Answers to some exercises from the book.

4.2

1. Rule of sum: $3+4+3=\boxed{10}$

2. Rule of product: $3\cdot4\cdot3=\boxed{36}$

3. Rule of product:
   * PC: $3\cdot4=12$
   * PB: $3\cdot3=9$
   * CB: $4\cdot3=12$

   Rule of sum: $12+9+12=\boxed{33}$

4. Rule of product: $3\cdot4=\boxed{12}$

5. Rule of product: $2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2=2^7=\boxed{128}$

6. Rule of product: $10\cdot10\cdot10\cdot26\cdot26\cdot26=\boxed{17\ 576\ 000}$.
   (Note that the first letter is fixed, because the year is fixed).
   "in any one year"

7. With 5 digits: letter 4 is the same as letter 2 and letter 5 is the same as letter 1. Hence, rule of product: $26\cdot26\cdot26=\boxed{17576}$
   So, we only need to choose letter 1, 2 and 3.

   With 6 digits: we only need to choose letter 1, 2 and 3. (Because letter 4 = letter 3, letter 5 = letter 2, letter 6 = letter 1). So, again $\boxed{17576}$.

8. $5\cdot4\cdot3\cdot2\cdot1=5!=\boxed{120}$

9. Rule of product:
   * length 1: 2
   * length 2: $2\cdot2=4$
   * length 3: $2\cdot2\cdot2=8$
   * length 4: $2\cdot2\cdot2\cdot2=16$.

   Rule of sum: $2+4+8+16=\boxed{30}$.

10. * 1 digit: 1
    * 2 digits: 7 on the left: $1*9$ (it's 9 because we can't repeat the 7)
      7 on the right: $8*1$ (it's 8 because we can't start with a 0 or 7)
    * 3 digits: 7 on the left: $1*9*9$
      7 in the middle: $8*1*9$
      7 on the right: $8*9*1$

    Rule of sum: $1+9+8+81+72+72=\boxed{243}$

4.3

1. a) Rule of product: $12 \cdot 2 \cdot 4 = \boxed{96}$
   b) Rule of product: $2 \cdot 4 = \boxed{8}$

Sorry, there is a mistake. It should be:
(a) 3*12*2*4=288
(b) 3*2*4=24

2. $k=4$, $n=7$ rep allowed? No. order important? No. So, $\binom{7}{4} = \boxed{35}$

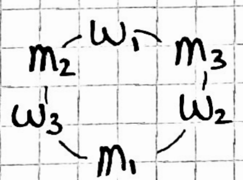
3. Skip this question.

4. For each nucleotide, we have 4 possibilies. Each chromosome contains 4×10^9 nucleotides $\Big\}$ => There are $4^{4\times10^9}$ possibilities for a chromosome.
Moreover, each human has 46 chromosomes. So, there are $\left(4^{4\times10^9}\right)^{46}$ genetically different human beings.

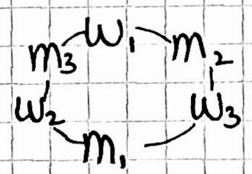
5. $\underbrace{A, B}, C, D, E, F$
They want to sit together, so consider them as one person. So, we have 5 people.
There are $\frac{5!}{5} = 4!$ ways of seating these 5 people (see Example 23).
We still need to multiply this number with 2 because either A is to the left of B or the other way around. So, $4! \cdot 2 = \boxed{48}$.

6. a) 3M, 3W. # permutations of the 3 women: $3!$; # permutations of the 3 men: $3!$ $\Big\}$ # permutations where the sexes alternate: $3!3!$ and we start with a woman.
Hence, $\frac{3!3!}{3} = \boxed{12}$
(Divide by 3 because $W_1M_1W_2M_2W_3M_3$ and $W_2M_2W_3M_3W_1M_1$ give the same seating plan as $W_3M_3W_1M_1W_2M_2$).

   b) $\boxed{2}$ namely



or



7. $k=8$, $n=2$ rep allowed? yes order important? yes So, $2^8 = \boxed{256}$

8. a) $3!4!2! = \boxed{288}$
   b) There are $3!$ permutations of $\{C, M, A\}$, and each permutation has 288 possibilities.
   So, $3! \cdot 288 = \boxed{1728}$.
   c) $(3+4+2)! = 9! = \boxed{362\,880}$.

9. a) Rule of product: $6 \cdot 9 \cdot 6 = \boxed{324}$
   b) Rule of product: $2 \cdot 2 \cdot 2 = \boxed{8}$

10. * 1 female member: Choosing 1 female member: $\binom{3}{1}$. Choosing 11 male members: $\binom{58}{11} \Rightarrow \binom{3}{1}\binom{58}{11}$
* 2 female members: $\binom{3}{2} \cdot \binom{58}{10}$
* 3 female members: $\binom{3}{3} \cdot \binom{58}{9}$.

Rule of sum: $\binom{3}{1}\binom{58}{11} + \binom{3}{2}\binom{58}{10} + \binom{3}{3}\binom{58}{9} = \boxed{850\,264\,180\,935}$

(11). ~~k≤7~~ ~~n=8~~ Skip this question (it's unclear whether, for example, whether the two doughnuts are the same or not...)

(12) a) $k=5$ rep. allowed? yes
$n=4$ order important? no.

$\binom{(n-1)+k}{k} = \binom{8}{5} = \boxed{56}$

b) Consider type I and type II. Then, at least 1 of type I and 1 of type II. So, $5-2=3$ cards are left, which we can choose from type I and type II.
$k=3$ $\binom{(n-1)+k}{k} = \binom{4}{3} = 4.$
$n=2.$
There ~~other combinations~~ are $\binom{4}{2}$ possibilities to choose the two types.
So, $\binom{4}{2}\cdot\binom{4}{3} = \boxed{24}$

c). As much variety as possible means at least one card of each type. So, we only need to ~~allocat~~ choose the 5^{th} card. For this we have $\boxed{4}$ possibilities.

(13). * length 1: 26 possible passwords.
* length 2: $26^2 = 676$ possible passwords.
* length 3: $26^3 = 17\,576$ possible passwords.
So, minimum length of the passwords is $\boxed{3}$.