# Lecture 27 Compare Two Populations - Proportion

BIO210 Biostatistics

Xi Chen

Spring, 2022

School of Life Sciences
Southern University of Science and Technology

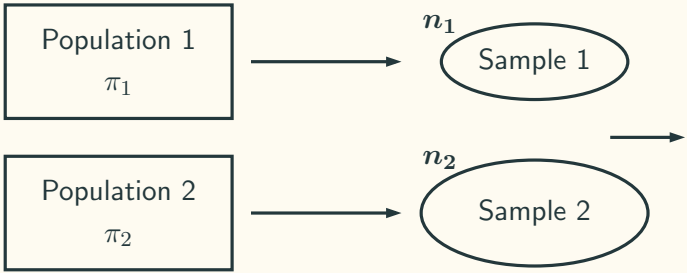
南方科技大学生命科学学院
SUSTech · SCHOOL OF
LIFE SCIENCES

# Compare two proportions

Whether the proportions of colour blindness are the same in two different populations (e.g. male vs female, Asian vs European) ?

Whether chemical A is better than chemical B for culturing cells in petri dishes (can be measured by percentage of cells that express Oct4) ?

Whether drug A is more efficient than drug B in terms of curing a certain disease (can be measured by percentage of cured patients) ?




$$H_0 :$$
$$\pi_1 = \pi_2,\ \pi_1 \geqslant \pi_2,\ \pi_1 \leqslant \pi_2$$
$$H_1 :$$
$$\pi_1 \neq \pi_2,\ \pi_1 < \pi_2,\ \pi_1 > \pi_2$$

*Clinical Infectious Diseases*

BRIEF REPORT

# Relationship Between the ABO Blood Group and the Coronavirus Disease 2019 (COVID-19) Susceptibility

**Jiao Zhao,[1,a] Yan Yang,[2,a] Hanping Huang,[3,a] Dong Li,[4,a] Dongfeng Gu,[1] Xiangfeng Lu,[5] Zheng Zhang,[2] Lei Liu,[2] Ting Liu,[3] Yukun Liu,[6] Yunjiao He,[1] Bin Sun,[1] Meilan Wei,[1] Guangyu Yang,[7,b] Xinghuan Wang,[8,b] Li Zhang,[3,b] Xiaoyang Zhou,[4,b] Mingzhao Xing,[1,b] and Peng George Wang[1,b]**

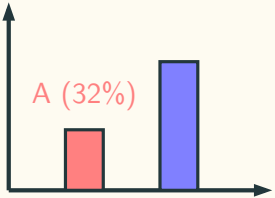
[1]School of Medicine, The Southern University of Science and Technology, Shenzhen,

# Type A blood in normal people and COVID-19 patients

$n_1 = 3694$

Normal Population

$\pi_1$

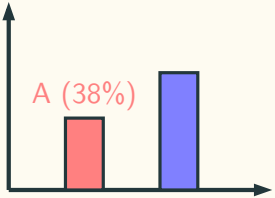



Is $\pi_1$ different from $\pi_2$ ?

$n_2 = 1775$

COVID-19 Population

$\pi_2$




$H_0 : \ \pi_1 = \pi_2$

$H_1 : \ \pi_1 \neq \pi_2$

# Strategy 1: Use One-sample Hypothesis Testing ??

**Two choices:**

- $H_0: \ \pi_1 = 0.38$

  $H_1: \ \pi_1 \neq 0.38$
- $H_0: \ \pi_2 = 0.32$

  $H_1: \ \pi_2 \neq 0.32$

**Two anwsers:**

- $z = -7.5$

  $p = 6.4 \times 10^{-14}$
- $z = 4.4$

  $p = 1.1 \times 10^{-5}$

# Strategy 2: Figure Out The Sampling Distribution of The Difference

- Let the random variable $\boldsymbol{P_1}$ represnt the proportion of blood type A in a sample ($n_1 = 3694$) drawn from normal people.
- Let the random variable $\boldsymbol{P_2}$ represnt the proportion of blood type A in a sample ($n_2 = 1775$) drawn from COVID-19 patients.

Normal
$\pi_1$

COVID-19
$\pi_2$

$$P_1 \sim \mathcal{N}\left(\mu_p = \pi_1,\, \sigma_p = \sqrt{\frac{\pi_1(1-\pi_1)}{n_1}}\right)$$

$$P_2 \sim \mathcal{N}\left(\mu_p = \pi_2,\, \sigma_p = \sqrt{\frac{\pi_2(1-\pi_2)}{n_2}}\right)$$

$$\boldsymbol{\delta = \pi_1 - \pi_2}$$

$$\boldsymbol{D} = P_1 - P_2$$

$$\boldsymbol{D} \sim \textbf{?}$$

# Sampling Distribution of The Difference of The Sample Proportion

**Question:** Two **r.v.** $P_1$ and $P_2$ are independent, $D = P_1 - P_2$, $D \sim ?(?, ?)$

$$
\begin{aligned}
E[D] &= E[P_1 - P_2] = E[P_1] - E[P_2] = \pi_1 - \pi_2 \\
var(D) &= var(P_1 - P_2) = var[P_1 + (-1) \cdot P_2] \\
&= var(P_1) + var[(-1) \cdot P_2] \\
&= var(P_1) + (-1)^2 \cdot var(P_2) \\
&= var(P_1) + var(P_2) \\
&= \frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2}
\end{aligned}
$$

$$
\boldsymbol{D} \sim \mathcal{N}\left(\pi_1 - \pi_2, \frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2}\right)
$$

**Extra Reading:**

- A linear function of a normal r.v. is still a normal r.v.
- The sum of two independent normal r.v. is also a normal r.v.

# Sampling Distribution of The Difference of The Sample Proportion

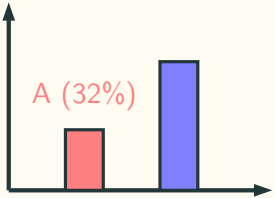
- $D \sim \mathcal{N}\left(\pi_1 - \pi_2, \dfrac{\pi_1(1-\pi_1)}{n_1} + \dfrac{\pi_2(1-\pi_2)}{n_2}\right)$
- $D = P_1 - P_2$ and $d = p_1 - p_2$ are the point estimator/estimate of $\delta$
- 95% CI: $(p_1 - p_2) \pm 1.96\sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$

# Type A blood in normal people and COVID-19 patients

$n_1 = 3694$

Normal Population

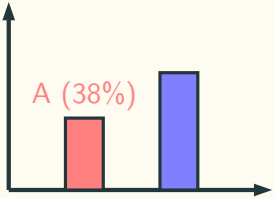
$\pi_1$




$n_2 = 1775$

COVID-19 Population

$\pi_2$




Is $\pi_1$ different from $\pi_2$ ?

$$H_0 : \ \pi_1 = \pi_2$$
$$H_1 : \ \pi_1 \neq \pi_2$$

$$H_0 : \ \delta = \pi_1 - \pi_2 = 0$$
$$H_1 : \ \delta = \pi_1 - \pi_2 \neq 0$$

# Two-sample Hypothesis Testing For Proportion

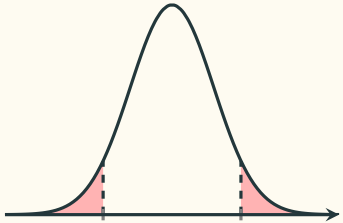
$$H_0: \ \delta = \pi_1 - \pi_2 = 0$$
$$H_1: \ \delta = \pi_1 - \pi_2 \neq 0$$
$$D \sim \mathcal{N}\left(\pi_1 - \pi_2, \ \frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2}\right)$$

if $H_0$ were true

$$D \sim \mathcal{N}\left(0, \left(\frac{1}{n_1} + \frac{1}{n_2}\right)\pi(1-\pi)\right)$$

1. What we observe is: $d = p_1 - p_2$
2. What is the probability of observing $d$ or more extreme?

$$z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\pi(1-\pi)}} = \frac{p_1 - p_2}{\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\pi(1-\pi)}}$$



What is the best estimate for $\pi$

# Two-sample Hypothesis Testing For Proportion

| | **Normal** | **COVID-19** |
|---|---|---|
| A | $a$ | $b$ |
| Non-A | $c$ | $d$ |
| **Total** | $n_1$ | $n_2$ |

Sample size: bigger is always better:

$$\pi : \frac{a+b}{n_1+n_2} = \frac{n_1 p_1 + n_2 p_2}{n_1+n_2} = p$$

The test statistic:

$$z = \frac{p_1 - p_2}{\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right) p(1-p)}}$$

**The test statistic:**

$$p = \frac{1188+670}{3694+1775} = 0.34,\ z = \frac{0.32-0.38}{\sqrt{\left(\frac{1}{3694} + \frac{1}{1775}\right) \times 0.34 \times 0.66}} = -4.4$$

# Example: Two-sample Hypothesis Testing For Proportion

**Myopia:** Researchers suspect that myopia, or nearsightedness, is becoming more common over time. A study from the year 2000 showed $139$ cases of myopia in $420$ randomly selected people. A separate study from 2015 showed $228$ cases in $600$ randomly selected people. Perform a hypothesis testing to see if the researchers' suspicion is true or not.

**Sample statistics:** $n_1 = 420$, $p_1 = \dfrac{139}{420} = 0.33$, $n_2 = 600$, $p_2 = \dfrac{228}{600} = 0.38$

**Pooled estimate for $\pi$:** $p = \dfrac{139 + 228}{420 + 600} = 0.36$

**The test statistics:** $z = \dfrac{p_1 - p_2}{\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right) p(1-p)}} = \dfrac{0.32 - 0.38}{\sqrt{\left(\frac{1}{420} + \frac{1}{600}\right) \times 0.36 \times 0.64}}$