Practice Exam Questions - Tutorial 1

1. Fill in the truth table for the following logical proposition.
   - $(p \vee (\neg q \Leftrightarrow r)) \wedge (\neg p \Rightarrow r)$
2. Fill in the truth table for the following logical proposition.
   - $((r \vee \neg q) \Rightarrow (q \Leftrightarrow p)) \wedge (r \vee p)$
3. Fill in the truth table for the following logical proposition.
   - $((q \wedge \neg p) \vee \neg r) \Leftrightarrow (r \Rightarrow \neg q)$
4. Fill in the truth table for the following logical proposition.
   - $((p \wedge \neg q) \Rightarrow \neg r) \Leftrightarrow (p \Rightarrow (q \vee r))$.
5. Consider the table below.

1		8	11	
	2			
3		4	5	10
	6		9	13
14	7	12		

For the benefit of those viewing in black and white: numbers 1,4,5,6,14 are *red*, numbers 3,8,9,13 are *green*, and numbers 2,7,10,11,12 are *blue*. For each of the statements listed, decide whether it is true or false, and then justify your answer as rigorously as you can. Note that, when I am talking about "numbers" and "colours", I mean *those in the table* – I don't write this explicitly simply to save space.

(a) All the numbers in the table are even if and only if all the numbers in the table are odd.

(b) For every blue number x, x is even or x is prime.

(c) There is a colour c such that for every prime number y, y does not have colour c.

(d) For every column d, if d contains three or more numbers, then d contains at least three colours.

(e) For every red number x and for every green number y, there is a blue number z such that $x < z < y$.

(f) There is an even number x and there is an odd number y such that x and y are both the same colour.

(g) For every even number x, there is a prime number y such that y is in the same row as x.

(h) For every column d, if d contains four or more numbers, then d contains at least four colours.

(i) For every red number x and for every green number y, if $x \leq y$ then there is a blue number z such that $x < z < y$.

(j) There is an odd number x, such that for every even number y, the column that y is in is strictly[1] to the left of the column that x is in.

6. Prove or disprove the following statements.

   (a) $(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})(\frac{(x+y)}{3} \notin \mathbb{N})$
   (b) $(\exists x \in \mathbb{N})(\forall y \in \mathbb{Z})(3x + 4y \neq 20)$
   (c) $\neg((\exists x \in \mathbb{N})(\exists y \in \mathbb{R})(2x - y \leq 10))$
   (d) $(\exists x \in \mathbb{N})(\forall y \in \mathbb{Z})(\exists z \in \mathbb{N})(x + y + z \geq -32)$

7. Prove or disprove the following statements.

   (a) $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})((x = y + 10) \vee (y \neq 0))$
   (b) $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{N})(y \geq \frac{1}{2}x^2 - 20)$
   (c) $\neg((\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x^2 - x + y \text{ is odd}))$
   (d) $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(\exists z \in \mathbb{N})(y < x < z)$.

8. Prove or disprove the following statements.

   (a) $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{N})((x > 0) \wedge (x + y^2 \text{ is even}))$,
   (b) $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{N})(\forall z \in \mathbb{N})(x - y < z - 10)$.

9. Prove or disprove the following statements.

   (a) $\neg((\forall x \in \mathbb{Z})(\exists y \in \mathbb{N})(-y < x < y))$
   (b) $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{N})(\forall z \in \mathbb{N})(x + y \neq z)$.

10. Prove or disprove the following statement.

$$(\forall x \in \mathbb{R})(x \in \mathbb{N} \Leftrightarrow (2x \in \mathbb{N} \wedge 3x \in \mathbb{N})).$$

11. Prove or disprove the following statement.

$$(\forall m \in \mathbb{Z})(\forall n \in \mathbb{Z})((mn \text{ is even}) \Leftrightarrow ((m \text{ is even}) \vee (n \text{ is even}))).$$

[1] I use *strictly* here in the same sense as strict inequality i.e. 1 is strictly less than 3, but 3 is not strictly less than 3.