

Department of Data Science and Knowledge Engineering

Discrete Mathematics 2021/2022 (resit) Exam Questions

— Do not turn this page before the official start of the exam! —

| First Name, Surname: | |
|----------------------|--|
| Student ID: | |

Program: Bachelor Data Science and Artificial Intelligence

Course code: KEN1130

Examiner: Dr. Steven Kelk, Dr. Marieke Musegaas and Dr. Stefan Maubach

Date/time: Wednesday 26th January 2022, 9:00h-11:00h

Format: Closed book exam

Allowed aids: Pens, simple (non-programmable) calculator from the DKE-list of allowed calculators.

Instructions to students:

- The exam consists of 8 questions on 16 pages (excluding the 1 cover page(s)).
- Fill in your name and student ID number on each page, including the cover page.
- Answer every question at the reserved space below the questions. If you run out of space, continue on the back side, and if needed, use the extra blank page.
- Ensure that you properly motivate your answers.
- Do not use red pens, and write in a readable way. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- Remember that $\mathbb{N} = \{1, 2, 3, ...\}.$
- · Good luck!

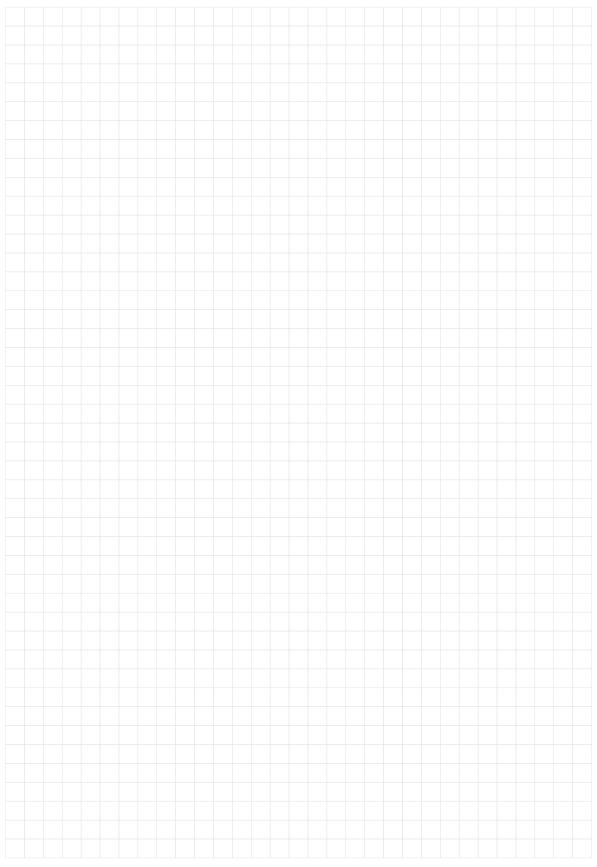
The following table will be filled by the examiner:

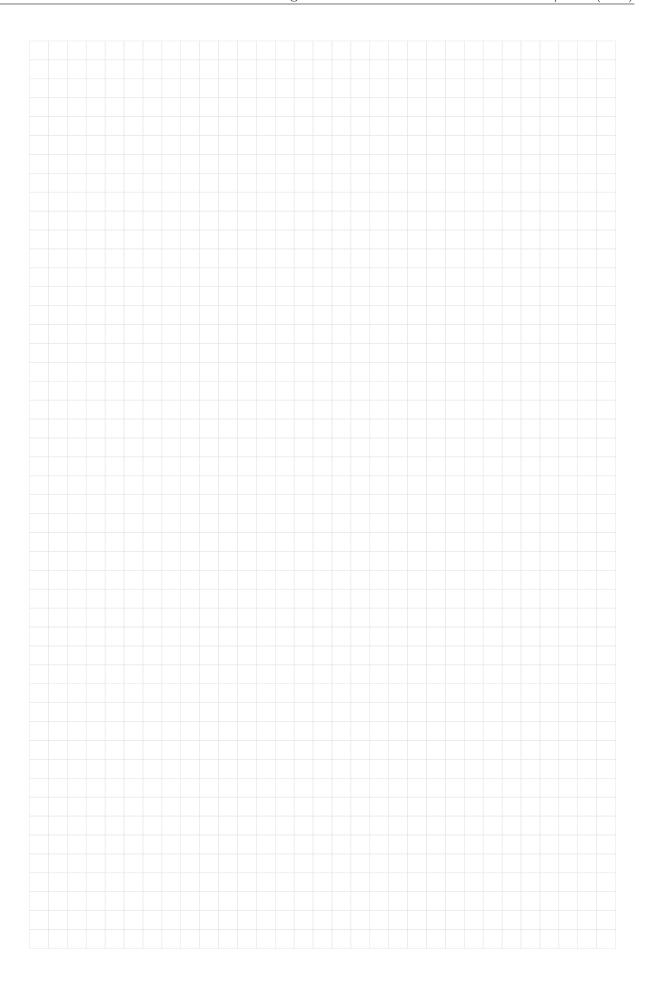
| Question: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
|-----------|----|----|----|----|----|----|----|---|-------|
| Points: | 10 | 15 | 15 | 15 | 15 | 10 | 15 | 5 | 100 |
| Score: | | | | | | | | | |

Question 1 (10 points)

Fill in the truth table for the following logical proposition.

•
$$(p \Rightarrow q) \Rightarrow (((q \Rightarrow r) \land \neg r) \Rightarrow \neg p)$$



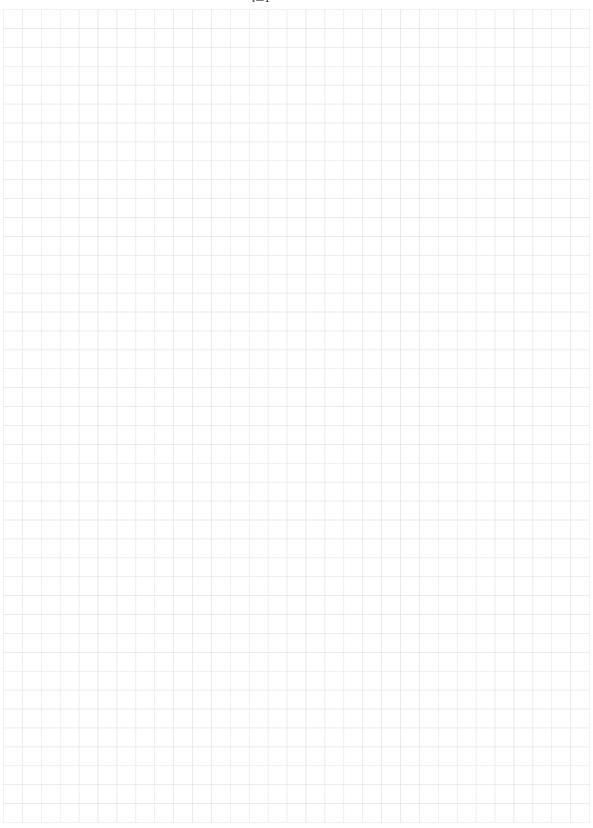


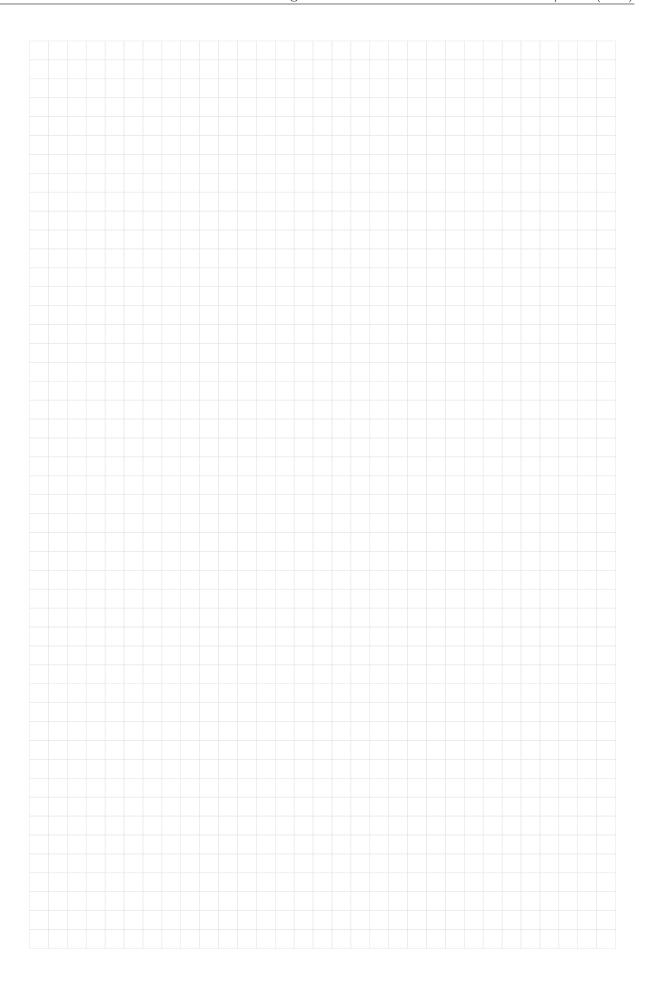
Question 2 (15 points)

Use induction to prove the following statement.

• For all integers $n \ge 1$,

$$\sum_{i=1}^{n} i^3 = \frac{1}{4}n^2(n+1)^2$$

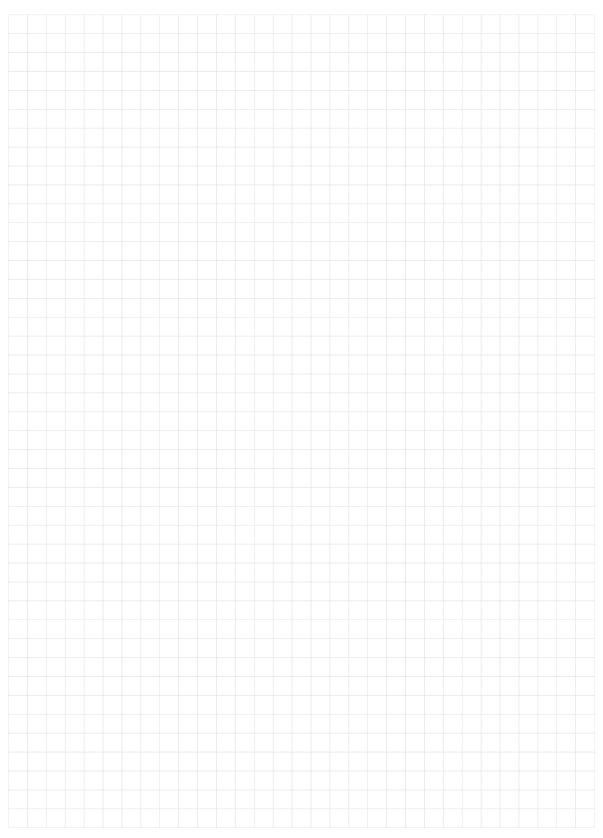


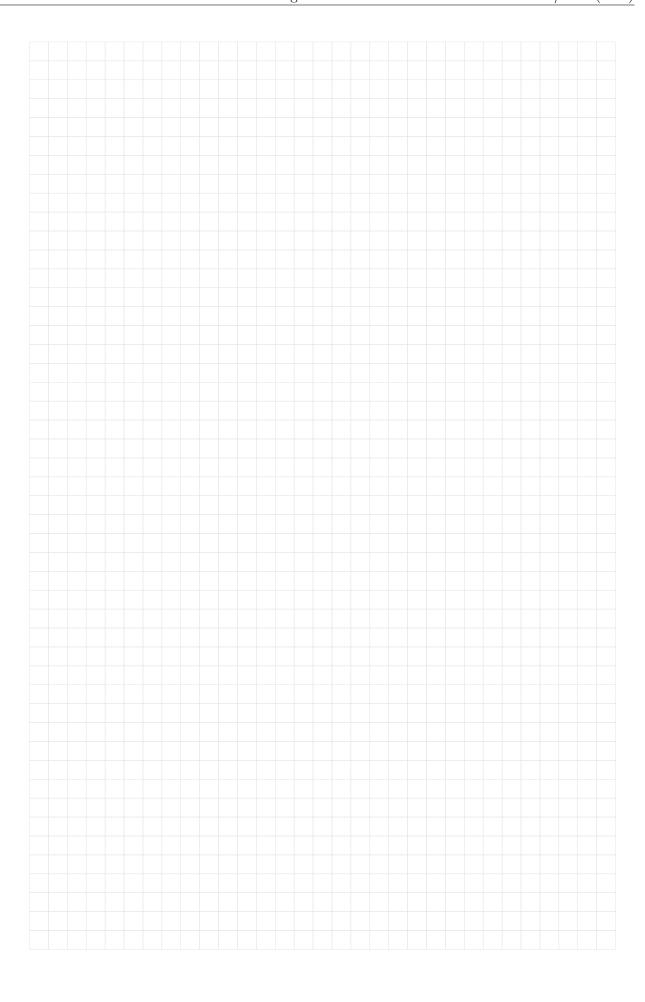


Question 3 (15 points)

Prove or disprove the following statements.

- (a) (7 points) For all sets A, B, and C, if $A \not\subseteq B$ and $B \not\subseteq C$ then $A \not\subseteq C$.
- (b) (8 points) For all sets A, B and C, if $A \subseteq B$ then $A \cap (B \cap C)^c = \emptyset$.





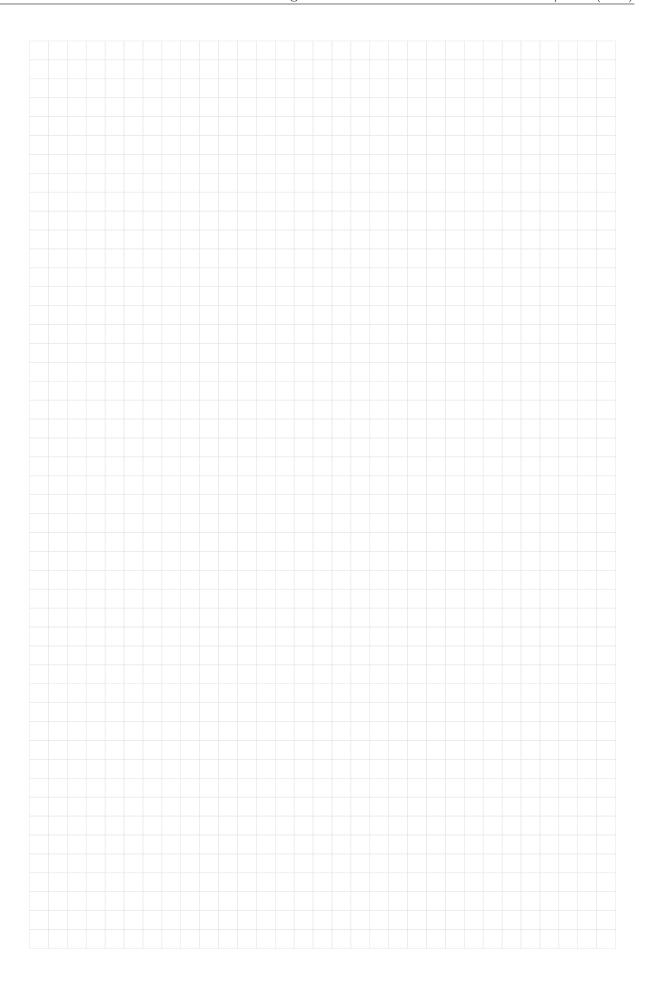
Question 4 (15 points)

Let $A = \mathbb{P}(\{a, b, c\})$. Let R be the relation on A defined as follows:

$$XRY$$
 means " $X \backslash Y \neq \emptyset$ and $Y \backslash X \neq \emptyset$ ".

Is R reflexive? Symmetric? Transitive? Anti-symmetric? For each of these properties, prove or disprove that it has that property.

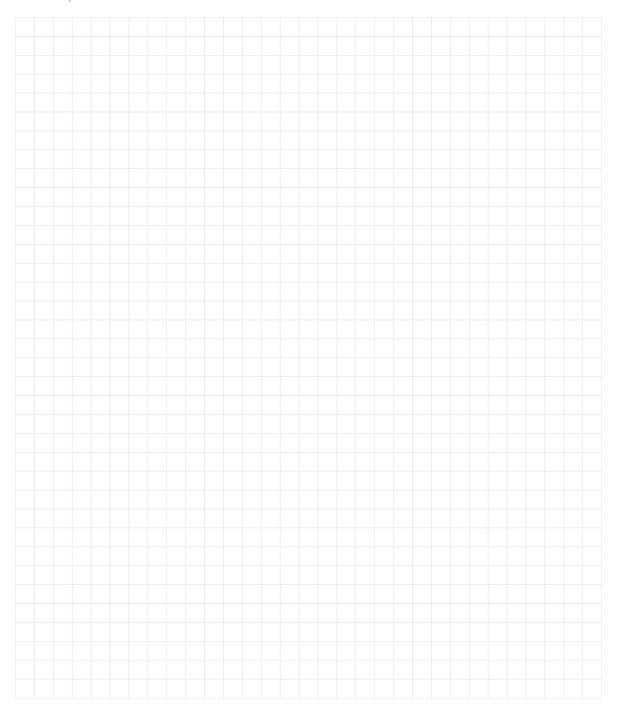


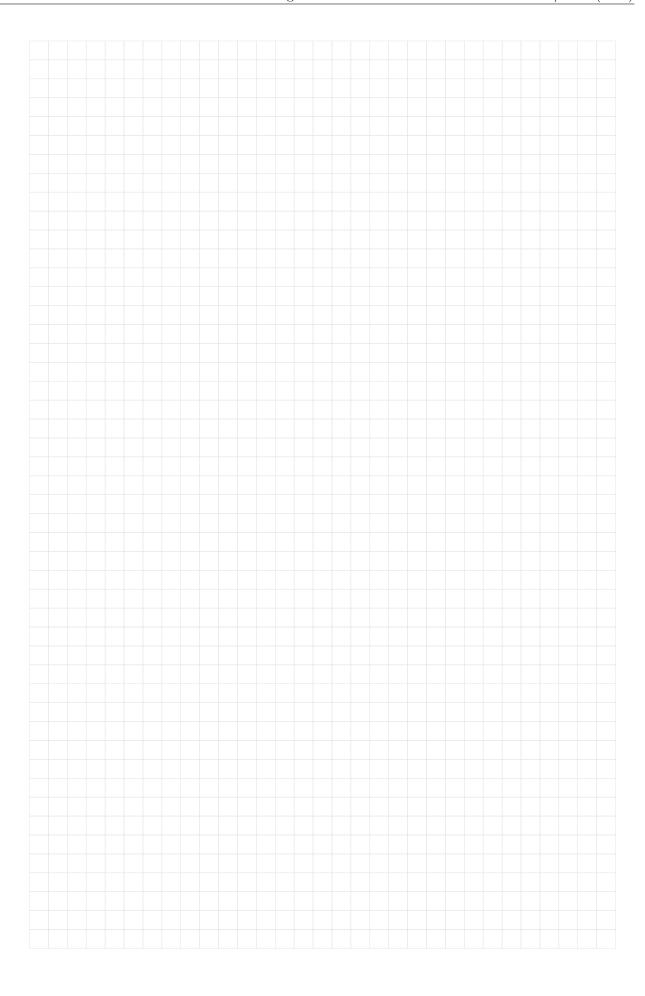


Question 5 (15 points)

All the following questions are about *counting*. For all questions please give an exact number at the end (i.e. don't just leave your answer as a counting equation).

- (a) (3 points) When you decide to order pizza, you must first choose the type of crust: thin crust or deep dish. Next, you choose one topping: cheese, pepperoni, or sausage. How many possibilities are there for your pizza?
- (b) (6 points) How many bit sequences of length eleven are there which start with one of the two bit sequences 101 or 010?
- (c) (6 points) A student possesses six distinct books on economics, and six distinct books on mathematics. The books are arranged in a sequence on a book shelf. How many options are there, if the books on mathematics should be on the left?



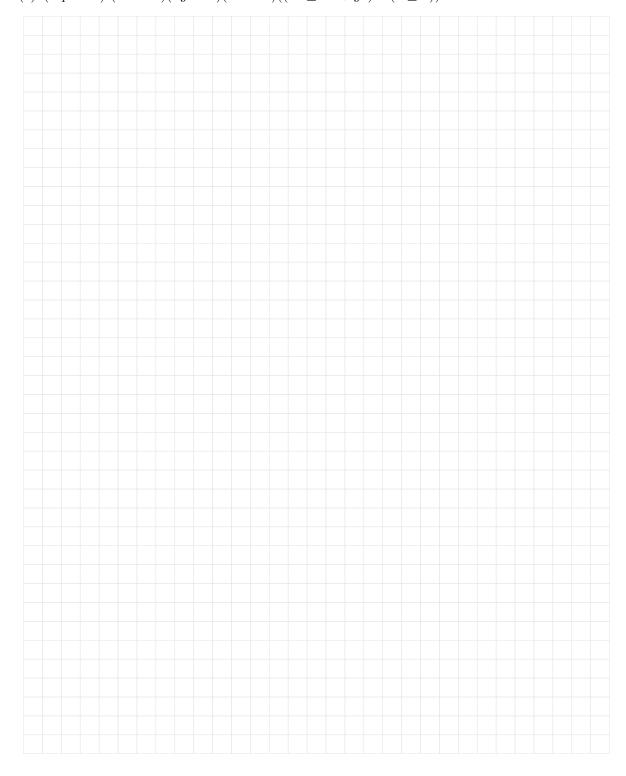


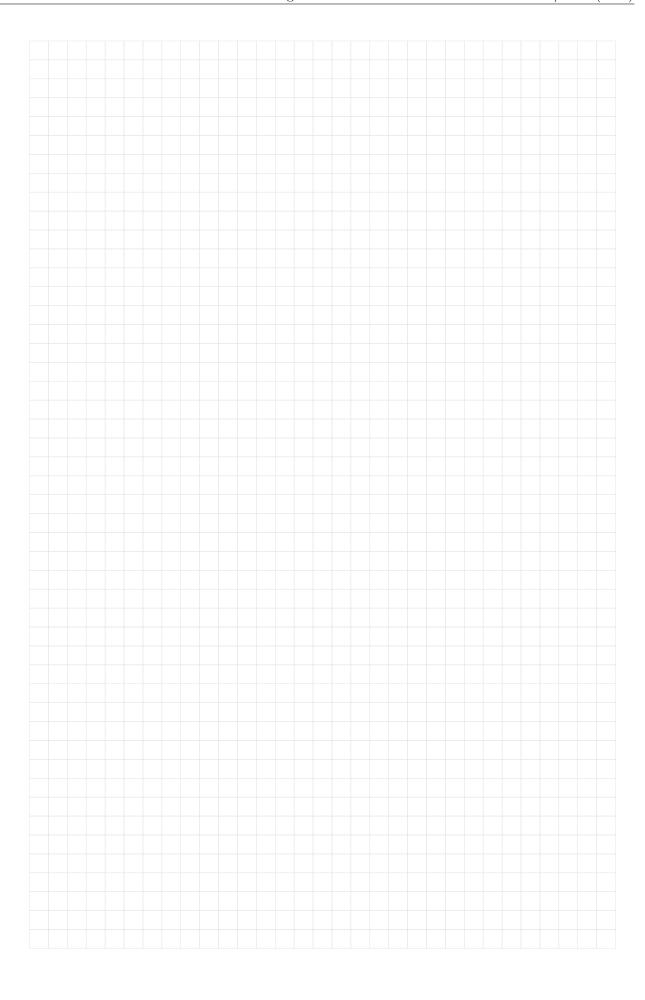
Question 6 (10 points)

Prove or disprove the following statements.

Let
$$X = \{-2, -1, 0, 1, 2\}$$
 and $Y = \{-2, -1, 0, 1, 2\}$.

- (a) $(2 \text{ points}) (\forall x \in X)(\exists y \in Y)(x+y=0).$
- (b) $(2 \text{ points}) (\exists x \in X) (\forall y \in Y) (x + y = y).$
- (c) (6 points) $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{N})((z^2 \ge x^2 + y^2) \land (z \ge 5)).$





Question 7 (15 points)

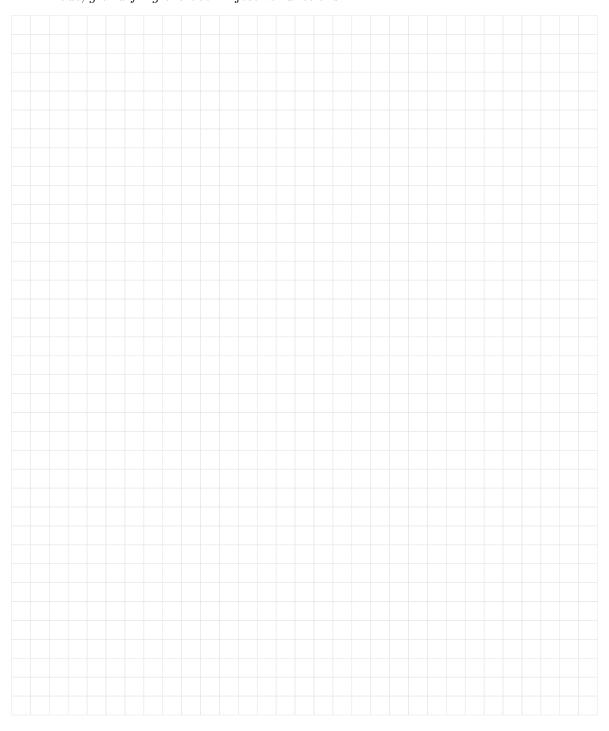
This is a question about functions.

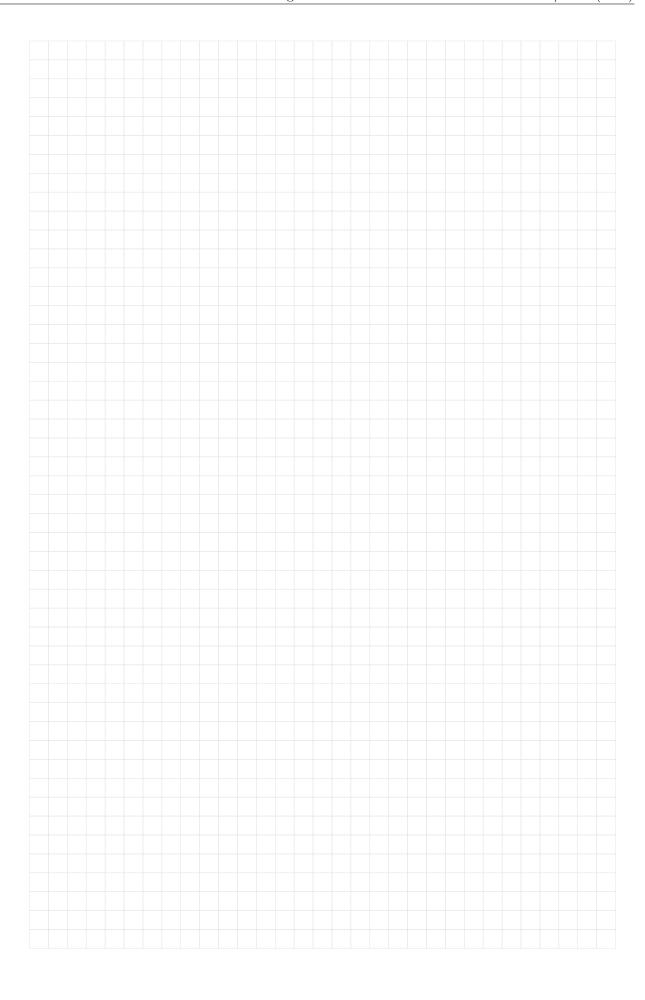
(a) (10 points) Let $f: \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus \{1\}$ be the function defined as follows:

$$f(x) = \frac{x+1}{x}.$$

Prove that f is a bijection.

- (b) (5 points) Construct sets A, B and functions $f:A\to B, g:B\to A$ such that
 - \bullet f is not an injective function,
 - but, g and $f \circ g$ are both injective functions.





Question 8 (5 points)

This is a question about set theory.

- (a) (2 points) Is $\{\{a,d,e\},\{b,c\},\{d,f\}\}$ a partition of $\{a,b,c,d,e,f\}$? Why/Why not?
- (b) (3 points) Let $A=\{1,2\},\,B=\{2,3\}$ and $C=\{3,4\}.$ Write down $(A\times B)\cap (A\times C).$

