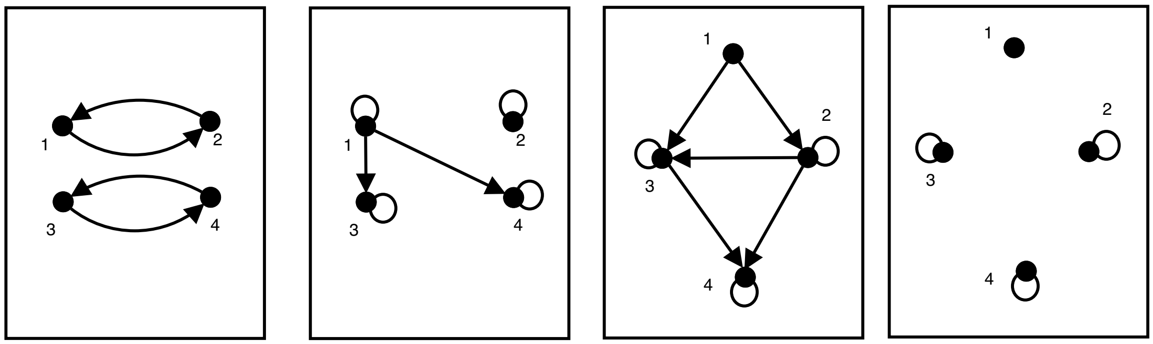
Practice Exam Questions - Tutorial 3

1. Let $A = \{2, 3\}$, $B = \{4, 5\}$, $C = \{6\}$, $D = \emptyset$, $E = \{2, \{2, 3\}, \emptyset\}$. Write down the elements in the following sets.
   (a) $B \times A$
   (b) $B \times D$
   (c) $\mathbb{P}(C) \times A$
   (d) $\mathbb{P}(B) \setminus B$
   (e) $\mathbb{P}(B \cup C)$
   (f) $(A \times B) \times C$
   (g) $\mathbb{P}(B) \times \mathbb{P}(D)$
   (h) $(E \setminus A) \times A$
2. Let $A = \{\emptyset\}$, $B = \{1, 2, \{3\}\}$, $C = \{2, 3\}$ and $D = \emptyset$.
   (a) Write down $A \times (B \setminus C)$.
   (b) Write down $\mathbb{P}(A \cap D)$.
3. Let $A = \emptyset$, $B = \{\emptyset\}$, $C = \{1, 2, 4\}$.
   (a) Write down $\mathbb{P}(B \setminus A) \cup A$.
   (b) Write down all partitions of C.
4. The statement $\mathbb{P}(A \cup B) = \mathbb{P}(A) \cup \mathbb{P}(B)$ is *not* true for all sets A and B.
   (a) Give an example of sets A and B for which the statement is not true.
   (b) What special property do A and B need to have to make the statement hold? In other words, you need to find a property M such that for all sets A and B,
   $$\Big(M \Leftrightarrow \big(\mathbb{P}(A \cup B) = \mathbb{P}(A) \cup \mathbb{P}(B)\big)\Big).$$
   You do not need to give a proof; you only need to find the property M.
5. (a) Let R be the relation on $\mathbb{Z}$ defined as follows: xRy means "$2x - 2y \leq 3$". Is R reflexive? Symmetric? Transitive? For each of these properties, prove or disprove that it has that property.
   (b) Let R be the relation on $\mathbb{R}$ defined as follows: xRy means "$x \geq |y|$" where "$|\cdot|$" means "absolute value". Is R reflexive? Symmetric? Transitive? For each of these properties, prove or disprove that it has that property.
   (c) Let R be the relation on $\mathbb{Z}$ defined as follows: xRy means "$(x - y) + 1$ is natural". Is R reflexive? Symmetric? Transitive? For each of these properties, prove or disprove that it has that property.

6. Let $A = \{1, 2, 3, 4\}$. For each of the four relations (a)-(d) shown below on A, state explicitly which of the properties it does and does not have: reflexive, symmetric, transitive. (No motivation is required).

(a) (b) (c) (d)



7. Let R be the relation on $\mathbb{Z}$ defined as follows: xRy means "$(x + \frac{1}{2})(y + \frac{1}{2}) \geq 0$". This is an equivalence relation. (You do not need to prove this). How many equivalence classes does R have? Describe briefly which elements of $\mathbb{Z}$ are in each equivalence class.