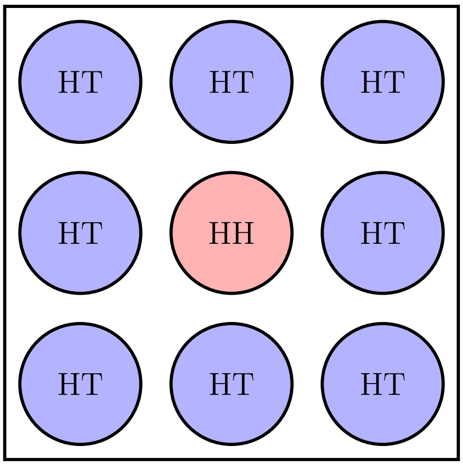
# Assignment 2
## Due on 10th Mar, 11 p.m.

1. **Independence of event complements (10 points)**: If events $A$ and $B$ are independent, are $A$ and $B^c$ independent? Prove your answer.

2. **A coin box with strange coins**: Li Lei has a coin box that contains 9 coins. 8 of them are fair, standard coins (heads and tails) and 1 coin has heads on both sides.




   **2.1)** **(10 points)** If Li Lei randomly chooses a coin and flip it, what is the probability of getting a head?

   **2.2)** **(20 points)** Li Lei randomly chose a coin without looking at it. He flipped it four times, and his friend Han Meimei told him that he got four heads. Now, if Li Lei flip the coin again, what is the probability of getting a head?

3. **Virus detection kits**: It is known that the ***prevalence*** of a virus is 0.01. There are two detection kits for this particular virus used by many hospitals. They are produced by different manufacturers and based on completely different assays. **Kit A** has a ***sensitivity*** of 0.9 and a ***specificity*** of 0.95. **Kit B** has a ***sensitivity*** of 0.95 and a ***specificity*** of 0.9. A random person goes to a hospital to get tested by both kits **A** and **B**. Let events $\boldsymbol{E_1}$ = { Kit **A** shows a positive result }, $\boldsymbol{E_2}$ = { Kit **B** shows a positive result } and $\boldsymbol{E_3}$ = { The person carries the virus }.

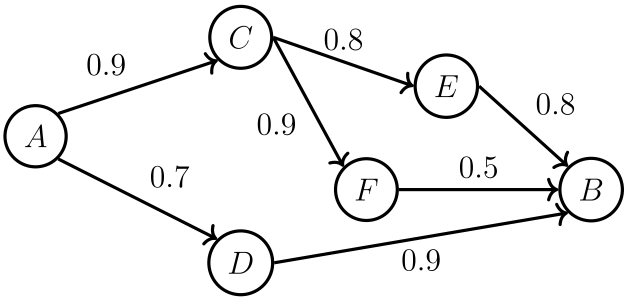
   **3.1)** **(10 points)** Calculate $P(\boldsymbol{E_1})$

**3.2)** **(10 points)** Calculate $P(\boldsymbol{E_2})$

**3.3)** **(10 points)** Are events $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$ independent? Why or why not? You can use the mathematical formula to prove your conclusion, or you can also use your own words to describe your intuition about this question.

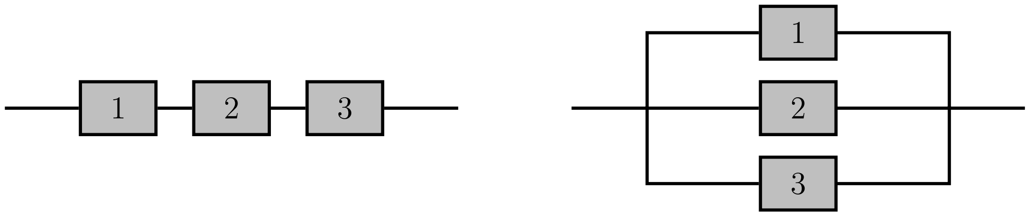
**3.4)** **(10 points)** Calculate $P(\boldsymbol{E_3} \mid \boldsymbol{E_1} \cap \boldsymbol{E_2})$

**4. Network reliability and signalling pathway (20 points)**: Let's start with a simple computer network connecting two nodes from $A$ to $B$ through intermediate nodes $C, D, E, F$ as shown in the picture below:




For every pair of directly connected nodes, say $i$ and $j$, there is a given probability $p_{ij}$ that the link from $i$ to $j$ is up. It is generally of interest to calculate the probability that there is a path connecting $A$ and $B$ in which all links are up.

This is a typical problem of assessing the reliability of a system consisting of components that can fail ***independently***. You will encounter this type of system very often in real life. Such a system can often be divided into subsystems, where each subsystem consists in turn of several components that are connected either in series or in parallel, like shown below:




Series connection

Parallel connection

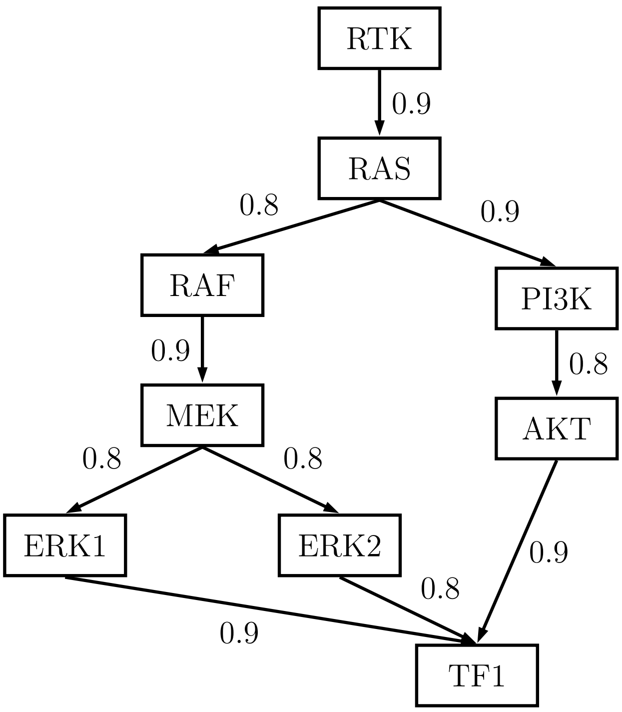
Let a subsystem consist of components $1, 2, 3, ..., m$, and let $p_i$ be the probability that component $i$ is up ("succeeds"). Then a series subsystem succeeds if ***all*** of its components are up, so its probability of success is the product of the probabilities of success of the corresponding components, *i.e.*

$$P(\text{series subsystem succeeds}) = \prod_{i=1}^{m} p_i = p_1\, p_2\, p_3 \cdots p_m$$

A parallel subsystem succeeds if ***any one*** of its components succeeds, so its probability of failure is the product of the probabilities of failure of the corresponding components, *i.e.*

$$\begin{aligned} P(\text{parallel subsystem succeeds}) &= 1 - P(\text{parallel subsystem fails}) \\ &= 1 - \prod_{i=1}^{m}(1-p_i) \\ &= 1 - (1-p_1)(1-p_2)(1-p_3)\cdots(1-p_m) \end{aligned}$$

Now let's look at a signalling pathway inside a cell, which is very important for the functions of a cell. The signalling pathway consists of a cascade of protein phosphorylation events. Many proteins are only activated when they are phosphorylated. In the following diagram depicting a simplified version of the MAP and PI3 kinase pathways, "$A \to B$" means "when $A$ is activated, it has a probability of phosphorylating $B$", and the probability of the phosphorylation is indicated on the edge:



Assume all phosphorylation events are independent and all proteins shown in the above picture must be activated first before they can phosphorylate their downstream proteins to activate them. Now, the protein RTK is activated, what is the probability that the protein TF1 is activated ?