# Class notes

## Exercises

(a) is $(q \wedge (p \to q)) \to p$ valid? (tautology)

remove $\to$, why? (we need to have 2 formulas)

$q \wedge (p \to q) \circ p$

remove $\wedge$ on LHS

$q, p \to q \circ p$

remove $\to$ on LHS

$q \circ p, p$ $\qquad\qquad$ $q, q \circ p$

If we have the same statement, we can ignore it

$q \circ p$ $\qquad\qquad$ $q \circ p$

Tableau is open $\to$ formula is not valid
↳ it would be false if
$V(q) = 1$, $V(p) = 0$, then $V((q \wedge (p \to q)) \to p) = 0$
$\to$ this is a counterexample
$\to$ if the question asks if a statement is a tautology, it is enough to expand on a single branch that leads to a counter example

## Terminology of tableaus

- **sequent** → each node of the tree
- **Closed branch** → on the end sequent there is the same formula on LHS & RHS
- **Closed tableau** → all the branches are closed
- **Open branch** →
- **Open tableau** → at least 1 open branch

## General tableau method

$$\frac{\varphi_1, \varphi_2, \ldots \varphi_n}{\psi} \text{ valid ?}$$

$\iff (\varphi_1, \varphi_2, \varphi_n) \to \psi$ valid

$\circ \; (\varphi_1 \wedge \varphi_2 \wedge \ldots \varphi_n) \to \psi$ ($\to$, right)

$\varphi_1 \wedge \varphi_2 \wedge \varphi_n \quad \circ \quad \psi$ ($\wedge$, left, n-1 times)

$\varphi_1, \varphi_2 \ldots \varphi_n \quad \circ \quad \psi$

If tableau closed → valid
If tableau open → find counter ex.

## Tableau use cases

(1) Check if a formula is valid
(2) Check if formula is satisfiable or contradiction
(3) Check if a set of formulas is satisfiable
(4) Check if 2 formulas are equivalent

(1) $\dfrac{\varphi \to \psi, \quad \psi}{\varphi}$ valid ?

$\varphi \to \psi, \quad \psi \circ \varphi$

(→, left)

$\psi \circ \varphi, \varphi \qquad\qquad \psi, \quad \psi \circ \varphi$

$\psi \circ \varphi \qquad\qquad\qquad \psi$

open branch:

$V(\psi) = 1, \quad V(\varphi) = 0$

→ complete example notes

(4) Check if $\varphi$ and $\psi$ are equivalent

→ $\varphi \models \psi \quad \wedge \quad \psi \models \varphi$

2 ways:

(1) $\varphi \circ \psi$ closed $\wedge$ $\psi \circ \varphi$ closed

(2) $\varphi \leftrightarrow \psi$ is tautology

$\circ \ \varphi \leftrightarrow \psi$ is closed

$\varphi \circ \psi \qquad \psi \circ \varphi$

## Notes on tableaus

- Tableau builds a model with specific requirem..
- Tableau method is complete for proving validity and for finding counterexamples
- Tablaus can generate all counterexamples
- Tableaus can be used with quantifiers too
  - ↳ problems with checking completeness

# Exercises in class

De Morgan law

(1) Is $\neg A \wedge \neg B \models \neg(A \vee B)$ valid?

$\neg A \wedge \neg B \circ \neg(A \vee B)$ ← is it closed?

why and didn't split?

$|$ $(\wedge, \text{left})$

$\neg A, \neg B \circ \neg(A \vee B)$

$|$ $(\neg, \text{right})$

$\neg A, \neg B, A \vee B \circ$

Don't do them at once!

$|$ $(\neg, \text{left})$

$\neg B, A \vee B \circ A$

order of these 2 steps doesn't matter

We CAN do it (But only with the NOT operator)

$|$

the $\vee$ splits → $A \vee B \circ A, B$

why $\vee$ split?

$A \circ A, B$ $\qquad$ $B \circ A, B$

X $\qquad$ X

Tableau is closed → valid inference

$\neg(A \vee B) \models \neg A \vee \neg B$

(2) $\neg(A \vee B) \models \neg A \wedge \neg B$

$\neg(A \vee B) \circ \neg A \wedge \neg B$

$|$ $(\neg, \text{left and right})$

$\circ \neg A \wedge \neg B, A, B$

$\circ \neg A, A, B$ $\qquad$ $\circ \neg B, A, B$

$|$ $\qquad$ $|$

$A \circ A, B$ $\qquad$ $B \circ A, B$

Why did the OR not split, but the AND did