



Fill in the truth table for the following logical proposition.

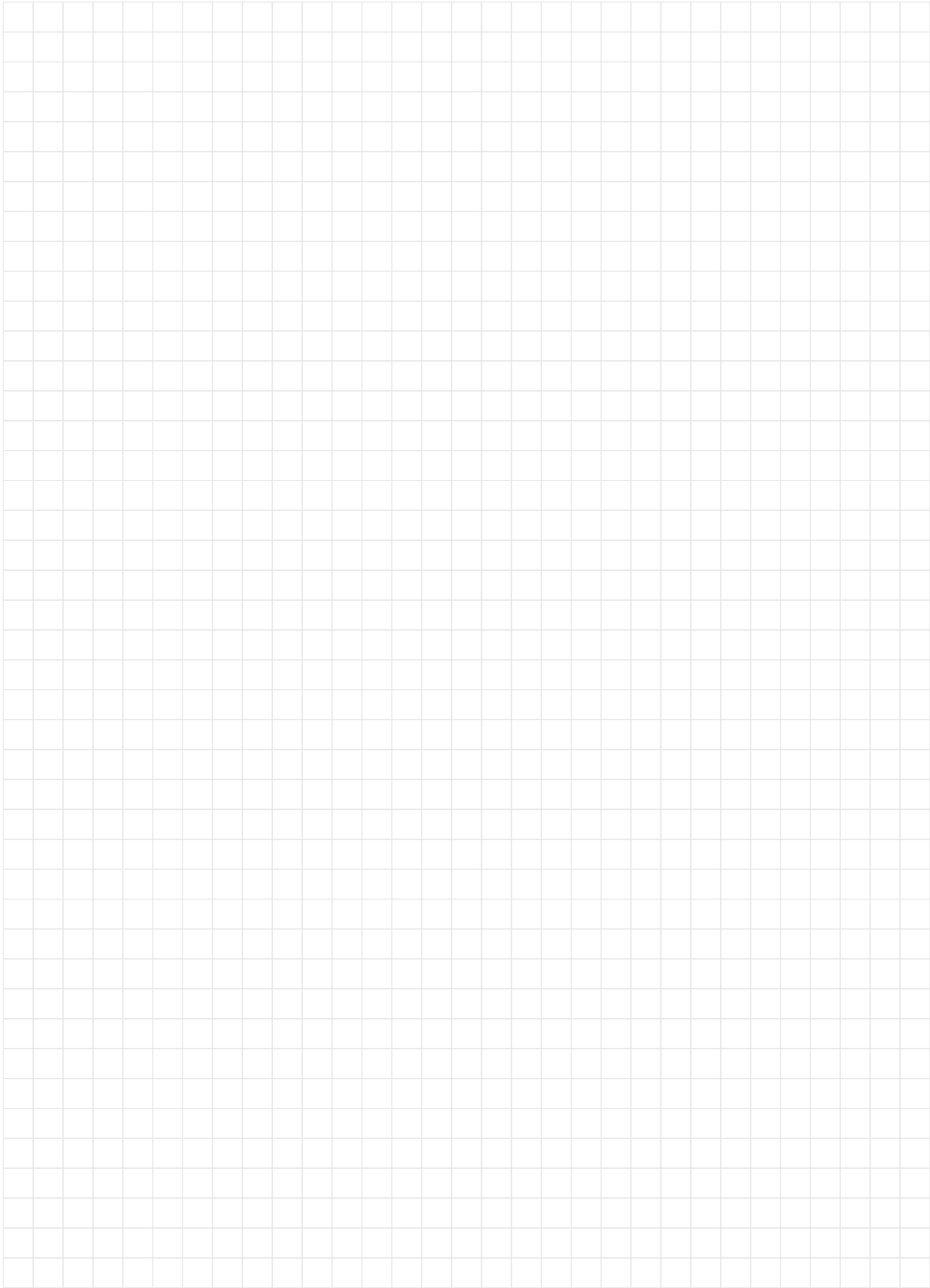


**Question 2** (15 points)

Use induction to prove the following statement.

- For all integers  $n \geq 1$ ,

$$\sum_{i=1}^n i^3 = \frac{1}{4}n^2(n+1)^2$$






**Question 3** (15 points)

Prove or disprove the following statements.

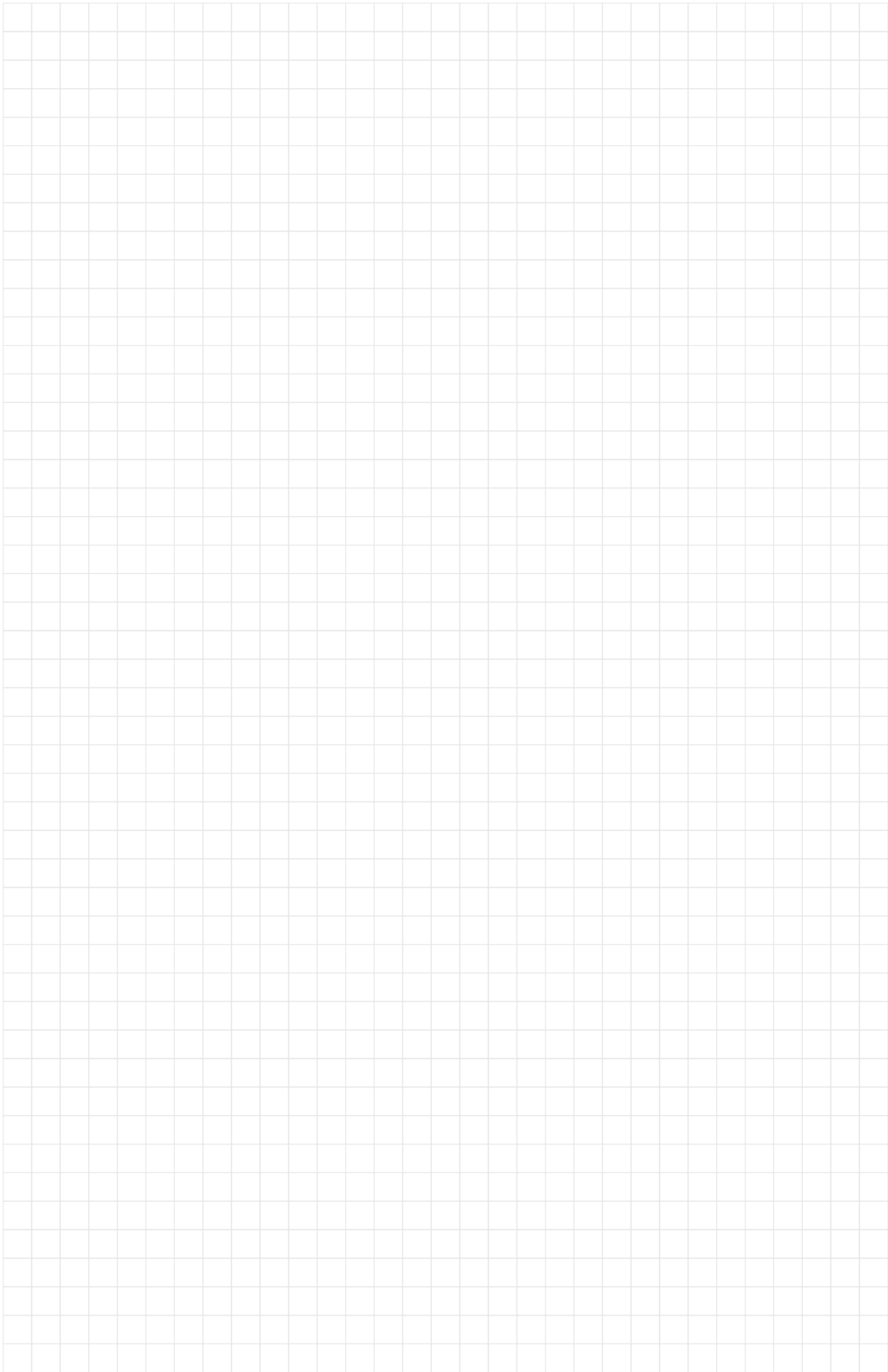
- (a) (7 points) For all sets  $A$ ,  $B$ , and  $C$ , if  $A \not\subseteq B$  and  $B \not\subseteq C$  then  $A \not\subseteq C$ .
- (b) (8 points) For all sets  $A$ ,  $B$  and  $C$ , if  $A \subseteq B$  then  $A \cap (B \cap C)^c = \emptyset$ .















**Question 6** (10 points)

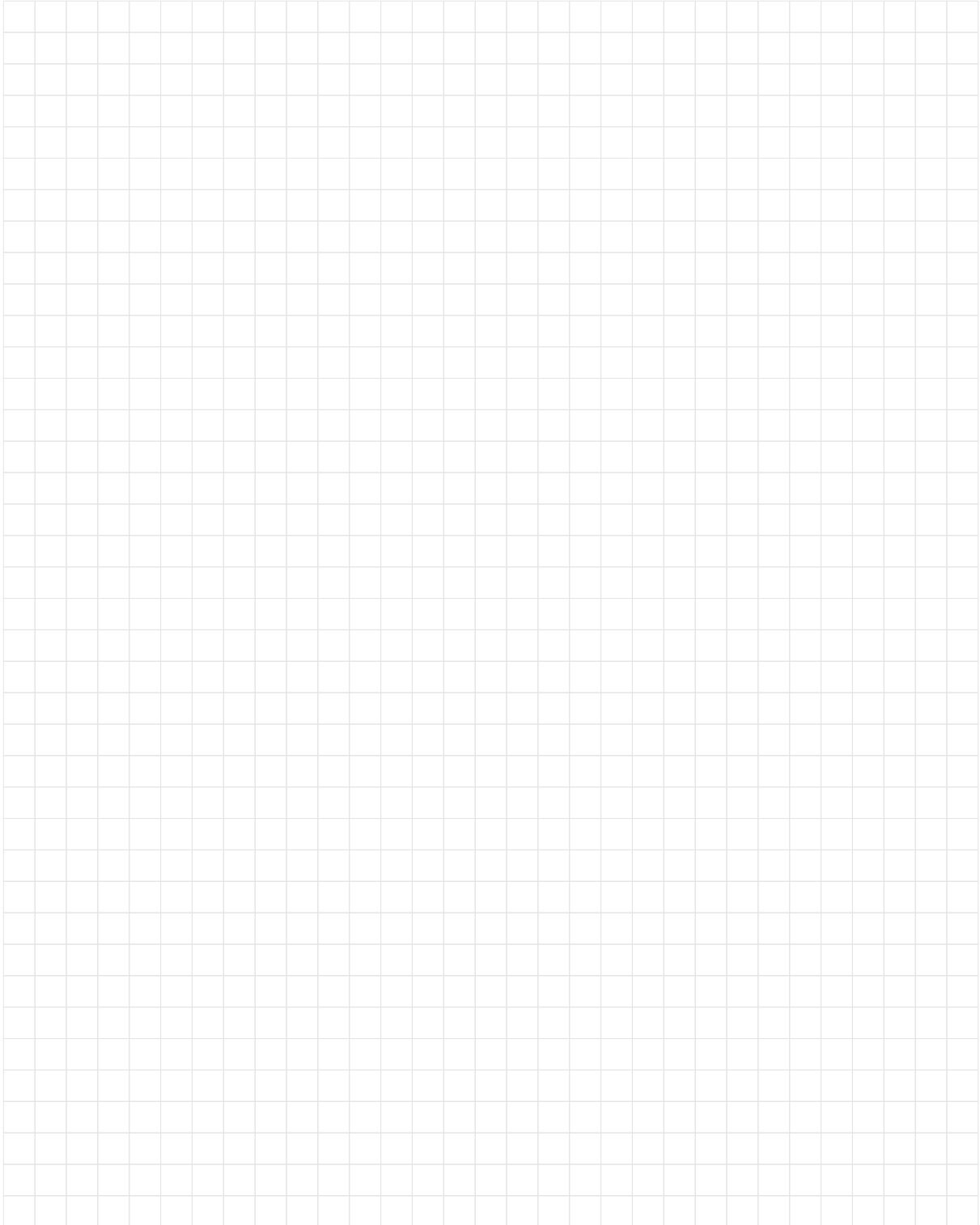
Prove or disprove the following statements.

Let  $X = \{-2, -1, 0, 1, 2\}$  and  $Y = \{-2, -1, 0, 1, 2\}$ .

(a) (2 points)  $(\forall x \in X)(\exists y \in Y)(x + y = 0)$ .

(b) (2 points)  $(\exists x \in X)(\forall y \in Y)(x + y = y)$ .

(c) (6 points)  $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})(\exists z \in \mathbb{N})((z^2 \geq x^2 + y^2) \wedge (z \geq 5))$ .





**Question 7** (15 points)

This is a question about *functions*.

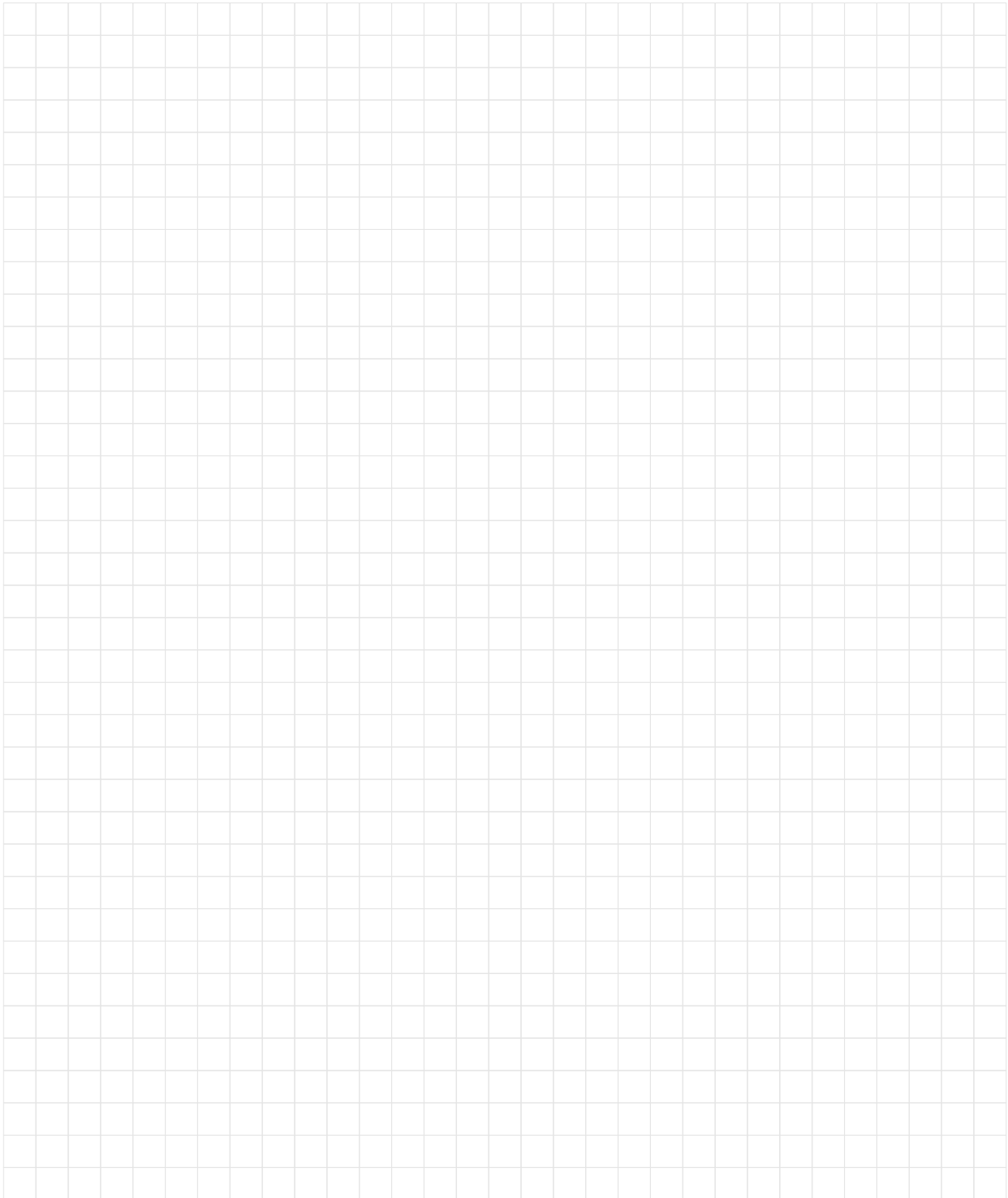
(a) (10 points) Let  $f : \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} \setminus \{1\}$  be the function defined as follows:

$$f(x) = \frac{x+1}{x}.$$

Prove that  $f$  is a bijection.

(b) (5 points) Construct sets  $A$ ,  $B$  and functions  $f : A \rightarrow B$ ,  $g : B \rightarrow A$  such that

- $f$  is not an injective function,
- but,  $g$  and  $f \circ g$  are both injective functions.





(5 points)

This is a question about *set theory*.

- (a) (2 points) Is  $\{\{a, d, e\}, \{b, c\}, \{d, f\}\}$  a partition of  $\{a, b, c, d, e, f\}$ ? Why/Why not?
- (b) (3 points) Let  $A = \{1, 2\}$ ,  $B = \{2, 3\}$  and  $C = \{3, 4\}$ . Write down  $(A \times B) \cap (A \times C)$ .



