Exercises

1	2	3	4	5	6	7	8	9

Surname, First name

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1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	3	3	3	3	3	3
4	4	4	4	4	4	4
5	5	5	5	5	5	5
6	6	6	6	6	6	6
7	7	7	7	7	7	7
8	8	8	8	8	8	8
9	9	9	9	9	9	9
0	0	0	0	0	0	0

KEN1130 Discrete Mathematics
Resit

a b c d e f → b
a b c d e f → c
a b c d e f → a

Fill in your answer(s) to the multiple-choice questions as shown above (circles = one correct answer).

Program: Data Science and Artificial Intelligence
Course code: KEN1130
Examiners: dr. Marieke Musegaas and dr. Stefan Maubach
Date/time: Wednesday 01.02.2023 9h00-11h00
Format: Closed book exam
Allowed aids: Pens, simple (non-programmable) calculator from the DACS-list of allowed calculators.
Instructions to students:

- The exam consists of 8 questions on 18 pages.
- Fill in your name and student ID number on the cover page and tick the corresponding numerals of your student number in the table (top right cover page).
- Answer every question in the reserved space below the question. **Do not write outside the reserved space or on the back of pages, this will not be scanned and will NOT be graded!** As a last resort if you run out of space, use the extra answer space at the end of the exam.
- *In no circumstance write on or near the QR code at the bottom of the page!*
- Ensure that you properly motivate your answers.
- Only use black or dark blue pens, and write in a readable way. Do not use pencils.
- Answers that cannot be read easily cannot be graded and may therefore lower your grade.
- If you think a question is ambiguous, or even erroneous, and you cannot ask during the exam to clarify this, explain this in detail in the space reserved for the answer to the question.
- If you have not registered for the exam, your answers will not be graded, and thus handled as invalid.
- You are not allowed to have a communication device within your reach, nor to wear or use a watch.
- You have to return all pages of the exam. You are not allowed to take any sheets, even blank, home.
- Good luck!

Question 1
Consider the following logical proposition.

$$((p \Rightarrow q) \wedge \neg r) \Leftrightarrow ((p \vee r) \wedge (q \Rightarrow r))$$

1.25p **1a** Suppose p is TRUE, q is TRUE and r is TRUE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1b** Suppose p is TRUE, q is TRUE and r is FALSE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1c** Suppose p is TRUE, q is FALSE and r is TRUE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1d** Suppose p is TRUE, q is FALSE and r is FALSE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1e** Suppose p is FALSE, q is TRUE and r is TRUE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1f** Suppose p is FALSE, q is TRUE and r is FALSE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1g** Suppose p is FALSE, q is FALSE and r is TRUE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

1.25p **1h** Suppose p is FALSE, q is FALSE and r is FALSE. Is the above logical proposition TRUE or FALSE?

(a) TRUE (b) FALSE

Question 2

7p **2a** Let P(n) be the statement “$1 + 3 + 5 + ... + (2n - 1) = n^2 + 10$". Which one of the answers is correct?

- (a) P(1) is true.
- (b) P(n) $\Rightarrow$ P($n + 1$) is true for all $n \in \mathbb{N}$.
- (c) P(n) is true for all $n \in \mathbb{N}$.
- (d) None of the above.

7p **2b** For all $n \in N$, $\left(1 + \frac{3}{1}\right) \cdot \left(1 + \frac{5}{4}\right) \cdot \left(1 + \frac{7}{9}\right) \cdot ... \cdot \left(1 + \left(\frac{2n+1}{n^2}\right)\right)$ is equal to

- (a) $\frac{(n+1)^2}{2}$
- (b) $\frac{(n+1)^3}{3}$
- (c) $(n + 1)^2$
- (d) None of the above.

Question 3

3p **3a** Evaluate the following statement and corresponding proof.

Theorem: $(A \cap B)^c \subseteq A^c \cap B^c$

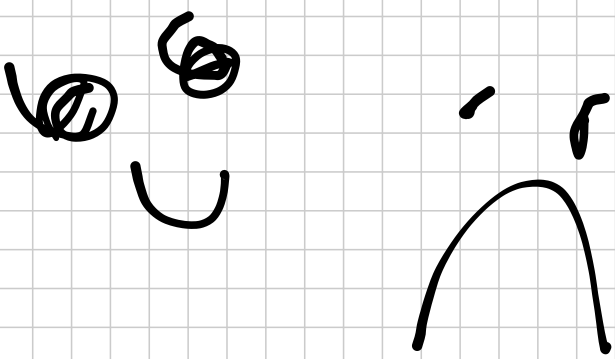
Proof: Let $x \in (A \cap B)^c$. Hence, $x \notin (A \cap B)$. So, $x \notin A$ ~~and~~ OR $x \notin B$. Therefore, $x \in A^c$ and $x \in B^c$. Hence, $x \in A^c \cap B^c$. □

- (a) The theorem and proof are both correct.
- (b) The theorem is correct, but the proof is incorrect.
- (c) The proof is correct, but the theorem is incorrect.
- (d) The theorem and the proof are both incorrect.

6p **3b** Prove or disprove the following statement. For all sets A, B, C and D,

$$\text{if } A \times B \subseteq C \times D, \text{ then } A \subseteq C \text{ and } B \subseteq D.$$

Va Jancub



6p **3c** Prove or disprove the following statement. For all sets A, B, C and D,

$$\text{if } A \subseteq C \text{ and } B \subseteq D \text{, then } A \times B \subseteq C \times D.$$

$A \subseteq C \wedge B \subseteq D \Rightarrow A \times B \subseteq C \times D$

~~Assume $A \subseteq C \wedge B \subseteq D$~~

let A, B, C, D be some arbitrary sets

Case 1: $A = \emptyset$ or $B = \emptyset$

then $A \times B = \emptyset$

and thus $A \times B \ (\emptyset) \subseteq C \times D$

Case 2: $A \neq \emptyset$ and $B \neq 0$

So $A \times B \neq \emptyset$

Assume $A \subseteq C \wedge B \subseteq D$

Let $(a, b) \in A \times B$

then $a \in A$, $b \in B$

since $A \subseteq C \Rightarrow a \in C$

since $B \subseteq D \Rightarrow b \in D$

so $(a, b) \in B \times D$ $\square$

Question 4

3p **4a** Let $A = \{1, 2, 3\}$. Which of the following relations on A is symmetric but neither reflexive nor transitive.

- (a) $R = \{(1, 2), (1, 3)\}$
- (b) $R = \{(1, 2), (2, 1)\}$
- (c) $R = \{(1, 1), (2, 2), (3, 3)\}$
- (d) $R = \{(1, 1), (1, 2), (2, 3)\}$
- (e) None of the above.

12p **4b** Let R be the relation on $\mathbb{N}$ defined as follows: xRy means "$3x - 5y$ is even". Prove or disprove that R is an equivalence relation.

xRy if $3x - 5y$ is even

Equivalence relation ?

- Reflexive ?

$\forall x \in \mathbb{N} : xRx$

let $x \in \mathbb{N}$: $3x - 5x = -2x$ (even)

so $x R x$

- Symmetric

$(\forall x \in \mathbb{N})(\forall y \in b) : xRy \rightarrow yRx$

Let $x, y \in \mathbb{N}$

Assume xRy, so $3x - 5x$ is even

$\Rightarrow 3x - 5y = 2k$ for $k \in \mathbb{Z}$

↳

Question 5

5p **5a** How many different length 12 strings can you make using exactly 8 “0" symbols and exactly 4 “1" symbols? (Two strings are considered equal if and only if they are identical in every position.)

- (a) 455
- (b) 495
- (c) 1365
- (d) 11880
- (e) 20736
- (f) 50388
- (g) 75582
- (h) 16777216
- (i) 19958400
- (j) 429981696
- (k) 68719476736
- (l) None of the above.

N= 8 × "0"
4 × "1"

No repetition
order is not imp.

n = 12 spots ← We assign 12 seats to 8 zeros

0 0 0 0 0 0 0 1 1 1 1 1
0 0 0 0 0 0 1 1 1 1 1 1

$\binom{12}{8}$

5p **5b** The national football association is selecting players for the national team. The national team plays with 3 attackers: left wing, centre forward, right wing. There are 8 players available in the country who can play as attacker. How many different attacking formations are possible? (Note that the same subset of 3 players can play in multiple different formations.)

- (a) 45
- (b) 56
- (c) 120
- (d) 336
- (e) 512
- (f) 6561
- (g) None of the above.

LW CF RW K = 3

n = 8

Orders matters
Repitition is not allowed

$= \frac{8!}{(5!)} = 8 \cdot 7 \cdot 6 = 336$

5p **5c** We have 3 colours of paint available, red, blue and green. There is an unlimited amount of each available, but we are only allowed to buy whole litres of paint. We want to mix these paints to obtain a total volume of 8 litres. The colour of the paint we obtain is uniquely determined by the amount of red, blue and green in it. (So, “Three litres red, two litres blue, three litres green" will generate one colour, and “Two litres red, five litres blue, one litre green" will generate another colour.) How many different colours can we obtain?

- a 45
- b 56
- c 120
- d 336
- e 512
- f 6561
- g None of the above.

3 colors, unlim. amount

8 liters

_ _ _ _ _ _ _ _ 8 slots

$K = 8$
$n = 3$

Order doesn't matter
Repetition is allowed

$$C_{n,k} = \binom{n+k-1}{8}$$

$$= \binom{3+7}{8} = \binom{10}{8}$$

$$= \frac{n!}{k!(n-k)!} = \frac{10!}{8!(2!)} = \frac{10 \cdot 9}{2}$$

$=$

Question 6

3p **6a** The negation of the statement

For every $x \in \mathbb{Z}$ there exists a $y \in \mathbb{Z}$ such that $y^2 > x$.

$(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z}) : (y^2 > x)$

is

- (a) For every $x \in \mathbb{Z}$ there does not exist a $y \in \mathbb{Z}$ such that $y^2 > x$.
- (b) For every $x \in \mathbb{Z}$ there does not exist a $y \in \mathbb{Z}$ such that $y^2 \le x$.
- (c) There exists an $x \in \mathbb{Z}$ such that for all $y \in \mathbb{Z}$, $y^2 > x$.
- (d) There exists an $x \in \mathbb{Z}$ such that for all $y \in \mathbb{Z}$, $y^2 \le x$.
- (e) There exists a $y \in \mathbb{Z}$ such that for every $x \in \mathbb{Z}$, $y^2 > x$.
- (f) For all $y \in \mathbb{Z}$, there exists an $x \in \mathbb{Z}$ such that $y^2 \le x$.
- (g) None of the above.

$(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z}) : (y^2 \le x)$

3p **6b** The following statement is false.

$$\neg((\exists x \in \mathbb{N})(\exists y \in \mathbb{N})(xy \le 1))$$

Provide a counterexample.

$\equiv (\forall x \in \mathbb{N})(\forall y \in \mathbb{N})(xy > 1)$

$x = 1$

$y = 1$

so $x \cdot y = 1 \cdot 1 = 1$, which is not greater than 1

6p **6c** Prove or disprove the following statement.

$$(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})\left(\frac{x}{y} \notin \mathbb{N}\right)$$

Let $x \in \mathbb{N}$

$\frac{x}{x} = 1 \rightarrow \frac{x}{x+1} < 1$

choose $y = x+1 \in \mathbb{N}$

$x < x+1 \Rightarrow x < y$

$\Rightarrow \frac{x}{y} < 1$

hence $\frac{x}{y} \in \mathbb{N}$

Question 7

Let $f : \mathbb{N} \to \mathbb{Z}$ be the function defined as follows:

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ -\left(\frac{x-1}{2}\right) & \text{if } x \text{ is odd} \end{cases}$$

3p **7a** Is $f \circ f$ a well-defined function? Explain why or why not.

$f(1) = -\left(\frac{1-1}{2}\right) = 0 \notin \mathbb{N}$

Domain of f = codomain of g
Co-domain of f = domain of g
Range (g) ⊆ domain f

Domain f∘g -> domain of g
Co-domain of f∘g -> co-domain f

5p **7b** Prove or disprove that f is injective.

$f : \mathbb{N} \to \mathbb{Z}$

$$f(x) = \begin{cases} \frac{x}{2} & \text{if even} \\ -\left(\frac{x-1}{2}\right) & \text{if odd} \end{cases}$$

$f : A \to B$ is injective

$\iff (\forall x \in A)(\forall y \in B)(f(x) = f(y) \Rightarrow x = y)$

if x and y are even

$\frac{x}{2} = \frac{y}{2}$

$\Rightarrow x = y$ ✓

if both are odd

$-\left(\frac{x-1}{2}\right) = -\left(\frac{y-1}{2}\right)$

$x-1 = y-1$

$x = y$ ✓

if x even and y odd

$\frac{x}{2} = -\left(\frac{y-1}{2}\right)$

$x = -y+1$

so $f(x) \neq f(y) \Rightarrow x = y$ ✓

Let $g : \mathbb{N} \to \mathbb{Z}$ be the function defined as follows:

$$g(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ -\left(\frac{x+1}{2}\right) & \text{if } x \text{ is odd} \end{cases}$$

5p **7c** Prove or disprove that g is surjective.

$f: A \to B$: surj

$\iff (\forall y \in B)(\exists x \in A)(y = f(x))$

$g: N \to Z$ is surj.

$\iff (\forall y \in Z)(\exists x \in \mathbb{N})(y = f(x))$

Let $x \in \mathbb{N}$ and is even ✗ even

then $y = f(x) = \frac{x}{2} \geq \frac{2}{2} = 1$ ✓

Let $x \in N$ and is odd ✗ odd

then $y = f(x) = -\left(\frac{x+1}{2}\right) \leq -\left(\frac{2}{2}\right) = -1$

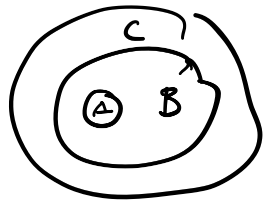
hence there is no $x \in \mathbb{N}$ such that $g(x) = 0$, so $f(x)$ is not surjective

Question 8

2p **8a** Let $A = \{a, b, c, \{a, b\}\}$. Which one of the following statements is **false**?

- (a) $\{a, b\} \in A$ ✓
- (b) $a \in A$ ✓
- (c) $\{a\} \in A$ ✗
- (d) $\{b, c\} \subseteq A$ ✓
- (e) None of the above.

2p **8b** If $A \subseteq B$ and $B \subseteq C$, then $|A \cup B \cup C|$ equals



- (a) $|A|$
- (b) $|B|$
- (c) $|C|$
- (d) None of the above.

2p **8c** Which one of the following statements is **false**?

- (a) $2 \in A \cup B$ implies that if $2 \notin A$ then $2 \in B$. ✓
- (b) If $\{2, 3\} \subseteq A$, then $2 \in A$ and $3 \in A$. ✓
- (c) If $\{2, 3\} \subseteq A \cap B$, then $\{2, 3\} \subseteq A$ and $\{2, 3\} \subseteq B$. ✓
- (d) If $\{3\} \subseteq A \backslash B$ and $\{2\} \subseteq B$, then $\{2, 3\} \subseteq A \cup B$. ✓
- (e) If $\{2\} \in A$ and $\{3\} \in A$, then $\{2, 3\} \subseteq A$. ✗
- (f) None of the above.

Extra space

If you use these extra answer boxes, **please mention clearly in your main answer that part of your answer can be found here!**

9a

9b

9c