Logic 2021-2022 — Bonus Assignment 1

April 19, 2023

- This assignment must be made individually.
- Include your name and student number on the first sheet.
- Hand in your solution by uploading **a single PDF file** via Canvas.
- The deadline for uploading your solution is monday April 24 at 23:59.This is a **hard** deadline, not uploading on time due to any reason (illness, computer or connectivity issues) cannot be taken into account (as grading has to start after this date, and we also want to publish solutions asap).
- If graded as sufficient, you earn 0.33 bonus points to your final mark for the course.

1 The assignment

1. For each of the following propositions, state whether it is a tautology or a contradiction, and whether the formula is satisfiable. No explanation necessary (and no particular method forced to use).

   (a) $\neg(\neg p \vee \neg(q \wedge p))$

   (b) $(\neg a \vee \neg b) \leftrightarrow \neg(a \wedge b)$

   (c) $\neg(a \to b) \to (a \to b)$

   (d) $\neg b \to (a \to b)$

2. Decide using truth tables whether the following inferences are valid or not. If not, specify a counter-example.

   (a) $a \models (b \wedge c) \to (a \to b)$

   (b) $\{p \vee q, q \vee r\} \models (p \wedge r) \to \neg q$

   (c) $\{p \to q, q \to r\} \models p \to r$

3. We consider the following three sentences all of whom are either true or false :

$P \to Q \equiv \neg P \vee Q$

P	$\to$	Q	$\neg$	P	$\vee$	Q
0	1	0	1	0	1	0
1	0	0	0	1	0	0
0	1	1	1	0	1	1
1	1	1	0	1	1	1

✓

(1)(a) $\neg[\neg p \lor \neg(q \land p)]$ $\qquad \neg(q \land p) \equiv \neg q \lor \neg p$

$\equiv \neg[\neg p \lor \neg q \lor \neg p]$

$\equiv \neg(\neg p \lor \neg q)$

$\equiv q \land p$

$\rightarrow$ the formula is satisfable $\qquad \neg(a \land b) \rightarrow (\neg a \lor \neg b)$

(b) $(\neg a \lor \neg b) \leftrightarrow \neg(a \land b)$

$\equiv [(\neg a \lor \neg b) \rightarrow \neg(a \land b)] \land [(\neg a \lor \neg b) \leftarrow \neg(a \land b)]$

$\equiv [\neg(\neg a \lor \neg b) \lor \neg(a \land b)] \land [(a \land b) \lor (\neg a \lor \neg b)]$

$\equiv [(a \land b) \lor (\neg a \lor \neg b)] \land [(a \land b) \lor (\neg a \lor \neg b)]$

$\equiv (a \land b) \lor (\neg a \lor \neg b)$

$\equiv (a \land b) \lor \neg(a \land b) \rightarrow$ Tautology

(c) $\neg(a \rightarrow b) \rightarrow (a \rightarrow b)$

$\equiv (a \rightarrow b) \lor (a \rightarrow b)$

$\equiv \neg a \lor b \rightarrow$ satisfiable

(d) $\neg b \rightarrow (a \rightarrow b)$

$\equiv b \lor (a \rightarrow b)$

$\equiv b \lor (\neg a \lor b)$

$\equiv b \lor \neg a \lor b \equiv b \lor \neg a \rightarrow$ satisfable

(2) (a) $a \models (b \land c) \rightarrow (a \rightarrow b)$

a	b	c	$a \rightarrow b$ (A)	$b \land c$ (B)	$B \rightarrow A$
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	1	0	1
1	0	0	0	0	1
1	0	1	0	0	1
0	1	1	1	1	1
1	1	0	1	0	1
1	1	1	1	1	1

The inference is valid (When a is true, the formula is also true)

(b) $\{p \vee q,\ q \vee r\} \models (p \wedge r) \to \neg q$

$\to (p \wedge r) \to \neg q$ is T $\leftrightarrow (p \vee q) \wedge (q \vee r)$ is T

p	q	r	$p \vee q$	$q \vee r$	$(p \wedge r)$	$(p \wedge r) \to q$
0	0	0	0	0	0	1
0	0	1	0	1	0	1
0	1	0	1	1	0	1
1	0	0	1	0	0	1
1	0	1	1	1	1	1
1	1	0	1	1	0	1
0	1	1	1	1	0	1
1	1	1	1	1	1	1

The inference is valid

(c) $\{p \to q,\ q \to r\} \models p \to r$

p	q	r	$p \to q$	$q \to r$	$p \to r$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	1
1	0	0	0	1	0
1	0	1	1	1	1
1	1	0	0	0	0
0	1	1	1	1	1
1	1	1	1	1	1

the inference is <u>not</u> valid

Counter example: p is F, q is T and r is F

(a) Sentence (b) is true.

(b) Sentence (c) is false.

(c) There are exactly two sentences correct

Formulate this information about the suspects in propositional logic. Use a, b, c as atomic propositions where a is true if and only if line (a) is true, etc. Use a truth table to find all configurations (if any!) of a, b, c.

4. Use the tableau method to check whether the following formulas are tautologies.
   (a) $p \to (p \wedge q)$
   (b) $(p \wedge q) \to p$
   (c) $\neg(p \to (p \wedge q))$
   (d) $\neg p \to (p \wedge q)$

5. Use the tableau method to check whether the following formulas are contradictions.
   (a) $(p \to \neg q) \wedge (q \to r) \wedge (r \to p)$
   (b) $(p \wedge \neg q) \wedge (p \to q)$
   (c) $(p \to (q \to \neg p)) \wedge p$

6. What is wrong with the following formula: $(p \to q) \wedge (q \to p) \to (p \vee \neg q)$

7. Use the tableau method to check whether the following are valid inferences
   (a) $\{p, q\} \models p \to q$
   (b) $\{p \to q, \neg q \to \neg r\} \models p \to r$.
   (c) $\{p \wedge q, \neg q\} \models r$

8. By means of natural deduction, prove the following:
   (a) from $\neg\neg p$, derive p.
   (b) from $p \to \neg q$ and q derive $\neg p$. (Hint: you may need the previous one in this)
   (c) derive $p \vee \neg p$.(Its useful to remember this one, so that you can use it whenever you need it)
   (d) from $p \vee \neg q$ derive $q \to p$.
   (e) from $p \to q$ derive $\neg q \to \neg p$.
   (f) from $a \vee \neg a$ derive $(\neg p \to \neg q) \to (q \to p)$. (There's no typo in this exercise.)

(3)

(a) Sentence (b) is true.

(b) Sentence (c) is false.

(c) There are exactly two sentences correct

Formulate this information about the suspects in propositional logic. Use a, b, c as atomic propositions where a is true if and only if line (a) is true, etc. Use a truth table to find all configurations (if any!) of a, b, c.

. . .

(4) (a) $p \to (p \wedge q)$

$\circ \; p \to (p \wedge q)$

$\to$ RHS

$p \circ p \wedge q$

$\wedge$ RHS

$p \circ p$ | $p \circ q$

X | open

$\to$ inference is not valid

(b) $(p \wedge q) \to p$

$\circ \; (p \wedge q) \to p$

$\to$ RHS

$p \wedge q \circ p$

$\wedge$ LHS

$p, q \circ p$

X

$\to$ inference is valid

(c) $\neg(p \to (p \land q))$

$\circ \quad \neg(p \to (p \land q))$

$\neg$ RHS

$p \to (p \land q) \quad \circ$

$\circ \, p \qquad\qquad p \land q \quad \circ$

open

$\land$ LHS

$p, q \quad \circ$

open

inference is <u>not</u> valid

(d) $\neg p \to (p \land q)$

$\circ \quad \neg p \to (p \land q)$

$\to$ RHS

$\neg p \quad \circ \quad p \land q$

$\neg$, RHS

$\circ \quad p \land q, p$

$\land$, RHS

$\circ \, p, p \qquad\qquad \circ \, p, q$

open

$\circ \quad p$

open

$\longrightarrow$ inference is <u>not</u> valid

(5) (a) $\circ\ (p \to \neg q) \wedge (q \to r) \wedge (r \to p)$

(a) $(p \to \neg q) \wedge (q \to r) \wedge (r \to p)$
(b) $(p \wedge \neg q) \wedge (p \to q)$
(c) $(p \to (q \to \neg p)) \wedge p$

$\wedge$, RHS

$\circ\ (p \to \neg q)$

$\to$ RHS

$p \circ \neg q$

open

$\circ\ (q \to r) \wedge (r \to p)$

$\wedge$, RHS

$\circ\ q \to r$

$q \circ r$

open

$\circ\ r \to p$

$r \circ p$

open

All branches are open $\to$ contraddiction

(b) $\circ\ (p \wedge \neg q) \wedge (p \to q)$

$\wedge$ RHS

$\circ\ p \wedge \neg q$

$\circ\ p$

open

$\circ\ \neg q$

open

$\circ\ p \to q$

$p \circ q$

open

$\to$ All branches are open

$\hookrightarrow$ Contraddiction

(c) $\circ\ (p \to (q \to \neg p)) \wedge p$

$\wedge$ RHS

$\circ\ p \to (q \to \neg p)$

$p \circ q \to \neg p$

$\to$ RHS

$p, q \circ \neg p$

$\neg$ RHS

$p, q \circ$

open

$\circ\ p$

open

All branches are open

$\hookrightarrow$ Contraddiction

(6) $(p \to q) \land (q \to p) \to (p \lor \neg q)$

↳ This statement is not clear due to improper use of parenthesis. it could be interpreted as either:

(a) $[(p \to q) \land (q \to p)] \to (p \lor \neg q)$

(b) $(p \to q) \land [(q \to p) \to (p \lor \neg q)]$

(7) (a) $\{p, q\} \models p \to q$

(a) $\{p, q\} \models p \to q$
(b) $\{p \to q, \neg q \to \neg r\} \models p \to r.$
(c) $\{p \land q, \neg q\} \models r$

$p, q \circ p \to q$

→ RHS

$p, q \circ q$

X

Closed tableau → valid inference

(b) $p \to q, \neg q \to \neg r \circ p \to r$

→ RHS

$p, p \to q, \neg q \to \neg r \circ r$

→ LHS

$p, \neg q \to \neg r \circ r, q$ | $p, \neg q \to \neg r, q \circ r$

→ LHS

$p \circ r, q, \neg q$ | $p, \neg r \circ r, q$ | $p, q \circ \neg q, r$ | $p, q, \neg r \circ r$

$\neg$ | $\neg$ | $\neg$ | $\neg$

$p \circ r, q$ | $p \circ r, q$ | $p \circ r, q$ | $p, q \circ r$

Tableau is open → contradiction

(c) $\{p \land q, \neg q\} \models r$

$p \land q, \neg q \circ r$

| $\neg$ LHS

$p \land q \circ r, q$

|

$p, \underline{q} \circ r\ \underline{q}$

X

Tableau is closed $\rightarrow$ valid inference

(8) (a) from $\neg\neg p$ derive p

1. $\neg\neg p$ (given)

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. . . .

I did not exercise enough natural induction in time for the deadline of this assignment

(a) from $\neg\neg p$, derive p.

(b) from $p \rightarrow \neg q$ and q derive $\neg p$. (Hint: you may need the previous one in this)

(c) derive $p \vee \neg p$.(Its useful to remember this one, so that you can use it whenever you need it)

(d) from $p \vee \neg q$ derive $q \rightarrow p$.

(e) from $p \rightarrow q$ derive $\neg q \rightarrow \neg p$.

(f) from $a \vee \neg a$ derive $(\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p)$. (There's no typo in this exercise.)