EXERCISES 5.5

Evaluate the definite integrals in Exercises 1–20.

1. $\displaystyle\int_0^2 x^3\,dx$ 2. $\displaystyle\int_0^4 \sqrt{x}\,dx$

3. $\displaystyle\int_{1/2}^1 \frac{1}{x^2}\,dx$ 4. $\displaystyle\int_{-2}^{-1}\left(\frac{1}{x^2}-\frac{1}{x^3}\right)dx$

5. $\displaystyle\int_{-1}^2 (3x^2-4x+2)\,dx$ 6. $\displaystyle\int_1^2\left(\frac{2}{x^3}-\frac{x^3}{2}\right)dx$

7. $\displaystyle\int_{-2}^2 (x^2+3)^2\,dx$ 8. $\displaystyle\int_4^9\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx$

9. $\displaystyle\int_{-\pi/4}^{-\pi/6}\cos x\,dx$ 10. $\displaystyle\int_0^{\pi/3}\sec^2\theta\,d\theta$

11. $\displaystyle\int_{\pi/4}^{\pi/3}\sin\theta\,d\theta$ 12. $\displaystyle\int_0^{2\pi}(1+\sin u)\,du$

13. $\displaystyle\int_{-\pi}^{\pi}e^x\,dx$ 14. $\displaystyle\int_{-2}^{2}(e^x-e^{-x})\,dx$

15. $\displaystyle\int_0^e a^x\,dx \quad (a>0)$ 16. $\displaystyle\int_{-1}^1 2^x\,dx$

17. $\displaystyle\int_{-1}^1 \frac{dx}{1+x^2}$ 18. $\displaystyle\int_0^{1/2}\frac{dx}{\sqrt{1-x^2}}$

19. $\displaystyle\int_{-1}^1 \frac{dx}{\sqrt{4-x^2}}$ 20. $\displaystyle\int_{-2}^0 \frac{dx}{4+x^2}$

Find the area of the region R specified in Exercises 21–32. It is helpful to make a sketch of the region.

21. Bounded by $y=x^4$, $y=0$, $x=0$, and $x=1$
22. Bounded by $y=1/x$, $y=0$, $x=e$, and $x=e^2$
23. Above $y=x^2-4x$ and below the x-axis
24. Bounded by $y=5-2x-3x^2$, $y=0$, $x=-1$, and $x=1$
25. Bounded by $y=x^2-3x+3$ and $y=1$
26. Below $y=\sqrt{x}$ and above $y=\dfrac{x}{2}$
27. Above $y=x^2$ and to the right of $x=y^2$
28. Above $y=|x|$ and below $y=12-x^2$
29. Bounded by $y=x^{1/3}-x^{1/2}$, $y=0$, $x=0$, and $x=1$
30. Under $y=e^{-x}$ and above $y=0$ from $x=-a$ to $x=0$
31. Below $y=1-\cos x$ and above $y=0$ between two consecutive intersections of these graphs
32. Below $y=x^{-1/3}$ and above $y=0$ from $x=1$ to $x=27$

Find the integrals of the piecewise continuous functions in Exercises 33–34.

33. $\displaystyle\int_0^{3\pi/2}|\cos x|\,dx$ 34. $\displaystyle\int_1^3 \frac{\operatorname{sgn}(x-2)}{x^2}\,dx$

In Exercises 35–38, find the average values of the given functions over the intervals specified.

35. $f(x)=1+x+x^2+x^3$ over $[0,2]$
36. $f(x)=e^{3x}$ over $[-2,2]$
37. $f(x)=2^x$ over $[0,1/\ln 2]$
38. $g(t)=\begin{cases}0 & \text{if } 0\le t\le 1\\ 1 & \text{if } 1<t\le 3\end{cases}$ over $[0,3]$

Find the indicated derivatives in Exercises 39–46.

39. $\displaystyle\frac{d}{dx}\int_2^x \frac{\sin t}{t}\,dt$ 40. $\displaystyle\frac{d}{dt}\int_t^3 \frac{\sin x}{x}\,dx$

41. $\displaystyle\frac{d}{dx}\int_{x^2}^0 \frac{\sin t}{t}\,dt$ 42. $\displaystyle\frac{d}{dx}x^2\int_0^{x^2}\frac{\sin u}{u}\,du$

43. $\displaystyle\frac{d}{dt}\int_{-\pi}^t \frac{\cos y}{1+y^2}\,dy$ 44. $\displaystyle\frac{d}{d\theta}\int_{\sin\theta}^{\cos\theta}\frac{1}{1-x^2}\,dx$

45. $\displaystyle\frac{d}{dx}F(\sqrt{x})$, if $\displaystyle F(t)=\int_0^t \cos(x^2)\,dx$

46. $H'(2)$, if $\displaystyle H(x)=3x\int_4^{x^2}e^{-\sqrt{t}}\,dt$

47. Solve the integral equation $\displaystyle f(x)=\pi\left(1+\int_1^x f(t)\,dt\right)$.

48. Solve the integral equation $\displaystyle f(x)=1-\int_0^x f(t)\,dt$.

49. Criticize the following erroneous calculation:

$$\int_{-1}^1 \frac{dx}{x^2}=-\frac{1}{x}\bigg|_{-1}^{1}=-1+\frac{1}{-1}=-2.$$

Exactly where did the error occur? Why is -2 an unreasonable value for the integral?

50. Use a definite integral to define a function $F(x)$ having derivative $\dfrac{\sin x}{1+x^2}$ for all x and satisfying $F(17)=0$.

51. Does the function $\displaystyle F(x)=\int_0^{2x-x^2}\cos\left(\frac{1}{1+t^2}\right)dt$ have a maximum or a minimum value? Justify your answer.

Evaluate the limits in Exercises 52–54.

52. $\displaystyle\lim_{n\to\infty}\frac{1}{n}\left(\left(1+\frac{1}{n}\right)^5+\left(1+\frac{2}{n}\right)^5+\cdots+\left(1+\frac{n}{n}\right)^5\right)$

53. $\displaystyle\lim_{n\to\infty}\frac{\pi}{n}\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\sin\frac{3\pi}{n}+\cdots+\sin\frac{n\pi}{n}\right)$

54. $\displaystyle\lim_{n\to\infty}\left(\frac{n}{n^2+1}+\frac{n}{n^2+4}+\frac{n}{n^2+9}+\cdots+\frac{n}{2n^2}\right)$

EXERCISES 5.6

Evaluate the integrals in Exercises 1–44. Remember to include a constant of integration with the indefinite integrals. Your answers may appear different from those in the Answers section but may still be correct. For example, evaluating $I=\int\sin x\cos x\,dx$ using the substitution $u=\sin x$ leads to $I=\frac{1}{2}\sin^2 x+C$; using $u=\cos x$ leads to $I=-\frac{1}{2}\cos^2 x+C$; and rewriting $I=\frac{1}{2}\int\sin(2x)\,dx$ leads to $I=-\frac{1}{4}\cos(2x)+C$. These answers are all equal except for different choices for the constant of integration C: $\frac{1}{2}\sin^2 x=-\frac{1}{2}\cos^2+\frac{1}{2}=-\frac{1}{4}\cos(2x)+\frac{1}{4}$.

You can always check your own answer to an indefinite integral by differentiating it to get back to the integrand. This is often easier than comparing your answer with the answer in the back of the book. You may find integrals that you can't do, but you should not make mistakes in those you can do because the answer is so easily checked. (This is a good thing to remember during tests and exams.)

1. $\displaystyle\int e^{5-2x}\,dx$ 2. $\displaystyle\int\cos(ax+b)\,dx$

3. $\displaystyle\int\sqrt{3x+4}\,dx$ 4. $\displaystyle\int e^{2x}\sin(e^{2x})\,dx$

19. $\displaystyle\int\tan x\ln\cos x\,dx$ 20. $\displaystyle\int\frac{x+1}{\sqrt{1-x^2}}\,dx$

21. $\displaystyle\int\frac{dx}{x^2+6x+13}$ 22. $\displaystyle\int\frac{dx}{\sqrt{4+2x-x^2}}$

23. $\displaystyle\int\sin^3 x\cos^5 x\,dx$ 24. $\displaystyle\int\sin^4 t\cos^5 t\,dt$

25. $\displaystyle\int\sin ax\cos^2 ax\,dx$ 26. $\displaystyle\int\sin^2 x\cos^2 x\,dx$

27. $\displaystyle\int\sin^6 x\,dx$ 28. $\displaystyle\int\cos^4 x\,dx$

29. $\displaystyle\int\sec^5 x\tan x\,dx$ 30. $\displaystyle\int\sec^6 x\tan^2 x\,dx$

31. $\displaystyle\int\sqrt{\tan x}\sec^4 x\,dx$ 32. $\displaystyle\int\sin^{-2/3}x\cos^3 x\,dx$

33. $\displaystyle\int\cos x\sin^4(\sin x)\,dx$ 34. $\displaystyle\int\frac{\sin^3\ln x\cos^3\ln x}{x}\,dx$

35. $\displaystyle\int\frac{\sin^2 x}{\cos^4 x}\,dx$ 36. $\displaystyle\int\frac{\sin^3 x}{\cos^4 x}\,dx$

5. $\displaystyle\int \frac{x\,dx}{(4x^2+1)^5}$

6. $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

7. $\displaystyle\int x\,e^{x^2}\,dx$

8. $\displaystyle\int x^2 2^{x^3+1}\,dx$

9. $\displaystyle\int \frac{\cos x}{4+\sin^2 x}\,dx$

10. $\displaystyle\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$

11. $\displaystyle\int \frac{e^x+1}{e^x-1}\,dx$

12. $\displaystyle\int \frac{\ln t}{t}\,dt$

13. $\displaystyle\int \frac{ds}{\sqrt{4-5s}}$

14. $\displaystyle\int \frac{x+1}{\sqrt{x^2+2x+3}}\,dx$

15. $\displaystyle\int \frac{t\,dt}{\sqrt{4-t^4}}$

16. $\displaystyle\int \frac{x^2\,dx}{2+x^6}$

17. $\displaystyle\int \frac{dx}{e^x+1}$

18. $\displaystyle\int \frac{dx}{e^x+e^{-x}}$

37. $\displaystyle\int \csc^5 x\cot^5 x\,dx$

38. $\displaystyle\int \frac{\cos^4 x}{\sin^8 x}\,dx$

39. $\displaystyle\int_0^4 x^3(x^2+1)^{-\frac{1}{2}}\,dx$

40. $\displaystyle\int_1^{\sqrt{e}} \frac{\sin(\pi\ln x)}{x}\,dx$

41. $\displaystyle\int_0^{\pi/2} \sin^4 x\,dx$

42. $\displaystyle\int_{\pi/4}^{\pi} \sin^5 x\,dx$

43. $\displaystyle\int_e^{e^2} \frac{dt}{t\ln t}$

44. $\displaystyle\int_{\frac{\pi^2}{16}}^{\frac{\pi^2}{9}} \frac{2^{\sin\sqrt{x}}\cos\sqrt{x}}{\sqrt{x}}\,dx$

45. Use the identities $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$ and $\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$ to help you evaluate the following:

$$\int_0^{\pi/2}\sqrt{1+\cos x}\,dx \quad\text{and}\quad \int_0^{\pi/2}\sqrt{1-\sin x}\,dx$$

EXERCISES 5.7

In Exercises 1–16, sketch and find the area of the plane region bounded by the given curves.

1. $y = x, \quad y = x^2$

2. $y = \sqrt{x}, \quad y = x^2$

3. $y = x^2 - 5, \quad y = 3 - x^2$

4. $y = x^2 - 2x, \quad y = 6x - x^2$

5. $2y = 4x - x^2, \quad 2y + 3x = 6$

6. $x - y = 7, \quad x = 2y^2 - y + 3$

7. $y = x^3, \quad y = x$

8. $y = x^3, \quad y = x^2$

9. $y = x^3, \quad x = y^2$

10. $x = y^2, \quad x = 2y^2 - y - 2$

11. $y = \dfrac{1}{x}, \quad 2x + 2y = 5$

12. $y = (x^2-1)^2, \quad y = 1 - x^2$

13. $y = \dfrac{1}{2}x^2, \quad y = \dfrac{1}{x^2+1}$

14. $y = \dfrac{4x}{3+x^2}, \quad y = 1$

15. $y = \dfrac{4}{x^2}, \quad y = 5 - x^2$

16. $x = y^2 - \pi^2, \quad x = \sin y$

Find the areas of the regions described in Exercises 17–28. It is helpful to sketch the regions before writing an integral to represent the area.

17. Bounded by $y = \sin x$ and $y = \cos x$, and between two consecutive intersections of these curves

18. Bounded by $y = \sin^2 x$ and $y = 1$, and between two consecutive intersections of these curves

19. Bounded by $y = \sin x$ and $y = \sin^2 x$, between $x = 0$ and $x = \pi/2$

20. Bounded by $y = \sin^2 x$ and $y = \cos^2 x$, and between two consecutive intersections of these curves

21. Under $y = 4x/\pi$ and above $y = \tan x$, between $x = 0$ and the first intersection of the curves to the right of $x = 0$

22. Bounded by $y = x^{1/3}$ and the component of $y = \tan(\pi x/4)$ that passes through the origin

23. Bounded by $y = 2$ and the component of $y = \sec x$ that passes through the point $(0, 1)$

24. Bounded by $y = \sqrt{2}\cos(\pi x/4)$ and $y = |x|$

25. Bounded by $y = \sin(\pi x/2)$ and $y = x$

26. Bounded by $y = e^x$ and $y = x + 2$

27. Find the total area enclosed by the curve $y^2 = x^2 - x^4$.

28. Find the area of the closed loop of the curve $y^2 = x^4(2+x)$ that lies to the left of the origin.

29. Find the area of the finite plane region that is bounded by the curve $y = e^x$, the line $x = 0$, and the tangent line to $y = e^x$ at $x = 1$.

30. Find the area of the finite plane region bounded by the curve $y = x^3$ and the tangent line to that curve at the point $(1, 1)$. *Hint:* Find the other point at which that tangent line meets the curve.

Review Exercises

Evaluate the integrals in Exercises 23–30.

23. $\displaystyle\int x^2\cos(2x^3+1)\,dx$

24. $\displaystyle\int_1^e \frac{\ln x}{x}\,dx$

25. $\displaystyle\int_0^4 \sqrt{9t^2+t^4}\,dt$

26. $\displaystyle\int \sin^3(\pi x)\,dx$

27. $\displaystyle\int_0^{\ln 2} \frac{e^u}{4+e^{2u}}\,du$

28. $\displaystyle\int_1^{\sqrt[4]{e}} \frac{\tan^2 \pi \ln x}{x}\,dx$

29. $\displaystyle\int \frac{\sin\sqrt{2s+1}}{\sqrt{2s+1}}\,ds$

30. $\displaystyle\int \cos^2\frac{t}{5}\sin^2\frac{t}{5}\,dt$

EXERCISES 6.1

Evaluate the integrals in Exercises 1–28.

1. $\displaystyle\int x\cos x\,dx$

2. $\displaystyle\int (x+3)e^{2x}\,dx$

3. $\displaystyle\int x^2\cos\pi x\,dx$

4. $\displaystyle\int (x^2-2x)e^{kx}\,dx$

5. $\displaystyle\int x^3\ln x\,dx$

6. $\displaystyle\int x(\ln x)^3\,dx$

7. $\displaystyle\int \tan^{-1}x\,dx$

8. $\displaystyle\int x^2\tan^{-1}x\,dx$

9. $\displaystyle\int x\sin^{-1}x\,dx$

10. $\displaystyle\int x^5e^{-x^2}\,dx$

11. $\displaystyle\int_0^{\pi/4}\sec^5 x\,dx$

12. $\displaystyle\int \tan^2 x\sec x\,dx$

13. $\displaystyle\int e^{2x}\sin 3x\,dx$

14. $\displaystyle\int xe^{\sqrt{x}}\,dx$

15. $\displaystyle\int_{1/2}^{1}\frac{\sin^{-1}x}{x^2}\,dx$

16. $\displaystyle\int_0^1 \sqrt{x}\sin(\pi\sqrt{x})\,dx$

17. $\displaystyle\int x\sec^2 x\,dx$

18. $\displaystyle\int x\sin^2 x\,dx$

EXERCISES 6.2

Evaluate the integrals in Exercises 1–28.

1. $\displaystyle\int \frac{2\,dx}{2x-3}$

2. $\displaystyle\int \frac{dx}{5-4x}$

3. $\displaystyle\int \frac{x\,dx}{\pi x+2}$

4. $\displaystyle\int \frac{x^2}{x-4}\,dx$

5. $\displaystyle\int \frac{1}{x^2-9}\,dx$

6. $\displaystyle\int \frac{dx}{5-x^2}$

7. $\displaystyle\int \frac{dx}{a^2-x^2}$

8. $\displaystyle\int \frac{dx}{b^2-a^2x^2}$

9. $\displaystyle\int \frac{x^2\,dx}{x^2+x-2}$

10. $\displaystyle\int \frac{x\,dx}{3x^2+8x-3}$

11. $\displaystyle\int \frac{x-2}{x^2+x}\,dx$

12. $\displaystyle\int \frac{dx}{x^3+9x}$

13. $\displaystyle\int \frac{dx}{1-6x+9x^2}$

14. $\displaystyle\int \frac{x\,dx}{2+6x+9x^2}$

15. $\displaystyle\int \frac{x^2+1}{6x-9x^2}\,dx$

16. $\displaystyle\int \frac{x^3+1}{12+7x+x^2}\,dx$

17. $\displaystyle\int \frac{dx}{x(x^2-a^2)}$

18. $\displaystyle\int \frac{dx}{x^4-a^4}$

19. $\displaystyle\int \frac{x^3\,dx}{x^3-a^3}$

20. $\displaystyle\int \frac{dx}{x^3+2x^2+2x}$

21. $\displaystyle\int \frac{dx}{x^3-4x^2+3x}$

22. $\displaystyle\int \frac{x^2+1}{x^3+8}\,dx$

23. $\displaystyle\int \frac{dx}{(x^2-1)^2}$

24. $\displaystyle\int \frac{x^2\,dx}{(x^2-1)(x^2-4)}$

25. $\displaystyle\int \frac{dx}{x^4-3x^3}$

26. $\displaystyle\int \frac{dt}{(t-1)(t^2-1)^2}$

27. $\displaystyle\int \frac{dx}{e^{2x}-4e^x+4}$

28. $\displaystyle\int \frac{d\theta}{\cos\theta(1+\sin\theta)}$

In Exercises 29–30 write the form that the partial fraction decomposition of the given rational function takes. Do not actually evaluate the constants you use in the decomposition.

29. $\displaystyle\frac{x^5+x^3+1}{(x-1)(x^2-1)(x^3-1)}$

30. $\displaystyle\frac{123-x^7}{(x^4-16)^2}$

31. Write $\displaystyle\frac{x^5}{(x^2-4)(x+2)^2}$ as the sum of a polynomial and a partial fraction decomposition (with constants left undetermined) of a rational function whose numerator has smaller degree than the denominator.

32. Show that x^4+4x^2+16 factors to $(x^2+kx+4)(x^2-kx+4)$ for a certain positive constant k. What is the value of k? Now repeat the previous exercise for the rational function $\displaystyle\frac{x^4}{x^4+4x^2+16}$.

33. Suppose that P and Q are polynomials such that the degree of P is smaller than that of Q. If

$$Q(x)=(x-a_1)(x-a_2)\cdots(x-a_n),$$

where $a_i \ne a_j$ if $i \ne j\,(1 \le i, j \le n)$, so that $P(x)/Q(x)$ has partial fraction decomposition

$$\frac{P(x)}{Q(x)}=\frac{A_1}{x-a_1}+\frac{A_2}{x-a_2}+\cdots+\frac{A_n}{x-a_n},$$

show that

$$A_j=\frac{P(a_j)}{Q'(a_j)} \qquad (1 \le j \le n).$$

This gives yet another method for computing the constants in a partial fraction decomposition if the denominator factors completely into distinct linear factors.

EXERCISES 6.5

In Exercises 1–22, evaluate the given integral or show that it diverges.

1. $\displaystyle\int_2^\infty \frac{1}{(x-1)^3}\,dx$

2. $\displaystyle\int_3^\infty \frac{1}{(2x-1)^{2/3}}\,dx$

3. $\displaystyle\int_0^\infty e^{-2x}\,dx$

4. $\displaystyle\int_{-\infty}^{-1} \frac{dx}{x^2+1}$

5. $\displaystyle\int_{-1}^1 \frac{dx}{(x+1)^{2/3}}$

6. $\displaystyle\int_0^a \frac{dx}{a^2-x^2}$

7. $\displaystyle\int_0^1 \frac{1}{(1-x)^{1/3}}\,dx$

8. $\displaystyle\int_0^1 \frac{1}{x\sqrt{1-x}}\,dx$

9. $\displaystyle\int_0^{\pi/2} \frac{\cos x\,dx}{(1-\sin x)^{2/3}}$

10. $\displaystyle\int_0^\infty x e^{-x}\,dx$

11. $\displaystyle\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$

12. $\displaystyle\int_0^\infty \frac{x}{1+2x^2}\,dx$

13. $\displaystyle\int_0^\infty \frac{x\,dx}{(1+2x^2)^{3/2}}$

14. $\displaystyle\int_0^{\pi/2} \sec x\,dx$

15. $\displaystyle\int_0^{\pi/2} \tan x\,dx$

16. $\displaystyle\int_e^\infty \frac{dx}{x\ln x}$

17. $\displaystyle\int_1^e \frac{dx}{x\sqrt{\ln x}}$

18. $\displaystyle\int_e^\infty \frac{dx}{x(\ln x)^2}$

19. $\displaystyle\int_{-\infty}^\infty \frac{x}{1+x^2}\,dx$

20. $\displaystyle\int_{-\infty}^\infty \frac{x}{1+x^4}\,dx$

21. $\displaystyle\int_{-\infty}^\infty x e^{-x^2}\,dx$

22. $\displaystyle\int_{-\infty}^\infty e^{-|x|}\,dx$

23. Find the area below $y=0$, above $y=\ln x$, and to the right of $x=0$.

24. Find the area below $y=e^{-x}$, above $y=e^{-2x}$, and to the right of $x=0$.

25. Find the area of a region that lies above $y=0$, to the right of $x=1$, and under the curve $y=\dfrac{4}{2x+1}-\dfrac{2}{x+2}$.

26. Find the area of the plane region that lies under the graph of $y=x^{-2}e^{-1/x}$, above the x-axis, and to the right of the y-axis.

27. Prove Theorem 2(a) by directly evaluating the integrals involved.

28. Evaluate $\int_{-1}^1 (x\,\mathrm{sgn}\,x)/(x+2)\,dx$. Recall that $\mathrm{sgn}\,x = x/|x|$.

29. Evaluate $\int_0^2 x^2\,\mathrm{sgn}\,(x-1)\,dx$.

In Exercises 30–41, state whether the given integral converges or diverges, and justify your claim.

30. $\displaystyle\int_0^\infty \frac{x^2}{x^5+1}\,dx$

31. $\displaystyle\int_0^\infty \frac{dx}{1+\sqrt{x}}$

32. $\displaystyle\int_2^\infty \frac{x\sqrt{x}\,dx}{x^2-1}$

33. $\displaystyle\int_0^\infty e^{-x^3}\,dx$

34. $\displaystyle\int_0^\infty \frac{dx}{\sqrt{x}+x^2}$

35. $\displaystyle\int_{-1}^1 \frac{e^x}{x+1}\,dx$

36. $\displaystyle\int_0^\pi \frac{\sin x}{x}\,dx$

37. $\displaystyle\int_0^\infty \frac{|\sin x|}{x^2}\,dx$

38. $\displaystyle\int_0^{\pi^2} \frac{dx}{1-\cos\sqrt{x}}$

39. $\displaystyle\int_{-\pi/2}^{\pi/2} \csc x\,dx$

40. $\displaystyle\int_2^\infty \frac{dx}{\sqrt{x}\ln x}$

41. $\displaystyle\int_0^\infty \frac{dx}{xe^x}$

EXERCISES 2.10

In Exercises 1–14, find the given indefinite integrals.

1. $\displaystyle\int 5\,dx$

2. $\displaystyle\int x^2\,dx$

3. $\displaystyle\int \sqrt{x}\,dx$

4. $\displaystyle\int x^{12}\,dx$

5. $\displaystyle\int x^3\,dx$

6. $\displaystyle\int (x+\cos x)\,dx$

7. $\displaystyle\int \tan x\cos x\,dx$

8. $\displaystyle\int \frac{1+\cos^3 x}{\cos^2 x}\,dx$

9. $\displaystyle\int (a^2-x^2)\,dx$

10. $\displaystyle\int (A+Bx+Cx^2)\,dx$

11. $\displaystyle\int (2x^{1/2}+3x^{1/3})\,dx$

12. $\displaystyle\int \frac{6(x-1)}{x^{4/3}}\,dx$

13. $\displaystyle\int \left(\frac{x^3}{3}-\frac{x^2}{2}+x-1\right)dx$

14. $\displaystyle 105\int (1+t^2+t^4+t^6)\,dt$

In Exercises 15–22, find the given indefinite integrals. This may require guessing the form of an antiderivative and then checking by differentiation. For instance, you might suspect that $\int\cos(5x-2)\,dx = k\sin(5x-2)+C$ for some k. Differentiating the answer shows that k must be $1/5$.

15. $\displaystyle\int \cos(2x)\,dx$

16. $\displaystyle\int \sin\left(\frac{x}{2}\right)dx$

17. $\displaystyle\int \frac{dx}{(1+x)^2}$

18. $\displaystyle\int \sec(1-x)\tan(1-x)\,dx$

19. $\displaystyle\int \sqrt{2x+3}\,dx$

20. $\displaystyle\int \frac{4}{\sqrt{x+1}}\,dx$

21. $\displaystyle\int 2x\sin(x^2)\,dx$

22. $\displaystyle\int \frac{2x}{\sqrt{x^2+1}}\,dx$

Use known trigonometric identities such as $\sec^2 x = 1+\tan^2 x$, $\cos(2x) = 2\cos^2 x - 1 = 1-2\sin^2 x$, and $\sin(2x) = 2\sin x\cos x$ to help you evaluate the indefinite integrals in Exercises 23–26.

23. $\displaystyle\int \tan^2 x\,dx$

24. $\displaystyle\int \sin x\cos x\,dx$