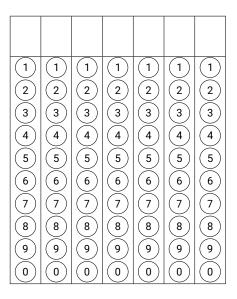
#### Surname, First name

Linear Algebra (KEN1410)

Resit





Fill in your answer(s) to the multiple-choice questions as shown above (circles = one correct answer).

**Program: Data Science and Artificial Intelligence** 

Course code: KEN1410

Examiners: Dr. Marieke Musegaas and Dr. Stefan Maubach

Date/time: Wednesday 5 July 2023, 09:00 - 11:00

Format: Closed book exam

Allowed aids: Pens, simple (non-programmable) calculator from the DACS-list of allowed calculators. Instructions to students:

- The exam consists of 7 questions on 20 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. Do <u>not</u> write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded! As a last resort if you run out of space, use the extra answer space at the end of the exam.
- In no circumstance write on or near the QR code at the bottom of the page!
- · Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- · Good luck!

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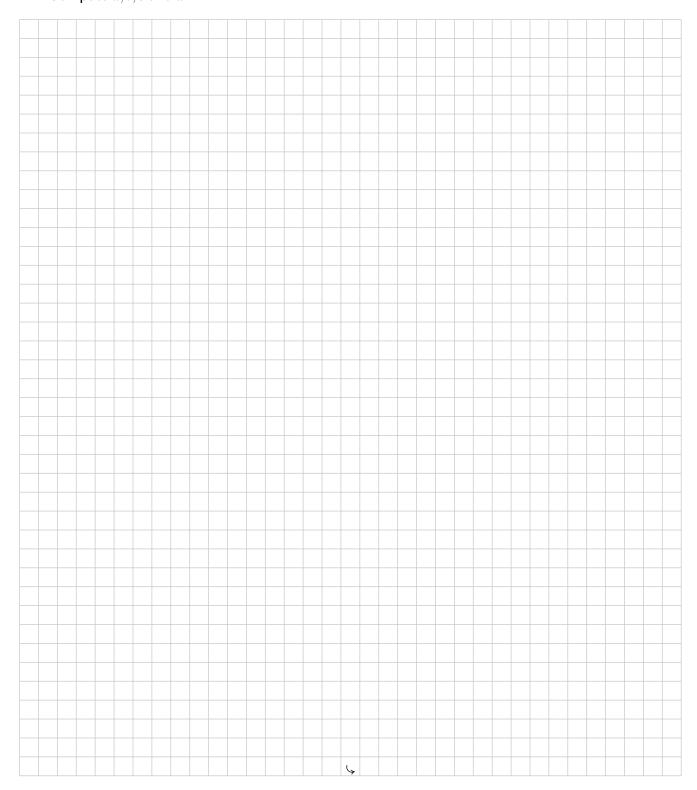




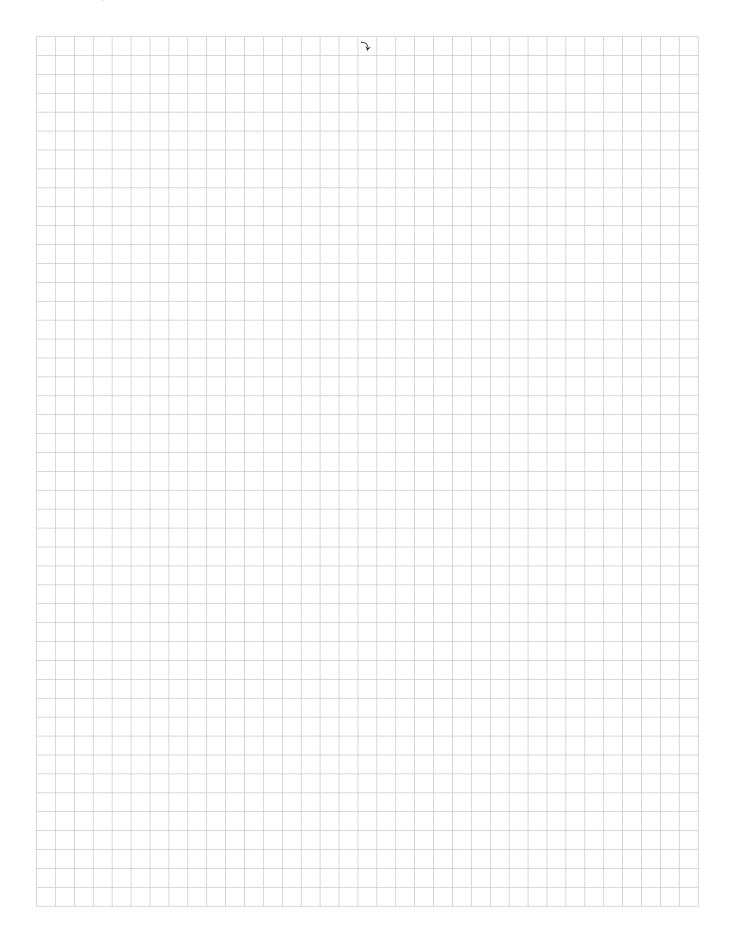
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#### **Question 1**

10p Consider the polynomial  $p(t) = at^3 + bt^2 + ct + d$ , where a,b,c and d are real numbers. It is given that p(-1) = 1, p(0) = 0, p(1) = 1 and p(2) = -1. Compute a,b,c and d.











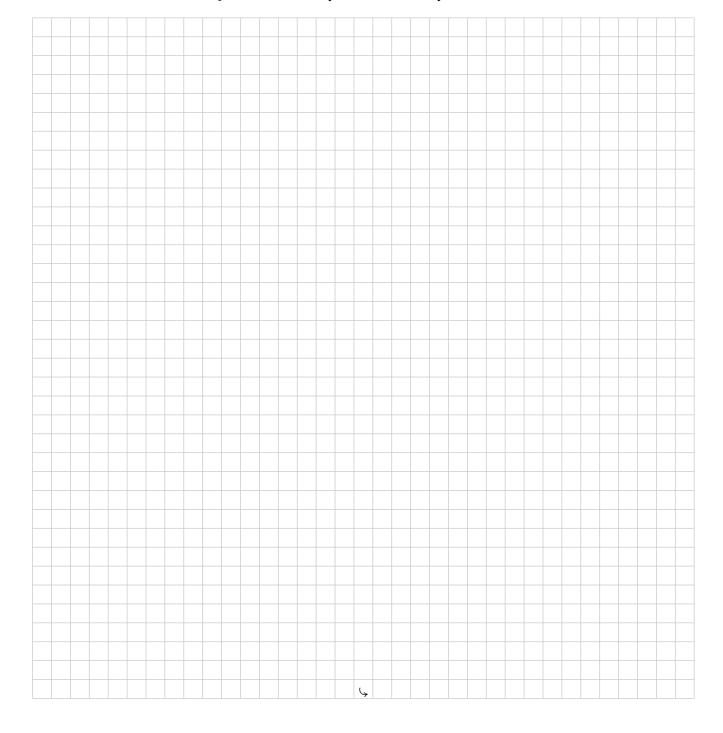
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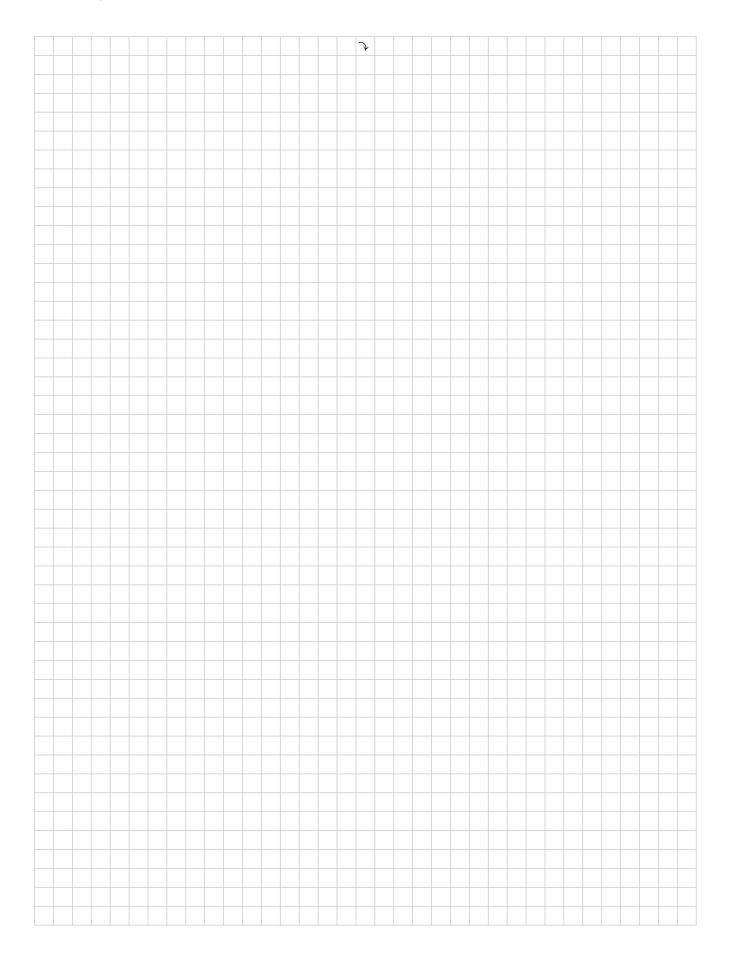
#### **Question 2**

10p **2** Consider the following system of linear equations depending on a parameter p:

$$x_1 + 2x_2 + 4x_3 = 2$$
  
 $2x_1 + (p+3)x_2 + 8x_3 = 2$   
 $x_1 + 2x_2 + p^2x_3 = p$ .

Determine the values of p for which this system has exactly one solution.







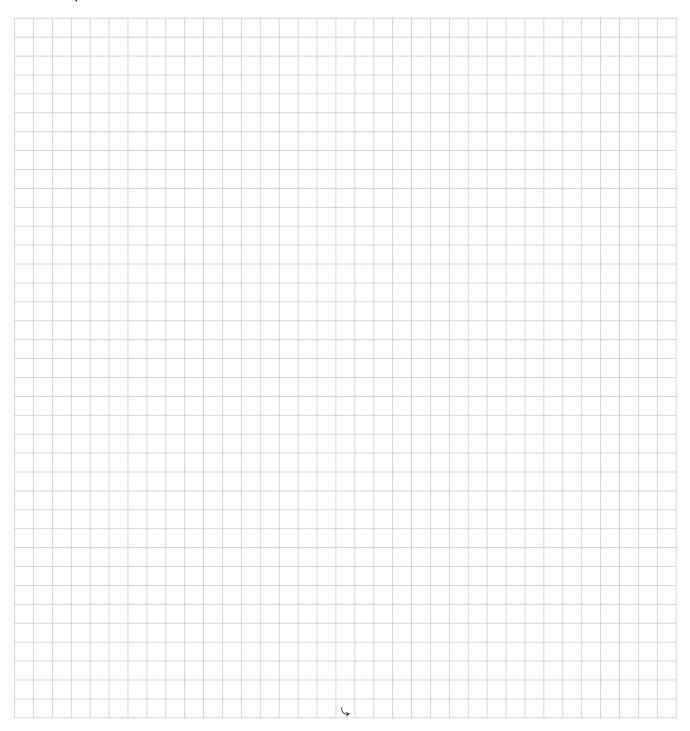


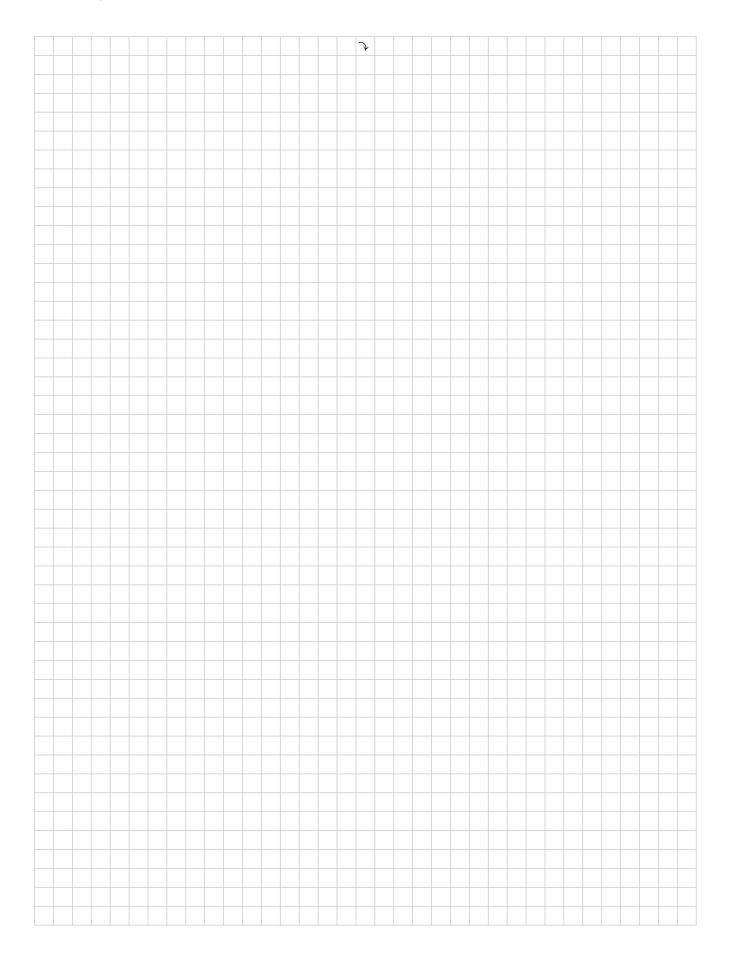
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## Question 3

10p

3 Let 
$$A = \begin{bmatrix} 3 & 3 & 6 & 6 & 7 \\ 4 & 0 & 4 & 4 & 0 \\ 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 5 & 6 & 7 & 1 & 8 \end{bmatrix}$$
. Compute det  $A$ .





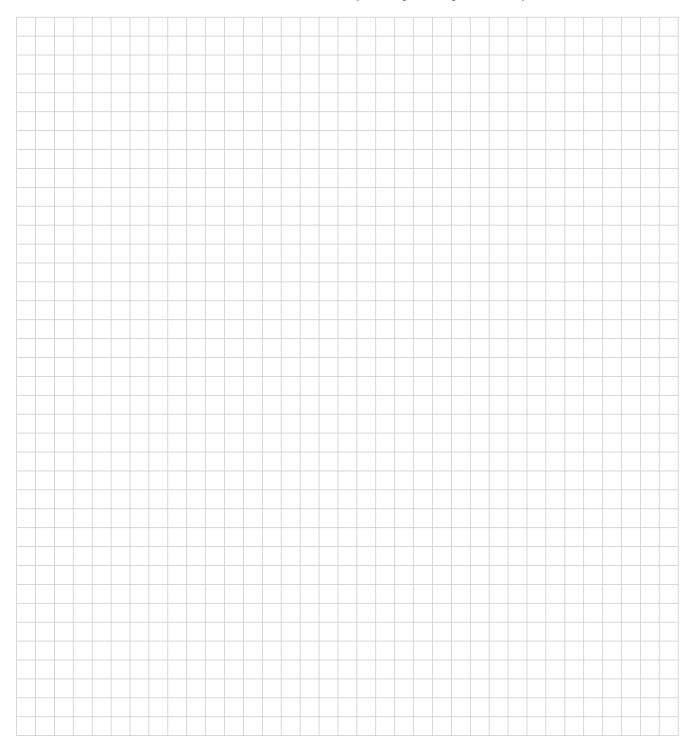


5р

4 Assume that  $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  and  $\begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$  are solutions of the equation  $A\mathbf{x} = \mathbf{b}$ , where A is a  $3 \times 3$  matrix and

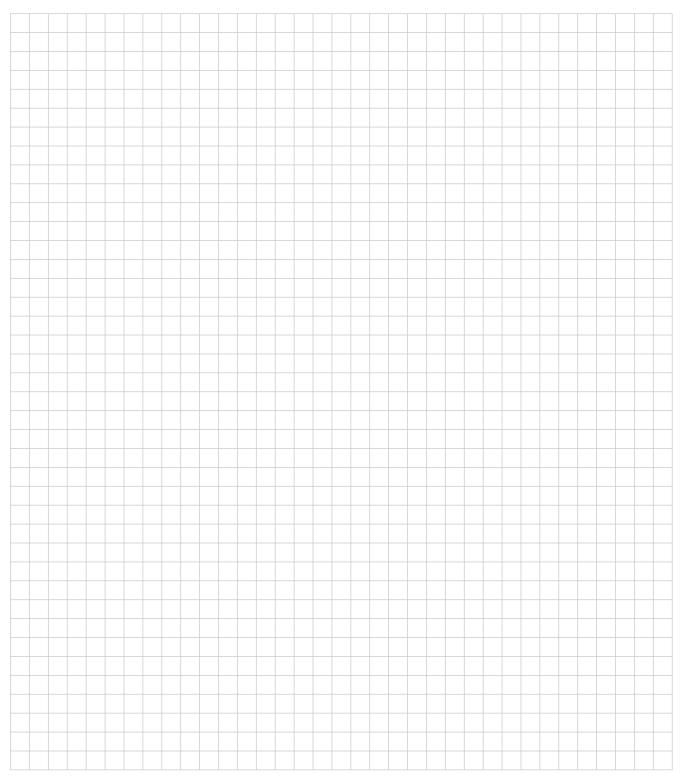
 $b \neq 0$ 

Determine three different solutions of the corresponding homogeneous equation  $A\mathbf{x} = \mathbf{0}$ .



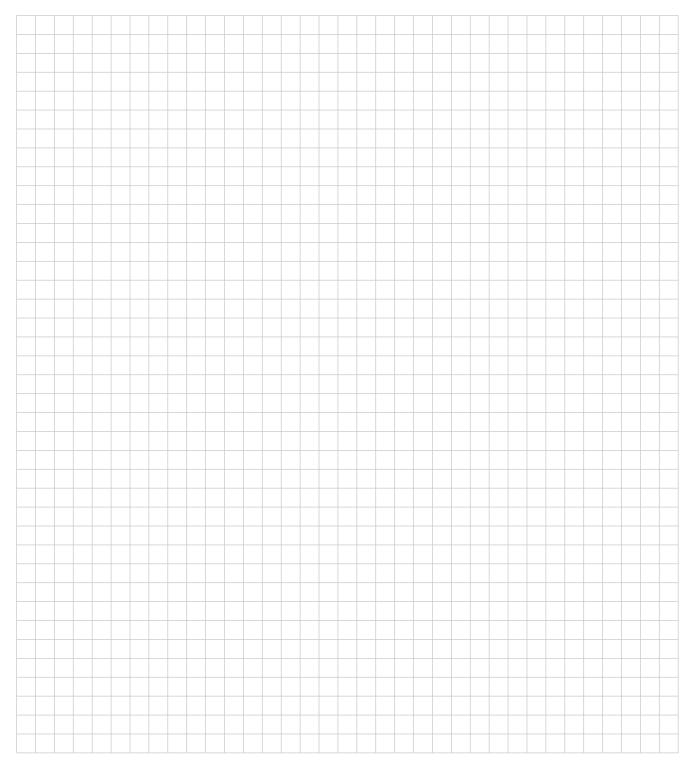
5р

**5** Let  $T: \mathbb{R}^2 \to \mathbb{R}^2$  be a transformation such that  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}$ . Is T a linear transformation? If yes, prove it. If not, explain why not.

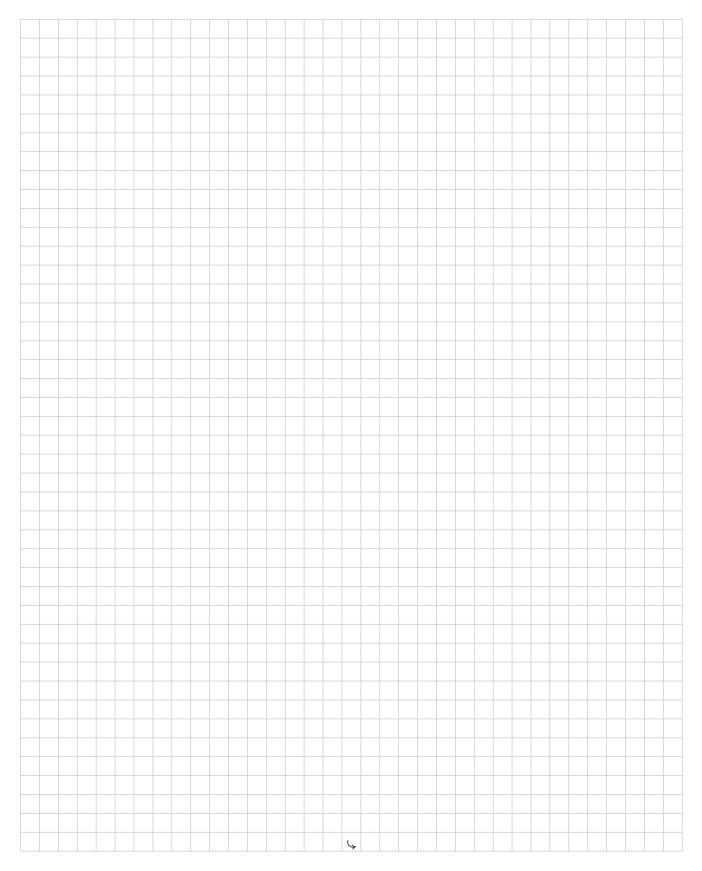


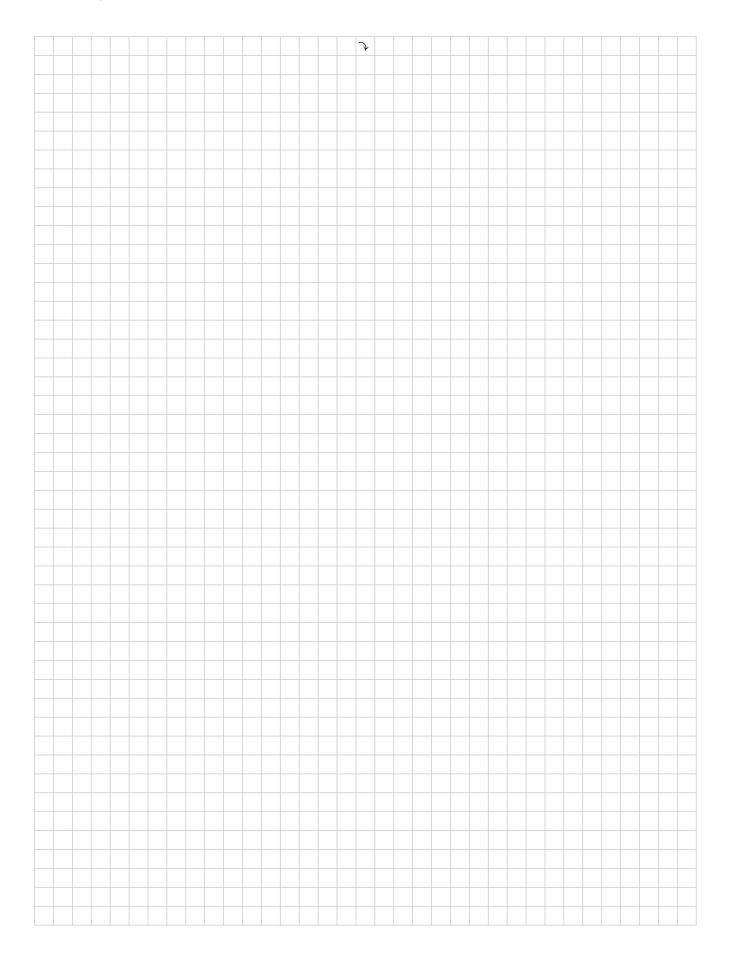
Let 
$$A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$
.

5p **6a** Show that the matrix A has eigenvalues 3 and 5.



10p **6b** Orthogonally diagonalize matrix A, i.e. find an orthogonal matrix P and a diagonal matrix D such that  $A = PDP^T$ .





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Answer the following multiple-choice questions.

An explanation is not required.

For every multiple-choice question, there is only one correct answer.

#### Please read the multiple choice instructions on the cover page!

- 2p **7a** Let A be a  $6 \times 4$  matrix. Is the following statement true or false? "The column space of A and the row space of A are orthogonal to each other."
  - (a) True (b) False
- 2p **7b** Let A be a  $6 \times 4$  matrix. Is the following statement true or false? "The row space of A and the nullspace of A are orthogonal to each other."
  - (a) True (b) False
- 2p **7c** Let A be a  $6 \times 4$  matrix. Is the following statement true or false? "The column space of A and the nullspace of A are orthogonal to each other."
  - (a) True (b) False
- 2p **7d** Let A be a  $6 \times 4$  matrix. What is the smallest possible dimension of Nul A?
  - (a) 0
  - (b) 2
  - (c) 4
  - (d) 6
  - (e) None of the above.
- 2p **7e** Let A be a  $6 \times 4$  matrix. What is the largest possible dimension of Col A?
  - (a) 0
  - (b) 2
  - (c) 4
  - (d) 6
  - (e) None of the above.

5р

7f Let 
$$A = \begin{bmatrix} 1 & 1 & -2 & -1 \\ -1 & 4 & -3 & 1 \\ 0 & 7 & 1 & -8 \end{bmatrix}$$
,  $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}$  and  $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}$ .

Does  ${\bf u}$  belong to Nul A? Does  ${\bf v}$  belong to Col A?

- (a)  $\mathbf{u}$  belongs to Nul A.  $\mathbf{v}$  belongs to Col A.
- (b)  $\mathbf{u}$  belongs to Nul A.  $\mathbf{v}$  does not belong to Col A.
- $\bigcirc$  u does not belong to Nul A. v belongs to Col A.
- (d)  $\mathbf{u}$  does not belong to Nul A.  $\mathbf{v}$  does not belong to Col A.

5р

**7g** What is the dot product (inner product) of  $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$  and  $\begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$ ?

- $\begin{array}{c}
   \begin{bmatrix}
   0 \\
   2 \\
   3
  \end{array}$
- (c) 0
- (d) 5
- (e) The dot product cannot be computed for these vectors.
- f None of the above.

5p

**7h** If  $\mathbf{y} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$  and  $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ , then the orthogonal projection of  $\mathbf{y}$  onto Span  $\{\mathbf{u}\}$  is

- - $\begin{array}{c|c}
    \hline
    c \\
    5 \\
    \hline
    d \\
    \hline
    \end{array}$
  - None of the above.

5р

- **7i** If A is a  $3 \times 3$  matrix such that  $A \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  and  $A \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ , then the product  $A \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}$  is

- $\begin{array}{c|c}
  \hline
   & 1 \\
  -1 \\
  0 \\
  \end{array}$
- $\begin{array}{|c|c|c|}
  \hline
  \mathbf{d} & 9 \\
  10 \\
  11
  \end{array}$
- (e) Not uniquely determined by the information given.

5р

7j For what value (or values) of p is the vector  $\begin{bmatrix} 1 \\ 2 \\ p \\ 5 \end{bmatrix}$  a linear combination of the vectors  $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ 

and 
$$\begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$
?

- a -1 only
- (b) 1 only
- c 3 only
- $\stackrel{\circ}{\text{(e)}}$  for infinitely many values of p
- (f) None of the above.

5p **7k** Recall that  $\mathbb{P}_2$  denotes the set of polynomials of degree at most 2. In other words,  $\mathbb{P}_2$  consists of all polynomials of the form

$$\mathbf{p}(t) = a_0 + a_1 t + a_2 t^2,$$

where the coefficients  $a_0, a_1, a_2$  and the variable t are real numbers.

Let V be the subset of  $\mathbb{P}_2$  made up of only those polynomials  $\mathbf{p}(t)$  such that  $\mathbf{p}(1)$  = 0, i.e.

$$V = \{\mathbf{p} \in \mathbb{P}_2 : \mathbf{p}(1) = 0\}.$$

Similarly, let W be the subset of  $\mathbb{P}_2$  made up of only those polynomials  $\mathbf{p}(t)$  such that  $\mathbf{p}(0)$  = 1, i.e.

$$W = \{ \mathbf{p} \in \mathbb{P}_2 : \mathbf{p}(0) = 1 \}.$$

Is V is a subspace of  $\mathbb{P}_2$ ? Is W is a subspace of  $\mathbb{P}_2$ ?

- (a) V is a subspace of  $\mathbb{P}_2$ . W is a subspace of  $\mathbb{P}_2$ .
- (b) V is a subspace of  $\mathbb{P}_2$ . W is not a subspace of  $\mathbb{P}_2$ .
- (c) V is not a subspace of  $\mathbb{P}_2$ . W is a subspace of  $\mathbb{P}_2$ .
- (d) V is not a subspace of  $\mathbb{P}_2$ . W is not a subspace of  $\mathbb{P}_2$ .
- 71 Let  $A = \begin{bmatrix} a & -3 \\ 2 & b \end{bmatrix}$ , where a, b are real numbers. Given is that the characteristic polynomial of A is  $\lambda^2 19$

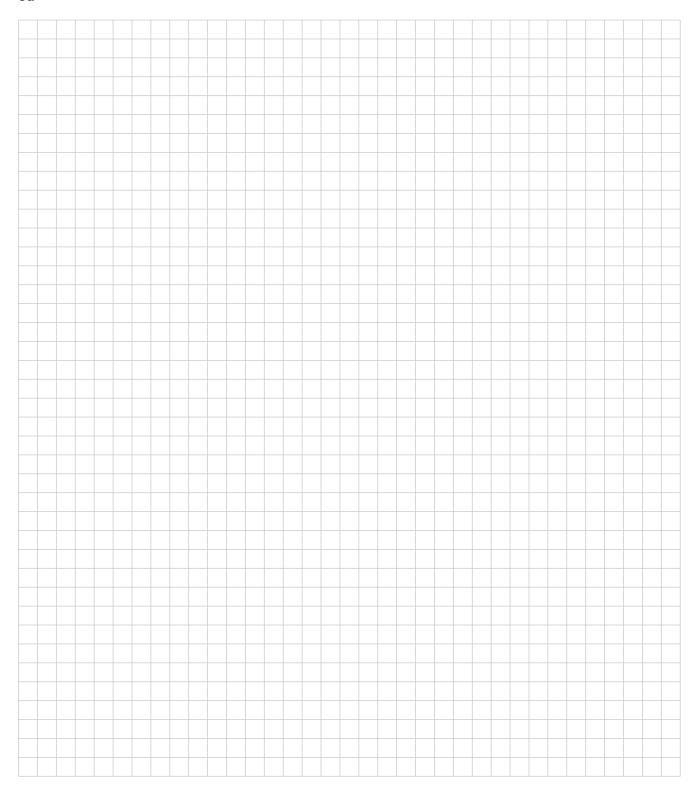
How many different values can a have?

- $\bigcirc$  There are at least three different values a can have.
- (b) There are exactly two values a can have.
- $\bigcirc$  There is only one value a can have.
- (d) There are no posisble values for a.

#### Extra space

If you use these extra answer boxes, please mention clearly in your main answer that part of your answer can be found here!

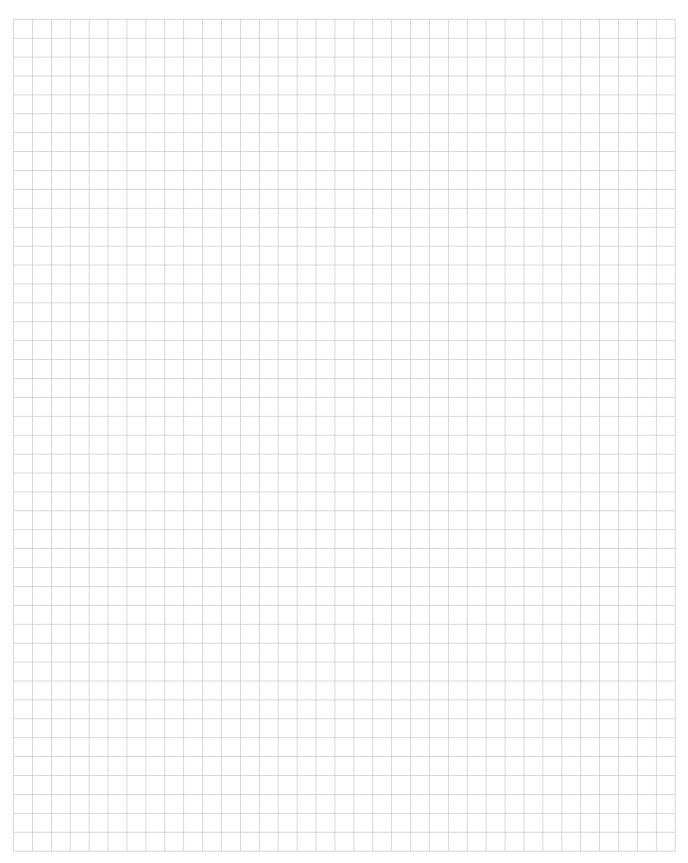
8a



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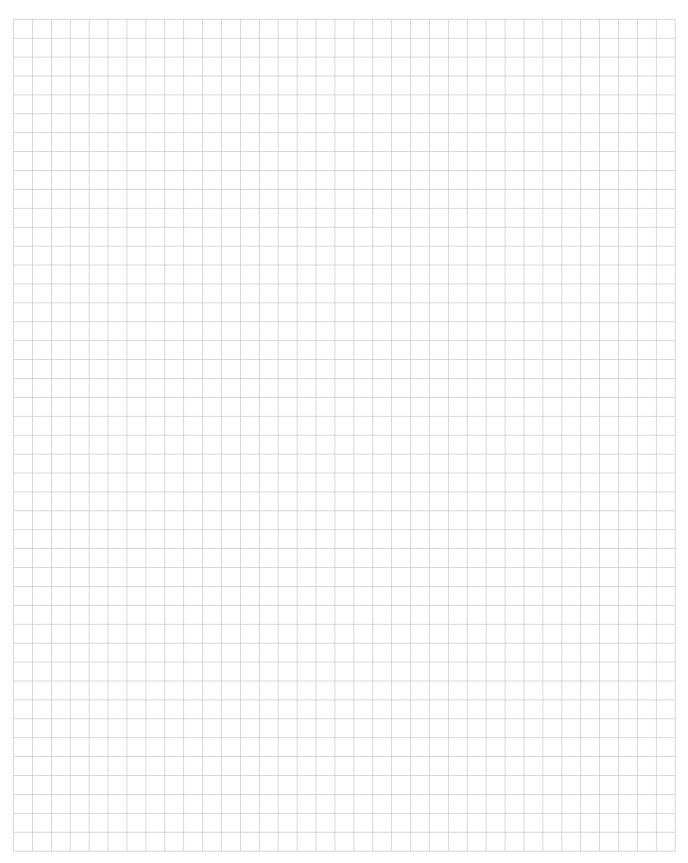
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8b





8c





8d

