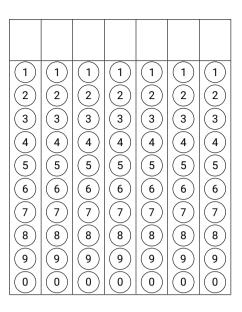
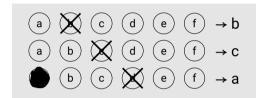
Surname, First name

Linear Algebra (KEN1410)

Exam





Fill in your answer(s) to the multiple-choice questions as shown above (circles = one correct answer).

Program: Data Science and Artificial Intelligence

Course code: KEN1410

Examiners: dr. Marieke Musegaas and dr. Stefan Maubach

Date/time: Tuesday 29 March 2022, 09:00 - 11:00

Format: Closed book exam

Allowed aids: Pens, simple (non-programmable) calculator from the DKE-list of allowed calculators.

Instructions to students:

- The exam consists of 8 questions on 18 pages.
- Fill in your name and student ID number on the cover page.
- Answer every question in the reserved space below the question. Do <u>not</u> write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded! As a last resort if you run out of space, use the extra answer space at the end of the exam.
- In no circumstance write on the QR code at the bottom of the page!
- Ensure that you properly motivate your answers.
- Before answering the questions, please first read all the exam questions, and then make a plan to spend the two hours.
- Only use black or dark blue pens, and write in a readable way. No Pencils. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- · You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- · Good luck!



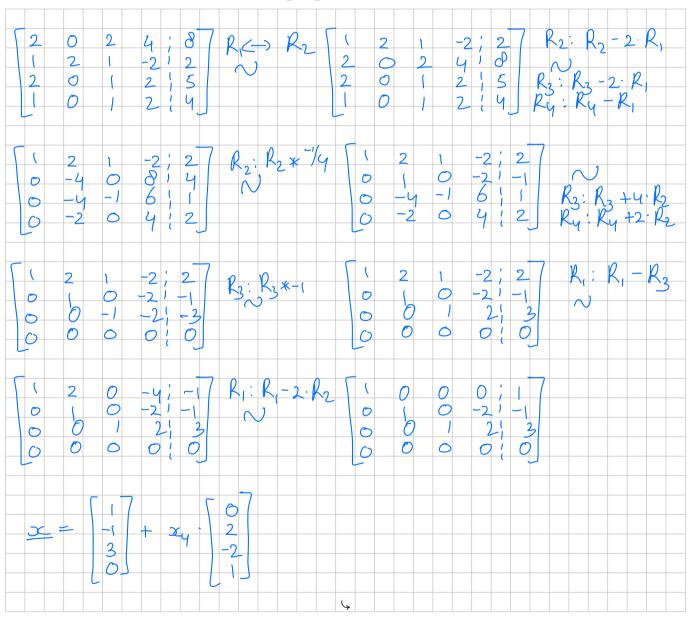
Consider the following matrix A:

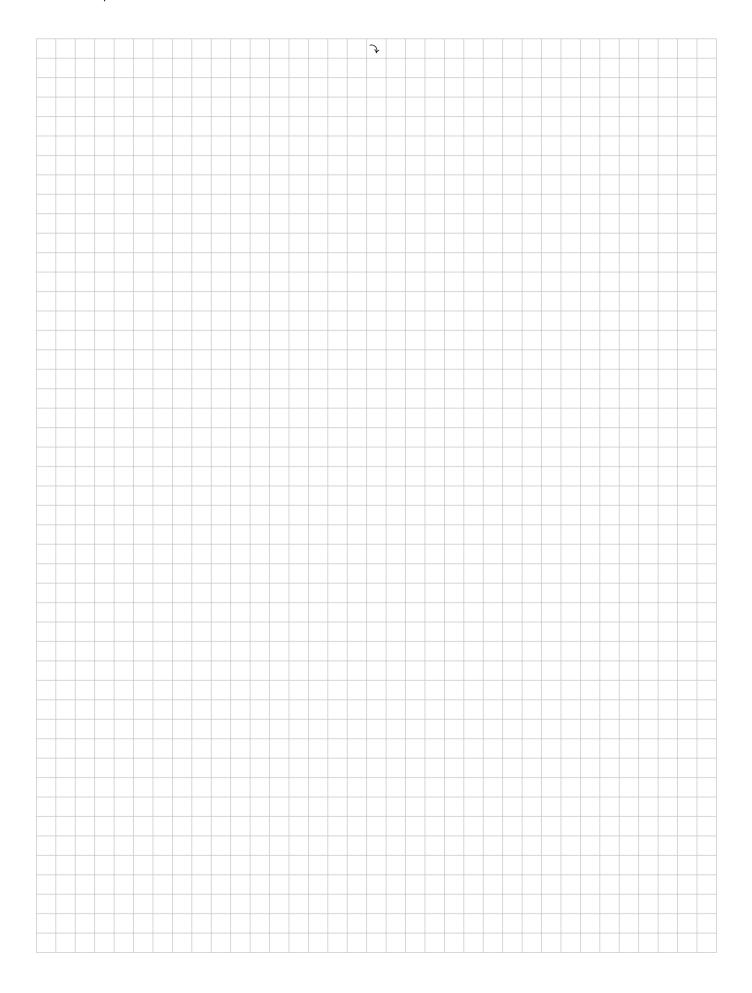
$$A = \left[\begin{array}{cccc} 2 & 0 & 2 & 4 \\ 1 & 2 & 1 & -2 \\ 2 & 0 & 1 & 2 \\ 1 & 0 & 1 & 2 \end{array} \right].$$

Define $T: \mathbb{R}^4 \to \mathbb{R}^4$ by $T(\mathbf{x}) = A\mathbf{x}$.

15p

1 Determine all $\mathbf{x} \in \mathbb{R}^4$ such that $T(\mathbf{x}) = \begin{bmatrix} 8 \\ 2 \\ 5 \\ 4 \end{bmatrix}$









Exercise 2

Let $a, b, c, d \in \mathbb{R}$ and consider the following system of linear equations

$$x_1 + ax_2 + bx_3 = 14$$

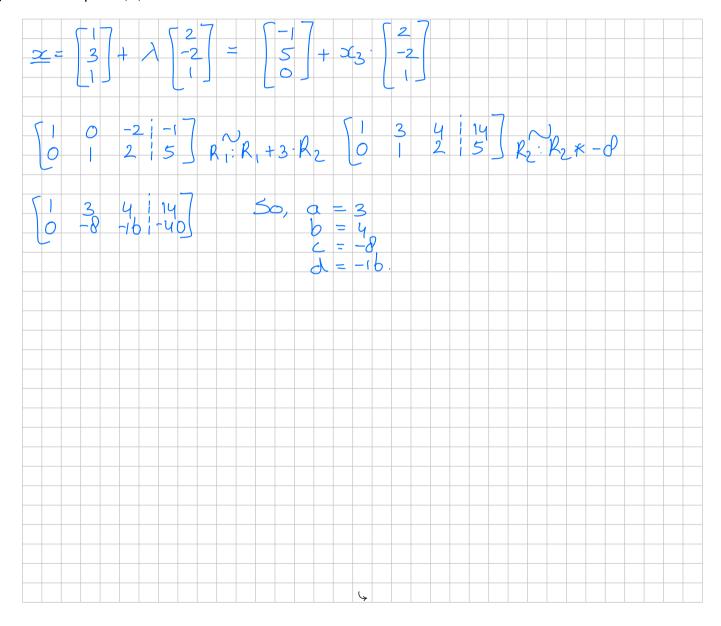
 $cx_2 + dx_3 = -40$

This system of linear equations has a solution set that looks like

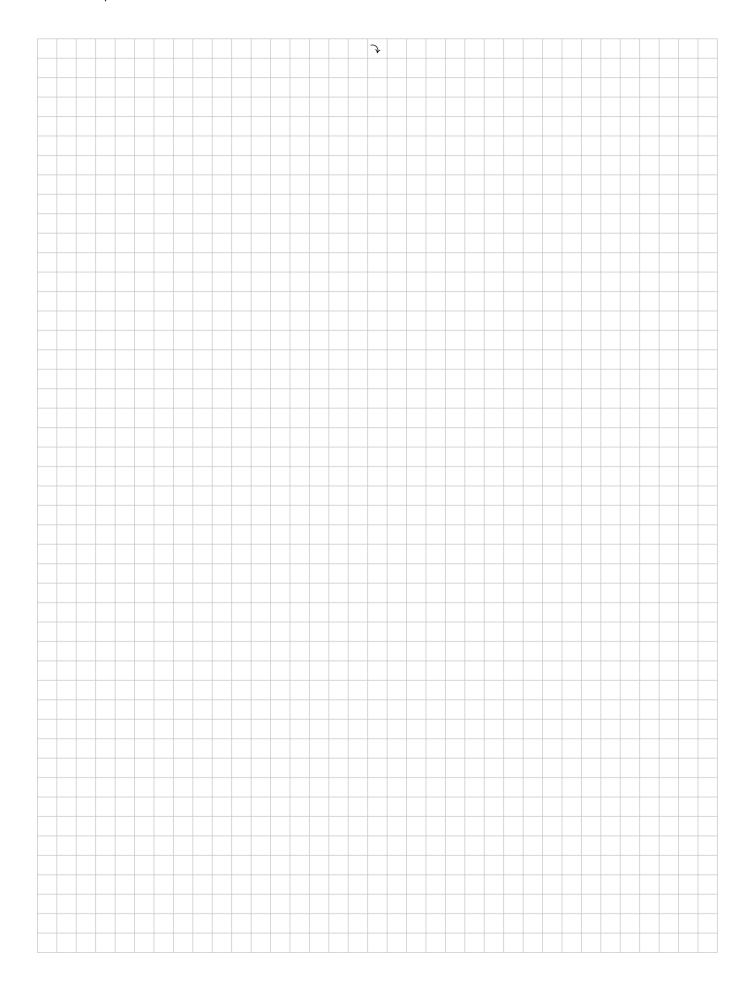
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix},$$

where $\lambda \in \mathbb{R}$.

15p **2** Compute a, b, c and d.







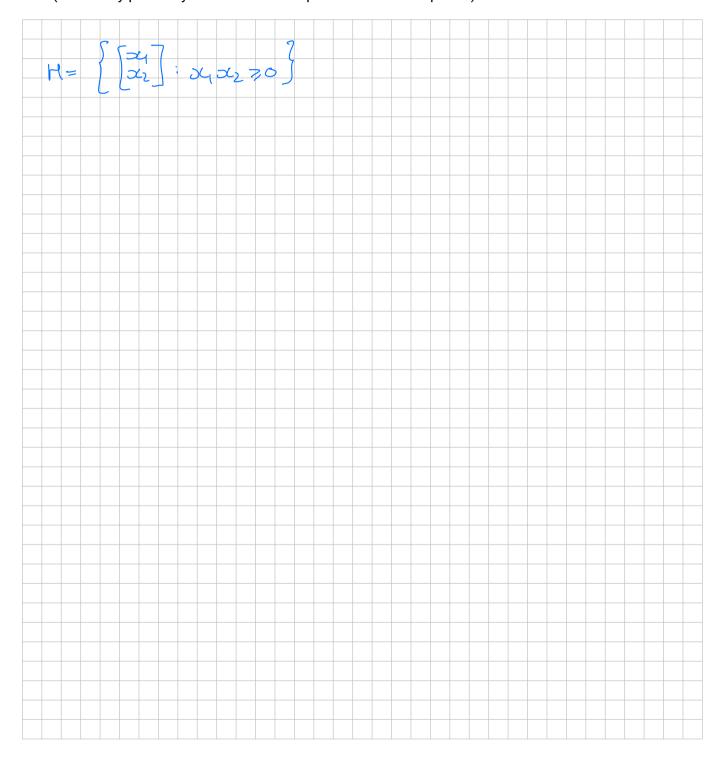




5р

- **3** Provide, **IF POSSIBLE**, an example of a subset H of \mathbb{R}^2 that has the following three properties:
 - the zero vector is in H,
 - H is **NOT** closed under vector addition,
 - ullet H is closed under multiplication by scalars.

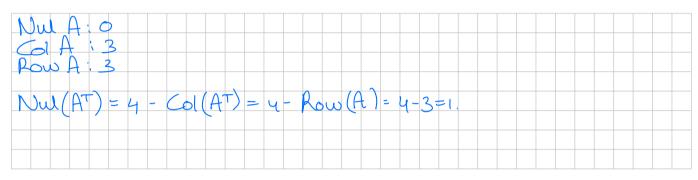
(Note: only provide your answer. An explanation is not required.)



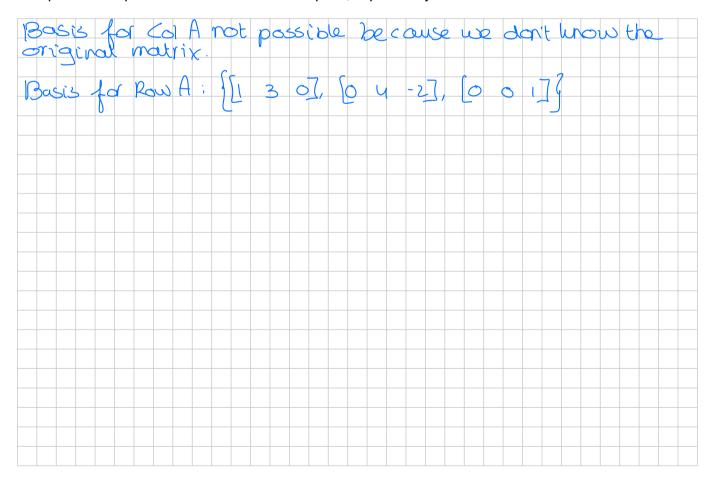
A matrix A has after a couple of row operations the following form

$$\left[\begin{array}{cccc}
1 & 3 & 0 \\
0 & 4 & -2 \\
0 & -6 & 3 \\
0 & 0 & 1
\end{array}\right].$$

5p **4a** Provide the dimensions of the following four vector spaces: Nul A, Col A, Row A and Nul A^T . (Note: only provide your answer. An explanation is not required.)



5p **4b** Provide, **IF POSSIBLE**, a basis of the following two vector spaces: Col A and Row A. If it is not possible to provide a basis for a vector space, explain why.



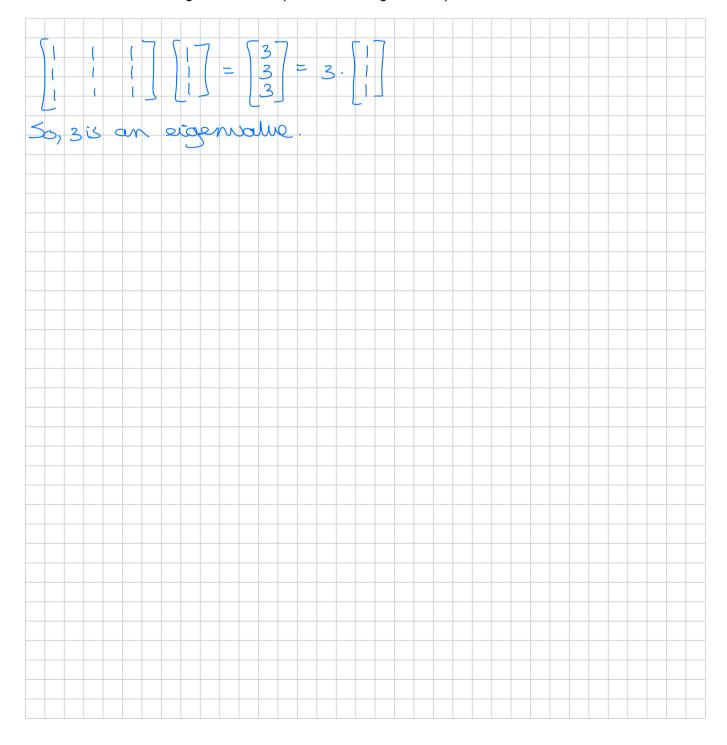


Exercise 5

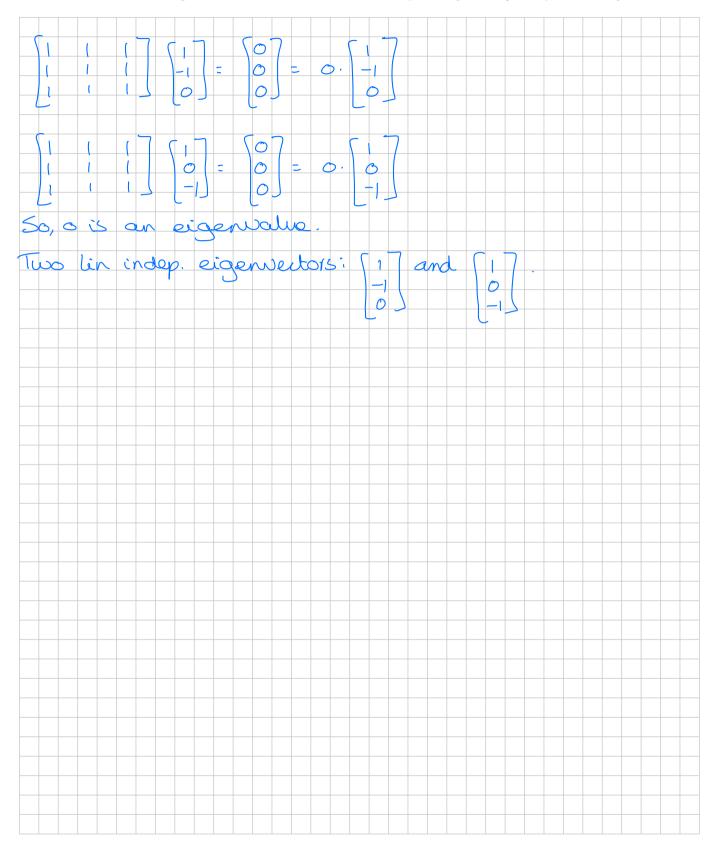
Consider the following matrix A:

$$A = \left[\begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right].$$

5p **5a** Show that 3 is an eigenvalue of A (hint: find an eigenvector).



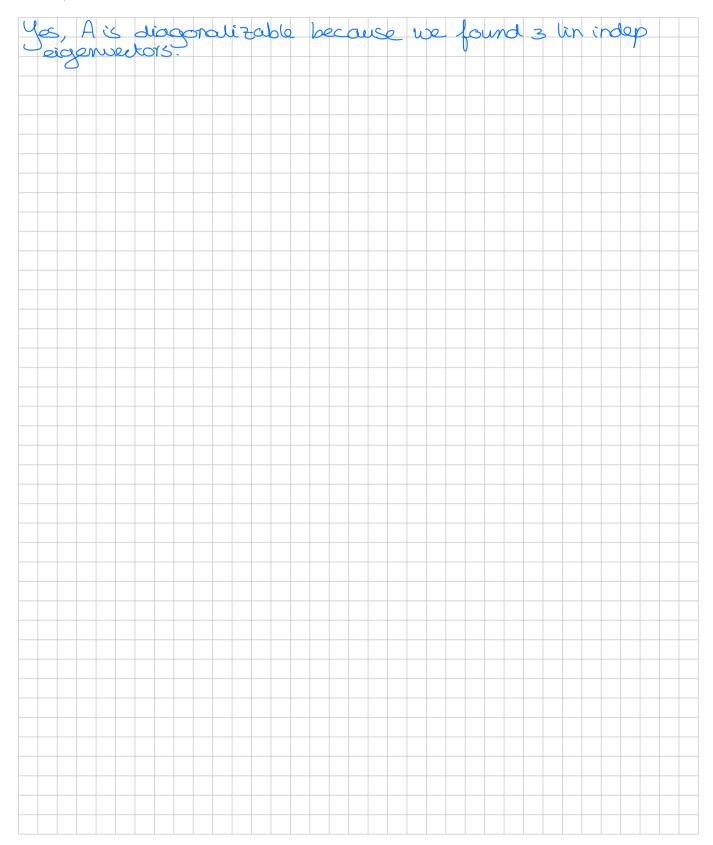
10p **5b** Show that 0 is an eigenvalue of A. And find two corresponding linearly independent eigenvectors.



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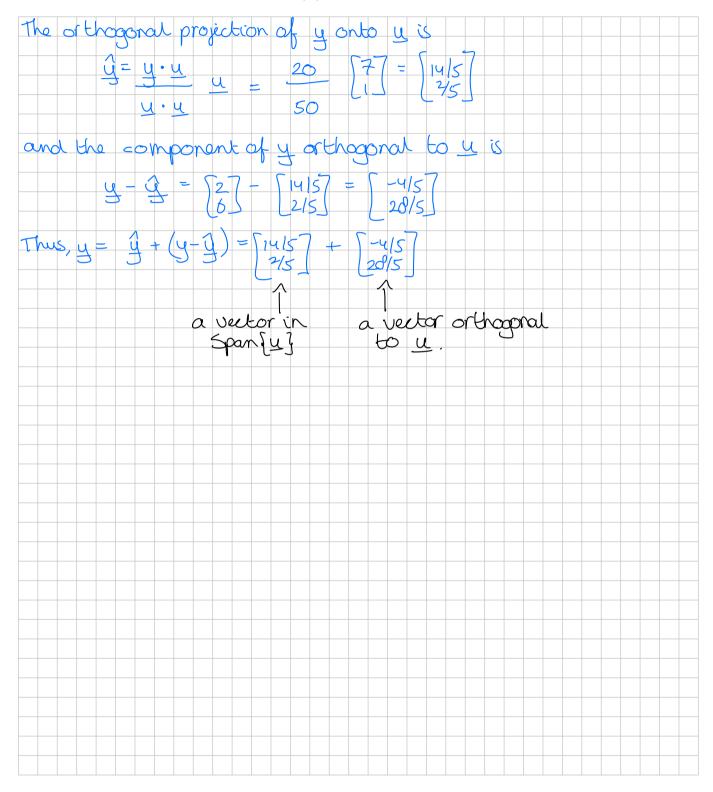


5p **5c** Is the matrix A diagonalizable? Briefly explain. (Note: you do not need to diagonalize A.)



Let
$$\mathbf{y} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$
 and $\mathbf{u} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$.

10p **6** Write ${\bf y}$ as the sum of a vector in ${\rm Span}\{{\bf u}\}$ and a vector orthogonal to ${\bf u}$.

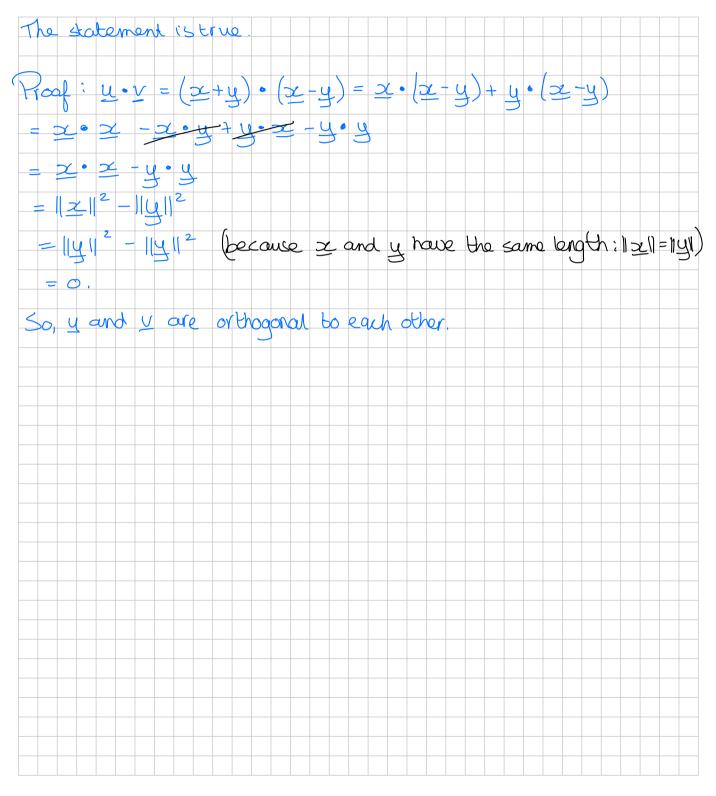




Exercise 7

10p **7** Prove or disprove the following statement.

Let \mathbf{x} and \mathbf{y} in \mathbb{R}^3 be two vectors that have the same length ($||\mathbf{x}|| = ||\mathbf{y}||$) and define $\mathbf{u} = \mathbf{x} + \mathbf{y}$ and $\mathbf{v} = \mathbf{x} - \mathbf{y}$. Then, \mathbf{u} and \mathbf{v} are orthogonal to each other.

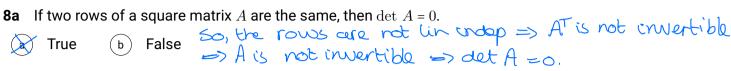




Exercise 8

True or False? For each of the statements below: state whether the statement is true or false. (Note: An explanation is not required. Please read the multiple choice instructions on the cover page!)

3р



8b If A is a 6×8 matrix, then it is possible that it has a 1-dimensional null space. 3р

False the null space has dimension at least 2 True

8c Two orthogonal vectors are automatically also linearly independent. 3р

[0], [1] are orthogonal but not lin indep. False True

Зр

8d If λ is an eigenvalue of A, then it is also an eigenvalue of A^T .

True (b) False $\det (A^T - \lambda T) = \det (A^T - \lambda T) = \det$ True

8e Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$. If $\mathbf{x} \cdot \mathbf{y} = 0$, then there does not exist a plane in \mathbb{R}^3 that contains both \mathbf{x} and \mathbf{y} . 3р

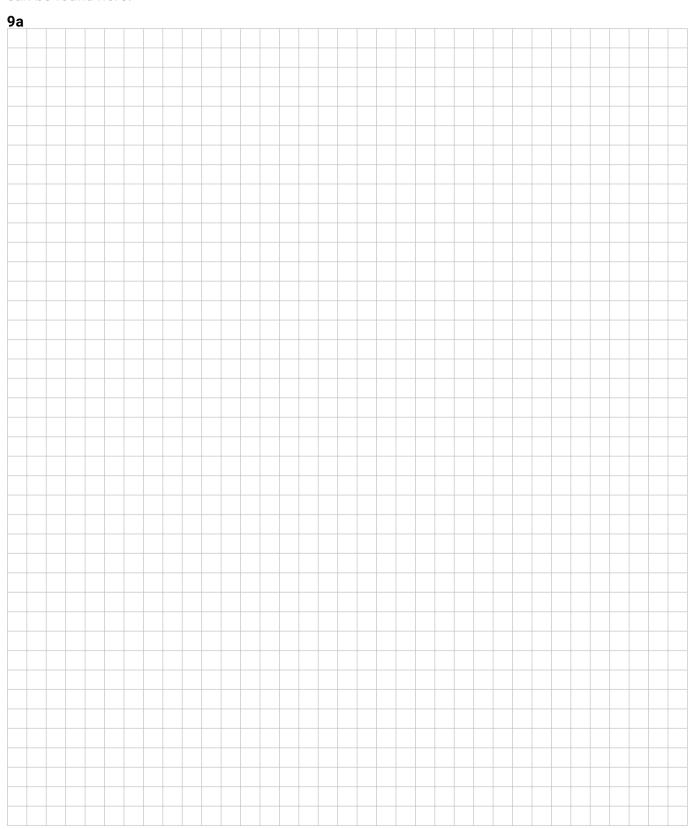
Consider $x=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $y=\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

Since x and y are lin. indep., they span a plane in R3. Mence, they both belong to this plane.



Extra space

If you use these extra answer boxes, please mention clearly in your main answer that part of your answer can be found here!



14 / 18



