Southern University of Science And Technology
School of Life Sciences
BIO210 Biostatistics
(Spring, 2022)

Assignment 3
Due on 19th Mar, 11 p.m.

1. **Mobile phones during an exam**: The mobile phones of n students are thrown into a box before an exam. The students then pick up their mobile phones at random after the exam ends (*i.e.* so that every assignment of the mobile phones to the students is equally likely). What is the probability that

   1.1) **(5 points)** every student gets his or her mobile phone back?

   1.2) **(5 points)** the first m students who picked mobile phones get their own mobile phones back?

   1.3) **(5 points)** everyone among the first m students to pick up the mobile phones gets back a mobile phone belonging to one of the last m students to pick up the mobile phones?

   Now assume, in addition, that every mobile phone thrown into the box has probability p of getting dirty (independently of what happens to the other mobile phones or who has dropped or picked it up). What is the probability that

   1.4) **(5 points)** the first m students will pick up clean mobile phones?

   1.5) **(5 points)** exactly m students will pick up clean mobile phones?

2. **Travelling**: Han Meimei is taking a probability course and want to think about her commutes using probabilistic modelling. She starts promptly at 8am. Han Meimei drives and thus is at the mercy of traffic lights. When all traffic lights on her route are green, the entire trip takes 18 minutes. Han Meimei's route includes 5 traffic lights, each of which is red with probability 1/3, independent of every other light. Each red traffic light that she encounters adds 1 minute to her commute (for slowing, stopping, and returning to speed).

   2.1) **(2.5 points)** Find the PMF, expectation, and variance of the length (in minutes) of Han Meimei's commute.

   2.2) **(5 points)** Given that Han Meimei's commute took her at most 19 minutes, what is the expected number of red lights that she encountered?

   2.3) **(2.5 points)** Given that Han Meimei encountered a total of three red lights, what is the probability that exactly two out of the first three lights were red?

3. Rolling dice: Two fair four-sided tetrahedral dice are rolled simultaneously. Let the random variable $\boldsymbol{X}$ be the absolute difference of the two rolls.

3.1) **(5 points)** Calculate the PMF, the expected value, and the variance of $\boldsymbol{X}$.

3.2) **(5 points)** Calculate and plot the PMF of $\boldsymbol{X}^2$.

4. Achieving a High Score: Suppose that a person plays a game in which his score must be one of the 50 numbers 1, 2, 3, ..., 50 and that each of these 50 numbers is equally likely to be his score. The first time he plays the game, his score is $\boldsymbol{X}$. He then continues to play the game until he obtains another score $\boldsymbol{Y}$, such that $\boldsymbol{Y} \geqslant \boldsymbol{X}$. It may be assumed that all plays of the game are independent.

4.1) **(5 points)** List all possible values that $\boldsymbol{Y}$ can take.

4.2) **(5 points)** What is the probability that $\boldsymbol{Y}$ takes the value 50 ?

4.3) **(5 points)** Compute the PMF of $\boldsymbol{Y}$.

5. Going To The Lab: Han Meimei lives in an old town without real-time traffic information. Her lab is located inside a city nearby. Han Meimei usually checks the traffic report in the city to decide how she is going to the lab. If the report is **"traffic jam"**, she will always ride her bike. If the report is **"no traffic jam"**, she will always drive her car. Now, we know that if the report is **"traffic jam"**, the probability of actually having a traffic jam is 0.8. On the other hand, if the report is **"no traffic jam"**, the probability of actually having a traffic jam is 0.1. On *Monday* and *Friday*, the report is **"traffic jam"** 70% of the time and on other days it is 20%.

5.1) **(7.5 points)** One day, Han Meimei missed the report and there is a traffic jam in the city. What is the probability that the report was **"traffic jam"** if it was on Monday or Friday? What is the probability that the report was **"traffic jam"** if it was on other days of the week?

5.2) **(7.5 points)** The probability of Han Meimei missing the report is equal to 0.2 on any day of the week. If she misses the report, Han Meimei will flip a fair coin to decide whether to ride her bike or drive her car. Let event A = { Han Meimei is riding her bike }, and event B = { The report is **"no traffic jam"** }. Are events A and B independent? Does your answer depend on the days of the week?

5.3) **(7.5 points)** Han Meimei is riding a bike and there is no traffic jam. What is the probability that she saw the report? Does it depend on the days of the week?

Han Meimei's university encourages low carbon travel. If she rides her bike to the lab, she gets 5 points on that day; if she drives her car to the lab, she gets 0 points on that day. Let the random variable $\boldsymbol{X}$ represent the points that Han Meimei gets on a day.

5.4) **(2.5 points)** Which of the following statements are correct about the random variable $\boldsymbol{X}$? Put a ✓ in front of the statements that you think are correct.

[ ] $\boldsymbol{X}$ is a discrete random variable
[ ] $\boldsymbol{X}$ is a Bernoulli random variable
[ ] $\boldsymbol{X}$ is a Binomial random variable
[ ] $\boldsymbol{X}$ is a continuous random variable

5.5) **(7.5 points)** Compute the **PMF** of $\boldsymbol{X}$.

5.6) **(7.5 points)** Compute $E[\boldsymbol{X}]$ and $var(\boldsymbol{X})$