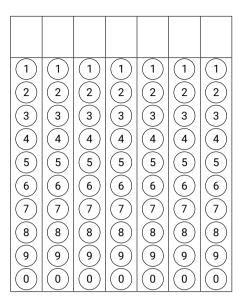
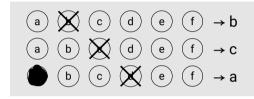
Surname, First name

Linear Algebra (KEN1410)

Resit





Fill in your answer(s) to the multiple-choice questions as shown above (circles = one correct answer).

Program: Data Science and Artificial Intelligence

Course code: KEN1410

Examiners: Dr. Marieke Musegaas and Dr. Stefan Maubach

Date/time: Wednesday 5 July 2023, 09:00 - 11:00

Format: Closed book exam

Allowed aids: Pens, simple (non-programmable) calculator from the DACS-list of allowed calculators. Instructions to students:

- The exam consists of 7 questions on 20 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. Do <u>not</u> write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded! As a last resort if you run out of space, use the extra answer space at the end of the exam.
- In no circumstance write on or near the QR code at the bottom of the page!
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- · You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- · Good luck!

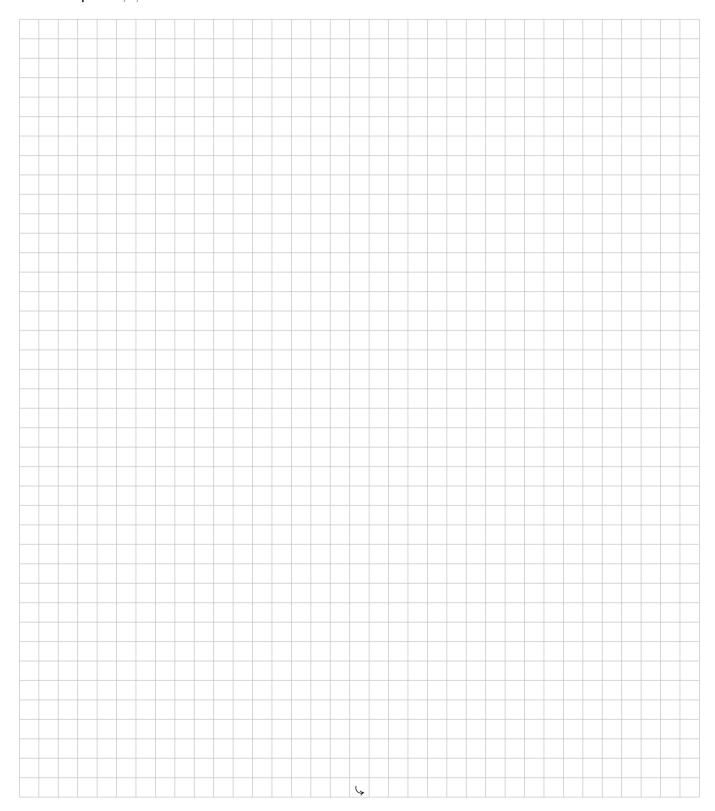
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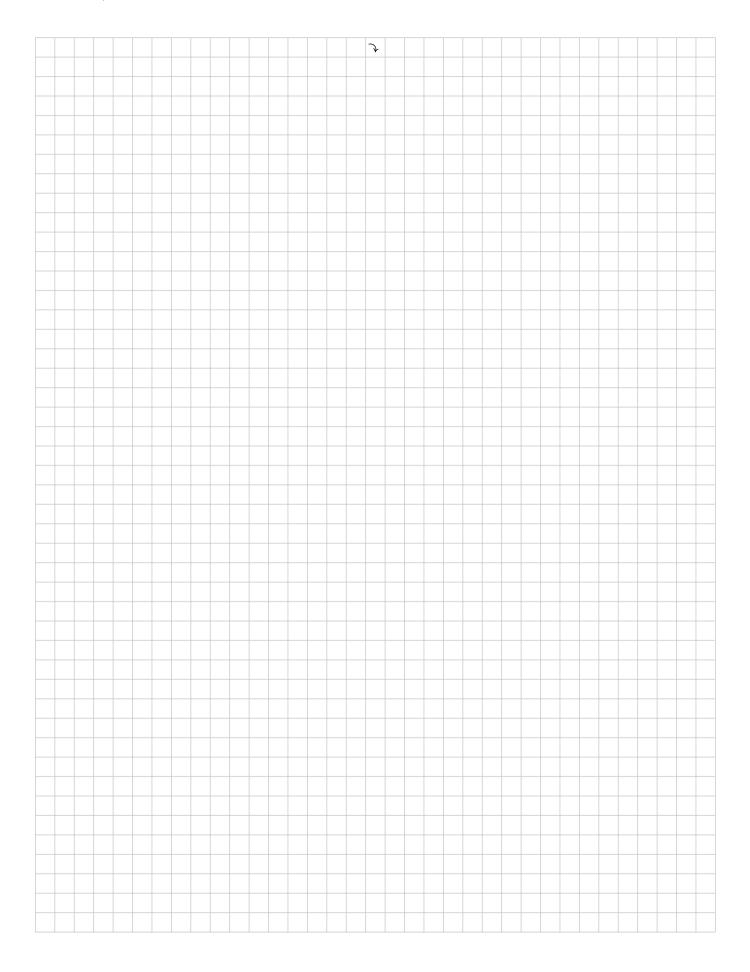


Question 1

10p Consider the polynomial $p(t) = at^3 + bt^2 + ct + d$, where a, b, c and d are real numbers. It is given that p(-1) = 1, p(0) = 0, p(1) = 1 and p(2) = -1. Compute a, b, c and d.









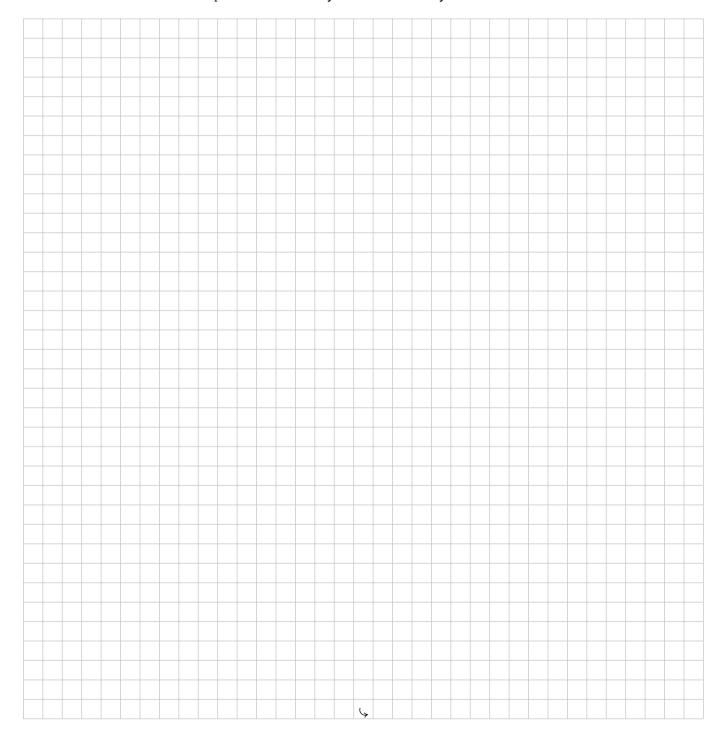


10p **2** Consider the following system of linear equations depending on a parameter *p*:

$$x_1 + 2x_2 + 4x_3 = 2$$

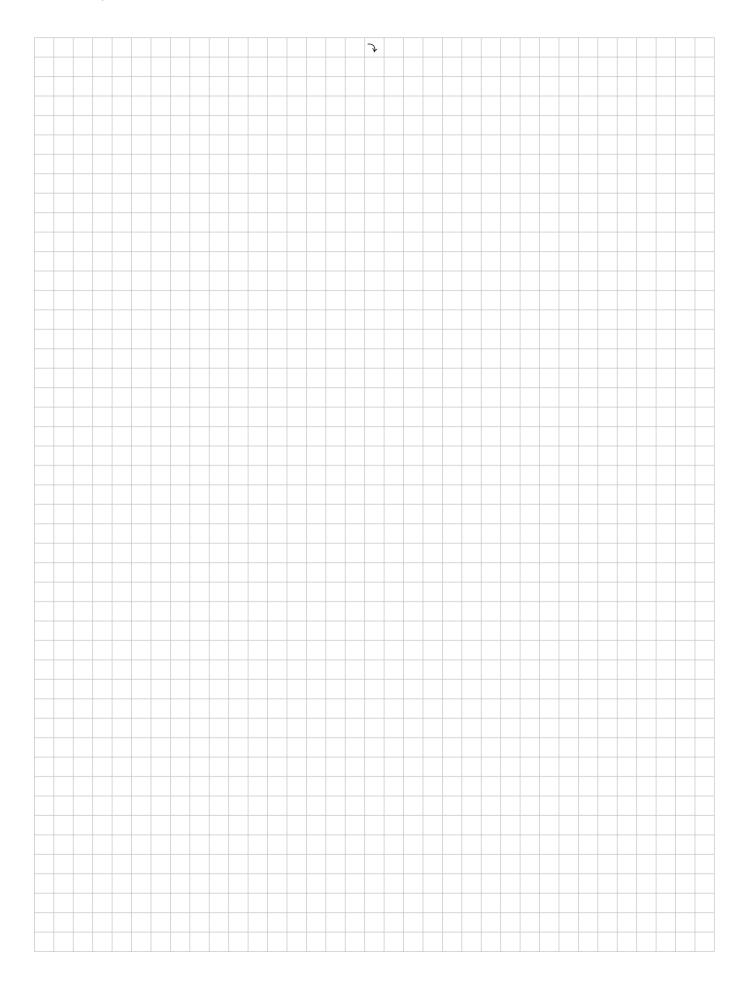
 $2x_1 + (p+3)x_2 + 8x_3 = 2$
 $x_1 + 2x_2 + p^2x_3 = p$.

Determine the values of p for which this system has exactly one solution.



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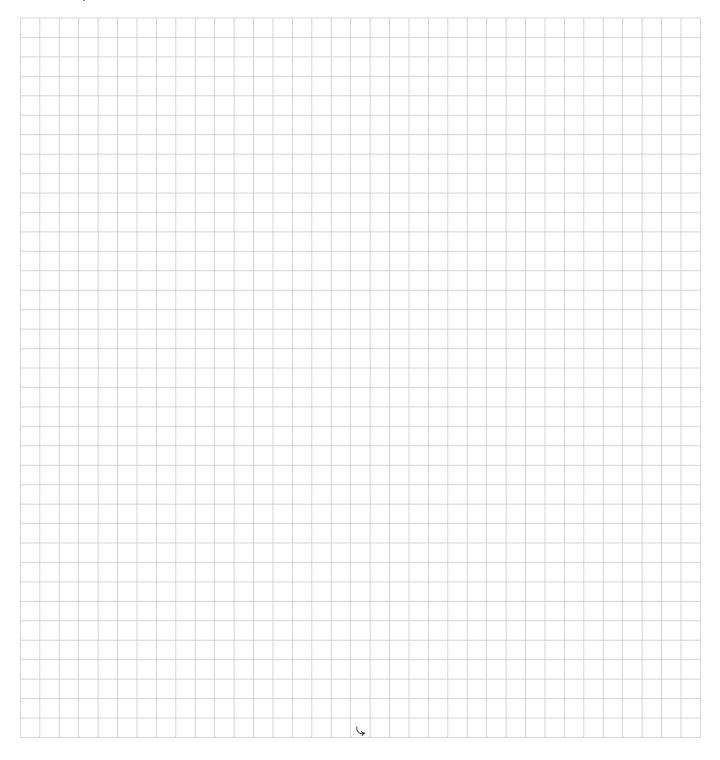




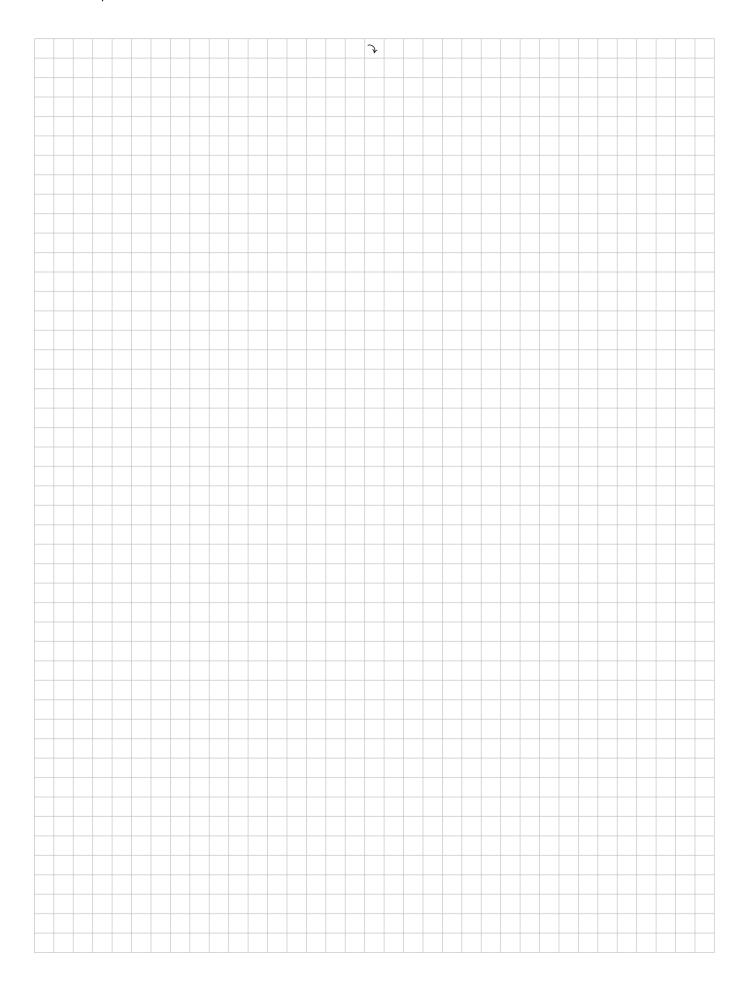
Question 3

10p

3 Let
$$A = \begin{bmatrix} 5 & 5 & 6 & 6 & 7 \\ 4 & 0 & 4 & 4 & 0 \\ 3 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 5 & 6 & 7 & 1 & 8 \end{bmatrix}$$
. Compute det A .



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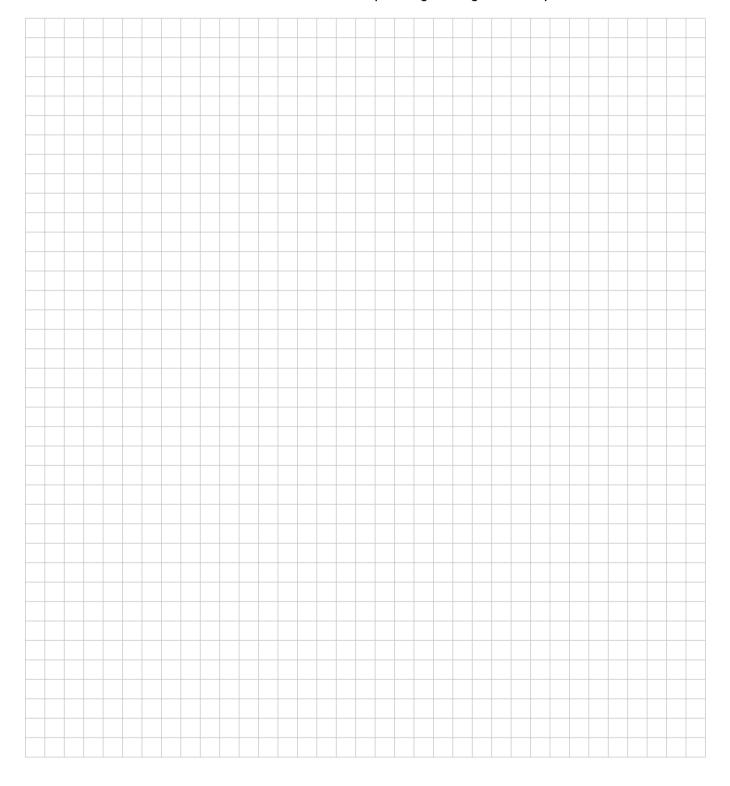
Question 4

5p

4 Assume that $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$ are solutions of the equation $A\mathbf{x} = \mathbf{b}$, where A is a 3×3 matrix and

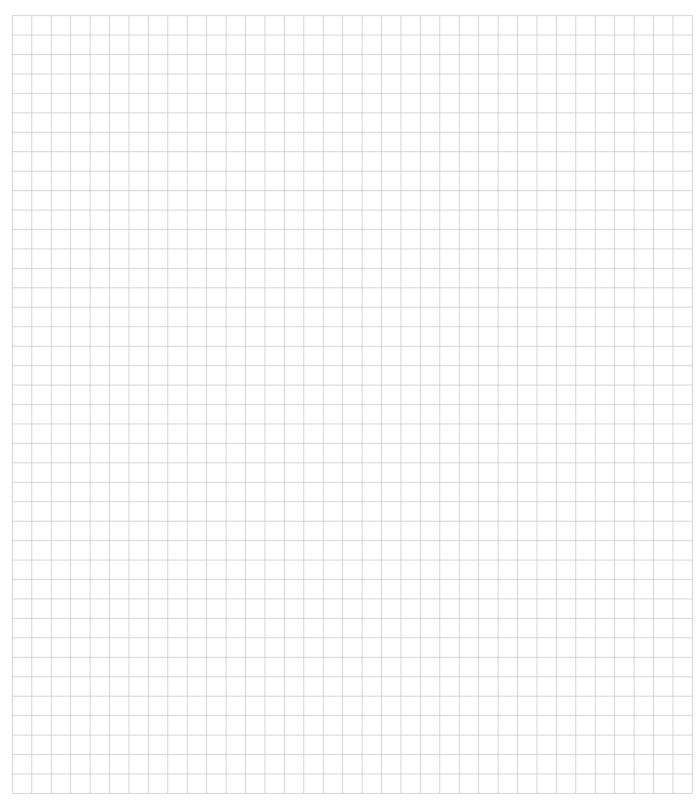
 $\mathbf{b} \neq \mathbf{0}$.

Determine three different solutions of the corresponding homogeneous equation $A\mathbf{x} = \mathbf{0}$.



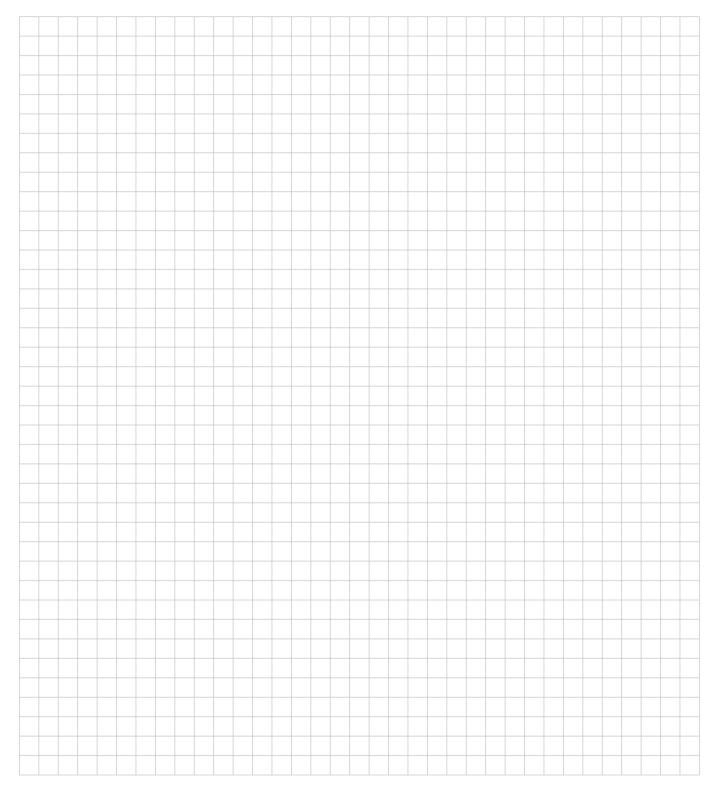
5р

5 Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a transformation such that $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}$. Is T a linear transformation? If yes, prove it. If not, explain why not.



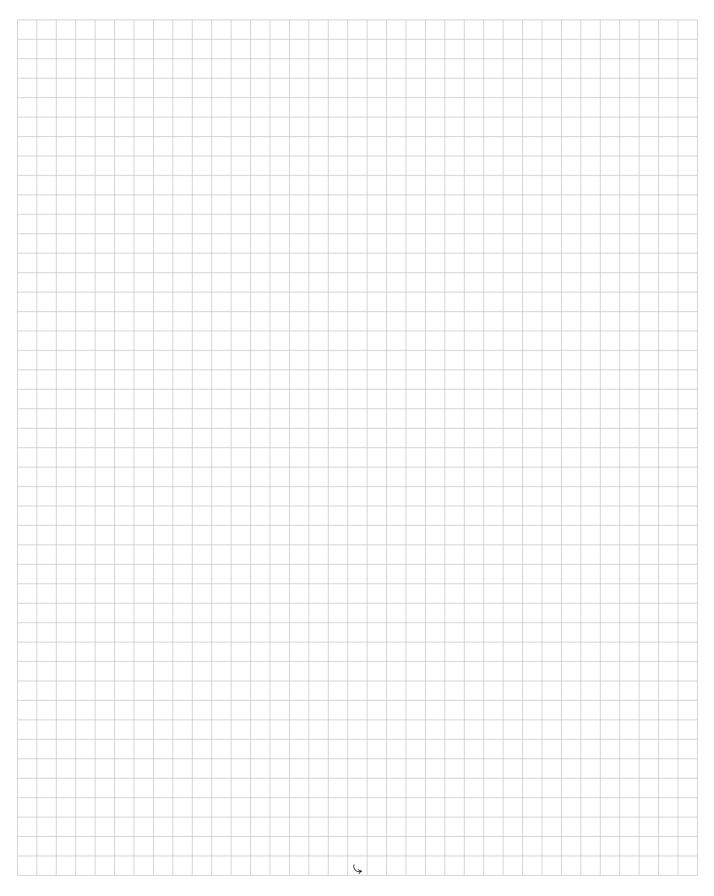
Let
$$A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$
.

5p **6a** Show that the matrix A has eigenvalues 3 and 5.

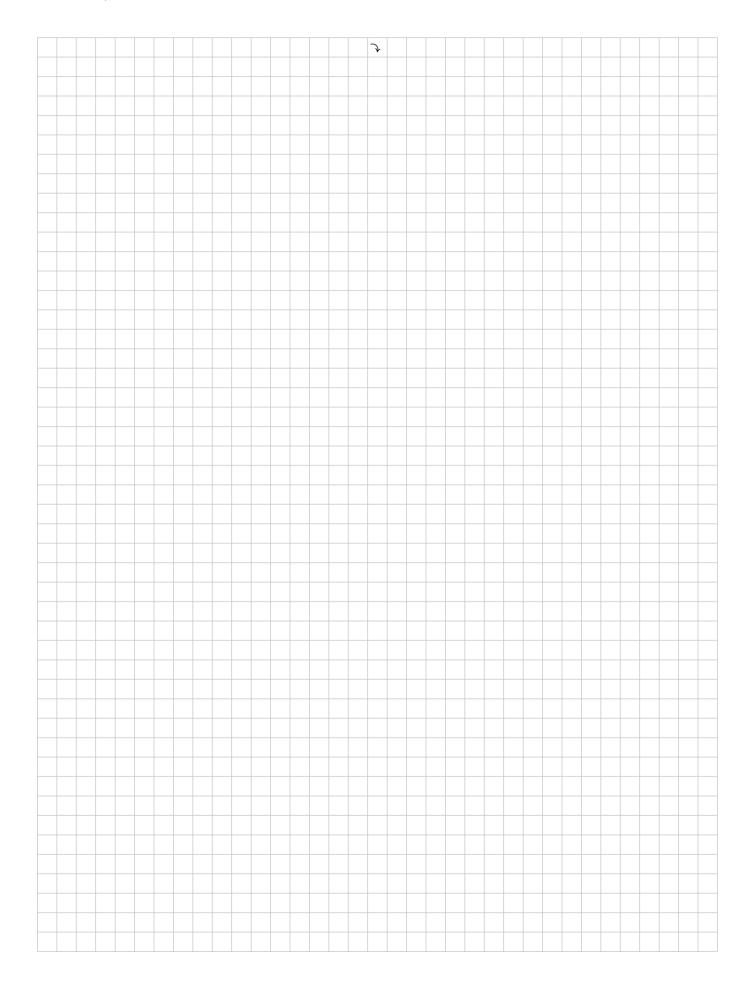




10p **6b** Orthogonally diagonalize matrix A, i.e. find an orthogonal matrix P and a diagonal matrix D such that $A = PDP^T$.









Answer the following multiple-choice questions.

An explanation is not required.

For every multiple-choice question, there is only **one correct answer**.

Please read the multiple choice instructions on the cover page!

- 2p **7a** Let A be a 6×4 matrix. Is the following statement true or false? "The column space of A and the row space of A are orthogonal to each other."
 - (a) True (b) False
- 2p **7b** Let A be a 6×4 matrix. Is the following statement true or false? "The row space of A and the nullspace of A are orthogonal to each other."
 - (a) True (b) False
- 2p **7c** Let A be a 6×4 matrix. Is the following statement true or false? "The column space of A and the nullspace of A are orthogonal to each other."
 - (a) True (b) False
- 2p **7d** Let A be a 6×4 matrix. What is the smallest possible dimension of Nul A?
 - (a) 0
 - (b) 2
 - (c) 4
 - (d) 6
 - None of the above.
- 2p **7e** Let A be a 6×4 matrix. What is the largest possible dimension of Col A?
 - (a) 0
 - (b) 2
 - (c) 4
 - (d) 6
 - (e) None of the above.

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5p

7f Let
$$A = \begin{bmatrix} 1 & 1 & -2 & -1 \\ -1 & 4 & -3 & 1 \\ 0 & 7 & 1 & -8 \end{bmatrix}$$
, $\mathbf{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}$.

Does u belong to Nul A? Does v belong to Col A?

- (a) \mathbf{u} belongs to Nul A. \mathbf{v} belongs to Col A.
- (b) u belongs to Nul A. v does not belong to Col A.
- \bigcirc u does not belong to Nul A. v belongs to Col A.
- (d) u does not belong to Nul A. \mathbf{v} does not belong to Col A.

5р

- **7g** What is the dot product (inner product) of $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$?
- $\begin{array}{c}
 \begin{bmatrix}
 0 \\
 2 \\
 3
 \end{array}$
- (c) 0
- (d) 5
- (e) The dot product cannot be computed for these vectors.
- f None of the above.

5r

- **7h** If $\mathbf{y} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, then the orthogonal projection of \mathbf{y} onto Span $\{\mathbf{u}\}$ is

 - $\begin{array}{c}
 \begin{bmatrix} 1 \\ \frac{3}{2} \\ \frac{-1}{2} \end{bmatrix}
 \end{array}$

 - None of the above.

5р

- 7i If A is a 3×3 matrix such that $A \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $A \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, then the product $A \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}$ is
- $\begin{array}{c|c}
 \hline
 0 \\
 0 \\
 0
 \end{array}$

- (e) Not uniquely determined by the information given.

5р

7j For what value (or values) of p is the vector $\begin{bmatrix} 1 \\ 2 \\ p \\ 5 \end{bmatrix}$ a linear combination of the vectors $\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

and
$$\begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$
?

- (a) -1 only
- (b) 1 only
- c 3 only
- f e for infinitely many values of p
- (f) None of the above.



7k Recall that \mathbb{P}_2 denotes the set of polynomials of degree at most 2. In other words, \mathbb{P}_2 consists of all polynomials of the form

$$\mathbf{p}(t) = a_0 + a_1 t + a_2 t^2,$$

where the coefficients a_0, a_1, a_2 and the variable t are real numbers.

Let V be the subset of \mathbb{P}_2 made up of only those polynomials $\mathbf{p}(t)$ such that $\mathbf{p}(1)$ = 0, i.e.

$$V = {\mathbf{p} \in \mathbb{P}_2 : \mathbf{p}(1) = 0}.$$

Similarly, let W be the subset of \mathbb{P}_2 made up of only those polynomials $\mathbf{p}(t)$ such that $\mathbf{p}(0)$ = 1, i.e.

$$W = \{ \mathbf{p} \in \mathbb{P}_2 : \mathbf{p}(0) = 1 \}.$$

Is V is a subspace of \mathbb{P}_2 ? Is W is a subspace of \mathbb{P}_2 ?

- (a) V is a subspace of \mathbb{P}_2 . W is a subspace of \mathbb{P}_2 .
- (b) V is a subspace of \mathbb{P}_2 . W is not a subspace of \mathbb{P}_2 .
- (c) V is not a subspace of \mathbb{P}_2 . W is a subspace of \mathbb{P}_2 .
- (d) V is not a subspace of \mathbb{P}_2 . W is not a subspace of \mathbb{P}_2 .
- 71 Let $A = \begin{bmatrix} a & -3 \\ 2 & b \end{bmatrix}$, where a, b are real numbers. Given is that the characteristic polynomial of A is $\lambda^2 19$.

How many different values can a have?

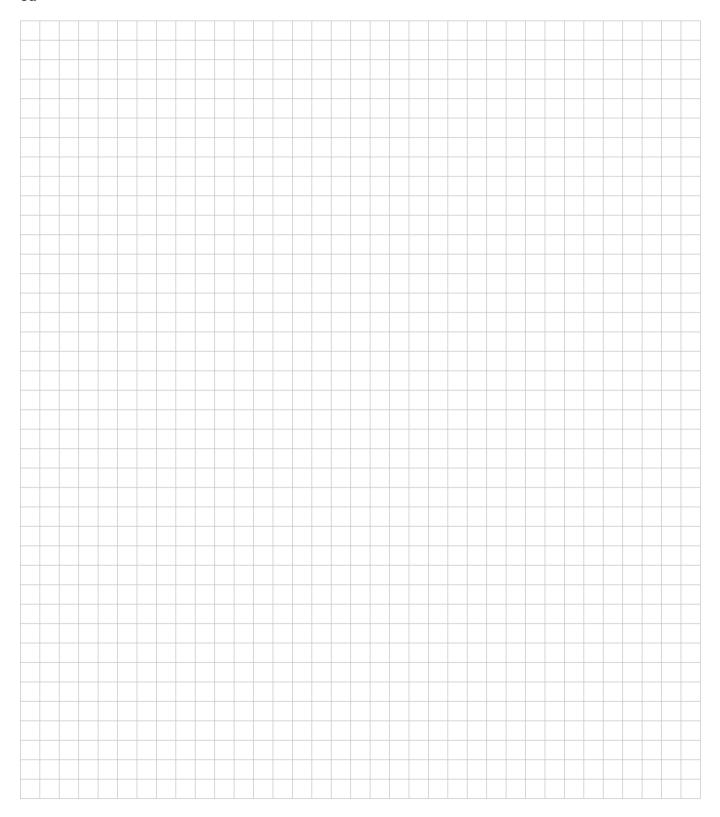
- (a) There are at least three different values a can have.
- $\stackrel{\smile}{(b)}$ There are exactly two values a can have.
- $\stackrel{\smile}{\text{(c)}}$ There is only one value a can have.
- (d) There are no posisble values for a.



Extra space

If you use these extra answer boxes, please mention clearly in your main answer that part of your answer can be found here!

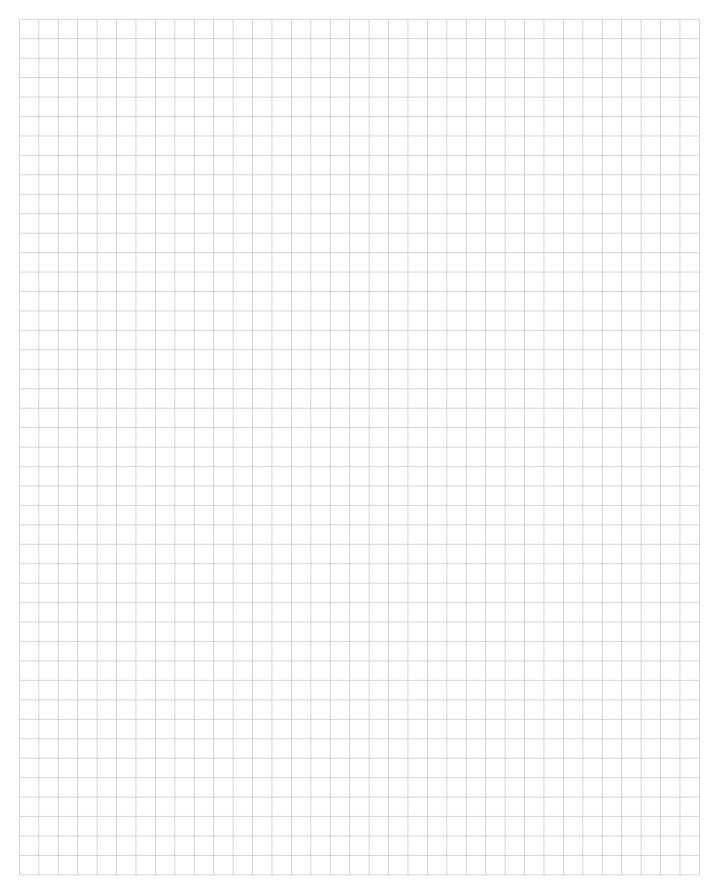
8a



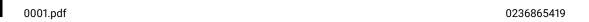




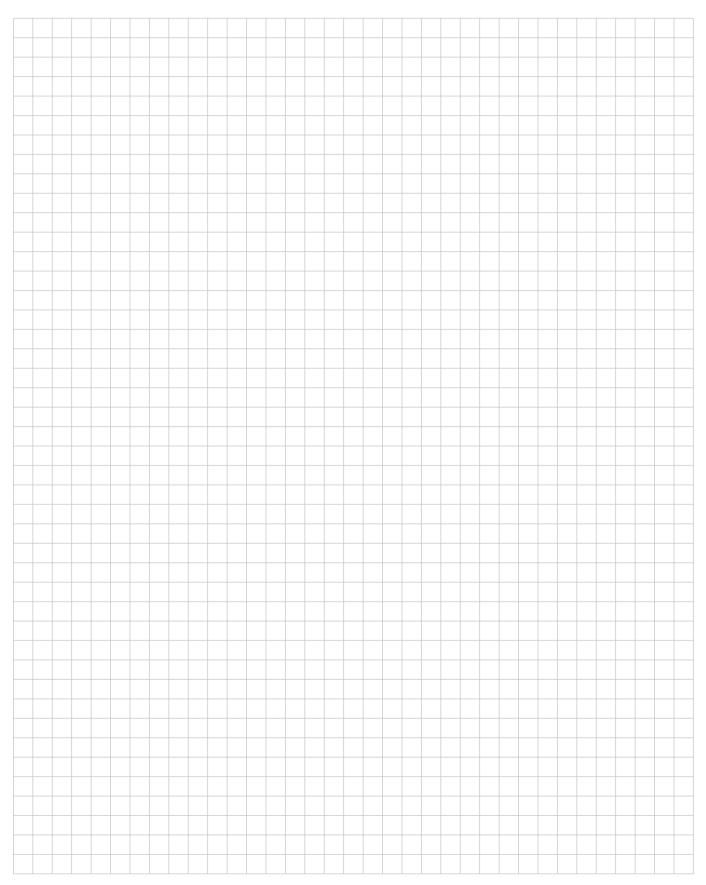
8b







8c







8d

