Lecture 40 Nonparametric Methods

BIO210 Biostatistics

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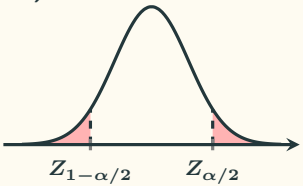
Parametric Tests

1. Specify what you are comparing.
2. Formulate hypotheses
3. Check assumptions
4. Determine significance level α
5. Compute the test statistic
6. Check significance
7. Make a decision about whether to reject H_0
8. Interpret findings

6a)



6b) Calculate the p-value.

6c) Construct $(1-\alpha)\times 100\%$ confidence interval to see if it covers the H_0 value.

Nonparametric Methods

Nonparametric tests do not rely on data following specific distribution ($e.g.$ normal). They are also called distribution-free methods and are often used when the assumptions of parametric tests are violated.

- Wilcoxon, Frank (1945) Individual comparisons by ranking methods. Biometrics Bulletin. 1 (6): 80–83.
- Wilcoxon Sign Test
- Wilcoxon Signed-Rank Test
- Wilcoxon Rank Sum Test (Mann-Whitney U Test)

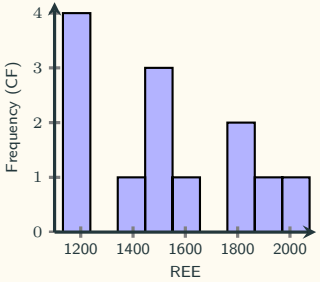
Nonparametric Test 1 - Wilcoxon Sign Test

Wilcoxon Sign Test:
A test for median of paired data.

- Compute the difference
- Record the sign
- Remove zeros and update sample size
- Record the number of positive signs (D)
- Use D to calculate the $p-$value

Resting energy expenditure (REE) for patients with cystic fibrosis and healthy individuals matched on age, sex, height and weight.

Pair	REE (kcal/day) CF	Healthy	Difference	Sign
1	1153	996	157	+
2	1132	1080	52	+
3	1165	1182	-17	-
4	1460	1452	8	+
5	1634	1162	472	+
6	1493	1619	-126	-
7	1358	1140	218	+
8	1453	1123	330	+
9	1185	1113	72	+
10	1824	1463	361	+
11	1793	1632	161	+
12	1930	1614	316	+
13	2075	1836	239	+



Nonparametric Test 1 - Wilcoxon Sign Test

1. The null/alternative hypotheses:
   - H_0: no difference in REE between CF and healthy people
   - H_1: there is a difference
2. If H_0 were true, we should expect similar number of "+" and "-" signs.
3. Under H_0:

$$D \sim B(n, 0.5) \begin{cases} D \sim \mathcal{N}\left(\frac{n}{2}, \frac{n}{4}\right), & \text{if the sample size is large enough} \\ & (i.e. \text{ both } D \text{ and } n - D \text{ are more than } 10) \\ D \sim B(n, 0.5), & \text{otherwise} \end{cases}$$

Nonparametric Test 2 - Wilcoxon Signed-Rank Test

Wilcoxon Signed-Rank Test: a test for median of paired data, but taking into account the magnitude of the difference.

Compute the difference

↓

Rank the difference by the absolute values (the smallest to the largest)

↓

Update the sign of the rank by the sign of the difference

↓

Sum the positive and negative ranks, separately and let T be the smaller sum of rank, ignoring the sign

↓

Use T to calculate the $p-$value

Pair	REE (kcal/day) CF	Healthy	Difference	Rank	Signed rank +	-
1	1153	996	157	6	6	
2	1132	1080	52	3	3	
3	1165	1182	-17	2		2
4	1460	1452	8	1	1	
5	1634	1162	472	13	13	
6	1493	1619	-126	5		5
7	1358	1140	218	8	8	
8	1453	1123	330	11	11	
9	1185	1113	72	4	4	
10	1824	1463	361	12	12	
11	1793	1632	161	7	7	
12	1930	1614	316	10	10	
13	2075	1836	239	9	9	
				Sum	84	7

Nonparametric Test 2 - Wilcoxon Signed-Rank Test

1. The null/alternative hypotheses:
   - H_0: no difference in REE between CF and healthy people
   - H_1: there is a difference
2. If H_0 were true, we should expect similar number of "+" and "-" signs, and the absolute values of the sum of positive ranks and the sum of negative ranks should be comparable.
3. Under H_0:

$$Z_T = \frac{T - \mu_T}{\sigma_T} \sim \mathcal{N}(0, 1)$$

where $\mu_T = \frac{n(n+1)}{4}$, and $\sigma_T = \sqrt{\frac{n(n+1)(2n+1)}{24}}$

Nonparametric Test 3 - Wilcoxon Rank Sum Test

- **Wilcoxon Rank Sum Test (Mann–Whitney U Test)**

- A test for median of two independent samples, taking into account the magnitude of the difference.

- Data: Normalised mental age (nMA) in two populations of children suffering from phenylketonuria (unable to metabolise phenylalanine). It has been suggested that an elevated level of serum phenylalanine increases a child's likelihood of mental deficiency.

Low Exposure				High Exposure			
34.5	47.5	54.0	37.5	28.0	45.5	52.0	35.0
48.7	54.0	39.5	49.0	46.0	53.0	37.0	48.0
55.0	40.0	51.0	56.5	53.0	37.0	48.3	54.0
45.5	51.0	57.0	47.0	43.5	48.7	54.0	44.0
52.0	58.5	47.0	53.0	51.0	55.0		
58.5							

- **Wilcoxon Rank Sum Test**

- Step 1: Treat two samples as one, rank by the magnitude (smallest as rank 1) while keep tracking the source of the data.

Data:	28.0	34.5	35.0	37.0	37.0	37.5	39.5	40.0	43.5	44.0	45.5	45.5	46.0
Rank:	1	2	3	4	5	6	7	8	9	10	11	12	13
Data:	47.0	47.0	47.5	48.0	48.3	48.7	48.7	49.0	51.0	51.0	51.0	52.0	52.0
Rank:	14	15	16	17	18	19	20	21	22	23	24	25	26
Data:	53.0	53.0	53.0	54.0	54.0	54.0	54.0	55.0	55.0	56.5	57.0	58.5	58.5
Rank:	27	28	29	30	31	32	33	34	35	36	37	38	39

- **Wilcoxon Rank Sum Test**

- Step 2: Identify tied values, and update the rank with the average of their ranks.

Data:	28.0	34.5	35.0	37.0	37.0	37.5	39.5	40.0	43.5	44.0	45.5	45.5	46.0
Rank:	1	2	3	4.5	4.5	6	7	8	9	10	11.5	11.5	13
Data:	47.0	47.0	47.5	48.0	48.3	48.7	48.7	49.0	51.0	51.0	51.0	52.0	52.0
Rank:	14.5	14.5	16	17	18	19.5	19.5	21	23	23	23	25.5	25.5
Data:	53.0	53.0	53.0	54.0	54.0	54.0	54.0	55.0	55.0	56.5	57.0	58.5	58.5
Rank:	28	28	28	31.5	31.5	31.5	31.5	34.5	34.5	36	37	38.5	38.5

- **Wilcoxon Rank Sum Test**

- Step 3: Sum the ranks in each group separately, and let W be the smaller sum:

Rank Sum (Low Exposure): 467
Rank Sum (High Exposure): 313
$\Rightarrow W = 313$

- Step 4: Compute the test statistics:

$$Z_W = \frac{W - \mu_W}{\sigma_W}$$

where $\mu_W = \frac{n_S(n_S + n_L + 1)}{2}$, and $\sigma_W = \sqrt{\frac{n_S n_L (n_S + n_L + 1)}{12}}$

n_S: is the sample size of the group that has the smaller rank sum

n_L: is the sample size of the group that has the larger rank sum

- **Wilcoxon Rank Sum Test**

- Step 5: calculate the $p-$value:

  5.1) The null/alternative hypotheses:
  H_0: no difference in nMA medians between low and high phenylalanine exposure
  H_1: there is a difference

  5.2) If H_0 were true, we should expect the ranks to be distributed randomly between the two groups. Therefore, the average ranks for each sample should be approximately equal.

  5.3) Under H_0:

$$Z_W \sim \mathcal{N}(0, 1)$$

  5.4) The $p-$value can be calculated using the above property.

Nonparametric Methods

- Advantages: fewer assumptions, population distribution don't need to be normally distributed, test statistics based on ranks are easier to calculate.

- Disadvantages: less power, need larger sample size, not considering all information ($i.e.$ only ranks).