

# Predicate Logic

## Introduction

Statement

Translation

John walks

$R_j$

Bob walks

$R_b \leftarrow$  object

$\uparrow$

property

## Syntax

- Symbols for constants (objects)  $\rightarrow a, b, c$
- Symbols for variables  $\rightarrow x, y, z$
- Symbols for predicates  $\rightarrow A, B, C$
- Logical operators  $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$
- Quantifiers  $\forall x, \exists x$

## Syllogisms

- All A are B :  $\forall x (Ax \rightarrow Bx)$
- Some A are B :  $\exists x (Ax \wedge Bx)$
- All A are not B :  $\forall x (Ax \rightarrow \neg Bx) (\neg \exists x (Ax \wedge Bx))$

## Relations

- John sees Mary  $\rightarrow S_{j.m}$
- John gives Mary the book  $\rightarrow G_{j.m.b}$

## Complex quantifier patterns

- Everyone sees someone  $\forall x \exists y (Sxy)$
- Everyone is seen by someone  $\forall x \exists y (Syx)$

## Examples

$Lxy \rightarrow$ x loves y		Every boy loves a girl
$Gx \rightarrow$ x is girl		$\forall x (Bx \rightarrow \exists y (Gy \wedge Lxy))$
$Bx \rightarrow$ x is boy		

Every girl who loves all boys does not love every girl:

$$\forall x ((Gx \wedge \varphi(x)) \rightarrow \psi(x))$$

## Intuitive validities

$$\neg \forall x \varphi x \equiv \exists x \neg \varphi x$$

$\uparrow$   
any predicate

$$\neg \exists x \varphi x \equiv \forall x \neg \varphi x$$

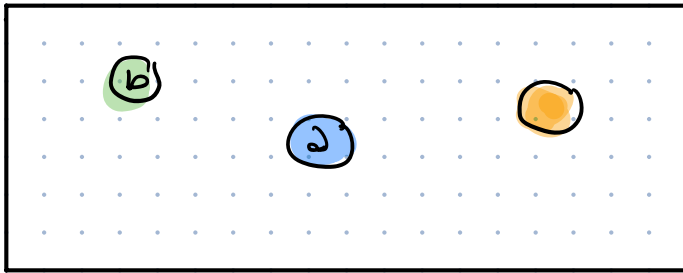
$$\forall x \varphi x \equiv \neg \exists x \neg \varphi x$$

$$\exists x \varphi x \equiv \neg \forall x \neg \varphi x$$

$$\neg \forall x (\varphi x \rightarrow \psi x) \equiv \exists x \neg (\varphi x \rightarrow \psi x) \\ \equiv \exists x (\varphi x \wedge \neg \psi x)$$

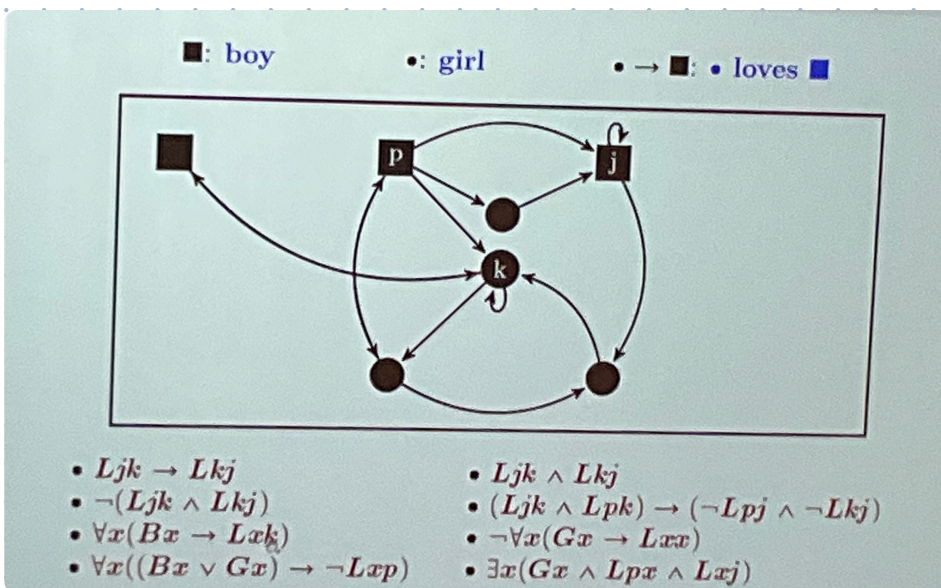
$$\forall x (\varphi x \wedge \psi x) \equiv \forall x \varphi x \wedge \forall x \psi x$$

## Evaluating formulas



Color is a property

- $Ba \rightarrow \text{true}$  (a is blue)
- $\exists x Sx \vee Cb$  (x is either square or circle)
- $Ra \rightarrow Sb$  true (a is not red)



## Language

- Term  $t$  is a variable / constant
- Formula
  - 1  $P t_1 t_2$
  - 2  $\varphi \wedge \psi$
  - 3  $\forall x \varphi$

## Substitution

- Inside a term  $\rightarrow$  replacing the occurrences of the variable  $y$  for the term  $t$  inside  $s$

$$(s)_t^y$$

For a constant:  $(c)_t^y := c$

$$(a)_c^x = a$$

For a variable: 
$$\begin{cases} (x)_t^y := x \\ (y)_t^y := t \end{cases}$$

- Inside a formula

$$\begin{array}{l} (P t_1 \dots t_n)_t^y := P(t_1)_t^y \dots (t_n)_t^y \\ (\neg \varphi)_t^y := \neg (\varphi)_t^y \\ (\varphi \wedge \psi)_t^y := (\varphi)_t^y \wedge (\psi)_t^y \\ (\varphi \vee \psi)_t^y := (\varphi)_t^y \vee (\psi)_t^y \\ (\varphi \rightarrow \psi)_t^y := (\varphi)_t^y \rightarrow (\psi)_t^y \\ (\varphi \leftrightarrow \psi)_t^y := (\varphi)_t^y \leftrightarrow (\psi)_t^y \end{array} \quad \begin{cases} (\forall x \varphi)_t^y := \forall x (\varphi)_t^y \\ (\forall y \varphi)_t^y := \forall y \varphi \\ (\exists x \varphi)_t^y := \exists x (\varphi)_t^y \\ (\exists y \varphi)_t^y := \exists y \varphi \end{cases}$$

## Models

A model is a tuple  $\mathcal{M} = \langle D, I, g \rangle$

- $D$  is the domain  $\rightarrow$  non-empty collection of obj.
- $I$  interpretation function  $\rightarrow$  assigns to each symbol
  - $\uparrow$  a relation