

Reminder:

$$f: S \rightarrow S$$

function

is also a relation

①

$$S = \{s_1, s_2, s_3, s_4\}$$

ex:

$$f(s_1) = s_1 \iff f_{s_1 s_1}$$

$$f(s_2) = s_3 \iff f_{s_2 s_3}$$

$$f(s_3) = s_4$$

$$f(s_4) = s_1$$

$$f = \{ (s_1, s_1), (s_2, s_3), (s_3, s_4), (s_4, s_1) \}$$

$$R_a = \{ (s_1, s_1), (s_2, s_1) \}$$

$$R_{?(p \vee q)} = \{ (s_1, s_1), (s_3, s_3) \}$$

$$R_{?\neg(p \vee q)} = \{ (s_2, s_2) \}$$

$$R_{?\neg(p \vee q); a; ?(p \vee q)} = R_{?\neg(p \vee q)} ; R_a ; R_{?(p \vee q)} \\ = \{ (s_2, s_1) \}$$

$$(\cancel{s_2, s_2})$$

prove

$$[\alpha^*]\varphi \rightarrow [\alpha][\alpha^*]\varphi$$

(2)

1.	$[\alpha^*]\varphi$	Ass.
2.	$[\alpha^*]\varphi \rightarrow (\varphi \wedge [\alpha][\alpha^*]\varphi)$	Mix Axiom
3.	$\varphi \wedge [\alpha][\alpha^*]\varphi$	$E \rightarrow (1, 2)$
4.	$[\alpha][\alpha^*]\varphi$	$E \wedge (3)$
5.	$[\alpha^*]\varphi \rightarrow [\alpha][\alpha^*]\varphi$	

$$([\alpha](\varphi \rightarrow \psi) \wedge [\alpha]\varphi \wedge [\beta]\varphi) \rightarrow ([\alpha \cup \beta]\varphi \wedge [\alpha]\psi)$$

(3)

1.	$[\alpha](\varphi \rightarrow \psi) \wedge [\alpha]\varphi \wedge [\beta]\varphi$	
2.	$([\alpha]\varphi \wedge [\beta]\varphi) \rightarrow [\alpha \cup \beta]\varphi$	Choice axiom
3.	$[\alpha]\varphi \wedge [\beta]\varphi$	$E \wedge (1)$
4	$[\alpha \cup \beta]\varphi$	$E \rightarrow (2, 3)$
5	$[\alpha](\varphi \rightarrow \psi)$	$E \wedge (1)$
6.	$[\alpha](\varphi \rightarrow \psi) \rightarrow ([\alpha]\varphi \rightarrow [\alpha]\psi)$	principle (2)
7.	$[\alpha]\varphi \rightarrow [\alpha]\psi$	$E \rightarrow (5, 6)$
8.	$[\alpha]\varphi$	$E \wedge (3)$
9	$[\alpha]\psi$	$E \rightarrow (7, 8)$
10	$[\alpha \cup \beta]\varphi \wedge [\alpha]\psi$	$I \wedge (9, 4)$
11	Statement (too lazy to copy)	
		$I \rightarrow (1, 10)$