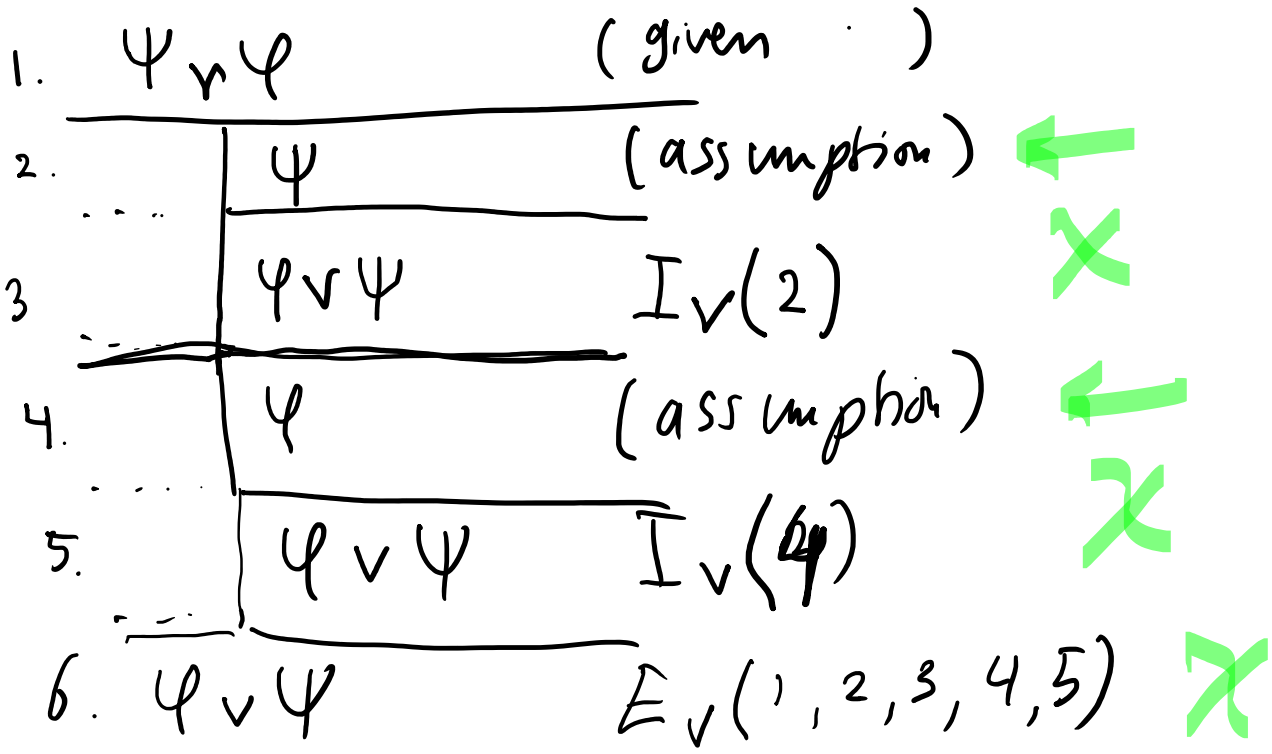
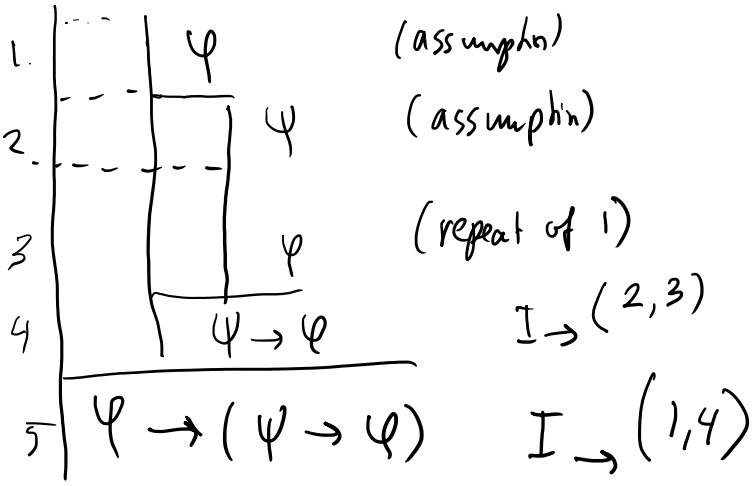
Deduce $\varphi \vee \psi$ from $\psi \vee \varphi$





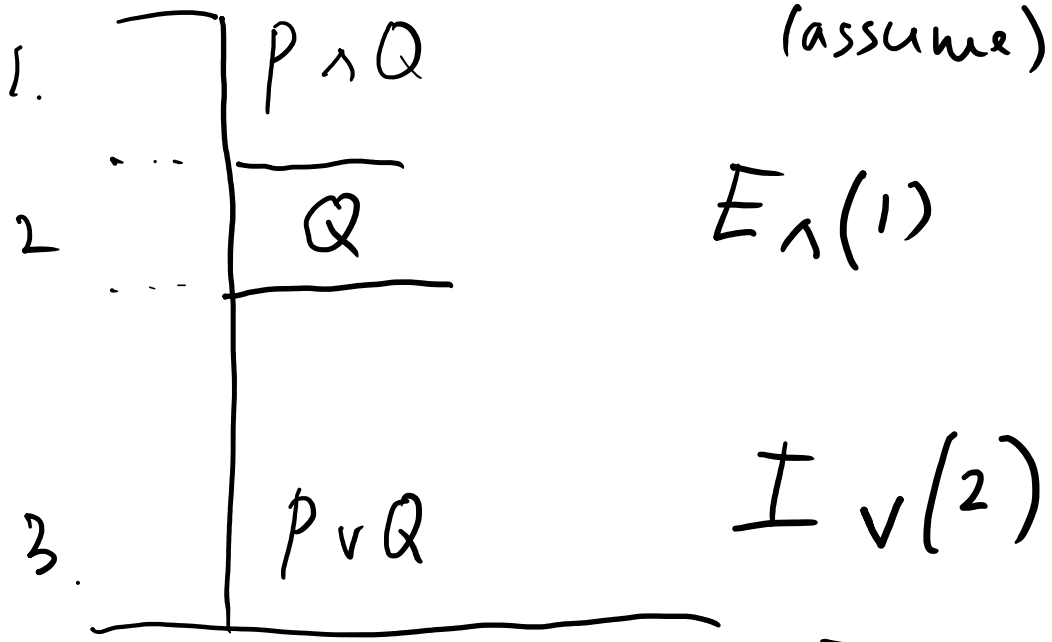
(Intermezzo:

$\neg\varphi$ = "$\varphi \to \bot$"

$\bot$ = "$p \wedge \neg p$"

but please forget that ;-))

Prove $(P \wedge Q) \rightarrow (P \vee Q)$



4. $(P \wedge Q) \rightarrow (P \vee Q)$ $I\rightarrow(1,3)$

Prove $\neg(\neg P \wedge \neg Q)$ from $(P \vee Q)$

1.	$P \vee Q$		(given)
2.	$\neg P \wedge \neg Q$		(assumption)
3.		$\neg P$	$E_\wedge(2)$
4.		$\neg Q$	$E_\wedge(2)$
5.		P	(assumption)
6.		$\bot$	$E_\neg(5, 3)$
7.		Q	(assumption)
8.		$\bot$	$E_\neg(7, 4)$
9.	$\bot$		$E_\vee(1, 5, 6, 7, 8)$
10.	$\neg(\neg P \wedge \neg Q)$		$I_\neg(2, 9)$

Forgotten rule:

$$\frac{\bot}{\varphi}$$ "$\bot$"

Give proofs using natural deduction:

(1) deduce $(R \to P) \to (R \to Q)$ from $P \to Q$

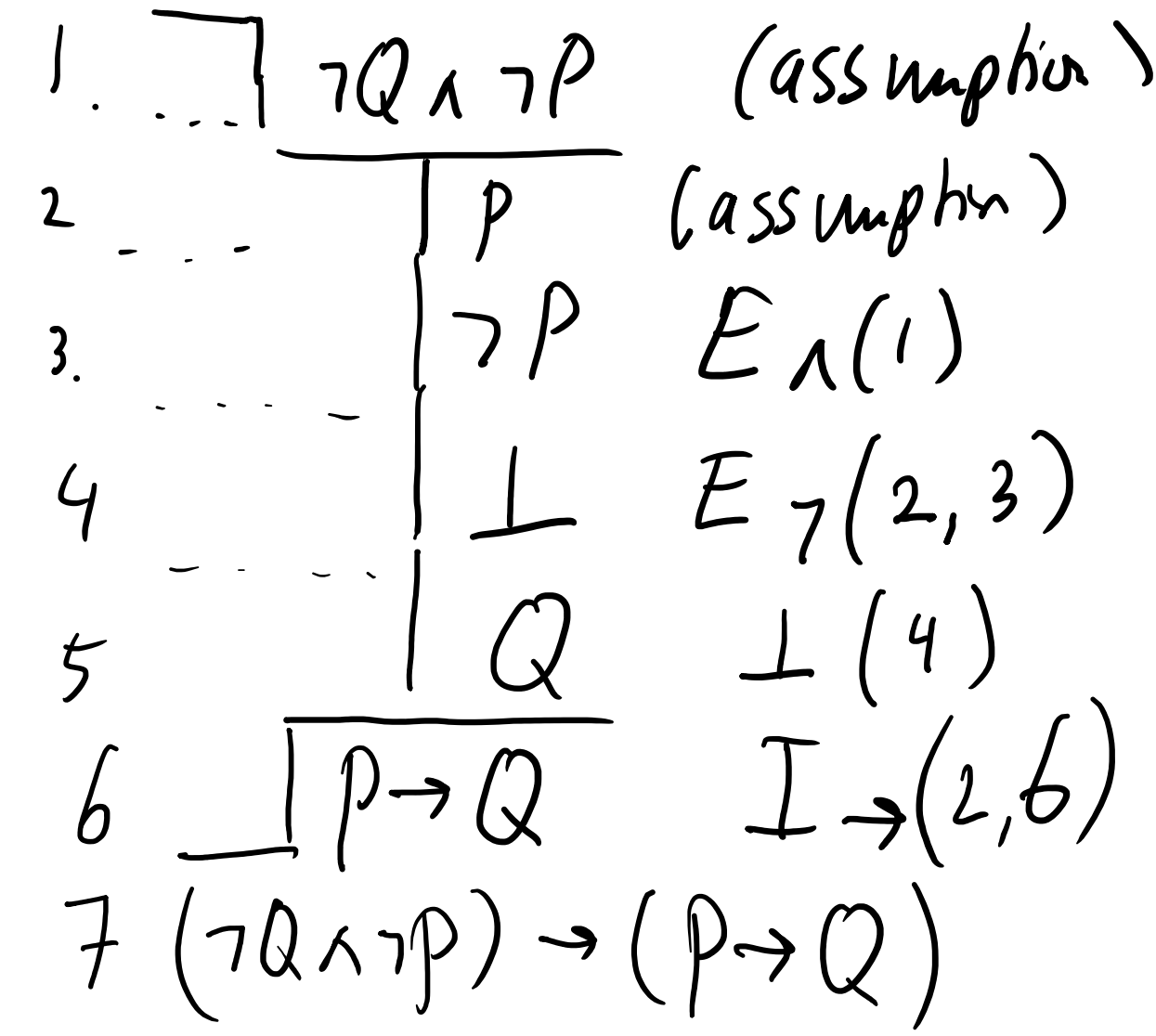
(2) $((P \vee Q) \wedge R) \to ((P \vee R) \wedge (Q \vee R))$

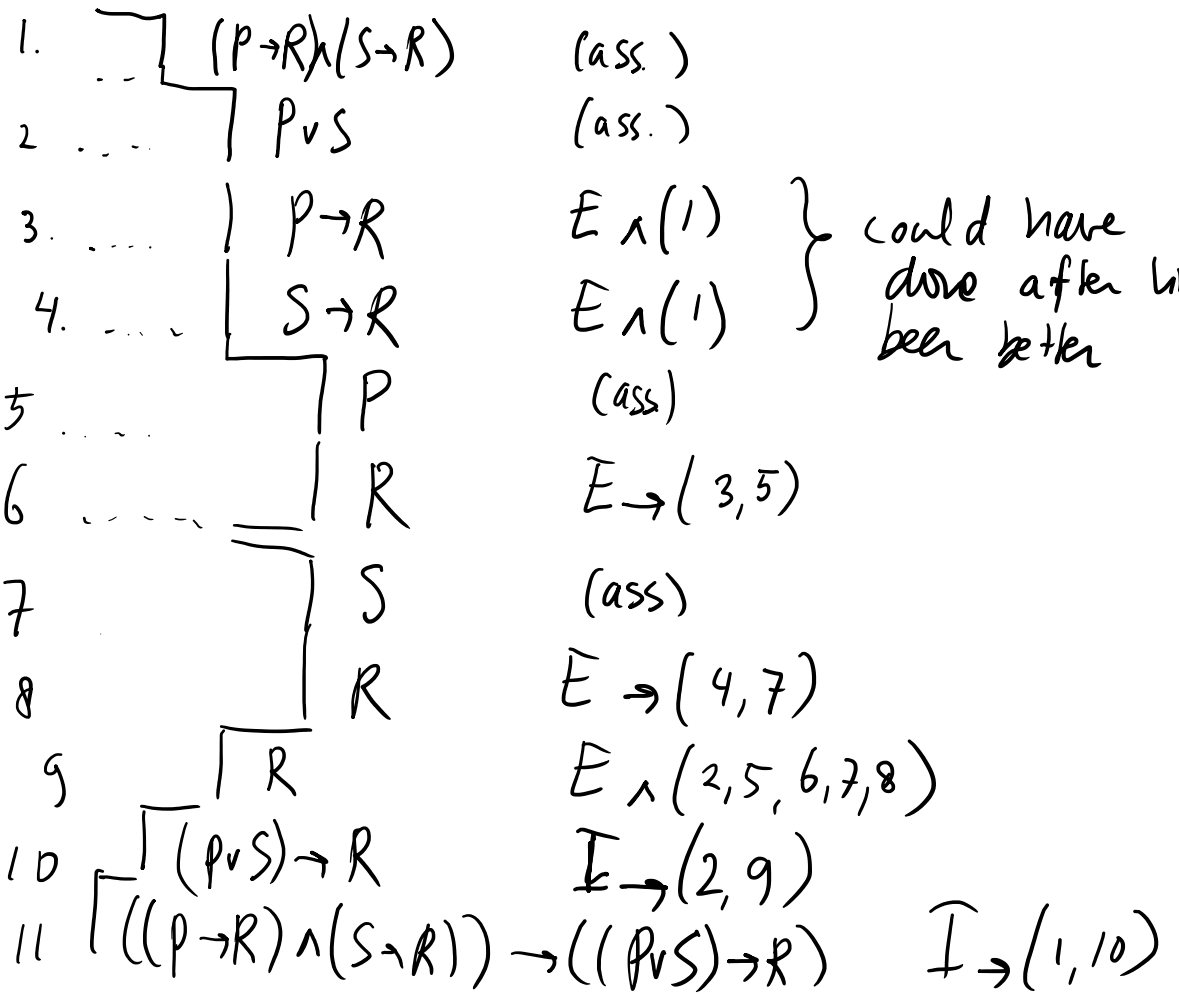
(3) $P \vee \neg P$ (Hard.)

(4) $((P \to R) \wedge (S \to R)) \to ((P \vee S) \to R)$

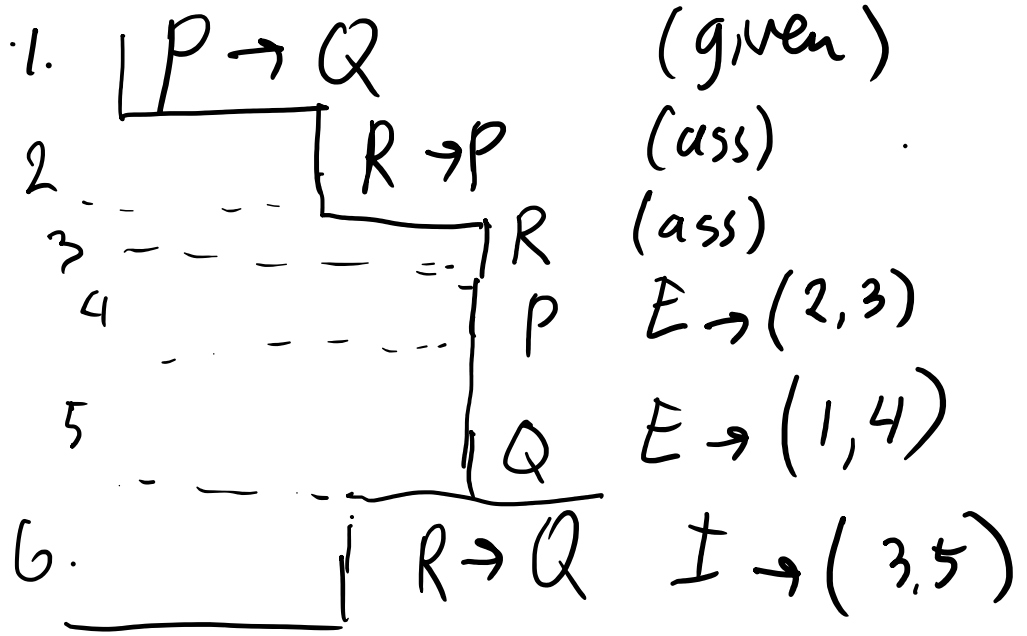
(5) $(\neg Q \wedge \neg P) \to (P \to Q)$

(3):

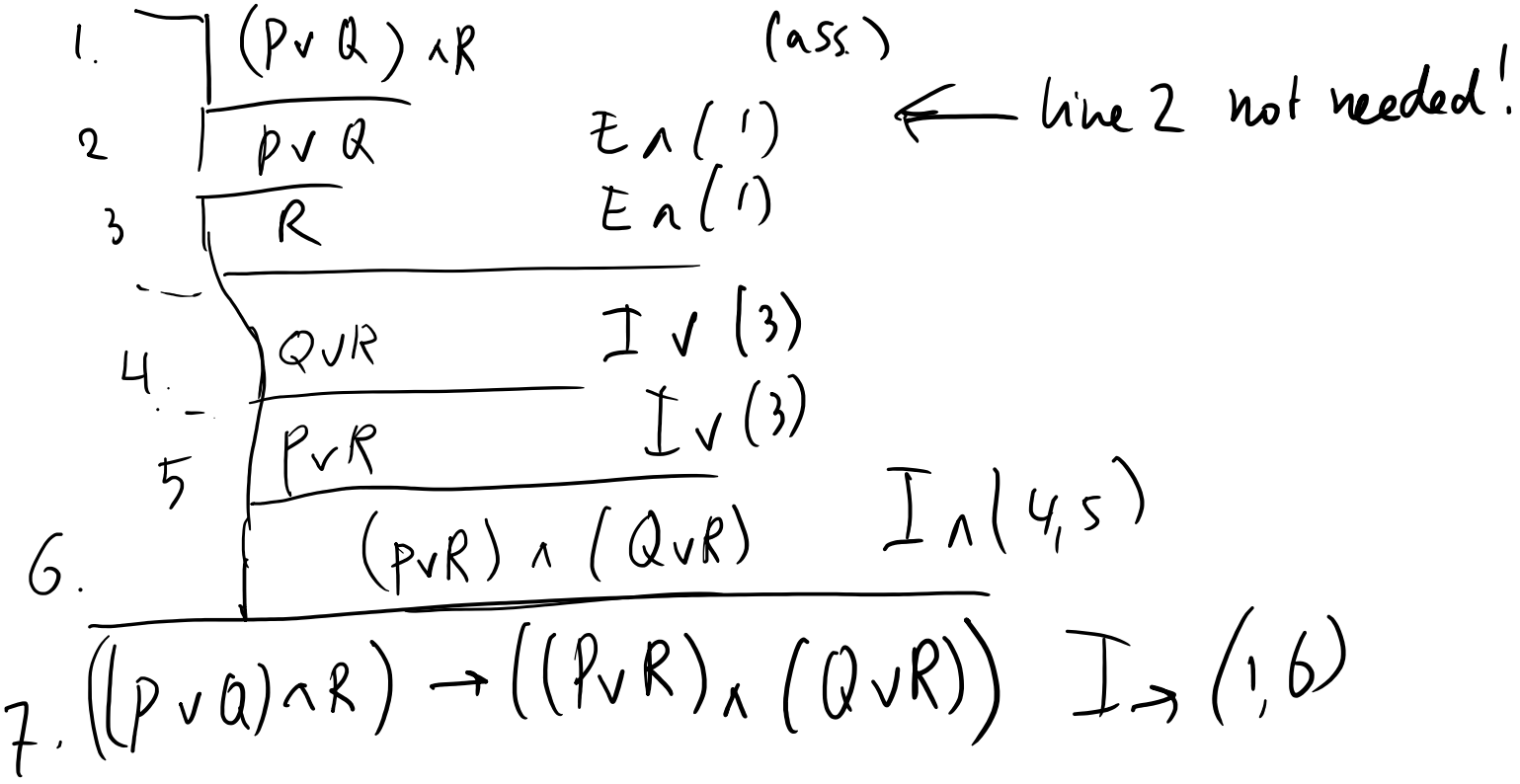



1. (P→R)∧(S→R) (ass.)
2. P∨S (ass.)
3. P→R E∧(1)
4. S→R E∧(1)
5 P (ass)
6 R E→(3,5)
7 S (ass)
8 R E→(4,7)
9 R E∨(2,5,6,7,8)
10 (P∨S)→R I→(2,9)
11 ((P→R)∧(S→R))→((P∨S)→R) I→(1,10)
could have done after line 1, would've been better

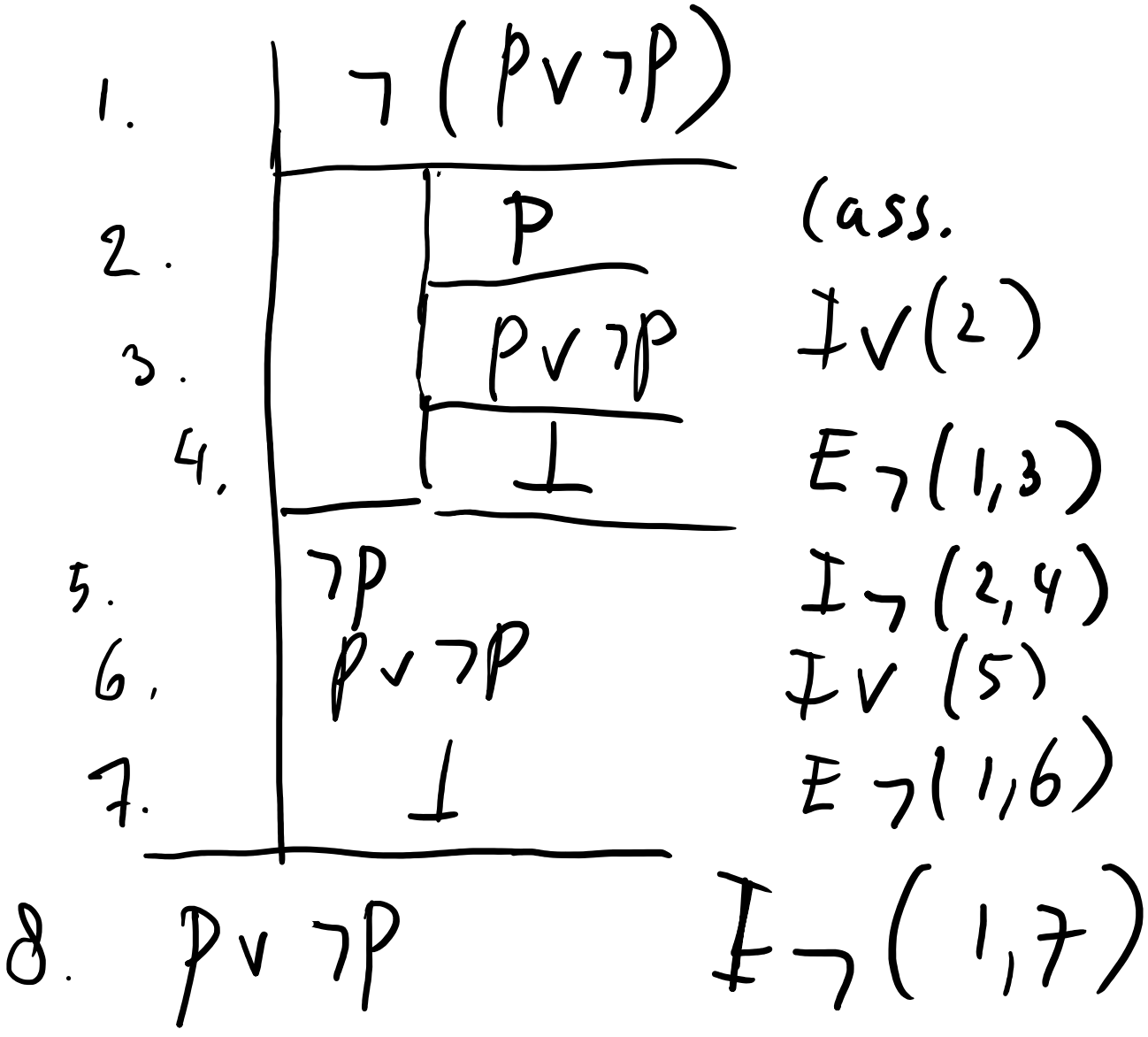
① $(P \to Q)$ implies $((R \to P) \to (R \to Q))$



7. $(R \to P) \to (R \to Q)$ I → (2,6)



← line 2 not needed!


1. ¬(P∨¬P)
2. P (ass.
3. P∨¬P ∨I(2)
4. ⊥ ¬E(1,3)
5. ¬P ¬I(2,4)
6. P∨¬P ∨I(5)
7. ⊥ ¬E(1,6)
8. P∨¬P ¬I(1,7)