The four rules in predicate logic proofs

Easy are:

$\boxed{E_\forall}$

$$\frac{\forall x\, \varphi(x)}{\varphi(c)}\ E_\forall$$

example:

$$\frac{\forall x\,(Px \to Qxx)}{Pc \to Qcc}\ E_\forall$$

and

$\boxed{I_\exists}$

$$\frac{\varphi(c)}{\exists x\, \varphi(x)}\ I_\exists$$

example:

$$\frac{Pc \to Qcc}{\exists x\,(Px \to Qxx)}\ I_\exists$$

There is no indent when introducing the constants. The following two rules do, however

$\boxed{I_\forall}$

$$\begin{array}{l|l}
 & c \quad \text{(universal constant)} \\
\hline
 & \vdots \\
\hline
 & \varphi(c) \\
\hline
\multicolumn{2}{l}{\forall x\, \varphi(x) \qquad I_\forall}
\end{array}$$

example:

1. $\forall x(Px \to Qx)$		(given ass.)
2. $\forall x\, Px$		(given ass.)
3	$Pc \to Qc$, c univ. constant	$E_\forall(1)$
4.	Pc	$E_\forall(2)$
5	Qc	$E_\to(3,4)$
6. $\forall x\, Qx$		$I_\forall(3,5)$

(note: line 3 could have been split into

3	c univ. constant	
4	$Pc \to Qc$,	$E_\forall(1)$)

$\boxed{E_\exists}$ $\exists x\, \varphi(x)$

$\varphi(c)$	c exist. constant
$\vdots$	
ψ (no c in ψ)	

ψ $\quad E_\exists$

example

1.	$\forall x (Px \to Qx)$	(given ass.)
2.	$\exists x\, Px$	(given ass.)
3	Pc	c exist. constant (2)
4.	$Pc \to Qc$	$E_\forall (1)$
5	Qc	$E_\to (3,4)$
6	$\exists y\, Qy$	$I_\exists (5)$
7.	$\exists y\, Qy$	$E_\exists$

(so this proves

$$\forall x (Px \to Qx), \exists x\, Px \models \exists y\, Qy$$)

Perhaps to remember:

$$\frac{\forall x\, \varphi(x)}{\varphi(c)} \quad E_\forall$$

(no indent) if $\forall$ is before/above)

c … $\varphi(c)$

$\forall x \varphi(x)$ $\quad I_\forall$

(indent happens if $\forall x$ is below)

$$\frac{\varphi(c)}{\exists x \varphi(x)} \quad I_\exists$$

(no indent if $\exists$ is below)

$\exists x\, \varphi(x)$

$\varphi(c)$
ψ

ψ

(indent if $\exists$ is before)