Practice Exam Questions - Tutorial 5

1. There are 200 students in a class. There are 3 bands (A)ha, (B)auhaus and (C)hvrches. 5 students like all 3 bands. Every student likes at least one band. 179 students like Aha (and possibly other bands). 179 students like Bahaus (and possibly other bands). 17 students like Chvrches (and possibly other bands). The number of students who like *only* Aha (and no other bands) is equal to the number of students who like *only* Bahaus (and no other bands), and this in turn is equal to the number of students who like *only* Chvrches (and no other bands). How many students like *exactly two* bands?

2. These are questions about combinatorics. For all questions please give an exact number at the end (i.e. don't just leave your answer as a counting equation).

   (a) We sell shares, in blocks of 1000 euro, in four different risk categories: safe, low risk, medium risk, high risk. A customer wants to invest 10,000 euro, and wishes to know how many different investment portfolios are possible, where "4000 euro safe, 1000 euro low risk, 3000 euro medium risk, 2000 euro high risk" and "1000 euro safe, 5000 euro low risk, 3000 euro medium risk, 1000 euro high risk" are examples of investment portfolios. How many different investment portfolios are possible?

   (b) Let A and B be sets such that $|A| = 3$ and $|B| = 7$. How many injective functions are there from A to B?

   (c) How many length 10 binary strings can you make using "0" and "1" symbols, such that exactly 4 of the symbols are 0, and 6 of the symbols are 1? (Two strings are considered identical if and only if they are identical in every position).

   (d) Let A be a finite set of size n. How many different relations can be constructed on A? Note that the labels of the elements of A are important here. So, for example, if $A = \{0, 1\}$, then the relation $\{(0, 0), (0, 1)\}$ is considered different to the relation $\{(1, 1), (1, 0)\}$ (even though they have the "same shape").

3. These are questions about combinatorics. For all questions please give an exact number at the end (i.e. don't just leave your answer as a counting equation).

   (a) Let A be a set containing 9 elements. How many subsets of A contain exactly 3 elements?

   (b) In a shortened form of the game cricket a team bowls for 15 "overs". There are 4 types of over possible: *fast*, *seam*, *off-spin* or *leg-spin*. (Don't worry about the meaning of these terms). A cricket strategy consists of deciding how many overs of each type to bowl. Two examples of cricket strategies are "Four fast, eight seam, one off-spin, two leg-spin" and "One fast, one seam, seven off-spin and six leg-spin". How many different cricket strategies are possible?

   (c) How many different length 14 strings can you make where the first 4 positions consist of a permutation of the letters $\{a, b, c, d\}$ and in the remaining part of the string there are exactly 5 e letters and exactly 5 f letters? Note that two strings are considered identical if and only if they are identical in every position. So $abcdefefefefef$ is one example, $bcadefffffeeee$ would be another, and so on.

4. These are questions about combinatorics. For all questions please give an exact number at the end (i.e. don't just leave your answer as a counting equation).

   (a) We have 3 colours of paint available: red, blue and green. There is an unlimited amount of each available, but we are only allowed to buy whole litres of paint. We want to mix these paints to obtain a total volume of 9 litres. The colour of the paint we obtain is uniquely determined by the amount of red, blue and green in it. So *"three litres red, two litres blue, four litres green"* will generate one colour, and *"two litres red, six litres blue, one litre green"* will generate another colour. How many different colours can we obtain?

   (b) Let A be a set containing 8 elements. How many subsets of A contain *at least* 6 elements?

   (c) To create a password we are allowed to use the letters $\{a, b, c, d, e\}$ and the numbers $\{0, 1, 2\}$. As you would expect, order matters (so $ab1c2a$ is a different password to $12abca$). We are allowed to use letters and numbers more than once, if desired. A valid password has length 6, *starts with a letter*, and must include *at least one number*. How many valid passwords are there?

5. Let S be a set $\{1, 2, 3, \ldots, k\}$ where k is equal to the length of your surname. So for Steven (whose surname is Kelk), $S = \{1, 2, 3, 4\}$. How many functions are there from S to S that are *not* bijective?

6. All the following questions are about *counting*. For all questions please give an exact number at the end (i.e. don't just leave your answer as a counting equation).

   (a) How many different length 5 strings can you make using "0", "1" and "2" symbols, such that each string uses all three symbols (i.e. each string contains at least one "0", at least one "1" and at least one "2"). Note that two strings are considered identical if and only if they are identical in every position. *Hint:* inclusion-exclusion might help you here.

   (b) Let A and B be finite sets such that $|A| = |B| = 7$. We say that a function $f : A \to B$ is *imperfect* if it is not a bijection. How many imperfect functions are there from A to B?

   (c) We have 4 colours of paint available, *red*, *blue*, *green* and *white*. There is an unlimited amount of each available, but we are only allowed to buy whole litres of paint. We want to mix these paints to obtain a total volume of 8 litres. The colour of the paint we obtain is uniquely determined by the amount of red, blue, green and white paint used to make it. So *"Three litres red, two litres blue, two litres green, one litre white"* will generate one colour, and *"Two litres red, three litres blue, one litre green and two litres white"* will generate another colour. How many different colours can we obtain?

   (d) A sports club has, in total, 15 players available. A team, however, has exactly 11 players in it. How many different teams can be formed from the players available at the club?