

Practice Exam Questions Week 4, **Linear Algebra**

1. Consider the following matrix:

$$M = \begin{bmatrix} -2 & 6 & 0 & -2 & 2 \\ -1 & 3 & 0 & -1 & 1 \\ 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

Compute bases for the two subspaces $\text{Col}(A)$ and $\text{Nul}(A)$.

2. True or false? If the given statement is true, briefly explain why. If it is false, give a counterexample.

- \mathbb{R}^2 is a subspace of \mathbb{R}^3 .
- A vector is an arrow in three-dimensional space.
- A subset H of a vector space V is a subspace of V if the zero vector is in H .
- A subspace is also a vector space.
- A vector space is also a subspace.
- The null space of an $m \times n$ matrix is in \mathbb{R}^m .
- The column space of an $m \times n$ matrix is in \mathbb{R}^m .

3. Determine all values of $p \in \mathbb{R}$ for which the null space of the following matrix has dimension 1.

$$G = \begin{bmatrix} 2 & -5 & 8 \\ -2 & -7 & p \\ 4 & 2 & 7 \end{bmatrix}$$

4. True or false? If the given statement is true, briefly explain why. If it is false, give a counterexample.

- If a 3×3 matrix A has rank 3, then its rows form a basis for \mathbb{R}^3 .
- If the first column of a 7×5 matrix B provides a basis for the column space, then the null space of B has dimension 4.
- The number of variables in the equation $A\mathbf{x} = \mathbf{0}$ equals the dimensions of $\text{Nul}(A)$.
- If $\dim V = n$ and if S spans V , then S is a basis of V .