

Logic 2022-2023 — Bonus Assignment 2

May 2, 2023

- This assignment must be made individually.
- Include your name and student number on the first sheet.
- Hand in your solution by uploading a **single PDF file** via Canvas.
- The deadline for uploading your solution is May 10th at 23:59.
- If graded as sufficient, you earn 0.33 bonus points to your final mark for the course.

1 The assignment

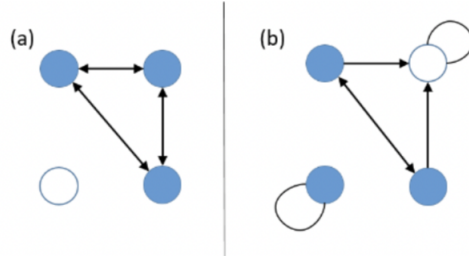
1. Consider the following syllogisms:

(a)
$$\begin{array}{l} \text{Some inferences are valid} \\ \text{Every valid inference is provable.} \\ \hline \text{Some provable inferences are valid} \quad \therefore \end{array}$$

(b)
$$\begin{array}{l} \text{Some DACS students are logicians} \\ \text{Some logicians are smart.} \\ \hline \text{Some DACS students are smart} \quad \therefore \end{array}$$

- (i) Demonstrate with Venn diagrams whether these syllogisms are valid.
 - (ii) Convert the two syllogisms into predicate logic formulas and check their validity using a tableau.
2. Consider the models (a) and (b) shown below. Both models have a unary predicate B and a binary predicate R . Blue objects have property B , a \rightarrow indicates that two

objects are R -related, a \leftrightarrow link indicates that the R relation runs in both directions. Loops indicate that an object is R -related to itself.



- (i) State two closed formulas that are true in model (a) and false in model (b).
 - (ii) State two closed formulas that are true in both models.
 - (iii) State two closed formulas that are false in both models.
3. Use the tableau method to find out whether the following inferences are valid.
- (i) $\forall x(Px \rightarrow Qx) \models \forall x(Px \vee Qx)$
 - (ii) $Pa, \forall x(Px \rightarrow Qx) \models \exists xQx$
 - (iii) $\top \models (\forall xPx) \leftrightarrow (\neg \exists x \neg Px)$
 - (iv) $\exists x \exists y(Rxy \vee Ryx) \models \exists x \exists y Rxy$
4. The following inferences/formulas are valid. Prove this by natural deduction.
- (i) $\forall x(Ax \rightarrow Bx) \models (\exists xAx) \rightarrow (\exists xBx)$
 - (ii) $Pa \rightarrow \forall x(Qx \rightarrow Qb), Qa, \neg Qb \models \neg Pa$
 - (iii) $\exists xPx, \forall xPx \rightarrow Qx \models \neg \forall x \neg Qx$