Practice Exam Questions Week 2, **Linear Algebra,** UCM

1. Consider the following matrix A and vector $\mathbf{b}$:

$$A = \begin{bmatrix} -1 & 2 & 3 & -4 & 8 \\ 3 & -6 & -4 & 7 & -9 \\ 2 & -4 & 1 & 1 & 5 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 7 \\ -6 \\ 7 \end{bmatrix}.$$

   (a) Are the columns of A linearly independent?

   (b) Compute the solution set of the associated linear system of equations $A\mathbf{x} = \mathbf{b}$ and express it in parametric vector form.

2. True or false? If the given statement is true, briefly explain why. If it is false, give a counterexample.

   (a) If the columns of an augmented matrix are linearly independent, then the associated linear system of equations is inconsistent.

   (b) Four different vectors in $\mathbb{R}^3$ always span $\mathbb{R}^3$.

   (c) The effect of adding a vector $\mathbf{p}$ to a vector $\mathbf{v}$ is to move the vector $\mathbf{v}$ in a direction parallel to $\mathbf{p}$.

   (d) If the augmented matrix of a linear system of equations has more rows than columns, then it cannot have infinitely many solutions.

   (e) If A and B are matrices for which the product AB and the sum $A + B$ are both well defined, then the product BA is also well defined.

   (f) If all the rows of an augmented matrix have a pivot, then the associated linear system of equations is inconsistent.

   (g) If S and T are 2×2 matrices such that $ST = 0$, then also $TS = 0$.

   (h) If the vectors $\mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathbb{R}^3$ are such that $\{\mathbf{x}, \mathbf{y}\}$ is linearly independent, $\{\mathbf{x}, \mathbf{z}\}$ is linearly independent, and $\{\mathbf{y}, \mathbf{z}\}$ is linearly independent, then also $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly independent.

(1) (ב)

$$A = \begin{bmatrix} -1 & 2 & 3 & -4 & 8 \\ 3 & -6 & -4 & 7 & -9 \\ 2 & -4 & 1 & 1 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 7 \\ -6 \\ 7 \end{bmatrix}.$$

$$\begin{bmatrix} -1 & 2 & 3 & -4 & 8 \\ 3 & -6 & -4 & 7 & -9 \\ 2 & -4 & 1 & 1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 & 3 & -4 & 8 \\ 0 & 0 & 5 & 5 & 15 \\ 0 & 0 & 7 & 9 & 21 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 & 3 & -4 & 8 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 7 & 9 & 21 \end{bmatrix}$$

what is this shame

$$\begin{bmatrix} -1 & 2 & 3 & -4 & 8 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -3 & 4 & -8 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -3 & 4 & -8 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{matrix} 7 \\ -6 \\ 7 \end{matrix}$$

free variables $\longrightarrow$ linearly dependent

$x_1 - 2x_2 + x_5 =$