# Appendix

# Game Predictions

**Denote:**  $P_j$  is any player that is not  $P_i$ .

# **Game Predictions**

Game of 2 firms:

Table 1: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj^1}$	$ m Vj(Sj,Si)^2$
$\sum_{i=1}^{n} S_i < K$	0	0.0	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	1	0.5	$[ \ 0 \ , 1 \ ]$	$[\ 0\ ,\ 0.5\ ]$
$\sum_{i=1}^{n} S_i < K$	2	1.5	$[ \ 0 \ , 0 \ ]$	$[\ 0\ ,0\ ]$
$\sum_{i=1}^{n} S_i \geq K$	1	0.0	[2, 2]	$[\ 0.5\ ,\ 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$			$[\; 2\;, 2\; ]$	$[\ 0.5\ ,\ 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	0	0.0	$[\; 2\; ,  2\; ]$	$[\ 0.5\ ,\ 0.5\ ]$

 $<sup>\</sup>frac{\sum_{i=1}^{n} i - j}{1}$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = 0$ , with payoff  $V_j(S_j, S_i) = 0$ . With 1 other firm choosing 0 and viceversa.

That is: 1 firms choose 0 units, with a payoff of 0. 1 firm chooses 2 units, with a payoff of 1.5.

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

<sup>&</sup>lt;sup>2</sup> Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

# Game of 3 firms:

Table 2: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj}^1$	$Vj(Sj, Si)^2$
$\sum_{i=1}^{n} S_i < K$	0	0.000	$[\ 0\ , 2\ ]$	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$			$[\ 0\ , \ 2\ ]$	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	2	1.500	[0, 1]	$[\ 0\ , 0.5\ ]$
$\sum_{i=1}^{n} S_i \geq K$	1	-0.167	[1, 2]	[-0.167, 0.167]
$\sum_{i=1}^{n} S_i \ge K$	2	0.167	[1, 2]	[-0.167, 0.167]
$\sum_{i=1}^{n} S_i \ge K$	0	0.000	[1, 2]	[ -0.167 , 0.167 ]

 $<sup>^{1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with vi(Si, S\_i) = 1.5, if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 1]$ , with payoff  $V_j(S_j, S_i) = [0, 0.5]$ . With 1 other firm choosing 0 and 1 other firm choosing 1 and viceversa.

That is: 1 firms choose 0 units, with a payoff of 0 . 1 firm chooses 2 units, with a payoff of 1.5 . 1 firms choose 1 units, with a payoff of 0.5 .

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^{n} S_i \ge K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

 $<sup>^{2}</sup>$  Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

#### Game of 4 firms:

Table 3: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj}^1$	$ m Vj(Sj,Si)^2$
$\sum_{i=1}^{n} S_i < K$	0	0.0	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	1	0.5	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	2	1.5	[0, 2]	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i \geq K$	1	0.0	[1, 2]	$[\ 0\ ,\ 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	2	0.5	$[\ 1\ , \ 2\ ]$	$[\ 0\ , 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	0	0.0	$[\; 1\;, 2\; ]$	$[\ 0\ , 0.5\ ]$

 $<sup>^{1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 2]$ , with payoff  $V_j(S_j, S_i) = [0, 1.5]$ . With 1 other firm choosing 2; 0 other firms choosing 1, and 2 other firms choosing 0, and viceversa.

That is: 2 firms choose 0 units, with a payoff of 0. 0 firms choose 1 units, with a payoff of 0. 2 firms choose 2 units, with a payoff of 1.5.

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

 $<sup>^{2}</sup>$  Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

#### Game of 5 firms:

Table 4: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj}^1$	$ m Vj(Sj,Si)^2$
$\sum_{i=1}^{n} S_i < K$	0	0.0	$[\ 0\ , 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	1	0.5	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	2	1.5	[0, 2]	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n-1} S_i \ge K$	1	-0.1	[1, 2]	[-0.1, 0.3]
$\sum_{i=1}^{n} S_i \ge K$			$[\ 1\ , \ 2\ ]$	[-0.1, 0.3]
$\sum_{i=1}^{n} S_i \ge K$	0	0.0	$[\; 1\;, 2\; ]$	[-0.1, 0.3]

 $<sup>^{1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 2]$ , with payoff  $V_j(S_j, S_i) = [0, 1.5]$ . With 1 other firm choosing 2; 1 other firm choosing 1, and 2 other firms choosing 0, and viceversa.

That is: 2 firms choose 0 units, with a payoff of 0. 1 firms choose 1 units, with a payoff of 0. 2 firms choose 2 units, with a payoff of 1.5.

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

<sup>&</sup>lt;sup>2</sup> Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

#### Game of 6 firms:

Table 5: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj^1}$	$ m Vj(Sj,Si)^2$
$\sum_{i=1}^{n} S_i < K$	0	0.0	[0, 2]	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	1	0.5	[0, 2]	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$			[0, 2]	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i \geq K$	1	0.0	[1, 2]	$[\ 0\ , 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	2	0.5	$[\ 1\ ,\ 2\ ]$	$[\ 0\ , 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	0	0.0	$[\; 1\;, 2\; ]$	$[\ 0\ , 0.5\ ]$

 $<sup>^{-1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 2]$ , with payoff  $V_j(S_j, S_i) = [0, 1.5]$ . With 2 other firms choosing 2; 0 other firms choosing 1, and 3 other firms choosing 0, and viceversa.

That is: 3 firms choose 0 units, with a payoff of 0. 0 firms choose 1 units, with a payoff of 0. 3 firms choose 2 units, with a payoff of 1.5.

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

 $<sup>^{2}</sup>$  Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

# Game of 7 firms:

Table 6: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj}^1$	$Vj(Sj, Si)^2$
$\sum_{i=1}^{n} S_i < K$	0	0.000	$[\ 0\ , 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	1	0.500	$[\ 0\ , \ 2\ ]$	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	2	1.500	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i \geq K$	1	-0.071	[1, 2]	[-0.071, 0.357]
$\sum_{i=1}^{n} S_i \ge K$		0.357	$[\ 1\ , \ 2\ ]$	[-0.071, 0.357]
$\sum_{i=1}^{n} S_i \ge K$	0	0.000	$[\ 1\ ,\ 2\ ]$	[ -0.071 , 0.357 ]

 $<sup>^{-1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 2]$ , with payoff  $V_j(S_j, S_i) = [0, 1.5]$ . With 2 other firms choosing 2; 1 other firm choosing 1, and 3 other firms choosing 0, and viceversa.

That is: 3 firms choose 0 units, with a payoff of 0. 1 firms choose 1 units, with a payoff of 0. 3 firms choose 2 units, with a payoff of 1.5.

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

 $<sup>^{2}</sup>$  Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

#### Game of 8 firms:

Table 7: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj}^1$	$ m Vj(Sj,Si)^2$
$\sum_{i=1}^{n} S_i < K$	0	0.0	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n-1} S_i < K$	1	0.5	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	2	1.5	[0, 2]	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i \geq K$			[1, 2]	$[\ 0\ , 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	2	0.5	$[\ 1\ , \ 2\ ]$	$[\ 0\ , 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	0	0.0	$[\; 1\;, 2\; ]$	$[\ 0\ , 0.5\ ]$

 $<sup>^{1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 2]$ , with payoff  $V_j(S_j, S_i) = [0, 1.5]$ . With 3 other firms choosing 2; 0 other firms choosing 1, and 4 other firms choosing 0, and viceversa.

That is: 4 firms choose 0 units, with a payoff of 0 . 0 firms choose 1 units, with a payoff of 0 . 4 firms choose 2 units, with a payoff of 1.5 .

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

 $<sup>^{2}</sup>$  Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

#### Game of 9 firms:

Table 8: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj^1}$	$ m Vj(Sj,Si)^2$
$\sum_{i=1}^{n} S_i < K$			$[\ 0\ , 2\ ]$	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	1	0.500	$[\ 0\ , 2\ ]$	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$		1.500	$[\ 0\ , \ 2\ ]$	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i \geq K$	1	-0.056	[1, 2]	[-0.056, 0.389]
$\sum_{i=1}^{n} S_i \ge K$	2	0.389	[1, 2]	[-0.056, 0.389]
$\sum_{i=1}^{n} S_i \ge K$	0	0.000	$[\ 1\ , 2\ ]$	[-0.056, 0.389]

 $<sup>^{1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 2]$ , with payoff  $V_j(S_j, S_i) = [0, 1.5]$ . With 3 other firms choosing 2; 1 other firm choosing 1, and 4 other firms choosing 0, and viceversa.

That is: 4 firms choose 0 units, with a payoff of 0 . 1 firms choose 1 units, with a payoff of 0 . 4 firms choose 2 units, with a payoff of 1.5 .

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

 $<sup>^{2}</sup>$  Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

#### Game of 10 firms:

Table 9: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj}^1$	$ m Vj(Sj,Si)^2$
$\sum_{i=1}^{n} S_i < K$	0	0.0	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	1	0.5	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	2	1.5	[0, 2]	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i \geq K$	1	0.0	[1, 2]	$[\ 0\ ,\ 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	2	0.5	$[\ 1\ , \ 2\ ]$	$[\ 0\ , 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	0	0.0	$[\; 1\;, 2\; ]$	$[\ 0\ , 0.5\ ]$

 $<sup>^{-1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 2]$ , with payoff  $V_j(S_j, S_i) = [0, 1.5]$ . With 4 other firms choosing 2; 0 other firms choosing 1, and 5 other firms choosing 0, and viceversa.

That is: 5 firms choose 0 units, with a payoff of 0 . 0 firms choose 1 units, with a payoff of 0 . 5 firms choose 2 units, with a payoff of 1.5 .

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

 $<sup>^{2}</sup>$  Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

#### Game of 11 firms:

Table 10: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj}^1$	$ m Vj(Sj,Si)^2$
$\sum_{i=1}^{n} S_i < K$	0	0.000	$[\ 0\ , 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n-1} S_i < K$	1	0.500	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	2	1.500	[0, 2]	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i \geq K$	1	-0.045	[1, 2]	[-0.045, 0.409]
$\sum_{i=1}^{n} S_i \ge K$	2	0.409	$[\ 1\ , \ 2\ ]$	[-0.045, 0.409]
$\sum_{i=1}^{n} S_i \ge K$	0	0.000	$[\; 1\;, 2\; ]$	[ -0.045 , 0.409 ]

 $<sup>^{1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 2]$ , with payoff  $V_j(S_j, S_i) = [0, 1.5]$ . With 4 other firms choosing 2; 1 other firm choosing 1, and 5 other firms choosing 0, and viceversa.

That is: 5 firms choose 0 units, with a payoff of 0 . 1 firms choose 1 units, with a payoff of 0 . 5 firms choose 2 units, with a payoff of 1.5 .

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

 $<sup>^{2}</sup>$  Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

# Game of 12 firms:

Table 11: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj}^1$	$ m Vj(Sj,Si)^2$
$\sum_{i=1}^{n} S_i < K$	0	0.0	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n-1} S_i < K$	1	0.5	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	2	1.5	[0, 2]	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i \geq K$			[1, 2]	$[\ 0\ , 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	2	0.5	$[\ 1\ , \ 2\ ]$	$[\ 0\ , 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	0	0.0	$[\; 1\;, 2\; ]$	$[\ 0\ , 0.5\ ]$

 $<sup>^{1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 2]$ , with payoff  $V_j(S_j, S_i) = [0, 1.5]$ . With 5 other firms choosing 2; 0 other firms choosing 1, and 6 other firms choosing 0, and viceversa.

That is: 6 firms choose 0 units, with a payoff of 0 . 0 firms choose 1 units, with a payoff of 0 . 6 firms choose 2 units, with a payoff of 1.5 .

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

 $<sup>^{2}</sup>$  Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

#### Game of 13 firms:

Table 12: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj}^1$	$ m Vj(Sj,Si)^2$
$\sum_{i=1}^{n} S_i < K$	0	0.000	$[\ 0\ , 2\ ]$	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n-1} S_i < K$	1	0.500	$[\ 0\ , 2\ ]$	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	2	1.500	[0, 2]	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i \geq K$	1	-0.038	[1, 2]	[-0.038, 0.423]
$\sum_{i=1}^{n} S_i \ge K$	2	0.423	[1, 2]	[-0.038, 0.423]
$\sum_{i=1}^{n} S_i \ge K$	0	0.000	$[\ 1\ , \ 2\ ]$	[ -0.038 , 0.423 ]

 $<sup>^{1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 2]$ , with payoff  $V_j(S_j, S_i) = [0, 1.5]$ . With 5 other firms choosing 2; 1 other firm choosing 1, and 6 other firms choosing 0, and viceversa.

That is: 6 firms choose 0 units, with a payoff of 0 . 1 firms choose 1 units, with a payoff of 0 . 6 firms choose 2 units, with a payoff of 1.5 .

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

 $<sup>^{2}</sup>$  Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

#### Game of 14 firms:

Table 13: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj}^1$	$ m Vj(Sj,Si)^2$
$\sum_{i=1}^{n} S_i < K$			$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$			[0, 2]	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$			[0, 2]	$[\ 0\ ,\ 1.5\ ]$
$\sum_{i=1}^{n} S_i \geq K$	1	0.0	[1, 2]	$[\ 0\ , 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	2	0.5	$[\ 1\ , \ 2\ ]$	$[\ 0\ , 0.5\ ]$
$\sum_{i=1}^{n} S_i \ge K$	0	0.0	$[\; 1\;, 2\; ]$	$[\ 0\ , 0.5\ ]$

 $<sup>^{1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 2]$ , with payoff  $V_j(S_j, S_i) = [0, 1.5]$ . With 6 other firms choosing 2; 0 other firms choosing 1, and 7 other firms choosing 0, and viceversa.

That is: 7 firms choose 0 units, with a payoff of 0 . 0 firms choose 1 units, with a payoff of 0 . 7 firms choose 2 units, with a payoff of 1.5 .

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

 $<sup>^{2}</sup>$  Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

#### Game of 15 firms:

Table 14: Game Payoffs

Scenarios	Si	Vi(Si, Sj)	$\mathrm{Sj^1}$	$ m Vj(Sj,Si)^2$
$\sum_{i=1}^{n} S_i < K$	0	0.000	$[\ 0\ , 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$	1	0.500	$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i < K$			$[\ 0\ , \ 2\ ]$	$[\ 0\ , 1.5\ ]$
$\sum_{i=1}^{n} S_i \geq K$	1	-0.033	[1, 2]	[-0.033, 0.433]
$\sum_{i=1}^{n-1} S_i \ge K$	2	0.433	$[\ 1\ , \ 2\ ]$	[-0.033, 0.433]
$\sum_{i=1}^{n} S_i \ge K$	0	0.000	$[\; 1\;, 2\; ]$	[ -0.033 , 0.433 ]

 $<sup>^{1}</sup>$  Sj = Range of values that Player j can choose.

— Pareto Equilibrium — Pareto Optimality is reached when:  $S_i = 2$  with  $V_i(S_i, S_j) = 1.5$ , if  $\sum_{i=1}^n S_i < K$  &  $S_j = [0, 2]$ , with payoff  $V_j(S_j, S_i) = [0, 1.5]$ . With 6 other firms choosing 2; 1 other firm choosing 1, and 7 other firms choosing 0, and viceversa.

That is: 7 firms choose 0 units, with a payoff of 0 . 1 firms choose 1 units, with a payoff of 0 . 7 firms choose 2 units, with a payoff of 1.5 .

- Nash Equilibrium There are 2 Nash Equilibriums .
- ( 1 ) Nash Equilibrium 1 , for  $\sum_{i=1}^n S_i < K$  : 3 firms choose 1 units, each with a payoff of 0.5 .
- ( 2 ) Nash Equilibrium 2 , for  $\sum_{i=1}^n S_i \geq K$  : 3 firms choose 2 units, each with a payoff of 0.1666667 .

 $<sup>^{2}</sup>$  Vj(Sj, Si) = Range of payoffs for Player j, for their possible strategies.

# Reference for Students

Your payoff in each scenario, with 1 other firms:

```
You get = 0 , if you choose = 0 & the rest of the firms choose = [0, \infty]
You get = 0.5 , if you choose = 1 & the rest of the firms choose = [0, 1]
You get = 1.5 , if you choose = 2 & the rest of the firms choose = [0, 0]
You get = 0 , if you choose = 1 & the rest of the firms choose = [2, \infty]
You get = 0.5 , if you choose = 2 & the rest of the firms choose = [1, \infty]
```

Your payoff in each scenario, with 2 other firms:

```
You get = 0 , if you choose = 0 & the rest of the firms choose = [0, \infty]
You get = 0.5 , if you choose = 1 & the rest of the firms choose = [0, 2]
You get = 1.5 , if you choose = 2 & the rest of the firms choose = [0, 1]
You get = -0.1666667 , if you choose = 1 & the rest of the firms choose = [3, \infty]
You get = 0.1666667 , if you choose = 2 & the rest of the firms choose = [2, \infty]
```

Your payoff in each scenario, with 3 other firms:

```
You get = 0 , if you choose = 0 & the rest of the firms choose = [0, \infty]
You get = 0.5 , if you choose = 1 & the rest of the firms choose = [0, 3]
You get = 1.5 , if you choose = 2 & the rest of the firms choose = [0, 2]
You get = 0 , if you choose = 1 & the rest of the firms choose = [4, \infty]
You get = 0.5 , if you choose = 2 & the rest of the firms choose = [3, \infty]
```

Your payoff in each scenario, with 4 other firms:

```
You get = 0 , if you choose = 0 & the rest of the firms choose = [0, \infty]
You get = 0.5 , if you choose = 1 & the rest of the firms choose = [0, 4]
You get = 1.5 , if you choose = 2 & the rest of the firms choose = [0, 3]
You get = -0.1 , if you choose = 1 & the rest of the firms choose = [5, \infty]
You get = 0.3 , if you choose = 2 & the rest of the firms choose = [4, \infty]
```

Your payoff in each scenario, with 5 other firms:

```
You get = 0, if you choose = 0 & the rest of the firms choose = [0, \infty]
```

You get 
$$= 0.5$$
, if you choose  $= 1$  & the rest of the firms choose  $= [0, 5]$ 

You get 
$$= 1.5$$
, if you choose  $= 2 \&$  the rest of the firms choose  $= [0, 4]$ 

You get = 0, if you choose = 1 & the rest of the firms choose = 
$$[6, \infty]$$

You get = 
$$0.5$$
, if you choose =  $2 \&$  the rest of the firms choose =  $[5, \infty]$ 

Your payoff in each scenario, with 6 other firms:

```
You get = 0, if you choose = 0 & the rest of the firms choose = [0, \infty]
```

You get 
$$= 0.5$$
, if you choose  $= 1$  & the rest of the firms choose  $= [0, 6]$ 

You get 
$$= 1.5$$
, if you choose  $= 2$  & the rest of the firms choose  $= [0, 5]$ 

You get = 
$$-0.07142857$$
, if you choose = 1 & the rest of the firms choose =  $[7, \infty]$ 

You get = 
$$0.3571429$$
, if you choose =  $2 \&$  the rest of the firms choose =  $[6, \infty]$ 

Your payoff in each scenario, with 7 other firms:

```
You get = 0, if you choose = 0 & the rest of the firms choose = [0, \infty]
```

You get 
$$= 0.5$$
, if you choose  $= 1$  & the rest of the firms choose  $= [0, 7]$ 

You get 
$$= 1.5$$
, if you choose  $= 2 \&$  the rest of the firms choose  $= [0, 6]$ 

You get 
$$= 0$$
, if you choose  $= 1$  & the rest of the firms choose  $= [8, \infty]$ 

You get = 
$$0.5$$
, if you choose =  $2 \&$  the rest of the firms choose =  $[7, \infty]$ 

Your payoff in each scenario, with 8 other firms:

You get = 0, if you choose = 0 & the rest of the firms choose = 
$$[0, \infty]$$

You get 
$$= 0.5$$
, if you choose  $= 1 \&$  the rest of the firms choose  $= [0, 8]$ 

You get 
$$= 1.5$$
, if you choose  $= 2 \&$  the rest of the firms choose  $= [0, 7]$ 

You get = 
$$-0.05555556$$
, if you choose = 1 & the rest of the firms choose =  $[9, \infty]$ 

You get = 
$$0.3888889$$
, if you choose =  $2 \&$  the rest of the firms choose =  $[8, \infty]$ 

Your payoff in each scenario, with 9 other firms:

You get = 0, if you choose = 0 & the rest of the firms choose = 
$$[0, \infty]$$

You get 
$$= 0.5$$
, if you choose  $= 1$  & the rest of the firms choose  $= [0, 9]$ 

You get 
$$= 1.5$$
, if you choose  $= 2 \&$  the rest of the firms choose  $= [0, 8]$ 

You get = 0, if you choose = 1 & the rest of the firms choose = 
$$[10, \infty]$$

You get = 0.5, if you choose = 2 & the rest of the firms choose = 
$$[9, \infty]$$

Your payoff in each scenario, with 10 other firms:

```
You get = 0, if you choose = 0 & the rest of the firms choose = [0, \infty]
You get = 0.5, if you choose = 1 & the rest of the firms choose = [0, 10]
You get = 1.5, if you choose = 2 & the rest of the firms choose = [0, 9]
You get = -0.04545455, if you choose = 1 & the rest of the firms choose = [11, \infty]
You get = 0.4090909, if you choose = 2 & the rest of the firms choose = [10, \infty]
```

Your payoff in each scenario, with 11 other firms:

```
You get = 0 , if you choose = 0 & the rest of the firms choose = [0, \infty]
You get = 0.5 , if you choose = 1 & the rest of the firms choose = [0, 11]
You get = 1.5 , if you choose = 2 & the rest of the firms choose = [0, 10]
You get = 0 , if you choose = 1 & the rest of the firms choose = [12, \infty]
You get = 0.5 , if you choose = 2 & the rest of the firms choose = [11, \infty]
```

Your payoff in each scenario, with 12 other firms:

```
You get = 0 , if you choose = 0 & the rest of the firms choose = [0, \infty]
You get = 0.5 , if you choose = 1 & the rest of the firms choose = [0, 12]
You get = 1.5 , if you choose = 2 & the rest of the firms choose = [0, 11]
You get = -0.03846154 , if you choose = 1 & the rest of the firms choose = [13, \infty]
You get = 0.4230769 , if you choose = 2 & the rest of the firms choose = [12, \infty]
```

Your payoff in each scenario, with 13 other firms:

```
You get = 0 , if you choose = 0 & the rest of the firms choose = [0, \infty]
You get = 0.5 , if you choose = 1 & the rest of the firms choose = [0, 13]
You get = 1.5 , if you choose = 2 & the rest of the firms choose = [0, 12]
You get = 0 , if you choose = 1 & the rest of the firms choose = [14, \infty]
You get = 0.5 , if you choose = 2 & the rest of the firms choose = [13, \infty]
```

Your payoff in each scenario, with 14 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose =  $[0, \infty]$ 

You get = 0.5 , if you choose = 1 & the rest of the firms choose = [0, 14 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose = [0, 13 ]

You get = -0.03333333 , if you choose = 1 & the rest of the firms choose = [ 15 ,  $\infty$ ]

You get = 0.4333333 , if you choose = 2 & the rest of the firms choose = [ 14 ,  $\infty$ ]