

$$8f) \quad a \vee \neg a \models (\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p)$$

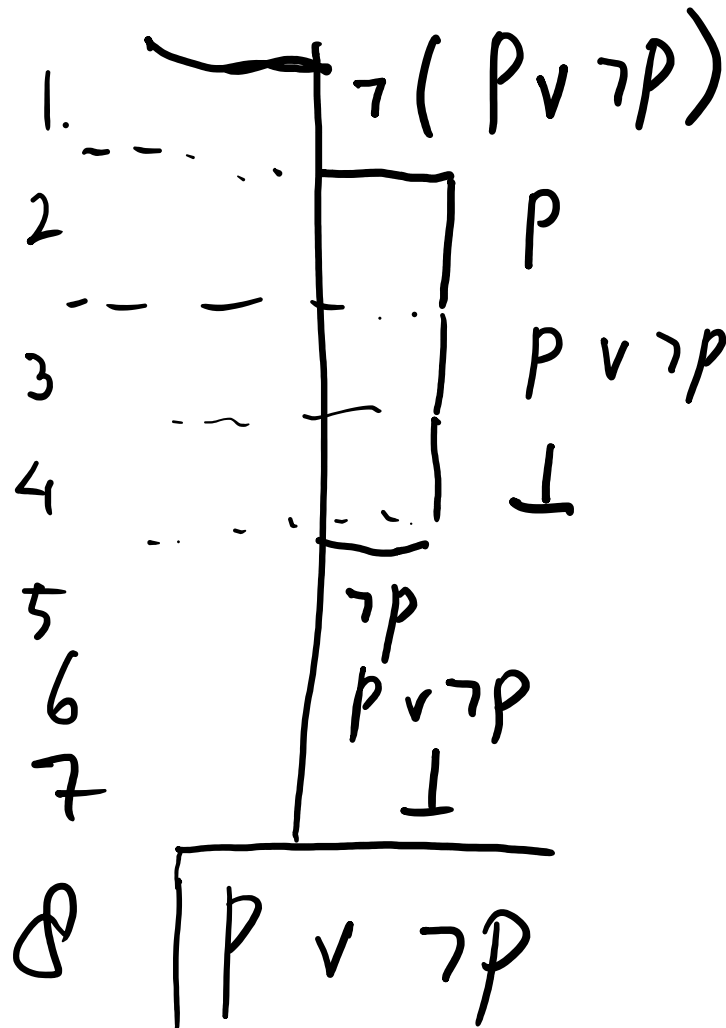
$ \begin{array}{l} 1. \mid a \vee \neg a \\ 2. \quad \mid \neg p \rightarrow \neg q \\ 3. \quad \quad \mid q \\ 4. \quad \quad \mid \neg p \\ 5. \quad \quad \mid \neg q \\ 6. \quad \quad \mid \perp \\ 7. \quad \quad \mid p \\ 8. \quad \mid q \rightarrow p \\ 9. \quad (\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p) \end{array} $	<p>(given)</p> <p>(assume)</p> <p>(assume)</p> <p>(assume)</p> <p>$E \rightarrow (2, 4)$</p> <p>$E \neg (3, 5)$</p> <p>$I \neg (4, 6)$</p> <p>$I \rightarrow (3, 7)$</p> <p>$I \rightarrow (2, 8)$</p>
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8.e	1.	$p \rightarrow q$	(given)
		$\underline{\quad}$	
2		$\neg q$	(ass.)
3		\vdots	(ass)
4.		p	$E \rightarrow (1, 3)$
5		q	$E \neg (2, 4)$
6		\perp	$I \neg (3, 5)$
7		$\neg p$	$I \rightarrow (2, 6)$
		$\neg q \rightarrow \neg p$	

8d)

1.	$P \vee \neg q$	Given
2.	P	ass.
3.	q	ass.
4.	P	repeat(2)
5.	$q \rightarrow P$	$I \rightarrow (3, 4)$
6.	$\neg q$	ass.
7.	q	ass.
8.	\perp	$E \neg (6, 7)$
9.	P	$\perp (8)$
10.	$q \rightarrow P$	$I \rightarrow (7, 9)$
11.	$q \rightarrow P$	$E \vee (2, 5, 6, 10)$

89)



ass.

ass.

$I_{\vee}(2)$

$E_{\neg}(1,3)$

$I_{\neg}(2,4)$

$I_{\vee}(5)$

$E_{\neg}(1,6)$

$I_{\neg}(1,7)$

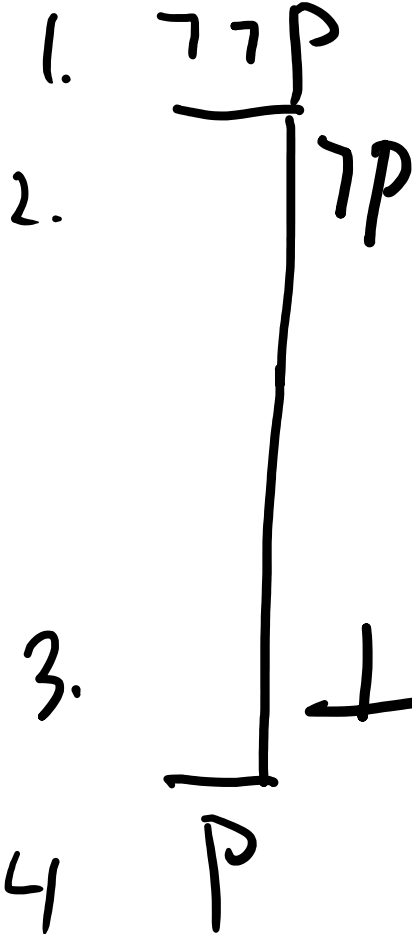
8. b)

1. $P \rightarrow \neg q$
2. q
3. $\neg P$
4. $\neg q$
5. \bot
6. $\neg P$

(given)
 (given)
 (ass.)
 $E \rightarrow (1, 3)$
 $E \neg (2, 4)$
 $I \neg (3, 5)$

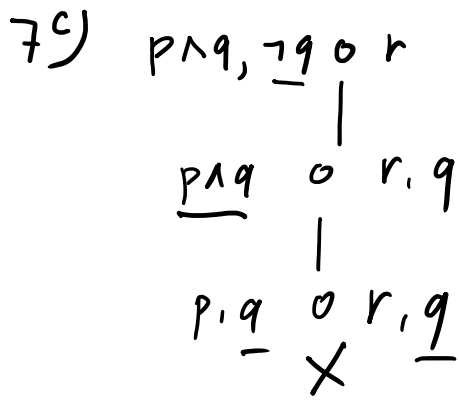
$\neg A$
 \vdots
 \bot
 $\neg A$

A
 \vdots
 \bot
 B
 $A \rightarrow B$

8⁹)

(giba)
(ass.)

$E_7(1,2)$
 $I_7(2,3).$



So inference is
valid

7b) $p \rightarrow q, \neg q \rightarrow \neg r \vee p \rightarrow r$

$p \rightarrow q, \neg q \rightarrow \neg r, p \vee r$

$p \rightarrow q, p, \neg r \vee r$

$p \rightarrow q, p \vee \neg q, r$

$p \rightarrow q, p \vee r$

$p, q \vee r$
open

$p \vee r, p$

Tableau open;
Inference
not valid!

$$7^a) \quad \begin{array}{c} p, q \circ p \rightarrow q \\ | \\ \hline p, q \circ q \\ \hline x \end{array}$$

6)

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

for matrix multiplication

$$(p \rightarrow q) \wedge r \not\equiv p \rightarrow (q \wedge r)$$

5^a) φ contradiction

$\varphi \circ$
closed

$\circ \rightarrow \varphi$
closed

$\neg \varphi$ tautology

$\leftrightarrow \varphi$ contradiction

$\circ \varphi$
closed

φ tautology

$\neg \varphi$ contradiction

5⁹) (cont'd)

$$\frac{(p \rightarrow \neg q) \wedge (q \rightarrow r) \wedge (r \rightarrow p) \circ}{p \rightarrow \neg q, (q \rightarrow r) \wedge (r \rightarrow p) \circ}$$

$$\frac{p \rightarrow \neg q, (q \rightarrow r) \wedge (r \rightarrow p) \circ}{p \rightarrow \neg q, \underline{q \rightarrow r}, r \rightarrow p \circ}$$

$$\frac{p \rightarrow \neg q, \underline{q \rightarrow r}, r \rightarrow p \circ}{p \rightarrow \neg q, r \rightarrow p \circ q}$$

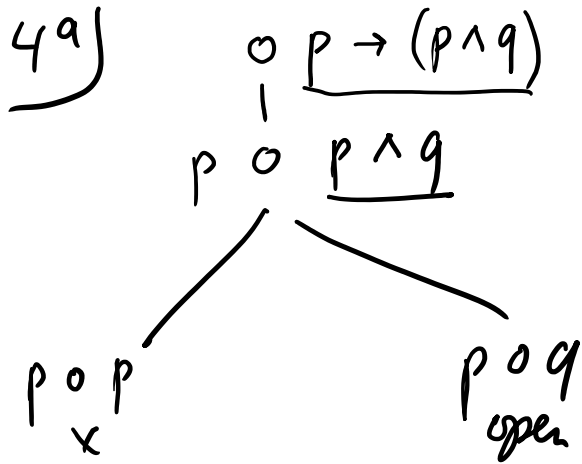
$$\frac{p \rightarrow \neg q, r \rightarrow p, r \circ}{p \rightarrow \neg q, r, p \circ}$$

$$\frac{p \rightarrow \neg q, r, p \circ}{r, p, \neg q \circ} \quad p \rightarrow \neg q, r \circ r$$

$$\frac{r, p, \neg q \circ}{r, p \circ q}$$

open

open, so $\neg \Phi$ not valid, so Φ is not a contradiction.



\Rightarrow tableau open, $\models p \rightarrow (p \wedge q)$
not valid, so
not a tautology.

(a) sentence (b) is true

(b) sentence (c) is false

(c) + there are exactly two sentences correct

$a = \text{true} \rightarrow \text{sentence 1 true}$
 $= \text{false} \rightarrow \text{sentence 1 false}$, b, c similar

$$a \leftrightarrow b$$

$$b \leftrightarrow \neg c$$

$$c \leftrightarrow ((a \wedge b \wedge \neg c) \vee (a \wedge \neg b \wedge c) \vee (\neg a \wedge b \wedge c))$$

a	b	c
1	1	0
0	0	0
1	1	1
0	0	1

$c \leftrightarrow ((a \wedge b \wedge \neg c) \vee (a \wedge \neg b \wedge c) \vee (\neg a \wedge b \wedge c))$
0
0
1
1

0
1
1
0

1
0
1
0

$$b \leftrightarrow \neg c$$

1
0
0
1