Assume that your company owns trucks with a cargo space of 16.5m long, 2.5m wide and 4m high. Assume that your company transports parcels of different sizes A, B and C.

- A: 1m x 1m x 2m
- B: 1m x 1.5m x 2m
- C: 1.5m x 1.5m x 1.5m

Is it possible to fill the complete cargo space with A, B and C parcels, without any gaps? If yes, show how. If no, give a proof why not.

(The first to send in a correct solution by email receives chocolate)

Revision

- Truth tables
- Proofs

  * proof by contradiction (know the principle!)
  * contrapositive
  * biconditional (= 2 implications)
  * induction (learn how to set up!)

* QUANTIFIERS : study how to $\begin{cases}\text{read} \\ \text{negate}\end{cases}$ quantifier statements

Sets:

- Definitions and notations: element, subset, intersection, union, complement, power set, product set, partition.
- Venn diagrams (to verify / understand statements with sets)
- Proofs with sets

→ to prove that $A \subseteq B$, you take $x \in A$, and show $x \in B$

→ to prove that $A = B$, you prove $A \subseteq B$ and $B \subseteq A$

tricks

→ typically, the proof consists of translating sets to logic

ex. $x \in A \cup B$, so $x \in A \vee x \in B$

→ proofs by { contradiction / contrapositive } often work well with empty sets

remember : $A \subseteq B : (\forall x)(x \in A \rightarrow x \in B)$ subset

$A \nsubseteq B : (\exists x)(x \in A \wedge x \notin B)$ not a subset

Relations on a set A

- **Relation diagrams**
- **Reflexive** $\forall x \in A: xRx$ (every element has a loop)

  not reflexive: $\exists x \in A: x \not{R} x$ $\longrightarrow$ find a counterexample x that is not related to itself.

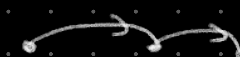
- **Symmetric** $\forall x,y \in A: xRy \rightarrow yRx$ (every arrow goes in two directions)

  not symmetric: $\exists x,y \in A: xRy \wedge y \not{R} x$

- **Transitive** $\forall x,y,z \in A: (xRy \wedge yRz) \longrightarrow xRz$

  not transitive $\exists x,y,z: xRy \wedge yRz \wedge x \not{R} z$

  OR

- **Anti-symmetric** $\forall x,y \in A: (xRy \wedge x \neq y) \rightarrow y \not{R} x$

  $(xRy \wedge yRx) \rightarrow x = y$

  not anti-sym: $\exists x,y \in A: xRy \wedge yRx \wedge x \neq y$

  or

Functions

- **Definition, domain, co-domain**

  $f : A \to B$, $f(x) \in B$ exists and is unique for all $x \in A$

- **Composition** for $f : A \to B$ and $g : C \to D$, $f \circ g : C \to B$ is well defined if range $(g) \subseteq$ domain (f)

- **Injective functions**

  $f : A \to B$ is injective if $\forall x, y \in A : x \neq y \to f(x) \neq f(y)$

  NOT injective : $\exists x, y \in A : x \neq y \wedge f(x) = f(y)$

- **Surjective functions**

  $f : A \to B$ is surjective if $\forall y \in B \; \exists x \in A : f(x) = y$

  NOT surjective if $\exists y \in B \; \forall x \in A \; f(x) \neq y$

- **Injective + surjective = bijective. Bijective = invertible.**

- **Inverse functions**

  $f : A \to B$ and $g : B \to A$ are inverse if

  $\forall x \in A \quad g(f(x)) = x$

  $\forall y \in B \quad f(g(y)) = y$

Combinatorics

- Selecting with/without repetition/order
- Inclusion-exclusion
- Practice a lot!

Good luck with your exams!!!