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8例 $a \vee \neg a \models (\neg p \to \neg q) \to (q \to p)$

1.	$a \vee \neg a$	(given)
2.	$\neg p \to \neg q$	(assume)
3	q	(assume)
4	$\neg p$	(assume)
5.	$\neg q$	$E_{\to}(2,4)$
6	$\perp$	$E_{\neg}(3,5)$
7.	p	$I_{\neg}(4,6)$
8	$q \to p$	$I_{\to}(3,7)$
9	$(\neg p \to \neg q) \to (q \to p)$	$I_{\to}(2,8)$

8.e

1.	$p \to q$	(given)
2	$\neg q$	(ass.)
3	p	(ass
4.	q	$E_{\to}(1,3)$
5	$\perp$	$E_{\neg}(2,4)$
6	$\neg p$	$I_{\neg}(3,5)$
7	$\neg q \to \neg p$	$I_{\to}(2,6)$

8 d)

1.	$p \vee \neg q$	given
2	p	ass.
3.	q	ass
4.	p	repeat(2)
5.	$q \to p$	$I_{\to}(3,4)$
6.	$\neg q$	ass.
7.	q	ass.
8.	$\perp$	$E_{\neg}(6,7)$
9	p	$\perp(8)$
10.	$q \to p$	$I_{\to}(7,9)$
11.	$q \to p$	$E_{\vee}(2,5,6,10)$

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89)

1.	$\neg(p \vee \neg p)$	ass.
2	p	ass.
3	$p \vee \neg p$	$I\vee(2)$
4	$\perp$	$E\neg(1,3)$
5	$\neg p$	$I\neg(2,4)$
6	$p \vee \neg p$	$I\vee(5)$
7	$\perp$	$E\neg(1,6)$
8	$p \vee \neg p$	$I\neg(1,7)$

8.b)

1.	$p \to \neg q$	(given)
2.	q	(given)
3	p	(ass.)
4	$\neg q$	$E_{\to}(1,3)$
5	$\perp$	$E_{\neg}(2,4)$
6.	$\neg p$	$I_{\neg}(3,5)$

A
$\vdots$
$\perp$
$\neg A$

A
$\vdots$
$\perp$
B
$A \to B$

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8 a)

1.	$\neg\neg p$	(given)
2.	$\quad \vert\ \neg p$	(ass.)
3.	$\quad \vert\ \bot$	$E_{\neg}(1,2)$
4	p	$I_{\neg}(2,3)$.

7c)

$p \wedge q, \underline{\neg q} \circ r$

$\underline{p \wedge q} \circ r, q$

$p, \underline{q} \circ r, \underline{q}$

$\times$

So inference is valid

7b)

$p \to q, \neg q \to \neg r \circ \underline{p \to r}$

$p \to q, \underline{\neg q \to \neg r}, p \circ r$

$p \to q, p, \underline{\neg r} \circ r$ $\qquad$ $p \to q, p \circ \neg q, r$

$\underline{p \to q}, p \circ r$

$p, q \circ r$ $\qquad$ $p \circ r, p$

open

Tableau open; Inference not valid!

7a) $p, q \circ \underline{p \to q}$

$|$

$p, \underline{q} \circ \underline{q}$

$\times$

6)

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

for matrix multiplication

$(p \to q) \land r \;\cancel{\mid\!\!=\!\!\mid}\; p \to (q \land r)$

5a) φ contradiction

$\varphi \circ$ closed

$\circ \varphi$ closed

$\circ \neg\varphi$ closed

φ tautology

$\neg\varphi$ contradiction

$\neg\varphi$ tautology

$\Leftrightarrow \varphi$ contradiction

5a) (cont'd)

$(p \to \neg q) \wedge (q \to r) \wedge (r \to p) \circ$

$p \to \neg q,\ (q \to r) \wedge (r \to p) \circ$

$p \to \neg q,\ q \to r,\ r \to p \circ$

$p \to \neg q,\ r \to p,\ r \circ$ | $p \to \neg q,\ r \to p \circ q$

$p \to \neg q,\ r,\ p \circ$ | $p \to \neg q,\ r \circ r$ $\times$

$r,\ p,\ \neg q \circ$ | $r,\ p \circ p$ $\times$

$r,\ p \circ q$

open

open, so $\models \neg\varphi$ not valid, so φ is not a contradiction.

4a)

$\circ\ p \to (p \wedge q)$

$p \circ p \wedge q$

$p \circ p$ ✗ $\qquad\qquad$ $p \circ q$ open

$\Rightarrow$ tableau open, $\not\models p \to (p \wedge q)$ not valid, so not a tautology.

(a) sentence (b) is true

(b) sentence (c) is false

(c) There are exactly two sentences correct

a = true → sentence 1 true
= false → sentence 1 false, b, c similar

$a \leftrightarrow b$

$b \leftrightarrow \neg c$

$c \leftrightarrow ((a \wedge b \wedge \neg c) \vee (a \wedge \neg b \wedge c) \vee (\neg a \wedge b \wedge c))$

$c \leftrightarrow ((a \wedge b \wedge \neg c) \vee (a \wedge \neg b \wedge c) \vee (\neg a \wedge b \wedge c))$

a	b	c	c		
1	1	0	0		1
0	0	0	0	1	0
1	1	1	1	1	1
0	0	1	1	0	0

$b \leftrightarrow \neg c$

1
0
0
1