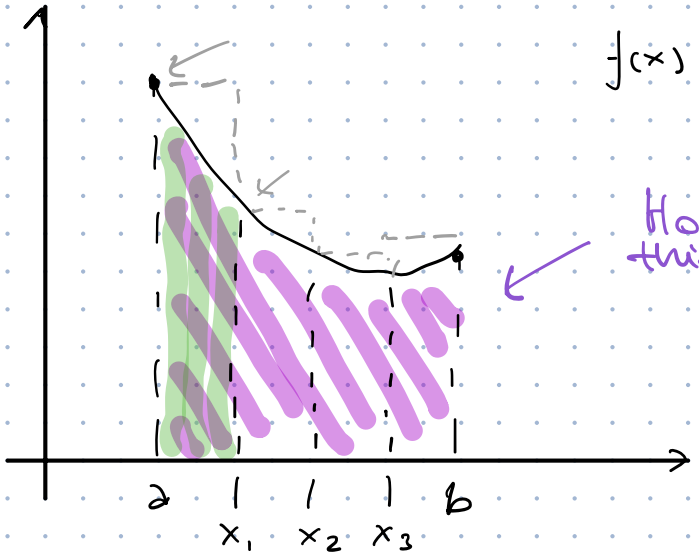
Integration

A. Calc. Chapters 5.2-4, 2.10

Definite Integrals



$f(x)$ is cont. on $[a, b]$

How do we calculate this area?

$a < x_0 < x_1 < x_2 < x_3 < b$ → partition of $[a, b]$ ↑ P

$\Delta x_n = x_{n+1} - x_n$

$U = \sum f(u_k) \cdot \Delta x > A$ upper boundary of Area below the curve

↑ max ↑ base of each rect

↓ the smaller the partitions are, the closer U will get to A

$A \geq L(f, P) = \sum_k f(l_k) \cdot \Delta x_k$ ← Rieman Sum

↳ as $\Delta x \to 0$, $L(f, P)$ converges to A

↳ as $\Delta x \to 0$, $U(f, P)$ converges to A

Definite integral:

A function is integrable

↑ complete def from slides

Integrable functions

- Piecewise continuous functions are integrable

Notation

$$\int_a^b f(x)\,dx$$

b ← upper bound

$f(x)$ ← Integrand

dx ← Integration variable

a ← lower bound

Things to note

- When is a integral negative?
  - $\int_a^b f(x)\,dx < 0$ if $f(x) < 0$ and $a < b$
  - $\int_b^a f(x)\,dx < 0$ if $f(x) > 0$ and $a < b$

Integrating even and odd functions

- Even functions:

- Odd functions

Average of a function

$$\langle f \rangle = \bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$