Lecture 20 Confidence Interval For The Variance

BIO210 Biostatistics

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Interval Estimation For The Variance

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$ Not normally distributed!

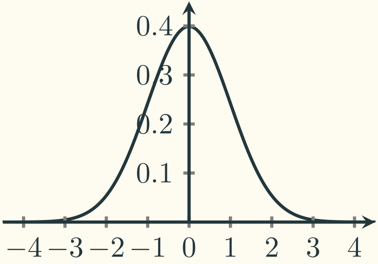
Sampling Distribution of The Sample Variance

$mean = 0$

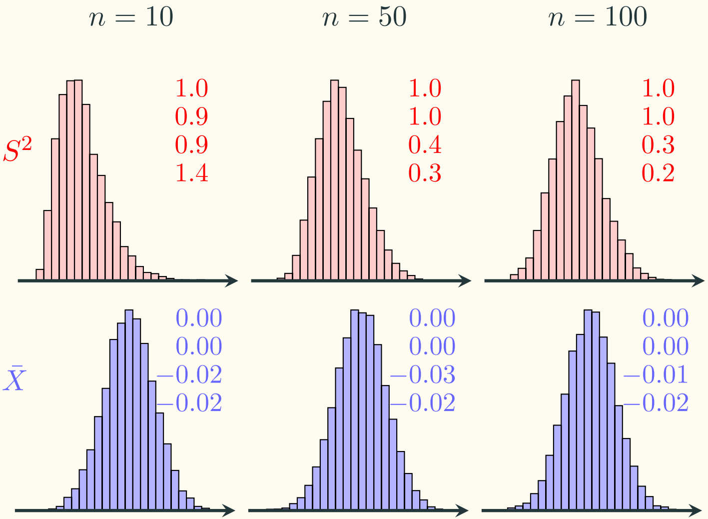
$median = 0$

$skew = 0$

$kurtosis = 0$

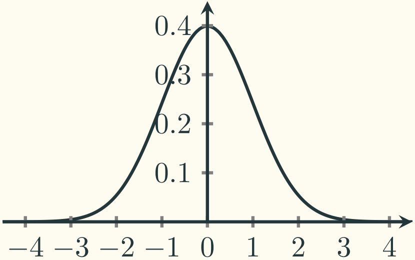


Population: $\mathcal{N}(0, 1)$



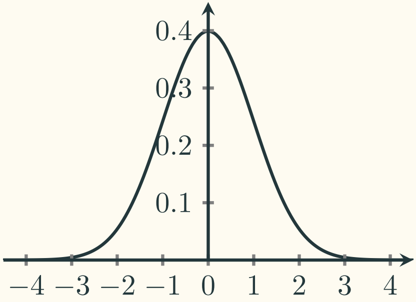
Chi-square Distribution

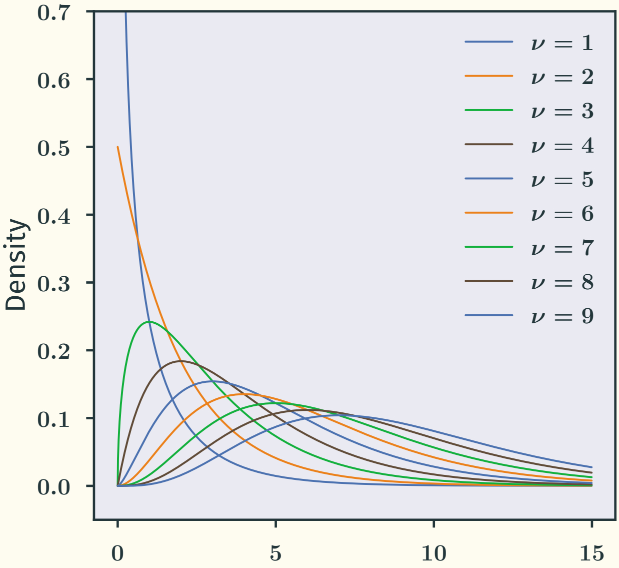
$$Q_1, Q_2, Q_3, ..., Q_n \sim \mathcal{N}(0,1)$$



$$\begin{aligned}
G_1 &= Q_1^2 & G_1 &\sim \chi_1^2 \\
G_2 &= Q_1^2 + Q_2^2 & G_2 &\sim \chi_2^2 \\
G_3 &= Q_1^2 + Q_2^2 + Q_3^2 & G_3 &\sim \chi_3^2 \\
&\vdots & &\vdots \\
G_n &= Q_1^2 + Q_2^2 + Q_3^2 + \cdots Q_n^2 & G_n &\sim \chi_n^2
\end{aligned}$$

Chi-square Distributions of Different ν





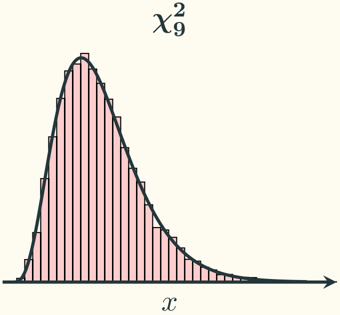
Sampling Distribution of The Sample Variance

Let $X_1, X_2, \ldots, X_n$ be a sample independently drawn from a normal distribution $\mathcal{N}(\mu, \sigma)$, and we define the sample mean and variance as:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i\,, \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

Then we have:

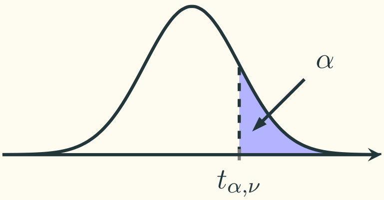
$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

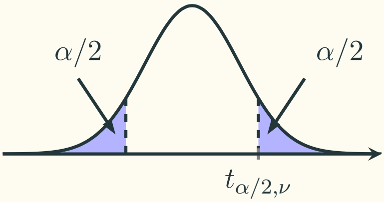


Confidence Interval For The Mean

Recall: construct $(1-\alpha)\times 100\%$ confidence interval for the mean.

- One-sided (lower bound): $P\left(\mu \geqslant \bar{X} - t_{\alpha,\nu}\cdot\frac{s}{\sqrt{n}}\right) = 1-\alpha$
- One-sided (upper bound): $P\left(\mu \leqslant \bar{X} + t_{\alpha,\nu}\cdot\frac{s}{\sqrt{n}}\right) = 1-\alpha$
- Two-sided: $P\left(\bar{X} - t_{\alpha/2,\nu}\cdot\frac{s}{\sqrt{n}} \leqslant \mu \leqslant \bar{X} + t_{\alpha/2,\nu}\cdot\frac{s}{\sqrt{n}}\right) = 1-\alpha$





Confidence Interval For Variance

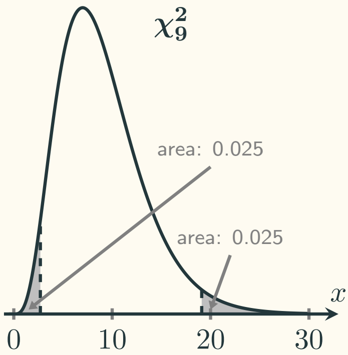
Goal: for a population with unknown variance σ^2, find a and b, such that $P(a \leqslant \sigma^2 \leqslant b) = 0.95$

$$P(\chi^2_{0.975,\nu} \leqslant \chi^2 \leqslant \chi^2_{0.025,\nu}) = 0.95$$

$$P\left[\chi^2_{0.975,\nu} \leqslant \frac{(n-1)S^2}{\sigma^2} \leqslant \chi^2_{0.025,\nu}\right] = 0.95$$

$$P\left[\frac{1}{\chi^2_{0.025,\nu}} \leqslant \frac{\sigma^2}{(n-1)S^2} \leqslant \frac{1}{\chi^2_{0.975,\nu}}\right] = 0.95$$

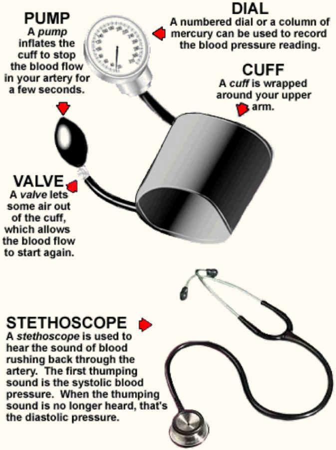
$$P\left(\frac{n-1}{\chi^2_{0.025,\nu}} \cdot S^2 \leqslant \sigma^2 \leqslant \frac{n-1}{\chi^2_{0.975,\nu}} \cdot S^2\right) = 0.95$$



95% CI for σ^2 :

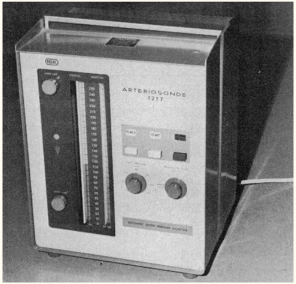
$$\left(\frac{n-1}{\chi^2_{0.025,\nu}} \cdot S^2 , \frac{n-1}{\chi^2_{0.975,\nu}} \cdot S^2\right)$$

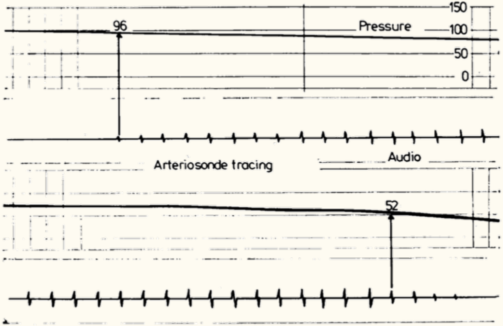
Sphygmomanometer



Confidence interval for variance

Hypertension: An Arteriosonde machine "prints" blood-pressure readings on a tape so that the measurement can be read rather than heard. A major argument for using such a machine is that the variability of measurements obtained by different observers on the same person will be lower than with a standard blood-pressure cuff.





Confidence interval for variance

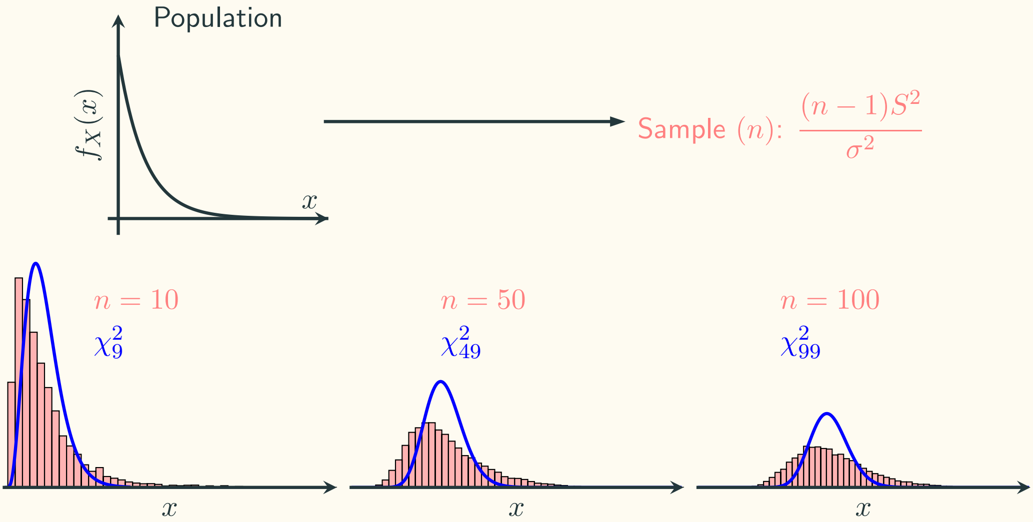
Persion (i)	Observer #1	Observer #2	Difference (d_i)
1	194	200	-6
2	126	123	3
3	130	128	2
4	98	101	-3
5	136	135	1
6	145	145	0
7	110	111	-1
8	108	107	1
9	102	99	3
10	126	128	-2

Does it make sense to construct a 95% CI for the variance?

95% CI for σ^2 :

$$\left(\frac{n-1}{\chi^2_{0.025,\nu}} \cdot S^2 , \frac{n-1}{\chi^2_{0.975,\nu}} \cdot S^2\right)$$

Sampling Distribution of The Sample Variance



Conditions For Valid Confidence Intervals For The Variance

1. Random Samples
2. Original population distribution must be normal
3. Independence ($n < 10\%$ population size)