Practice Exam Questions Week 3, **Linear Algebra**

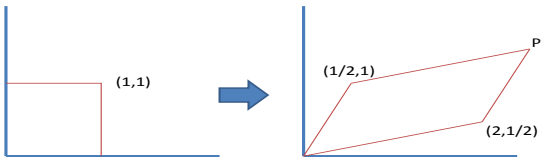
1. Consider the following matrix A:

$$A = \begin{bmatrix} 3 & -6 & 2 \\ 1 & -2 & 1 \\ -2 & 4 & -2 \end{bmatrix}.$$

   a. Are the columns of A linearly independent?

   b. Is A invertible? (Hint: use the answer from a.)

2. Consider the following transformation



   a. Construct the matrix which would give this transformation.

   b. Give the coordinates of point P.

3. Consider the following matrix A:

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix}.$$

   a. Compute the inverse of A.

   b. Let $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$ and find a solution to the linear system of equations $A\mathbf{x} = \mathbf{b}$.

   c. Give an example of an alternative bottom row for A which would make it singular.

4. Compute the determinant of the following matrix:

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}.$$

5. True or false? If the given statement is true, briefly explain why. If it is false, give a counterexample.

   a. If S is a 3×3 matrix such that $S^2 = 0$, then S^{-1} does not exist.

b. If the equation $A\mathbf{x} = \mathbf{b}$ is consistent, and the variable x_3 is a free variable in the reduced echelon form of A, then there is a solution with $x_3 = 4$.

c. If you take two vectors in $\mathbb{R}^3$ they will never be linearly dependent.

d. If F is (2×2) with $\det(F) = 0$ and g is a (2×1) vector, then the matrix equation $F\mathbf{x} = \mathbf{g}$ is always inconsistent.

e. If G is a (3×3) matrix for which $G^2 = I$, then $\det(G) = 1$.

f. If $\det(B) \neq 0$, then B^T is invertible.

g. If A is a (3×4) matrix, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^3$ onto $\mathbb{R}^4$.

h. An elementary row operation on A does not change the determinant of A.