The four rules in predicate logic proofs

Easy are:

$E_\forall$

$$\frac{\forall x\, \varphi(x)}{\varphi(c)}\ E_\forall$$

example:

$$\frac{\forall x\,(Px \to Qxx)}{Pc \to Qcc}\ E_\forall$$

and

$I_\exists$

$$\frac{\varphi(c)}{\exists x\, \varphi(x)}\ I_\exists$$

example:

$$\frac{Pc \to Qcc}{\exists x\,(Px \to Qxx)}\ I_\exists$$

There is no indent when introducing the constants. The following two rules do, however

$I_\forall$

	c	(universal constant)
	$\vdots$	
	$\varphi(c)$	
$\forall x\, \varphi(x)$		$I_\forall$

example:

1. $\forall x(Px \to Qx)$		(given ass.)
2. $\forall x\, Px$		(given ass.)
3	$Pc \to Qc$, c univ. constant	$E_\forall(1)$
4.	Pc	$E_\forall(2)$
5	Qc	$E_\to(3,4)$
6. $\forall x\, Qx$		$I_\forall(3,5)$

(note: line 3 could have been split into

3		c univ. constant
4	$Pc \to Qc$,	$E_\forall(1)$

)

$\boxed{E_\exists}$ $\exists x\, \varphi(x)$

$\varphi(c)$	c exist. constant
$\vdots$	
ψ (no c in ψ)	

ψ $\qquad E_\exists$

example

1.	$\forall x (P_x \to Q_x)$	(given ass.)
2.	$\exists x\, P_x$	(given ass.)
3	$\quad P_c$	c exist. constant (2)
4.	$\quad P_c \to Q_c$	$E_\forall (1)$
5	$\quad Q_c$	$E_\to (3,4)$
6	$\quad \exists y\, Q_y$	$I_\exists (5)$
7.	$\exists y\, Q_y$	$E_\exists$

(so this proves

$$\forall x (P_x \to Q_x),\ \exists x\, P_x \models \exists y\, Q_y$$)

Perhaps to remember:

$$\frac{\forall x\, \varphi(x)}{\varphi(c)}\ E_\forall$$

(no indent) if $\forall$ is before/above)

$\quad$ c

$\quad \vdots$

$\quad \varphi(c)$

$\forall x \varphi(x)$ $\qquad I_\forall$

(indent happens if $\forall x$ is below)

$$\frac{\varphi(c)}{\exists x \varphi(x)}\ I_\exists$$

(no indent if $\exists$ is below)

$\exists x\, \varphi(x)$

$\quad \varphi(c)$

$\quad \vdots$

$\quad \psi$

ψ

(indent if $\exists$ is before)