$\forall x\, \varphi(x) \models \exists x\, \varphi(x)$

1. $\forall x\, \varphi(x)$

$\exists x\, \varphi(x)$

$\forall x (P_x \to Q_x), \exists x P_x \models \exists_z Q_z$

1.	$\forall x (P_x \to Q_x)$	(given)
2.	$\exists x P_x$	(given)
3	P_c	(c, exist. constant (2))
4.	$P_c \to Q_c$	$E_\forall (1)$
5.	Q_c	$E_\to (3,4)$
6.	$\exists_z Q_z$	$I_\exists (5)$
7	$\exists_z Q_z$	$E_\exists (2,3,6)$

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$\forall x\, Px, \forall x\, Qx \vdash \forall x\,(Px \wedge Qx)$

1. $\forall x\, Px$ (given)
2. $\forall x\, Qx$ (given)
3. c, generic constant
4. Pc $E_\forall(1)$
5. Qc $E_\forall(2)$
6. $Pc \wedge Qc$ $I_\wedge(4,5)$
7. $\forall x\,(Px \wedge Qx)$ $I_\forall(3,6)$

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$$\exists y \forall x\, Rxy \models \forall x \exists y\, Rxy$$

1. $\exists y \forall x\, Rxy$
2. $\forall x\, Rxc$, (c exist. constant (1))
3. Rdc (d generic constant.
4. $\exists y\, Rdy$ $I_\exists(3;\ c=y)$
5. $\forall x \exists y\, Rxy$ $I_\forall(2,4)$
6. $\forall x \exists y\, Rxy$ $E_\exists(1,2,5)$

Reconvene at 17.03

① $\forall x \exists y Rxy \overset{?}{\models} \exists y \forall x Rxy$ (other way around)

② $\forall x \forall y Rxy \overset{?}{\models} \forall x Rxx$

③ $\exists x(Px \wedge Rx), \forall x(Px \to Qx) \overset{?}{\models} \exists x(Qx \wedge Rx)$

$\forall x \exists y R_{xy} \overset{?}{\vDash} \exists y \forall x R_{xy}$

1. $\forall x \exists y R_{xy}$ (given)
2. c , (universal constant)
3. $\exists y R_{cy}$ $E_{\forall}(1, x=c)$
4 R_{cd} $E_{\exists}(3)$, d exist. constant
5 $\forall x R_{xd}$ $I_{\forall}(2, 4)$
6 $\exists y \forall x R_{xy}$ $I_{\exists}(5)$
7 $\exists y \forall x R_{xy}$ $E_{\exists}(4, 6)$

This "proof" is WRONG!

Why? line 2
line 4
line 5 get rid of
line 7 get rid of
(Remember what I said with this example!)

$\forall x \forall y Rxy \vDash \forall x Rxx$

1. $\forall x \forall y Rxy$ (given)

2 | c (universal constant)

3 | $\forall y Rcy$ $E_\forall(1, x=c)$

4 | Rcc $E\forall(3, y=c)$

5. $\forall x Rxx$ $I_\forall(2,4)$

1. $\forall x(Px \to Qx)$ (given)

2. $\exists x(Px \wedge Rx)$ (given)

3. $Pc \wedge Rc$, $E_\exists(2, x=c)$, c exist. constant

4. $Pc \to Qc$ $E_\forall(1, x=c)$

5 Pc $E\wedge(3)$

6 Rc $E\wedge(3)$

7 Qc $E\to(4,5)$

8 $Qc \wedge Rc$ $I\wedge(6,7)$

9 $\exists x(Qx \wedge Rx)$ $I_\exists(8)$

10 $\exists x(Qx \wedge Rx)$ $E_\exists(2, 3, 9)$