

## Practice Exam Questions - Tutorial 3

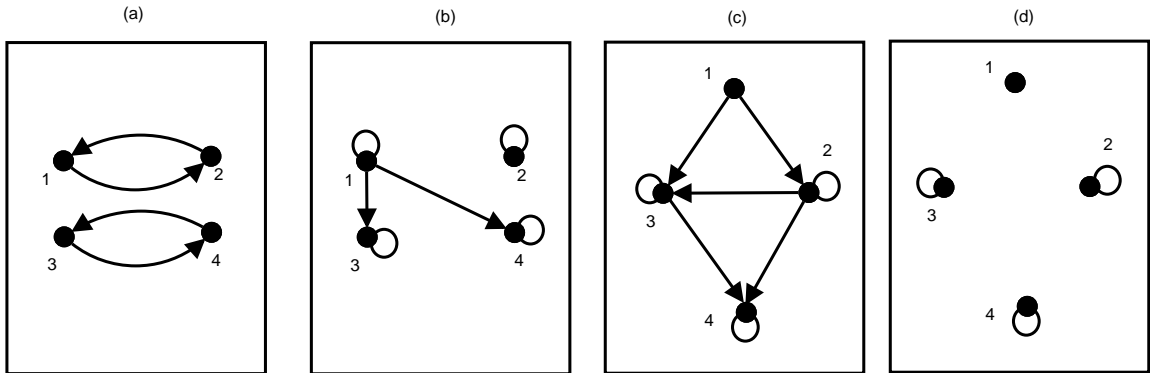
1. Let  $A = \{2, 3\}$ ,  $B = \{4, 5\}$ ,  $C = \{6\}$ ,  $D = \emptyset$ ,  $E = \{2, \{2, 3\}, \emptyset\}$ . Write down the elements in the following sets.
  - (a)  $B \times A$
  - (b)  $B \times D$
  - (c)  $\mathbb{P}(C) \times A$
  - (d)  $\mathbb{P}(B) \setminus B$
  - (e)  $\mathbb{P}(B \cup C)$
  - (f)  $(A \times B) \times C$
  - (g)  $\mathbb{P}(B) \times \mathbb{P}(D)$
  - (h)  $(E \setminus A) \times A$
2. Let  $A = \{\emptyset\}$ ,  $B = \{1, 2, \{3\}\}$ ,  $C = \{2, 3\}$  and  $D = \emptyset$ .
  - (a) Write down  $A \times (B \setminus C)$ .
  - (b) Write down  $\mathbb{P}(A \cap D)$ .
3. Let  $A = \emptyset$ ,  $B = \{\emptyset\}$ ,  $C = \{1, 2, 4\}$ .
  - (a) Write down  $\mathbb{P}(B \setminus A) \cup A$ .
  - (b) Write down all partitions of  $C$ .
4. The statement  $\mathbb{P}(A \cup B) = \mathbb{P}(A) \cup \mathbb{P}(B)$  is *not* true for all sets  $A$  and  $B$ .
  - (a) Give an example of sets  $A$  and  $B$  for which the statement is not true.
  - (b) What special property do  $A$  and  $B$  need to have to make the statement hold? In other words, you need to find a property  $M$  such that for all sets  $A$  and  $B$ ,

$$\left( M \Leftrightarrow \left( \mathbb{P}(A \cup B) = \mathbb{P}(A) \cup \mathbb{P}(B) \right) \right).$$

You do not need to give a proof; you only need to find the property  $M$ .

5.
  - (a) Let  $R$  be the relation on  $\mathbb{Z}$  defined as follows:  $xRy$  means “ $2x - 2y \leq 3$ ”. Is  $R$  reflexive? Symmetric? Transitive? For each of these properties, prove or disprove that it has that property.
  - (b) Let  $R$  be the relation on  $\mathbb{R}$  defined as follows:  $xRy$  means “ $x \geq |y|$ ” where “ $|\cdot|$ ” means “absolute value”. Is  $R$  reflexive? Symmetric? Transitive? For each of these properties, prove or disprove that it has that property.
  - (c) Let  $R$  be the relation on  $\mathbb{Z}$  defined as follows:  $xRy$  means “ $(x - y) + 1$  is natural”. Is  $R$  reflexive? Symmetric? Transitive? For each of these properties, prove or disprove that it has that property.

6. Let  $A = \{1, 2, 3, 4\}$ . For each of the four relations (a)-(d) shown below on  $A$ , state explicitly which of the properties it does and does not have: reflexive, symmetric, transitive. (No motivation is required).



7. Let  $R$  be the relation on  $\mathbb{Z}$  defined as follows:  $xRy$  means " $(x + \frac{1}{2})(y + \frac{1}{2}) \geq 0$ ". This is an equivalence relation. (You do not need to prove this). How many equivalence classes does  $R$  have? Describe briefly which elements of  $\mathbb{Z}$  are in each equivalence class.