# Appendix

## Game Estimations

***Denote*:**

- $P_j$: is any player that is not $P_i$
- *Column* $Sj$: Range of possible strategies for Player j, which whenever summed with the other players' strategies, satisfy the given scenario. It can take any value in that range that is also 0, 1 or 2. For example, in *Table 3: Game Payoffs* we have 3 firms playing the game, where the second row has $Sj = [0, 2]$, which is interpreted as the available strategies 0, 1 and 2 for all other players that are not Player i, such that whenever combined they satisfy $\sum_{i=1}^{n} S_i < K \leftrightarrow \sum_{i=1}^{n} S_i < 4$ (because K = number of firms + 1 and $Si = 1$). Thus, we define Player 1's strategy as $S_1 = 1$, Player 2's strategy as $S_2 = [0, 2]$ and Player 3's strategy as $S_3 = [0, 2]$; then $(S_1, S_2, S_3) = \{(1, 0, 0), (1, 1, 0), (1, 1, 1), (1, 0, 1), (1, 0, 2), (1, 2, 0)\}$ are suggested in $Sj = [0, 2]$ because they satisfy $S_1 + S_2 + S_3 < 4 \leftrightarrow \sum_{i=1}^{n} S_i < 4$.
- *Column* $Vj(Sj, Si)$: Range of possible payoffs for Player j.

## Game Predictions

Game of 2 firms:

Table 1: Game Payoffs

| **Scenarios** | **Si** | **Vi(Si, Sj)** | **Sj**[1] | **Vj(Sj, Si)**[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.0 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.5 | [ 0 , 1 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.5 | [ 0 , 0 ] | [ 0 , 0 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | 0.0 | [ 2 , 2 ] | [ 0.5 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.5 | [ 2 , 2 ] | [ 0.5 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.0 | [ 2 , 2 ] | [ 0.5 , 0.5 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j = 0$ , with payoff $V_j(S_j, S_i) = 0$ . With 1 other firm choosing 0 and viceversa.

That is: 1 firms choose 0 units, with a payoff of 0 . 1 firm chooses 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 3 firms:

Table 2: Game Payoffs

| Scenarios | Si | Vi(Si, Sj) | Sj[1] | Vj(Sj, Si)[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.000 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.500 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.500 | [ 0 , 1 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | -0.167 | [ 1 , 2 ] | [ -0.167 , 0.167 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.167 | [ 1 , 2 ] | [ -0.167 , 0.167 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.000 | [ 1 , 2 ] | [ -0.167 , 0.167 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with vi(Si, S_i) = 1.5 , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 1 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 0.5 ]. With 1 other firm choosing 0 and 1 other firm choosing 1 and viceversa.

That is: 1 firms choose 0 units, with a payoff of 0 . 1 firm chooses 2 units, with a payoff of 1.5 . 1 firms choose 1 units, with a payoff of 0.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 4 firms:

Table 3: Game Payoffs

| **Scenarios** | **Si** | **Vi(Si, Sj)** | **Sj**[1] | **Vj(Sj, Si)**[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.0 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | 0.0 | [ 1 , 2 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.5 | [ 1 , 2 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.0 | [ 1 , 2 ] | [ 0 , 0.5 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 2 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 1.5 ]. With 1 other firm choosing 2 ; 0 other firms choosing 1 , and 2 other firms choosing 0 , and viceversa.

That is: 2 firms choose 0 units, with a payoff of 0 . 0 firms choose 1 units, with a payoff of 0 . 2 firms choose 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 5 firms:

Table 4: Game Payoffs

| **Scenarios** | **Si** | **Vi(Si, Sj)** | **Sj**[1] | **Vj(Sj, Si)**[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.0 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | -0.1 | [ 1 , 2 ] | [ -0.1 , 0.3 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.3 | [ 1 , 2 ] | [ -0.1 , 0.3 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.0 | [ 1 , 2 ] | [ -0.1 , 0.3 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 2 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 1.5 ]. With 1 other firm choosing 2 ; 1 other firm choosing 1 , and 2 other firms choosing 0 , and viceversa.

That is: 2 firms choose 0 units, with a payoff of 0 . 1 firms choose 1 units, with a payoff of 0 . 2 firms choose 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 6 firms:

Table 5: Game Payoffs

| Scenarios | Si | Vi(Si, Sj) | Sj[1] | Vj(Sj, Si)[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.0 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | 0.0 | [ 1 , 2 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.5 | [ 1 , 2 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.0 | [ 1 , 2 ] | [ 0 , 0.5 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 2 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 1.5 ]. With 2 other firms choosing 2 ; 0 other firms choosing 1 , and 3 other firms choosing 0 , and viceversa.

That is: 3 firms choose 0 units, with a payoff of 0 . 0 firms choose 1 units, with a payoff of 0 . 3 firms choose 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 7 firms:

Table 6: Game Payoffs

| **Scenarios** | **Si** | **Vi(Si, Sj)** | **Sj**[1] | **Vj(Sj, Si)**[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.000 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.500 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.500 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | -0.071 | [ 1 , 2 ] | [ -0.071 , 0.357 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.357 | [ 1 , 2 ] | [ -0.071 , 0.357 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.000 | [ 1 , 2 ] | [ -0.071 , 0.357 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 2 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 1.5 ]. With 2 other firms choosing 2 ; 1 other firm choosing 1 , and 3 other firms choosing 0 , and viceversa.

That is: 3 firms choose 0 units, with a payoff of 0 . 1 firms choose 1 units, with a payoff of 0 . 3 firms choose 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 8 firms:

Table 7: Game Payoffs

| Scenarios | Si | Vi(Si, Sj) | Sj[1] | Vj(Sj, Si)[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.0 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | 0.0 | [ 1 , 2 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.5 | [ 1 , 2 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.0 | [ 1 , 2 ] | [ 0 , 0.5 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 2 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 1.5 ]. With 3 other firms choosing 2 ; 0 other firms choosing 1 , and 4 other firms choosing 0 , and viceversa.

That is: 4 firms choose 0 units, with a payoff of 0 . 0 firms choose 1 units, with a payoff of 0 . 4 firms choose 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 9 firms:

Table 8: Game Payoffs

| Scenarios | Si | Vi(Si, Sj) | Sj[1] | Vj(Sj, Si)[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.000 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.500 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.500 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | -0.056 | [ 1 , 2 ] | [ -0.056 , 0.389 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.389 | [ 1 , 2 ] | [ -0.056 , 0.389 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.000 | [ 1 , 2 ] | [ -0.056 , 0.389 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 2 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 1.5 ]. With 3 other firms choosing 2 ; 1 other firm choosing 1 , and 4 other firms choosing 0 , and viceversa.

That is: 4 firms choose 0 units, with a payoff of 0 . 1 firms choose 1 units, with a payoff of 0 . 4 firms choose 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 10 firms:

Table 9: Game Payoffs

| Scenarios | Si | Vi(Si, Sj) | Sj[1] | Vj(Sj, Si)[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.0 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | 0.0 | [ 1 , 2 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.5 | [ 1 , 2 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.0 | [ 1 , 2 ] | [ 0 , 0.5 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 2 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 1.5 ]. With 4 other firms choosing 2 ; 0 other firms choosing 1 , and 5 other firms choosing 0 , and viceversa.

That is: 5 firms choose 0 units, with a payoff of 0 . 0 firms choose 1 units, with a payoff of 0 . 5 firms choose 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 11 firms:

Table 10: Game Payoffs

| **Scenarios** | **Si** | **Vi(Si, Sj)** | **Sj**[1] | **Vj(Sj, Si)**[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.000 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.500 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.500 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | -0.045 | [ 1 , 2 ] | [ -0.045 , 0.409 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.409 | [ 1 , 2 ] | [ -0.045 , 0.409 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.000 | [ 1 , 2 ] | [ -0.045 , 0.409 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 2 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 1.5 ]. With 4 other firms choosing 2 ; 1 other firm choosing 1 , and 5 other firms choosing 0 , and viceversa.

That is: 5 firms choose 0 units, with a payoff of 0 . 1 firms choose 1 units, with a payoff of 0 . 5 firms choose 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 12 firms:

Table 11: Game Payoffs

| **Scenarios** | **Si** | **Vi(Si, Sj)** | **Sj**[1] | **Vj(Sj, Si)**[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.0 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | 0.0 | [ 1 , 2 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.5 | [ 1 , 2 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.0 | [ 1 , 2 ] | [ 0 , 0.5 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 2 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 1.5 ]. With 5 other firms choosing 2 ; 0 other firms choosing 1 , and 6 other firms choosing 0 , and viceversa.

That is: 6 firms choose 0 units, with a payoff of 0 . 0 firms choose 1 units, with a payoff of 0 . 6 firms choose 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 13 firms:

Table 12: Game Payoffs

| **Scenarios** | **Si** | **Vi(Si, Sj)** | **Sj**[1] | **Vj(Sj, Si)**[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.000 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.500 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.500 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | -0.038 | [ 1 , 2 ] | [ -0.038 , 0.423 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.423 | [ 1 , 2 ] | [ -0.038 , 0.423 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.000 | [ 1 , 2 ] | [ -0.038 , 0.423 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 2 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 1.5 ]. With 5 other firms choosing 2 ; 1 other firm choosing 1 , and 6 other firms choosing 0 , and viceversa.

That is: 6 firms choose 0 units, with a payoff of 0 . 1 firms choose 1 units, with a payoff of 0 . 6 firms choose 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 14 firms:

Table 13: Game Payoffs

| Scenarios | Si | Vi(Si, Sj) | Sj[1] | Vj(Sj, Si)[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.0 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.5 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | 0.0 | [ 1 , 2 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.5 | [ 1 , 2 ] | [ 0 , 0.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.0 | [ 1 , 2 ] | [ 0 , 0.5 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 2 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 1.5 ]. With 6 other firms choosing 2 ; 0 other firms choosing 1 , and 7 other firms choosing 0 , and viceversa.

That is: 7 firms choose 0 units, with a payoff of 0 . 0 firms choose 1 units, with a payoff of 0 . 7 firms choose 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

Game of 15 firms:

Table 14: Game Payoffs

| Scenarios | Si | Vi(Si, Sj) | Sj[1] | Vj(Sj, Si)[2] |
|---|---|---|---|---|
| $\sum_{i=1}^{n} S_i < K$ | 0 | 0.000 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 1 | 0.500 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i < K$ | 2 | 1.500 | [ 0 , 2 ] | [ 0 , 1.5 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 1 | -0.033 | [ 1 , 2 ] | [ -0.033 , 0.433 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 2 | 0.433 | [ 1 , 2 ] | [ -0.033 , 0.433 ] |
| $\sum_{i=1}^{n} S_i \geq K$ | 0 | 0.000 | [ 1 , 2 ] | [ -0.033 , 0.433 ] |

[1] Sj = Range of possible strategies for Player j.

[2] Vj(Sj, Si) = Range of possible payoffs for Player j.

—- Pareto Equilibrium — Pareto Optimality is reached when: $S_i = 2$ with $V_i(S_i, S_j) = 1.5$ , if $\sum_{i=1}^{n} S_i < K$ & $S_j =$ [ 0 , 2 ], with payoff $V_j(S_j, S_i) =$ [ 0 , 1.5 ]. With 6 other firms choosing 2 ; 1 other firm choosing 1 , and 7 other firms choosing 0 , and viceversa.

That is: 7 firms choose 0 units, with a payoff of 0 . 1 firms choose 1 units, with a payoff of 0 . 7 firms choose 2 units, with a payoff of 1.5 .

—- Nash Equilibrium — There are 2 Nash Equilibriums .

( 1 ) Nash Equilibrium 1 , for $\sum_{i=1}^{n} S_i < K$ : 3 firms choose 1 units, each with a payoff of 0.5 .

( 2 ) Nash Equilibrium 2 , for $\sum_{i=1}^{n} S_i \geq K$ : 3 firms choose 2 units, each with a payoff of 0.1666667 .

## Reference for Students

Your payoff in each scenario, with 1 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = [0, $\infty$]

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 1 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 0 ]

You get = 0 , if you choose = 1 & the rest of the firms choose in aggregate = [ 2 , $\infty$]

You get = 0.5 , if you choose = 2 & the rest of the firms choose in aggregate = [ 1 , $\infty$]

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Your payoff in each scenario, with 2 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = [0, $\infty$]

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 2 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 1 ]

You get = -0.1666667 , if you choose = 1 & the rest of the firms choose in aggregate = [ 3 , $\infty$]

You get = 0.1666667 , if you choose = 2 & the rest of the firms choose in aggregate = [ 2 , $\infty$]

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Your payoff in each scenario, with 3 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = [0, $\infty$]

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 3 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 2 ]

You get = 0 , if you choose = 1 & the rest of the firms choose in aggregate = [ 4 , $\infty$]

You get = 0.5 , if you choose = 2 & the rest of the firms choose in aggregate = [ 3 , $\infty$]

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Your payoff in each scenario, with 4 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = [0, $\infty$]

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 4 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 3 ]

You get = -0.1 , if you choose = 1 & the rest of the firms choose in aggregate = [ 5 , $\infty$]

You get = 0.3 , if you choose = 2 & the rest of the firms choose in aggregate = [ 4 , $\infty$]

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Your payoff in each scenario, with 5 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = [0, $\infty$]

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 5 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 4 ]

You get = 0 , if you choose = 1 & the rest of the firms choose in aggregate = [ 6 , $\infty$]

You get = 0.5 , if you choose = 2 & the rest of the firms choose in aggregate = [ 5 , $\infty$]

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Your payoff in each scenario, with 6 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = [0, $\infty$]

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 6 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 5 ]

You get = -0.07142857 , if you choose = 1 & the rest of the firms choose in aggregate = [ 7 , $\infty$]

You get = 0.3571429 , if you choose = 2 & the rest of the firms choose in aggregate = [ 6 , $\infty$]

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Your payoff in each scenario, with 7 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = [0, $\infty$]

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 7 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 6 ]

You get = 0 , if you choose = 1 & the rest of the firms choose in aggregate = [ 8 , $\infty$]

You get = 0.5 , if you choose = 2 & the rest of the firms choose in aggregate = [ 7 , $\infty$]

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Your payoff in each scenario, with 8 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = [0, $\infty$]

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 8 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 7 ]

You get = -0.05555556 , if you choose = 1 & the rest of the firms choose in aggregate = [ 9 , $\infty$]

You get = 0.3888889 , if you choose = 2 & the rest of the firms choose in aggregate = [ 8 , $\infty$]

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Your payoff in each scenario, with 9 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = [0, $\infty$]

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 9 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 8 ]

You get = 0 , if you choose = 1 & the rest of the firms choose in aggregate = [ 10 , $\infty$]

You get = 0.5 , if you choose = 2 & the rest of the firms choose in aggregate = [ 9 , $\infty$]

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Your payoff in each scenario, with 10 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = $[0, \infty]$

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 10 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 9 ]

You get = -0.04545455 , if you choose = 1 & the rest of the firms choose in aggregate = $[\ 11\ ,\ \infty]$

You get = 0.4090909 , if you choose = 2 & the rest of the firms choose in aggregate = $[\ 10\ ,\ \infty]$

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Your payoff in each scenario, with 11 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = $[0, \infty]$

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 11 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 10 ]

You get = 0 , if you choose = 1 & the rest of the firms choose in aggregate = $[\ 12\ ,\ \infty]$

You get = 0.5 , if you choose = 2 & the rest of the firms choose in aggregate = $[\ 11\ ,\ \infty]$

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Your payoff in each scenario, with 12 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = $[0, \infty]$

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 12 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 11 ]

You get = -0.03846154 , if you choose = 1 & the rest of the firms choose in aggregate = $[\ 13\ ,\ \infty]$

You get = 0.4230769 , if you choose = 2 & the rest of the firms choose in aggregate = $[\ 12\ ,\ \infty]$

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Your payoff in each scenario, with 13 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = $[0, \infty]$

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 13 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 12 ]

You get = 0 , if you choose = 1 & the rest of the firms choose in aggregate = $[\ 14\ ,\ \infty]$

You get = 0.5 , if you choose = 2 & the rest of the firms choose in aggregate = $[\ 13\ ,\ \infty]$

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Your payoff in each scenario, with 14 other firms:

You get = 0 , if you choose = 0 & the rest of the firms choose in aggregate = [0, $\infty$]

You get = 0.5 , if you choose = 1 & the rest of the firms choose in aggregate = [0, 14 ]

You get = 1.5 , if you choose = 2 & the rest of the firms choose in aggregate = [0, 13 ]

You get = -0.03333333 , if you choose = 1 & the rest of the firms choose in aggregate = [ 15 , $\infty$]

You get = 0.4333333 , if you choose = 2 & the rest of the firms choose in aggregate = [ 14 , $\infty$]

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