Exercises

1	2	3	4	5	6	7	8	9

Surname, First name

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

KEN1130 Discrete Mathematics
KEN1130 Discrete Mathematics Exam

a ~~b~~ c d e f → b
a b ~~c~~ d e f → c
a b c ~~d~~ e f → a

Fill in your answer(s) to the multiple-choice questions as shown above (circles = one correct answer).

Program: Data Science and Artificial Intelligence
Course code: KEN1130
Examiners: dr. Marieke Musegaas and dr. Otti D'Huys
Date/time: Monday 23.10.2023 17h00-19h00
Format: Closed book exam
Allowed aids: Pens, simple (non-programmable) calculator from the DACS-list of allowed calculators.
Instructions to students:

- The exam consists of 8 questions on 20 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

Question 1

Consider the following logical proposition.

$$(p \Rightarrow q) \Leftrightarrow ((p \wedge \neg q) \Rightarrow (q \wedge \neg q))$$

Answer the following questions about (the truth table of) the above proposition.

2.5p **1a** Suppose p is TRUE and q is TRUE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

2.5p **1b** Suppose p is TRUE and q is FALSE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

2.5p **1c** Suppose p is FALSE and q is TRUE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

2.5p **1d** Suppose p is FALSE and q is FALSE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

Question 2

12p **2a** Use induction to prove the following statement.

For every natural number $n \in \mathbb{N}$, $4^{3n} + 8$ is divisible by 9.

3p **2b** Disprove the following statement.

For every natural number $n \in \mathbb{N}$, $3^n > n!$

Question 3

6p **3a** Prove or disprove the following statement. For all sets A, B and C,

$$A \subseteq B \iff A \backslash C \subseteq B \backslash C.$$

6p **3b** Prove the following statement

$$(\{9^n : n \in \mathbb{N}\} \subseteq \{3^n : n \in \mathbb{N}\}) \wedge (\{9^n : n \in \mathbb{N}\} \neq \{3^n : n \in \mathbb{N}\})$$

In other words, you need to prove $\{9^n : n \in \mathbb{N}\} \subset \{3^n : n \in \mathbb{N}\}$

Question 4

5p **4a** Let R be the relation on the set $\mathbb{P}(\{1,2\})$ defined as follows: XRY means "$|X \setminus Y| = 1$". Draw the relation diagram.

8p **4b** Let R be the relation on the set $\mathbb{P}(\mathbb{N})$ defined as follows: XRY means "$|X \backslash Y| = 1$". Is R reflexive? Is R symmetric? Is R transitive? Is R anti-symmetric? For each of these properties, prove or disprove that it has that property.

3p **4c** Let R be the equivalence relation on $\mathbb{N}$ defined as follows: xRy means “$5x + y$ is even". How many equivalence classes does R have?

- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) 5
- (f) 6
- (g) None of the above, because R is not an equivalence relation.

Question 5

5p **5a** How many 8-digit binary strings contain exactly three ones?
(An example of a 8-digit binary string that contains exactly three ones is: 1 0 1 0 0 0 1 0)

- (a) 10
- (b) 21
- (c) 35
- (d) 45
- (e) 56
- (f) 60
- (g) 120
- (h) 125
- (i) 243
- (j) 336
- (k) 495
- (l) 512
- (m) 792
- (n) 6561
- (o) 6720
- (p) 32768
- (q) 390625
- (r) None of the above.

5p **5b** How many solutions are there for the equation $x + y + z = 10$, where $x, y, z \in \mathbb{N} \cup \{0\}$.
(For example, $x = 3$, $y = 4$, $z = 3$ is a solution.)

- (a) 35
- (b) 36
- (c) 66
- (d) 84
- (e) 120
- (f) 210
- (g) 220
- (h) 343
- (i) 720
- (j) 1000
- (k) 2187
- (l) 8008
- (m) 11440
- (n) 59049
- (o) 604800
- (p) 10000000
- (q) 282475249
- (r) None of the above.

5p **5c** A password must have five characters made from letters of the alphabet (which contains 26 letters). Both lower case letters and capital letters are allowed, but there must be at least one capital letter. How many different passwords are there?
(An example of a valid password is: hElLo)

- (a) 27405
- (b) 65780
- (c) 142506
- (d) 7893600
- (e) 11881376
- (f) 368322656
- (g) 380204032
- (h) None of the above.

Question 6

5p **6a** Prove or disprove the following statement.

$$(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})(\forall z \in \mathbb{Z})(x = yz \Rightarrow y = -z)$$

5p **6b** Prove or disprove the following statement.

$$(\forall n \in \mathbb{N})(\exists X \in \mathbb{P}(\mathbb{N}))(|X| < n)$$

3p **6c** Consider the following proof.

Proof: Assume a and b are odd integers. Then $a = 2k + 1$ and $b = 2l + 1$ for some $k, l \in \mathbb{Z}$.
Then $ab^2 = (2k+1)(2l+1)^2 = 8kl^2 + 8kl + 2k + 4l^2 + 4l + 1 = 2(4kl^2 + 4kl + k + 2l^2 + 2l) + 1$.
Since $4kl^2 + 4kl + k + 2l^2 + 2l \in \mathbb{Z}$, we see that ab^2 is odd. □

Determine which of the statements given below is being proved.
(Note: only one answer is the correct answer.)

- (a) If a or b is even, then ab^2 is even.
- (b) If a and b are even, then ab^2 is even.
- (c) If ab^2 is even, then a and b are even.
- (d) If ab^2 is even, then a is even or b is even.
- (e) None of the above.

Question 7

5p **7a** Let $f : \mathbb{Z} \to \mathbb{Z}$ be the function defined as follows:

$$f(x) = 3x^2 + 2x + 1.$$

Prove or disprove that f is a bijection.

4p **7b** Let $f : \mathbb{R}\backslash\{3\} \to \mathbb{R}\backslash\{2\}$ be the function defined as follows:

$$f(x) = \frac{4x-7}{2x-6}.$$

Which one of the following statements is true?
(Note: only one answer is the correct answer.)

- (a) f is not a well-defined function.
- (b) f is not injective.
- (c) f is not surjective.
- (d) f is a bijection, and the inverse of f is the function $f^{-1} : \mathbb{R}\backslash\{2\} \to \mathbb{R}\backslash\{3\}$ defined as: $f^{-1}(x) = \frac{6x-7}{2x-4}$.
- (e) f is a bijection, and the inverse of f is the function $f^{-1} : \mathbb{R}\backslash\{2\} \to \mathbb{R}\backslash\{3\}$ defined as: $f^{-1}(x) = \frac{2x-6}{4x-7}$.
- (f) f is a bijection, but f does not have an inverse.
- (g) None of the above.

4p **7c** Let $f : \mathbb{Z} \to \mathbb{Z}$ be the function defined as follows:

$$f(x) = \frac{x}{2} + \frac{1-(-1)^x}{4}.$$

Which one of the following statements is true?
(Note: only one answer is the correct answer.)

- (a) f is not a well-defined function, because $\frac{x}{2} \notin \mathbb{Z}$.
- (b) f is not injective and not surjective.
- (c) f is not injective but is surjective.
- (d) f is not surjective but is injective.
- (e) f is a bijection.
- (f) None of the above.

Question 8

3p **8a** Which one of the following statements is **false**?
(Note: only one answer is the correct answer.)

- a) $(\{2,3,4\} \subseteq A) \Rightarrow (2 \in A$ and $\{3,4\} \subseteq A)$
- b) $(\{2,3,4\} \in A$ and $\{2,3\} \in B) \Rightarrow (\{4\} \subseteq A \backslash B)$
- c) $(\{2,3,4\} \subseteq A \cap B) \Rightarrow (\{2,3,4\} \subseteq A$ and $\{2,3,4\} \subseteq B)$
- d) $(\{3,4\} \subseteq A \backslash B$ and $\{1,2\} \subseteq B) \Rightarrow (\{1,2,3,4\} \subseteq A \cup B)$
- e) $(\{2,3\} \subseteq A \cup B) \Rightarrow ((\{2,3\} \cap A = \varnothing) \Rightarrow (\{2,3\} \subseteq B))$
- f) None of the above.

3p **8b** Let $A = \{1,2\}$. Which one of the following statements is **false**?
(Note: only one answer is the correct answer.)

- a) $\{\{1,2\}\}$ is a partition of A.
- b) $\{\{1\},\{2\}\}$ is a partition of A.
- c) $\{\{(1,1),(2,2)\},\{(1,2),(2,1)\}\}$ is a partition of $A \times A$.
- d) $\{\{(1,1),(2,2),(1,2),(2,1)\}\}$ is a partition of $A \times A$.
- e) None of the above.

Question 9

If you use these extra answer boxes, **please mention clearly in your main answer that part of your answer can be found here!**

9a

9b

9c

9d

9e

9f