

The four rules in predicate logic proofs

(1)

Easy are:

E_V

example:

$$\frac{\forall x \varphi(x)}{\varphi(c)} E_V \quad \frac{\forall x (P_x \rightarrow Q_{xx})}{P_c \rightarrow Q_{cc}} E_V$$

and

I_I

example:

$$\frac{\varphi(c)}{\exists x \varphi(x)} I_I \quad \frac{P_c \rightarrow Q_{cc}}{\exists x (P_x \rightarrow Q_{xx})} I_I$$

There is no indent when introducing the constants. The following two rules do, however

I_V

$$\frac{\begin{array}{|l} \top c \\ \vdots \\ \varphi(c) \end{array}}{\forall x \varphi(x)} I_V \quad \text{(universal constant)}$$

example:

1.	$\forall x (P_x \rightarrow Q_x)$	(given ass.)
2.	$\forall x P_x$	(given ass.)
3.	$P_c \rightarrow Q_c$	c univ. constant $E_V(1)$
4.	P_c	$E_V(2)$
5.	Q_c	$E_{\rightarrow}(3,4)$
6.	$\forall x Q_x$	$I_V(3,5)$

(note: line 3 could have been split into

3.	c univ. constant
4.	$P_c \rightarrow Q_c$, $E_V(1)$

example

\exists	$\exists x \varphi(x)$
	$\varphi(c)$ c exist. constant
	\vdots
	ψ (no c in ψ)
ψ	E_{\exists}

1.	$\forall x (P_x \rightarrow Q_x)$	(given ass.)
2.	$\exists x P_x$	(given ass.)
3.	P_c	c exist. constant (2)
4.	$P_c \rightarrow Q_c$	$E_{\forall}(1)$
5.	Q_c	$E_{\rightarrow}(3,4)$
6.	$\exists y Q_y$	$I_{\exists}(5)$
7.	$\exists y Q_y$	E_{\exists}

(so this proves
 $\forall x (P_x \rightarrow Q_x), \exists x P_x \vdash \exists y Q_y$)

perhaps to remember:

$$\frac{\forall x \varphi(x)}{\varphi(c)} E_{\forall}$$

(no indent) If \forall is before/above

$$\frac{\varphi(c)}{\exists x \varphi(x)} I_{\exists}$$

(no indent) if \exists is below

$$\frac{\varphi(c)}{\forall x \varphi(x)} I_{\forall}$$

(indent happens if $\forall x$ is below)

$$\frac{\exists x \varphi(x)}{\varphi(c)} E_{\exists}$$

(indent if \exists is before)