Likelihood and the χ^2 statistic

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Assumption: obs. = model + (random) unc.

$$y_i = y_{m,i} + \eta_i; \eta_i \sim N(0, \sigma_i)$$

$$\epsilon_i \equiv \left(\frac{y_i - y_{m,i}}{\sigma_i} \right) \sim N(0, 1)$$

$$L = \prod_{i=1}^{N} \exp \left[- \left(\frac{y_i - y_{m,i}}{\sigma_i} \right)^2 \right]$$

$$\ln L \propto - \sum_{i=1}^{N} \left(\frac{y_i - y_{m,i}}{\sigma_i} \right)^2 \equiv -\chi^2$$

Properties of the χ^2 distribution

- χ^2 distribution of degree k
  Mean: k
  Variance: 2k

- Define reduced χ^2 by dividing by mean (= #DOF).
  Mean = 1
  Variance = 2

- Caution! The distribution is asymmetric!

What if errors are non-Gaussian (or unknown)?

- log-likelihood method will still work as long as model is kept fixed.
- Best χ^2 need not be close to 1.
- No big deal as long as model is fixed!