Exercises

1	2	3	4	5	6	7	8

Surname, First name

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

Linear Algebra (KEN1410)
Resit

a ~~b~~ c d e f → b
a b ~~c~~ d e f → c
a b c ~~d~~ e f → a

Fill in your answer(s) to the multiple-choice questions as shown above (circles = one correct answer).

Program: Data Science and Artificial Intelligence
Course code: KEN1410
Examiners: dr. Marieke Musegaas and dr. Stefan Maubach
Date/time: 27-06-2022 9:00 - 11:00
Format: Closed book exam
Allowed aids: Pens, simple (non-programmable) calculator from the DKE-list of allowed calculators.
Instructions to students:

- The exam consists of 7 questions on 20 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

Exercise 1

Let $\mathbf{a_1} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\mathbf{a_2} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{a_3} = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$.

15p **1** Find two different ways to express $\mathbf{b}$ as a linear combination of $\mathbf{a_1}$, $\mathbf{a_2}$ and $\mathbf{a_3}$.

Exercise 2

Let $T : \mathbb{R}^3 \to \mathbb{R}^5$ be a mapping such that

$$T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -3 \\ 6 \\ 2 \\ 5 \end{bmatrix}.$$

10p **2** Can T be a linear transformation? Explain.

Exercise 3
Consider the following matrix A depending on a parameter p:

$$A = \begin{bmatrix} p & 0 & 0 \\ 1 & p & 2 \\ 0 & -1 & 2 \end{bmatrix}.$$

10p **3** For which value(s) of p is A not invertible?

Exercise 4

A 3×3 matrix A has all its diagonal entries equal to 0. Moreover every row sum is equal to 3 and $\det A = 6$.

10p **4** Determine the eigenvalues of A.

Exercise 5
Consider the following matrix A:

$$A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}.$$

15p **5** Find two linearly independent vectors that are orthogonal to Nul A .

Exercise 6
Consider the following matrix A:

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \end{bmatrix}.$$

15p

6 Determine a vector $\mathbf{u}$ in Row A such that $\mathbf{u} - \begin{bmatrix} 3 \\ 2 \\ 9 \end{bmatrix}$ is orthogonal to Row A.

Exercise 7

For each of the questions below an explanation/derivation is not required; you only need to state the final answer.

5p

7a Determine two distinct vectors in $\mathbb{R}^3$ with length 1 that are orthogonal to both $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

5p

7b Let $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$ and $P = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & 0 \\ -2 & 0 & 1 \end{bmatrix}$.

Determine a diagonal matrix D such that $A = PDP^{-1}$.

5p **7c** Determine vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^2$ such that $\{\mathbf{u}, \mathbf{v}\}$ and $\{\mathbf{v}, \mathbf{w}\}$ are linearly independent, but $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly dependent.

5p **7d** Provide a matrix A that is not invertible and such that Nul $A = \{\mathbf{0}\}$.

5p **7e** Provide an example of a subset H of $\mathbb{R}^2$ that has the following three properties:

- the zero vector is in H,
- H is closed under vector addition,
- H is **NOT** closed under multiplication by scalars.

Extra space

If you use these extra answer boxes, **please mention clearly in your main answer that part of your answer can be found here!**

8a

8b

8c

8d

8e