Logic 2022-2023 — Bonus Assignment 2

May 2, 2023

- This assignment must be made individually.
- Include your name and student number on the first sheet.
- Hand in your solution by uploading **a single PDF file** via Canvas.
- The deadline for uploading your solution is May 10th at 23:59.
- If graded as sufficient, you earn 0.33 bonus points to your final mark for the course.

1 The assignment

1. Consider the following syllogisms:

(a)

Some inferences are valid
Every valid inference is provable.

Some provable inferences are valid $\therefore$

(b)

Some DACS students are logicians
Some logicians are smart.

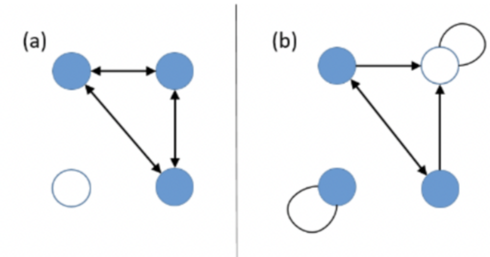
Some DACS students are smart $\therefore$

(i) Demonstrate with Venn diagrams whether these syllogisms are valid.

(ii) Convert the two syllogisms into predicate logic formulas and check their validity using a tableau.

2. Consider the models (a) and (b) shown below. Both models have a unary predicate B and a binary predicate R. Blue objects have property B, a $\rightarrow$ indicates that two

objects are R-related, a $\leftrightarrow$ link indicates that the R relation runs in both directions. Loops indicate that an object is R-related to itself.



(i) State two closed formulas that are true in model (a) and false in model (b).

(ii) State two closed formulas that are true in both models.

(iii) State two closed formulas that are false in both models.

3. Use the tableau method to find out whether the following inferences are valid.

   (i) $\forall x(Px \to Qx) \models \forall x(Px \vee Qx)$

   (ii) $Pa, \forall x(Px \to Qx) \models \exists x Qx$

   (iii) $\top \models (\forall x Px) \leftrightarrow (\neg\exists x \neg Px)$

   (iv) $\exists x\exists y(Rxy \vee Ryx) \models \exists x\exists y Rxy$

4. The following inferences/formulas are valid. Prove this by natural deduction.

   (i) $\forall x(Ax \to Bx) \models (\exists x Ax) \to (\exists x Bx)$

   (ii) $Pa \to \forall x(Qx \to Qb), Qa, \neg Qb \models \neg Pa$

   (iii) $\exists x Px, \forall x Px \to Qx \models \neg\forall x\neg Qx$