

Exam Calculus
April 3, 2013, 14:00-17:00

- The exam is a closed-book exam
- Use of calculators is forbidden during the exam
- You are allowed to use a sheet of hand-written A4 paper (only one side of it!) prepared in advance; it has to be written by hand and in a normal size (min. 12pt)
- You have 180 minutes for this exam
- Always explain your answer
- You can get maximally 80 points from this exam

1. Compute the following limits, if they exist, or explain why they do not exist:

- a. (2 points) $\lim_{x \rightarrow -3} \frac{x+3}{x-3}$
- b. (3 points) $\lim_{x \rightarrow 1} \frac{|x-1|}{x^2-1}$
- c. (3 points) $\lim_{x \rightarrow 0} \frac{2x^2}{e^x}$
- d. (4 points) $\lim_{x \rightarrow \infty} (\sqrt{9x^2 + 3x} - 3x)$

2. Let $f(x) = \frac{x+\sqrt{2}}{x^2-2}$

- a. (2 points) Determine the domain of f . Is f continuous on its domain? Why?
- b. (3 points) For what values of x is f increasing and decreasing? Does it have local minimum(s), maximum(s)? If so, at what values of x ?
- c. (4 points) For what values of x is f convex (=concave up) and concave (=concave down)? Does it have inflection points? If so, at what x ?
- d. (4 points) Does f have vertical, horizontal, and/or oblique asymptotes? Why?
- e. (2 point) Is f even or odd?
- f. (3 points) Based on your previous answers (a.-e.) and perhaps with drawing some extra points lying on the graph of f , sketch function f .

3. a. (3 points) Compute the derivative: $\left(a^{\sin(x^2)}\right)'$

b. (3 points) Compute the derivative: $\left(\frac{2x}{x^2-4}\right)''$

c. (3 points) Compute the derivative: $(|x-3|)'$

d. (4 points) Find equation of the tangent line of $f(x) = x^2 + 2x - 3$ at point $(4, f(4))$.

4. Compute the following integrals:

a. (3 points) $\int_0^1 e^x \sqrt{1+e^x} \, dx$

b. (3 points) $\int \arcsin(x) \, dx$

c. (4 points) $\int \frac{dx}{\sqrt{3-x^2}}$

d. (4 points) $\int_2^3 \frac{x^2}{x^2-1} \, dx$

5. Determine whether the series

- a. (3 points) $\sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$ converges or diverges by using any appropriate test
- b. (3 points) $\sum_{n=1}^{\infty} \frac{77^n}{n!}$ converges or diverges by using any appropriate test
- c. (3 points) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\pi^n}$ converges absolutely, converges conditionally, or diverges, by using any appropriate test

6. Let $f(x, y) = xy + 5$.

- a. (4 points) Find equation of the tangent plane of f at point $(1, 2, f(1, 2))$.
- b. (3 points) Compute $\frac{\partial^2 f(x, y)}{\partial x \partial y}$
- c. (4 points) Evaluate $\int \int_T f(x, y) \, dx \, dy$ over the triangle T with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$.
What is the interpretation of this integral?
- d. (3 points) Evaluate $\int \int_T 1 \, dx \, dy$ over the same triangle. What is the interpretation of this integral?