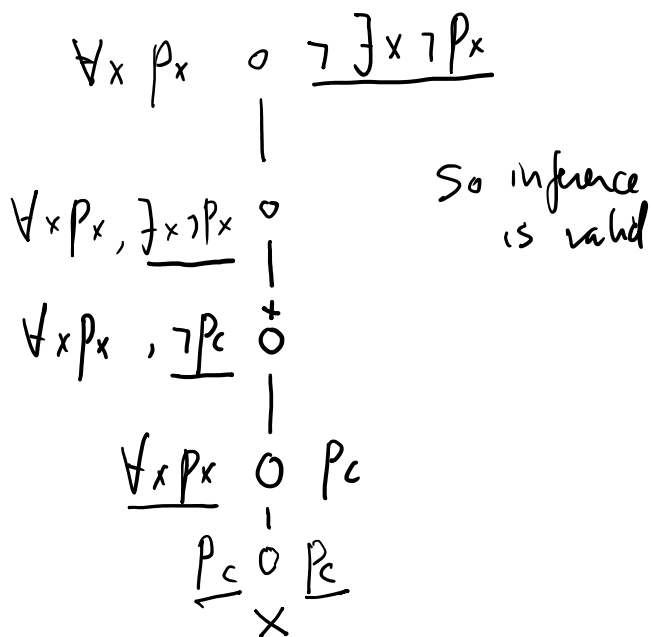


$$\forall x (P_x \rightarrow Q_x) \models \exists x (P_x \wedge \neg Q_x)$$



$$\forall x (P_x \rightarrow Q_x) \models \exists x (P_x \wedge \neg Q_x)$$

$$\underline{\forall x (P_x \rightarrow Q_x)} \circ \exists x (P_x \wedge \neg Q_x)$$

$$P_c \rightarrow Q_c \circ \exists x (P_x \wedge \neg Q_x)$$

$$P_c \rightarrow Q_c \circ P_c \wedge \neg Q_c$$

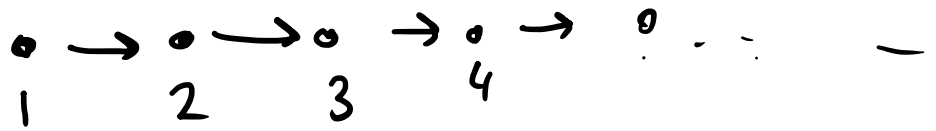
$$\circ P_c, P_c \wedge \neg Q_c \quad Q_c \circ P_c \wedge \neg Q_c$$

$$\begin{array}{l} \circ P_c, P_c \\ \hline \text{open} \end{array} \quad \circ P_c, \neg Q_c$$

first time, introduce constant;

already a constant, so no introducing new ones!

counter example:
 P_c does not hold
 (Q_c doesn't matter)
 $M = (D, \mathcal{I}, g) \quad D = \{c\}$



$$R \times (x+1)$$

$$\forall y \exists x Rxy \models \exists x \forall y Rxy$$

$$\forall y \exists x Rxy \circ \exists x \forall y Rxy \quad \text{universal step}$$

$$\exists x Rxc \circ \exists x \forall y Rxy \quad \text{existential step}$$

$$\forall y \exists x Rxy, Rdc \circ \exists x \forall y Rxy \quad \text{universal step}$$

$$" " \circ \forall y Rcy, \forall y Rdy \quad \text{existential}$$

$$\forall y \exists x Rxy, Rdc \circ Rce, \forall y Rdy, \exists x \forall y Rxy \quad \text{exist.}$$

$$\forall y \exists x Rxy, Rdc \circ Rce, Rdf, \exists x \forall y Rxy \quad \text{unus}$$

$$\exists x Rxc, \exists x Rxd, \exists x Rxe, \exists x Rxf, Rdc \circ Rce, Rdf, \exists x \forall y Rxy$$

you need extended rules to solve this.

$$\exists x \forall y Rxy \models \forall y \exists x Rxy$$

$$\begin{array}{c}
 \exists x \forall y R_{xy} \models \forall y \exists x R_{xy} \\
 \hline
 \exists x \forall y R_{xy} \quad \circ \quad \forall y \exists x R_{xy} \quad) \text{exist.} \\
 \quad \quad \quad | \\
 \forall y R_{cy} \quad \overset{+}{\circ} \quad \forall y \exists x R_{xy} \quad) \text{exist.} \\
 \quad \quad \quad | \\
 \forall y R_{cy} \quad \overset{+}{\circ} \quad \exists x R_{xd} \quad) \text{univ} \\
 \quad \quad \quad | \\
 R_{cc}, R_{cd} \quad \circ \quad \exists x R_{xd} \quad) \text{univ} \\
 \quad \quad \quad | \\
 R_{cc}, \underline{R_{cd}} \quad \circ \quad \underline{R_{cd}}, R_{dd} \\
 \quad \quad \quad \times
 \end{array}$$

So inference is valid.

$$\underline{\exists x \forall y (P_x \rightarrow Q_y)} \quad \circ \quad \forall x \exists y (P_y \rightarrow Q_x) \quad) \text{exist.}$$

$$\forall y (P_c \rightarrow Q_y) \quad \circ \quad \underline{\quad \quad \quad} \quad) \text{exist}$$

$$\underline{\forall y (P_c \rightarrow Q_y)} \quad \circ \quad \exists y (P_y \rightarrow Q_d) \quad) \text{unc}$$

$$P_c \rightarrow Q_c, P_c \rightarrow Q_d \quad \circ \quad \underline{\quad \quad \quad} \quad) \text{unc}$$

$$\quad \quad , \quad \underline{P_c \rightarrow Q_d} \quad \circ \quad \underline{P_c \rightarrow Q_d, P_d \rightarrow Q_d}$$

X

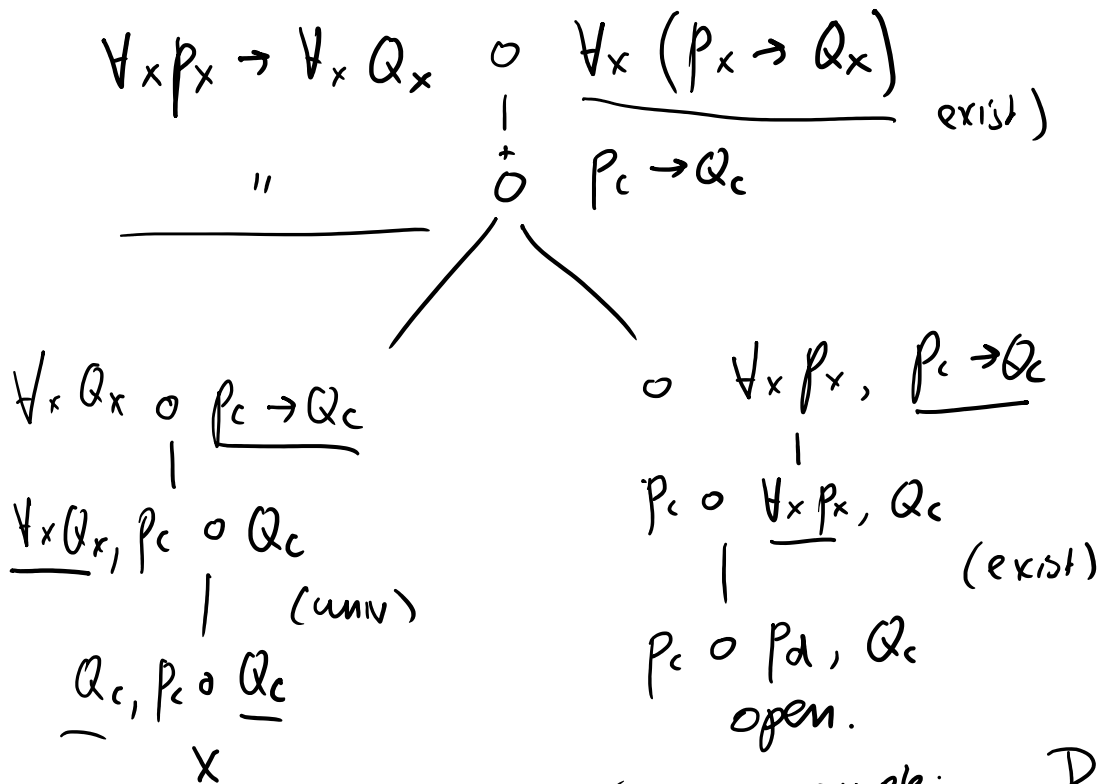
$$\begin{array}{c} \forall x (P_x \rightarrow \exists y Q_y) \quad \circ \quad \exists x Q_x \vee \neg \exists x P_x \\ | \\ " \quad \circ \quad \exists x Q_x, \quad \neg \exists x P_x \end{array}$$

$$\begin{array}{c} \forall x (P_x \rightarrow \exists y Q_y), \exists x P_x \quad \circ \quad \exists x Q_x \quad \text{(exist.)} \\ | \\ " \quad , \quad P_c \quad \circ \quad \text{"} \end{array}$$

$$\begin{array}{c} " \quad , \quad P_c \quad \circ \quad \text{"} \\ | \\ " \quad , \quad P_c \quad \circ \quad Q_c \quad \text{univ.} \\ | \\ P_c \rightarrow \exists y Q_y, P_c \quad \circ \quad Q_c \quad \text{univ.} \end{array}$$

So:
valid

$$\begin{array}{c} P_c \rightarrow \exists y Q_y, P_c \quad \circ \quad Q_c \\ | \\ P_c \circ P_c, Q_c \quad \text{X} \quad \forall x (P_x \rightarrow \exists y Q_y), P_c, Q_d \quad \circ \quad Q_c \quad \text{(exist.)} \\ | \\ " \quad , P_c, Q_d \quad \circ \quad Q_c, \exists x Q_x \\ | \\ " \quad , P_c, Q_d \quad \circ \quad Q_c, Q_d \\ \text{X} \end{array}$$



Thus:
Not
Valid



Counterexample:

$$\begin{aligned}
 D &= \{c, d\} \\
 P &= \{c\} \\
 Q &= \emptyset
 \end{aligned}$$