Practice Exam Questions Week 1, **Linear Algebra,**

1. Consider the following linear system of equations:

$$\begin{aligned}
&2x_1 - 3x_2 - 7x_3 + 13x_4 + x_5 = -3 \\
&x_1 + x_2 - x_3 + 4x_4 + 2x_5 = 4 \\
&-x_1 + x_2 + 3x_3 - 6x_4 - x_5 = 0
\end{aligned}$$

   (a) Determine the augmented matrix of this system and compute its reduced row echelon form.

   (b) Compute the whole solution set.

2. True or false? If the given statement is true, briefly explain why. If it is false, give a counterexample.

   T (a) If a system of linear equations has a unique solution, then the number of basic variables is larger than the number of free variables.

   T (b) If a linear system of four equations in four variables has a coefficient matrix with a pivot in each column, then the system has a unique solution.

   T (c) A consistent system of linear equations with fewer equations than unknowns (also called underdetermined system) can never have an unique solution.

   T (d) Suppose a (3×5) coefficient matrix for a system has three pivot columns. Then the system is consistent.

T there cannot be a row $0000 = b$ $b \neq 0$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{bmatrix} 1 & & & \\ 0 & 1 & & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{array}$$

(1) $\left[\begin{array}{ccccc|c} 2 & -3 & -7 & 13 & 1 & -3 \\ 1 & 1 & -1 & 4 & 2 & 4 \\ -1 & 1 & +3 & -6 & -1 & 0 \end{array}\right]$

$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 2 & 4 \\ 2 & -3 & -7 & 13 & 1 & -3 \\ -1 & 1 & +3 & -6 & -1 & 0 \end{array}\right]$

$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 2 & 4 \\ 0 & -5 & -5 & 5 & -3 & -11 \\ 0 & 2 & 2 & -2 & 1 & 4 \end{array}\right]$

$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 2 & 4 \\ 0 & -5 & -5 & 5 & -3 & -11 \\ 0 & 0 & 0 & 0 & -1/5 & -2/5 \end{array}\right]$

$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 2 & 4 \\ 0 & 1 & 1 & -1 & +3/5 & 11/5 \\ 0 & 0 & 0 & 0 & -1/5 & -2/5 \end{array}\right]$

$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 2 & 4 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1/5 & -2/5 \end{array}\right]$

$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 2 & 4 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$

$\left[\begin{array}{ccccc|c} 1 & 1 & -1 & 4 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$

(2) $\left[\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & 0 & -1 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$

$$\begin{aligned} x_1 \quad - \quad -2x_3 + 5x_4 \quad - \quad &= -1 \\ x_2 + x_3 - x_4 \quad - \quad &= 1 \\ x_5 &= 2 \end{aligned}$$

(b) $$\begin{cases} x_1 = -1 + 2x_3 + 5x_4 \\ x_2 = 1 - x_3 + x_4 \\ x_5 = 2 \\ x_3 \text{ free} \\ x_4 \text{ free} \end{cases}$$

(2)

$$\begin{bmatrix} 1 & b & c & d & x \\ 0 & 1 & g & x & x \\ 0 & 0 & 1 & x & x \end{bmatrix}$$