

Surname, First name

Linear Algebra (KEN1410)

Exam

1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

a	<input checked="" type="radio"/>	c	d	e	f	→ b
a	b	<input checked="" type="radio"/>	d	e	f	→ c
<input checked="" type="radio"/>	b	c	<input checked="" type="radio"/>	e	f	→ a

Fill in your answer(s) to the multiple-choice questions as shown above (circles = one correct answer).

**Program:** Data Science and Artificial Intelligence

**Course code:** KEN1410

**Examiners:** dr. Marieke Musegaas and dr. Stefan Maubach

**Date/time:** Tuesday 29 March 2022, 09:00 - 11:00

**Format:** Closed book exam

**Allowed aids:** Pens, simple (non-programmable) calculator from the DKE-list of allowed calculators.

**Instructions to students:**

- The exam consists of 8 questions on 18 pages.
- Fill in your name and student ID number on the cover page.
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Before answering the questions, please first read all the exam questions, and then make a plan to spend the two hours.
- Only use black or dark blue pens, and write in a readable way. No Pencils. Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

**Exercise 1**

Consider the following matrix  $A$ :

$$A = \begin{bmatrix} 2 & 0 & 2 & 4 \\ 1 & 2 & 1 & -2 \\ 2 & 0 & 1 & 2 \\ 1 & 0 & 1 & 2 \end{bmatrix}.$$

Define  $T: \mathbb{R}^4 \rightarrow \mathbb{R}^4$  by  $T(\mathbf{x}) = A\mathbf{x}$ .

15p

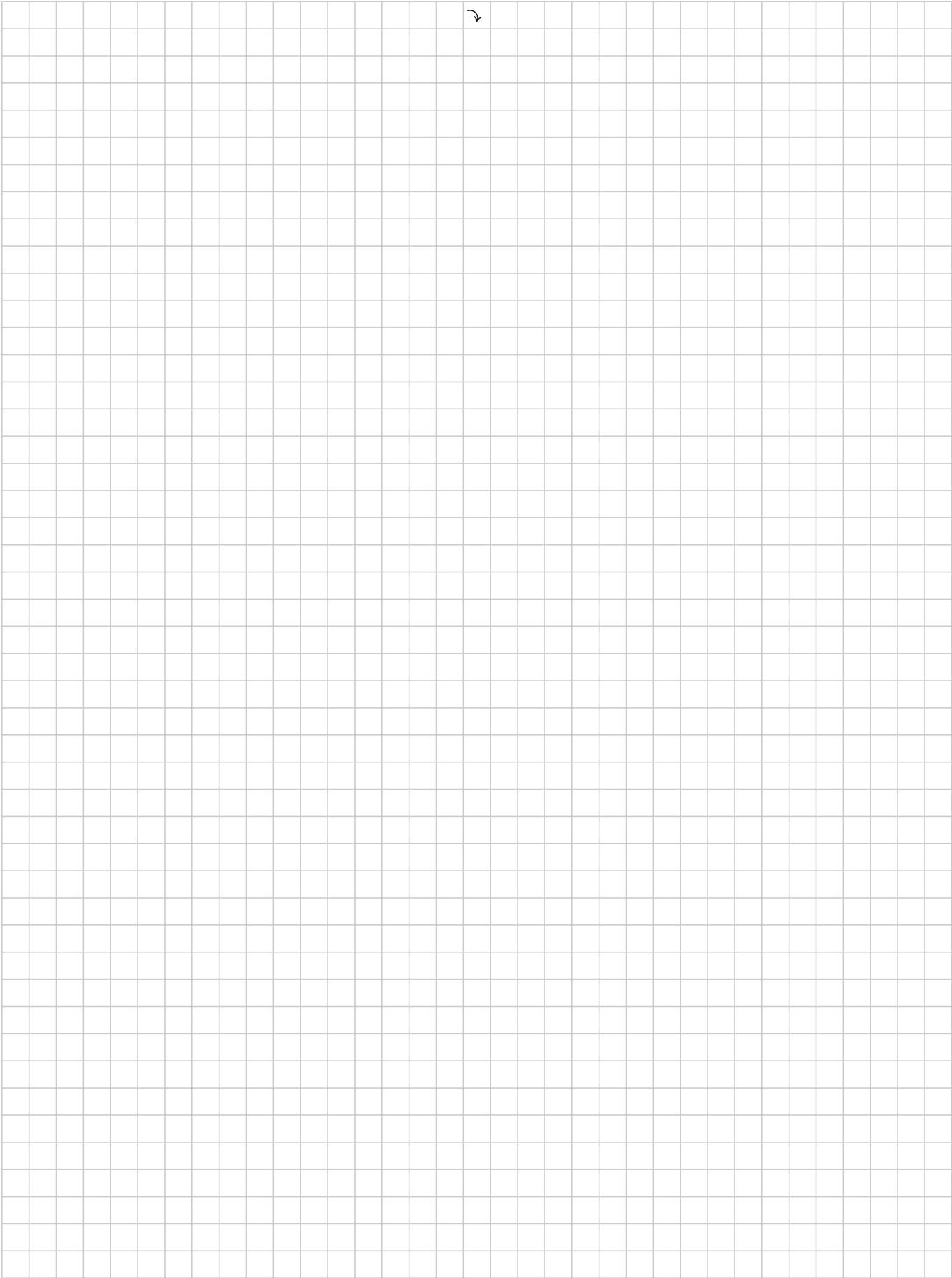
1 Determine all  $\mathbf{x} \in \mathbb{R}^4$  such that  $T(\mathbf{x}) =$

$$\begin{bmatrix} 8 \\ 2 \\ 5 \\ 4 \end{bmatrix}$$

$$T(\mathbf{x}) = \begin{bmatrix} 8 \\ 2 \\ 5 \\ 4 \end{bmatrix} \quad T(\mathbf{x}) = A\mathbf{x}$$

$$\begin{bmatrix} 2 & 0 & 2 & 4 \\ 1 & 2 & 1 & -2 \\ 2 & 0 & 1 & 2 \\ 1 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}$$



**Exercise 2**

Let  $a, b, c, d \in \mathbb{R}$  and consider the following system of linear equations

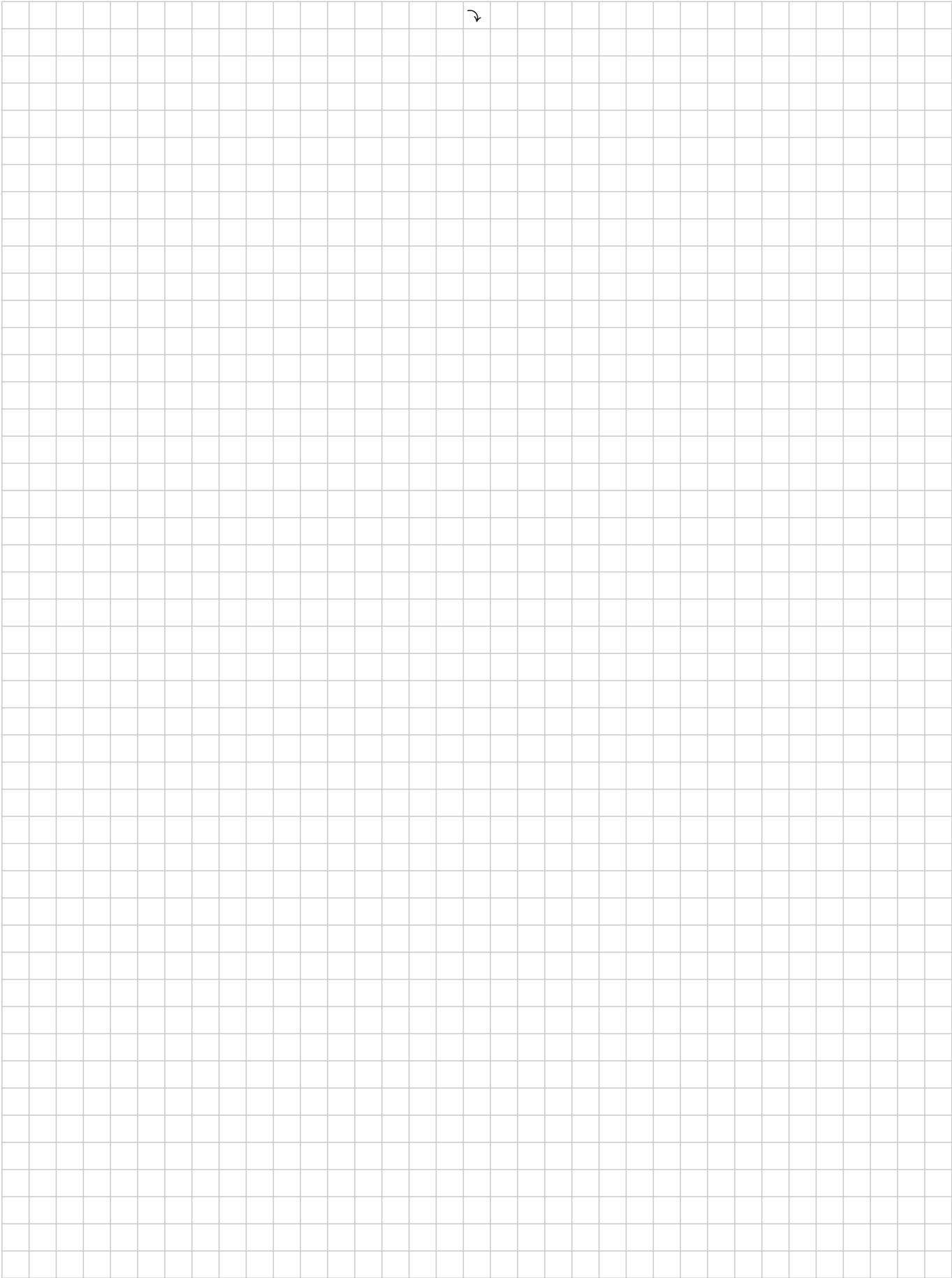
$$\begin{aligned}x_1 + ax_2 + bx_3 &= 14 \\ cx_2 + dx_3 &= -40\end{aligned}$$

This system of linear equations has a solution set that looks like

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix},$$

where  $\lambda \in \mathbb{R}$ .

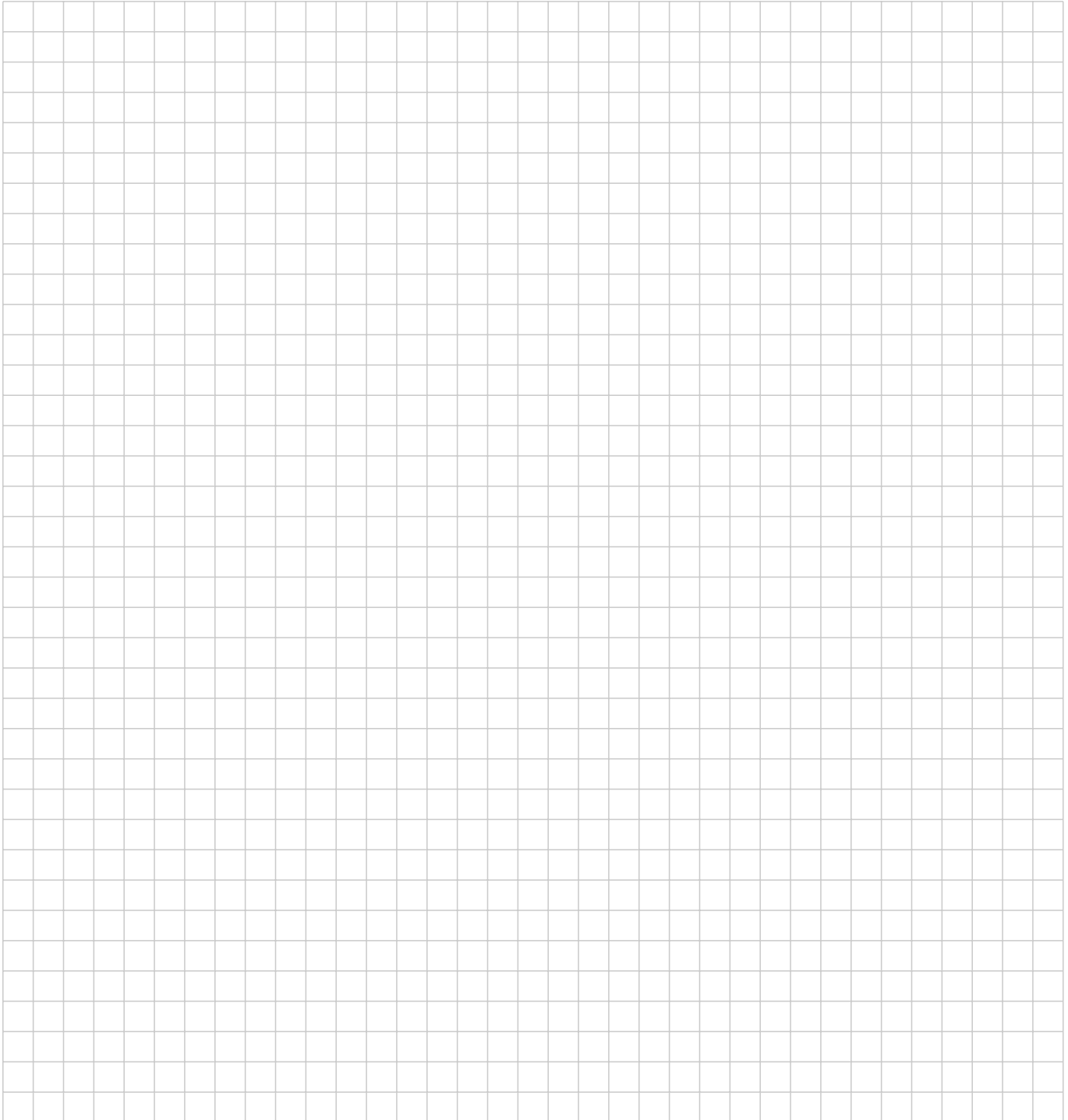
15p **2** Compute  $a, b, c$  and  $d$ .



**Exercise 3**

- 5p **3** Provide, **IF POSSIBLE**, an example of a subset  $H$  of  $\mathbb{R}^2$  that has the following three properties:
- the zero vector is in  $H$ ,
  - $H$  is **NOT** closed under vector addition,
  - $H$  is closed under multiplication by scalars.

(Note: only provide your answer. An explanation is not required.)



A matrix  $A$  has after a couple of row operations the following form

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 4 & -2 \\ 0 & -6 & 3 \\ 0 & 0 & 1 \end{bmatrix}.$$

**Exercise 5**

Consider the following matrix  $A$ :

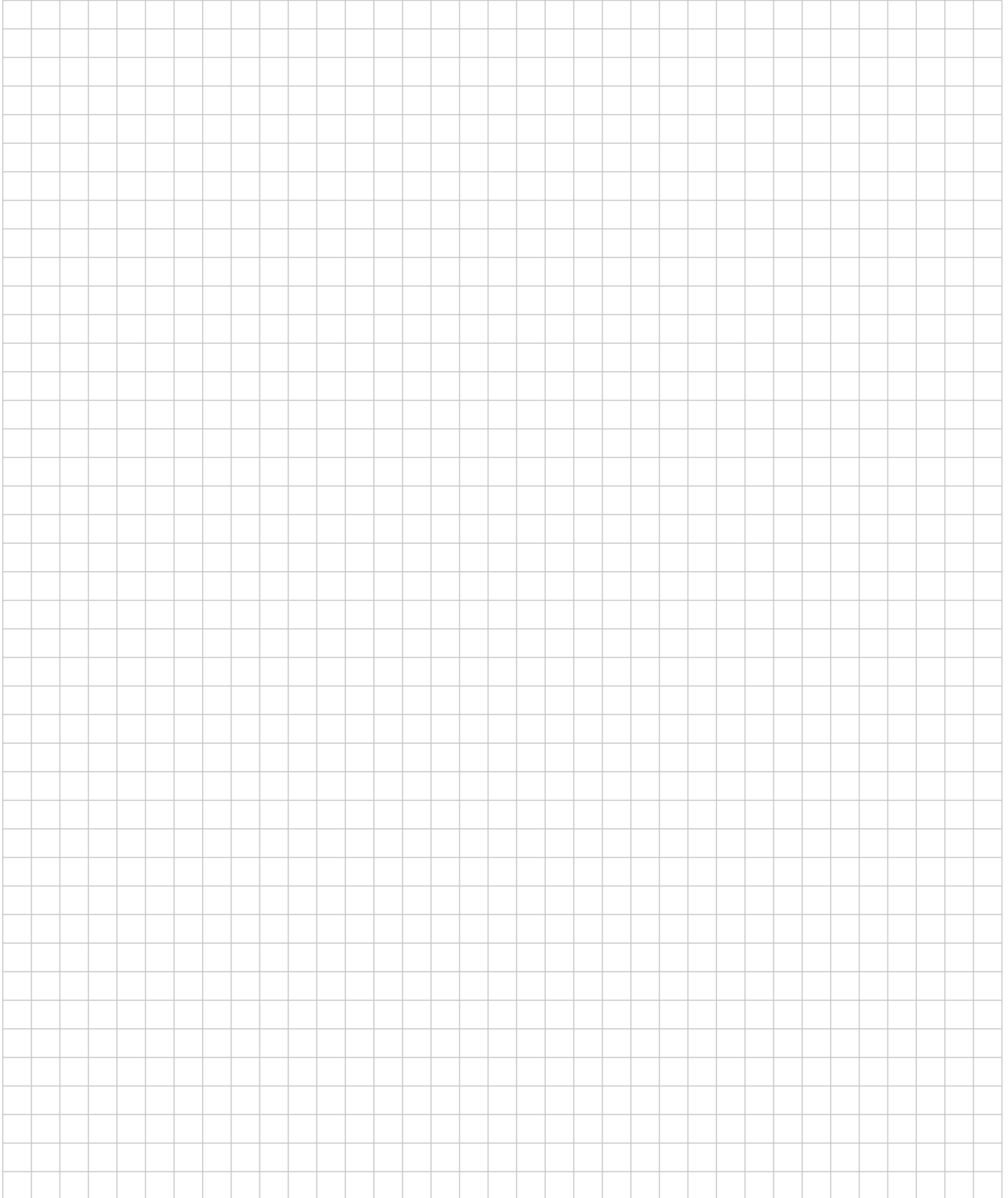
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

5p **5a** Show that 3 is an eigenvalue of  $A$  (hint: find an eigenvector).

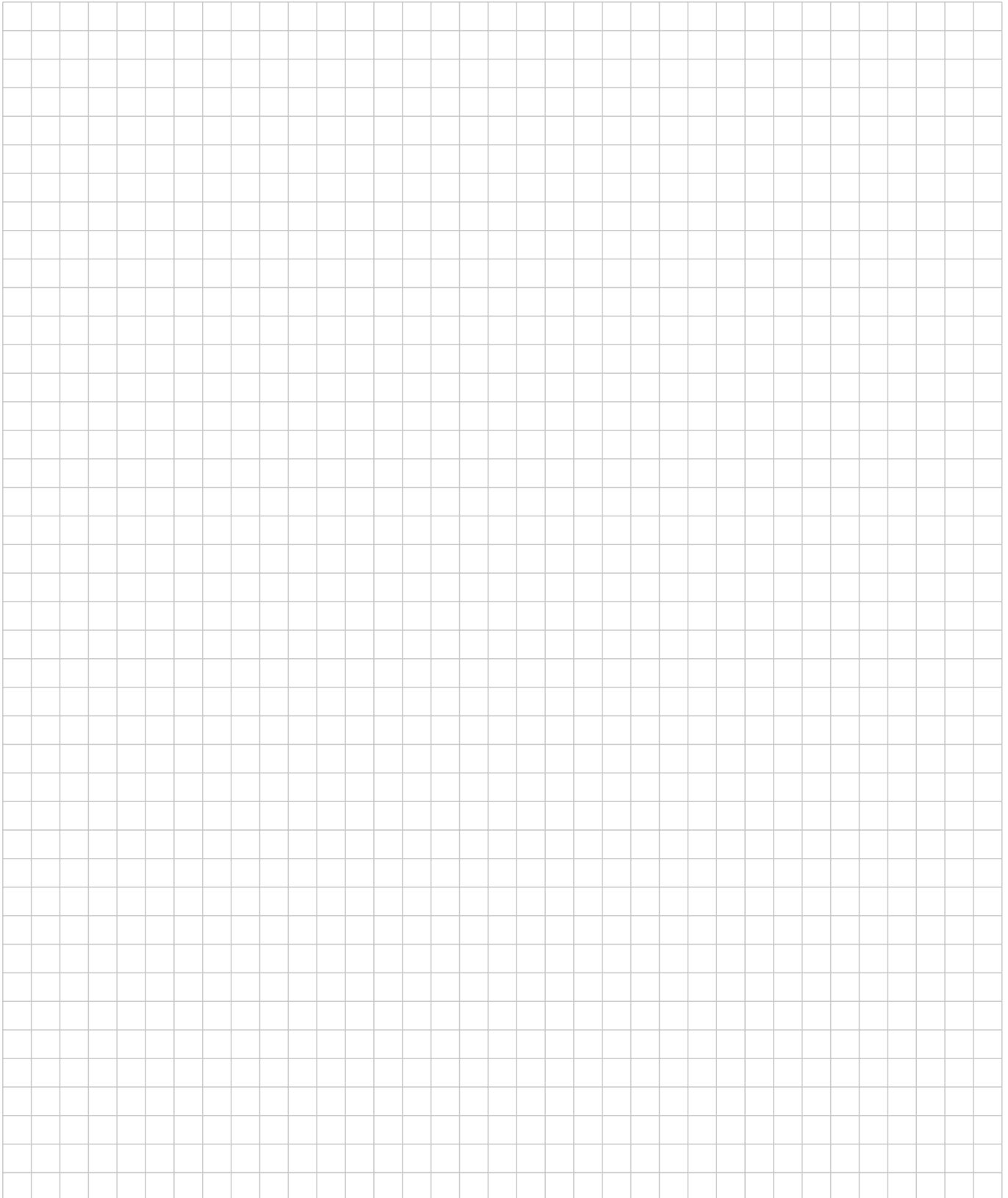




10p **5b** Show that 0 is an eigenvalue of  $A$ . And find two corresponding linearly independent eigenvectors.



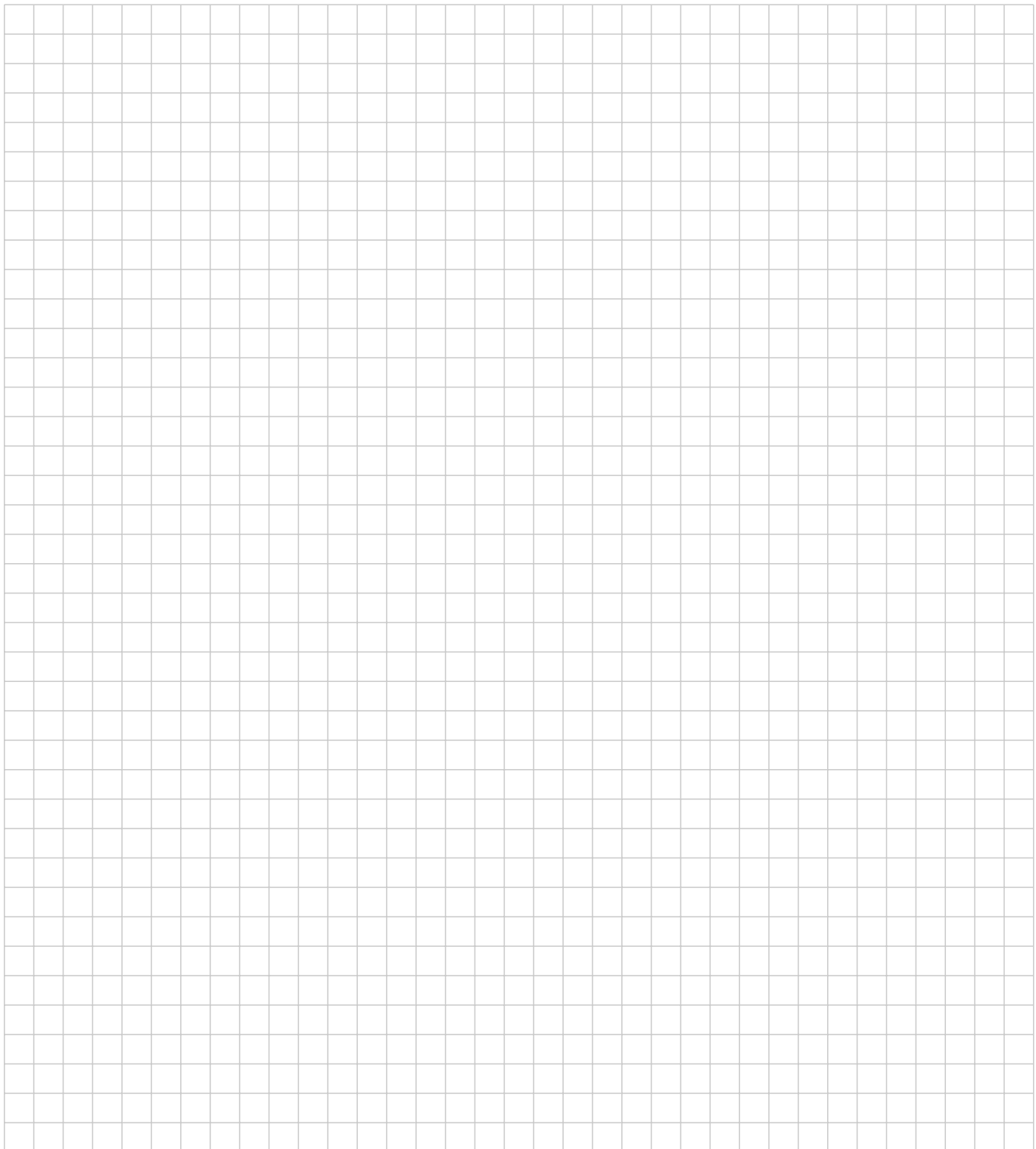
- 5p **5c** Is the matrix  $A$  diagonalizable? Briefly explain.  
(Note: you do not need to diagonalize  $A$ . You only need to state whether it is possible to diagonalize  $A$ .)



**Exercise 6**

Let  $\mathbf{y} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$  and  $\mathbf{u} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$ .

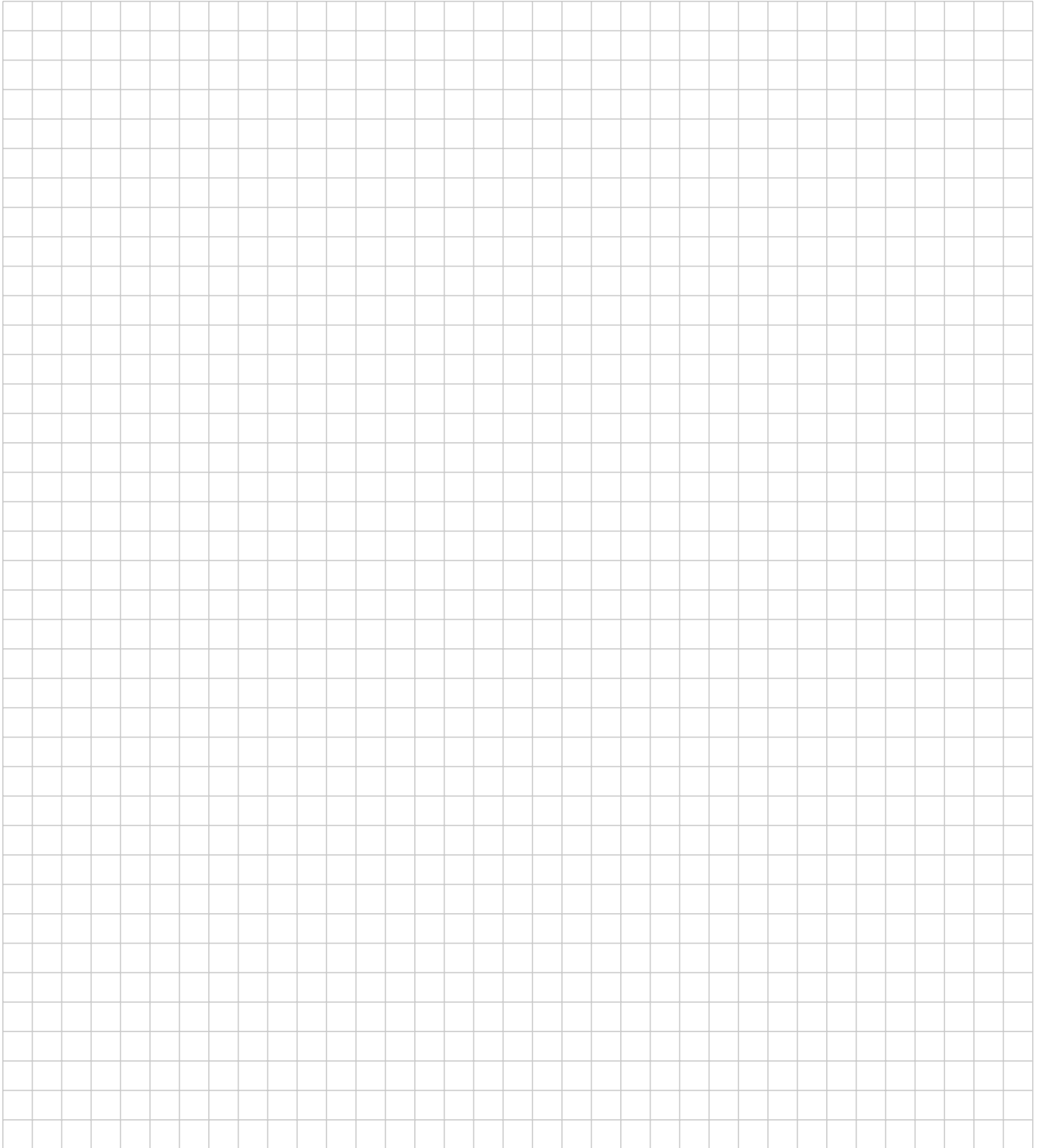
10p **6** Write  $\mathbf{y}$  as the sum of a vector in  $\text{Span}\{\mathbf{u}\}$  and a vector orthogonal to  $\mathbf{u}$ .



**Exercise 7**

10p **7** Prove or disprove the following statement.

Let  $\mathbf{x}$  and  $\mathbf{y}$  in  $\mathbb{R}^3$  be two vectors that have the same length ( $\|\mathbf{x}\| = \|\mathbf{y}\|$ ) and define  $\mathbf{u} = \mathbf{x} + \mathbf{y}$  and  $\mathbf{v} = \mathbf{x} - \mathbf{y}$ . Then,  $\mathbf{u}$  and  $\mathbf{v}$  are orthogonal to each other.



**Exercise 8**

True or False? For each of the statements below: state whether the statement is true or false. (Note: An explanation is not required. **Please read the multiple choice instructions on the cover page!**)

- 3p **8a** If two rows of a square matrix  $A$  are the same, then  $\det A = 0$ .  
☐ a True ☐ b False
- 3p **8b** If  $A$  is a  $6 \times 8$  matrix, then it is possible that it has a 1-dimensional null space.  
☐ a True ☐ b False
- 3p **8c** Two orthogonal vectors are automatically also linearly independent.  
☐ a True ☐ b False
- 3p **8d** If  $\lambda$  is an eigenvalue of  $A$ , then it is also an eigenvalue of  $A^T$ .  
☐ a True ☒ b False
- 3p **8e** Let  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$ . If  $\mathbf{x} \cdot \mathbf{y} = 0$ , then there does not exist a plane in  $\mathbb{R}^3$  that contains both  $\mathbf{x}$  and  $\mathbf{y}$ .  
☐ a True ☐ b False

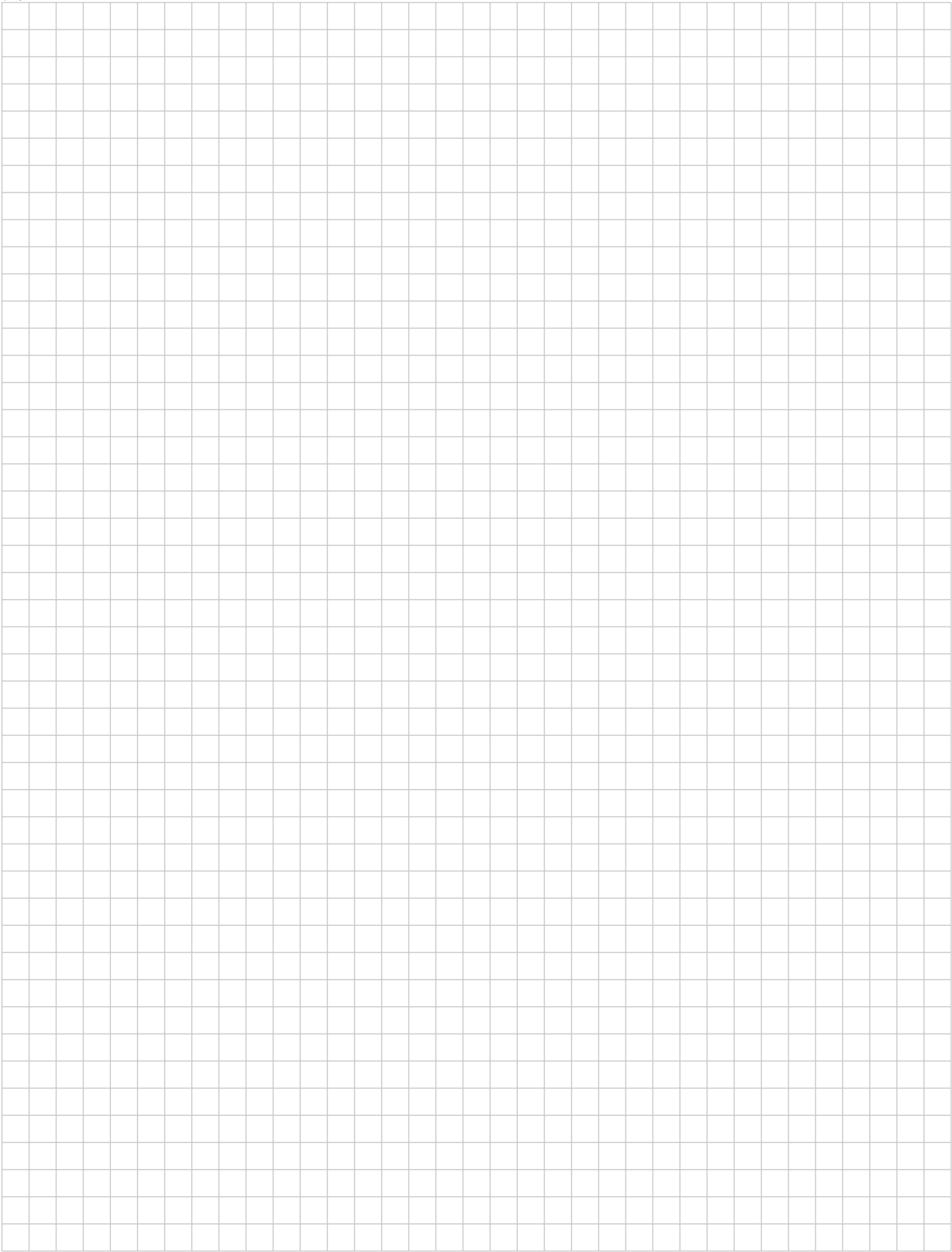
### Extra space

If you use these extra answer boxes, **please mention clearly in your main answer that part of your answer can be found here!**

**9a**

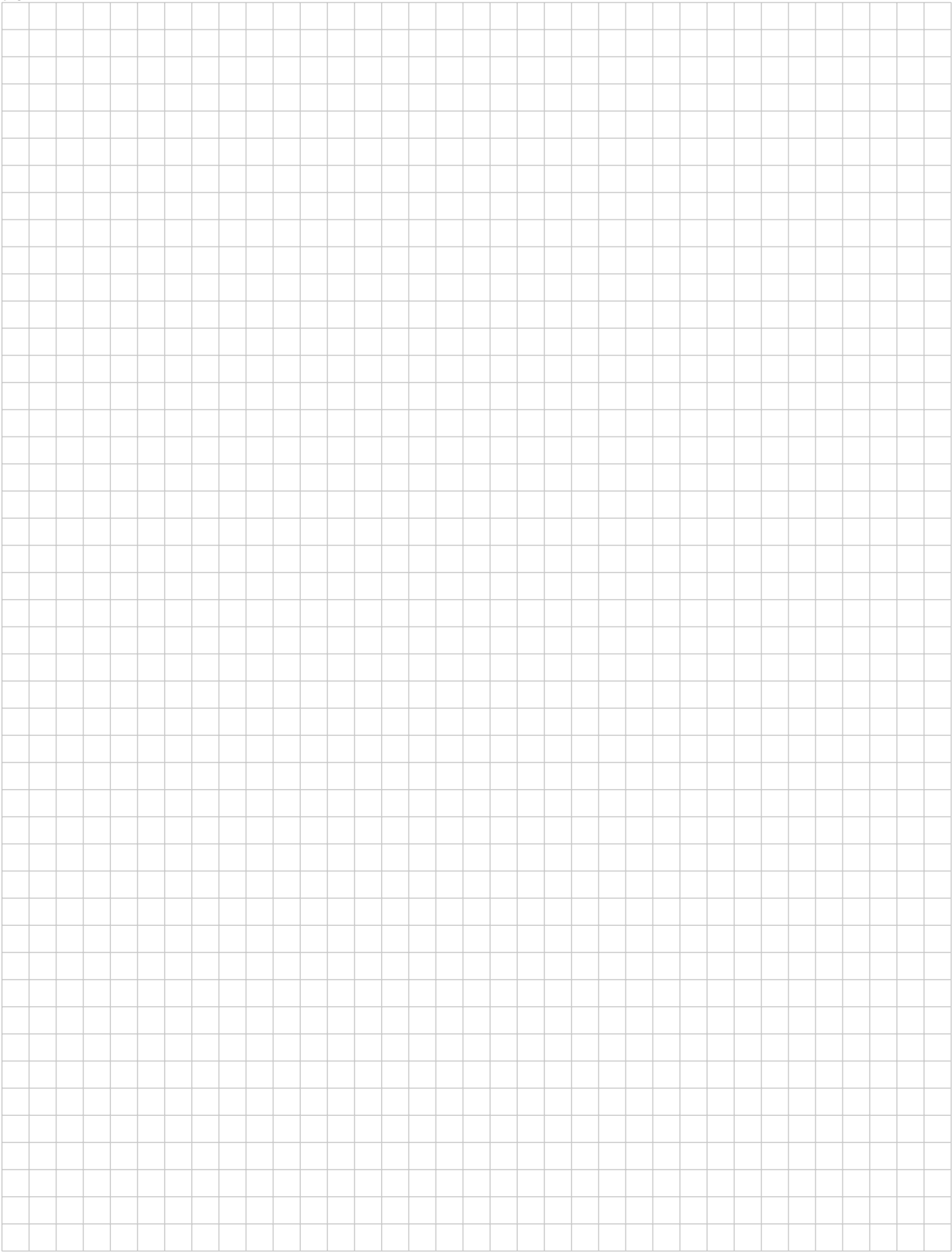


9b





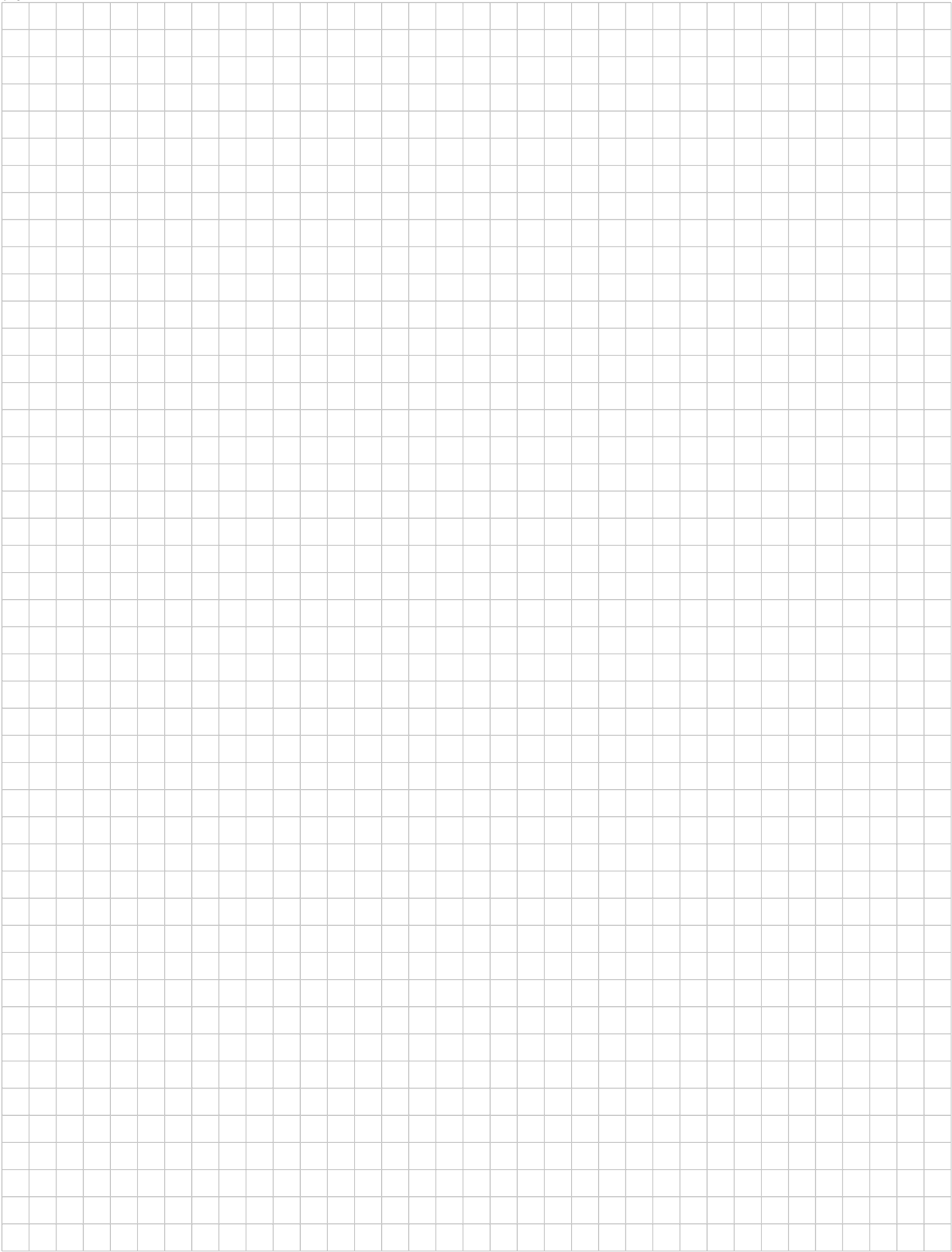
9c







9d



9e

