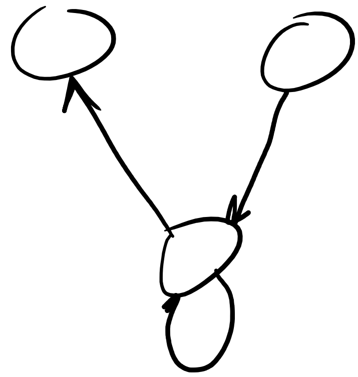
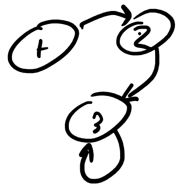
$$\{(1,1),(2,2),(3,3),(2,3),(3,1),(2,1)\}$$



$$(\ \_\ \ \_\ )^*$$



$R = \{ (1,2), (2,3), (3,3)$

$\subseteq S \times S$

S

$\subseteq S$

$S \times S \times S$