

$$8f) \quad a \vee \neg a \models (\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p)$$

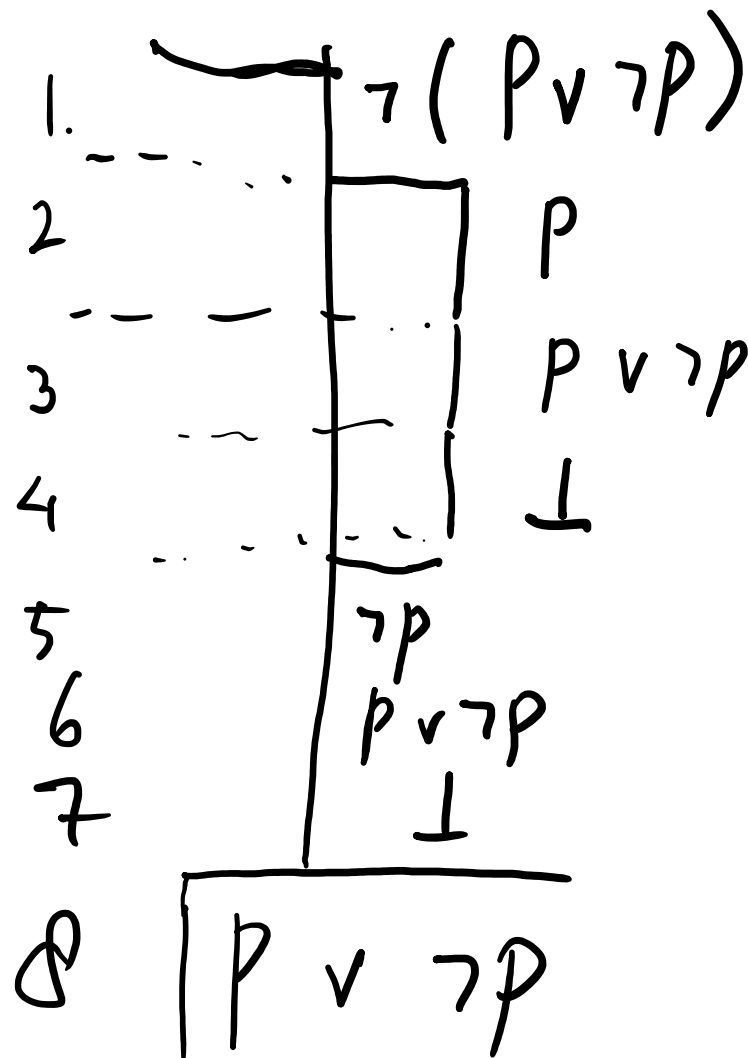
1.	a	\vee	$\neg a$	(given)
2.			$\neg p \rightarrow \neg q$	(assume)
3.			q	(assume)
4.			$\neg p$	(assume)
5.			$\neg q$	$E \rightarrow (2, 4)$
6.			\perp	$E \neg (3, 5)$
7.			p	$I \neg (4, 6)$
8.			$q \rightarrow p$	$I \rightarrow (3, 7)$
9.	$(\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p)$			$I \rightarrow (2, 8)$

8.e]	1.	$P \rightarrow q$	(given)
		$\frac{\quad}{\neg q}$	(ass.)
2		\vdots	
3		\vdots	(ass)
4.		\vdots	$E \rightarrow (1, 3)$
5		\vdots	$E \neg (2, 4)$
6		\vdots	$I \neg (3, 5)$
7		\vdots	$I \rightarrow (2, 6)$
		$\neg q \rightarrow \neg p$	

8d)

1.	$P \vee \neg q$	Given
2.	P	ass.
3.	q	ass.
4.	P	repeat(2)
5.	$q \rightarrow P$	$I \rightarrow (3, 4)$
6.	$\neg q$	ass.
7.	q	ass.
8.	\perp	$E \neg (6, 7)$
9.	P	$\perp (8)$
10.	$q \rightarrow P$	$I \rightarrow (7, 9)$
11.	$q \rightarrow P$	$E \vee (2, 5, 6, 10)$

89)



ass.

ass.

$I_v(2)$

$E_7(1, 3)$

$I_7(2, 4)$

$I_v(5)$

$E_7(1, 6)$

$I_7(1, 7)$

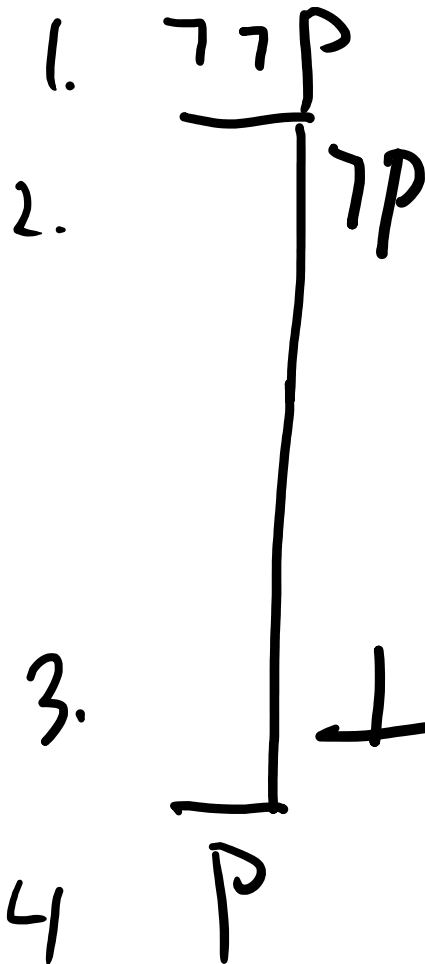
8. b)

1. $p \rightarrow \neg q$
2. q
3. p
4. $\neg q$
5. \perp
6. $\neg p$

(given)
(given)
(ass.)
 $E \rightarrow (1, 3)$
 $E \neg (2, 4)$
 $I \neg (3, 5)$

$\neg A$
 \vdots
 \perp
 $\neg A$

$\neg A$
 \vdots
 \perp
 B
 $A \rightarrow B$

8^a)

(giba)
(ass.)

$E_7(1,2)$
 $I_7(2,3).$

$$79) \quad \begin{array}{c} p, q \circ \underline{p \rightarrow q} \\ | \\ \hline p, q \circ q \\ \hline x \end{array}$$

6)

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

for matrix multiplication

$$(p \rightarrow q) \wedge r \not\equiv p \rightarrow (q \wedge r)$$

5^a) φ contradiction

$\varphi \circ$
closed

$\circ \rightarrow \varphi$
closed

$\neg \varphi$ tautology

$\leftrightarrow \varphi$ contradiction

$\circ \varphi$

closed

φ tautology

$\neg \varphi$ contradiction

5⁹) (cont'd)

$$\frac{(p \rightarrow \neg q) \wedge (q \rightarrow r) \wedge (r \rightarrow p) \circ}{p \rightarrow \neg q, (q \rightarrow r) \wedge (r \rightarrow p) \circ}$$

$$\frac{p \rightarrow \neg q, (q \rightarrow r) \wedge (r \rightarrow p) \circ}{p \rightarrow \neg q, \underline{q \rightarrow r}, r \rightarrow p \circ}$$

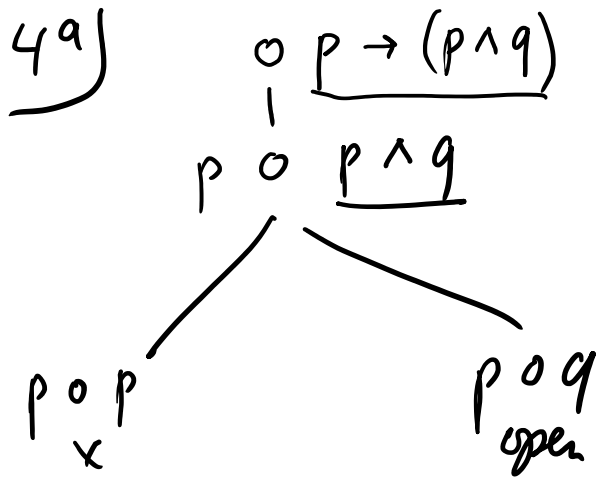
$$\frac{p \rightarrow \neg q, \underline{q \rightarrow r}, r \rightarrow p \circ}{p \rightarrow \neg q, r \rightarrow p \circ \quad \text{open, so } \neg \phi \text{ not valid, so } \phi \text{ is not a contradiction.}}$$

$$\frac{p \rightarrow \neg q, \underline{r \rightarrow p}, r \circ}{p \rightarrow \neg q, r, p \circ}$$

$$\frac{p \rightarrow \neg q, r, p \circ}{r, p, \neg q \circ} \quad \left| \quad p \rightarrow \neg q, r \circ \quad \text{open, so } \neg \phi \text{ not valid, so } \phi \text{ is not a contradiction.} \right.$$

$$\frac{r, p, \neg q \circ}{r, p \circ \quad \text{open}}$$

open



\Rightarrow tableau open, $\models p \rightarrow (p \wedge q)$
not valid, so
not a tautology.

(a) sentence (b) is true

(b) sentence (c) is false

(c) There are exactly two sentences correct

$a = \text{true} \rightarrow \text{sentence 1 true}$
 $= \text{false} \rightarrow \text{sentence 1 false}$, b, c similar

$$a \leftrightarrow b$$

$$b \leftrightarrow \neg c$$

$$c \leftrightarrow ((a \wedge b \wedge \neg c) \vee (a \wedge \neg b \wedge c) \vee (\neg a \wedge b \wedge c))$$

a	b	c
1	1	0
0	0	0
1	1	1
0	0	1

$c \leftrightarrow ((a \wedge b \wedge \neg c) \vee (a \wedge \neg b \wedge c) \vee (\neg a \wedge b \wedge c))$
0
1
1
0

$$b \leftrightarrow \neg c$$

1
0
0
1