Lecture 6 The Bayes' Theorem

BIO210 Biostatistics

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Spring, 2022

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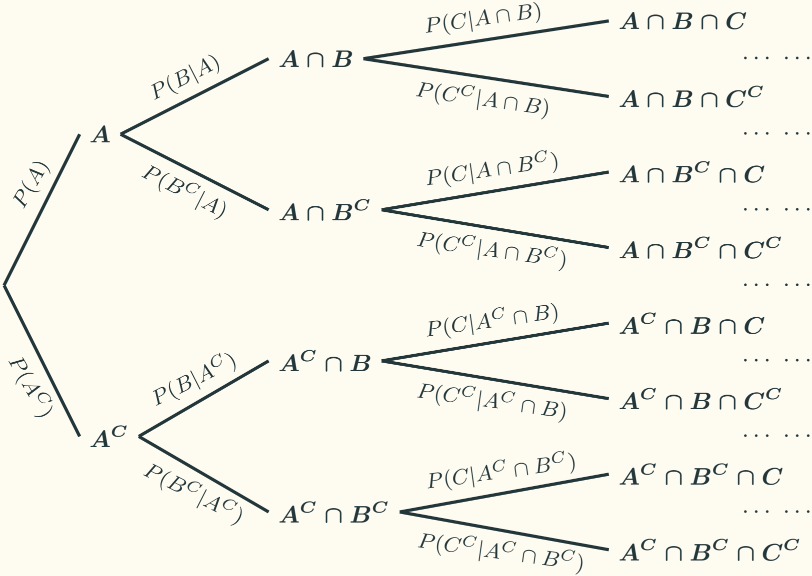
Basic components

Three basic components in conditional probability:

1. $P(A \cap B)$
2. $P(B)$
3. $P(A|B)$

Generalisation of $P(A \cap B \cap C \cap \cdots)$



The multiplication rule:

$$P\left(\bigcap_{i=1}^{n} A_i\right) =$$

$$P(A_1)\cdot$$

$$P(A_2|A_1)\cdot$$

$$P(A_3|A_1 \cap A_2)\cdot$$

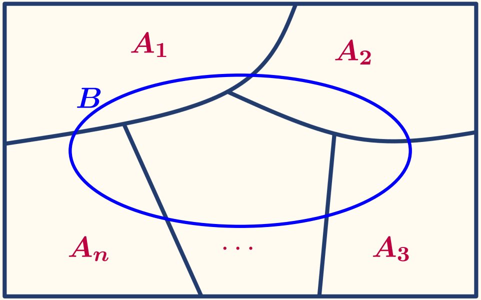
$$P(A_4|A_1 \cap A_2 \cap A_3)\cdot$$

$$\cdots$$

$$P\left(A_n|\bigcap_{i=1}^{n-1} A_i\right)$$

Generalisation of $P(B)$

Sample space Ω



$$A_i \cap A_j = \emptyset\,,\ \forall\ i \neq j,\ \ i,j = 1,2,3,\cdots,n$$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = 1$$

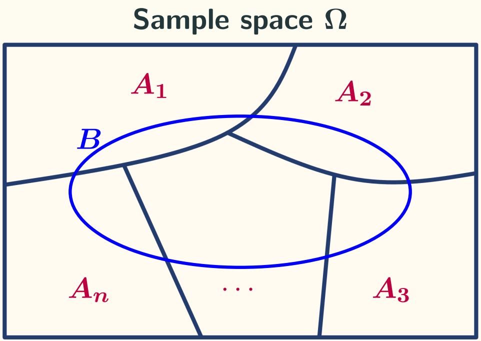
$P(A_i),\ P(B|A_i)$ are given

$$\begin{aligned}
P(B) =& P[(A_1 \cap B) \cup (A_2 \cap B) \cup \cdots \cup (A_n \cap B)] \\
=& P(A_1 \cap B) + P(A_2 \cap B) + \cdots + P(A_n \cap B) \\
=& P(A_1)P(B|A_1) + \\
& P(A_2)P(B|A_2) + \\
& \vdots \\
& P(A_n)P(B|A_n)
\end{aligned}$$

The total probability theorem:

$$P(B) = \sum_{i=1}^{n} P(B \cap A_i) = \sum_{i=1}^{n} P(A_i)P(B|A_i)$$

Generalisation of $P(A|B)$



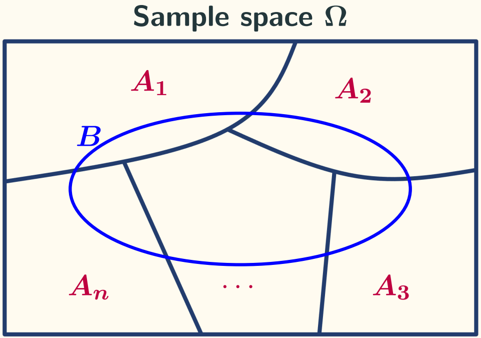
$A_i \cap A_j = \emptyset\,,\ \forall\ i \neq j,\ \ i, j = 1, 2, 3, \cdots, n$

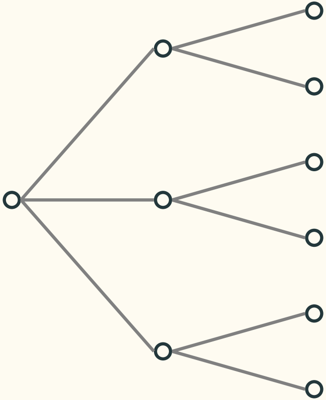
$P\left(\bigcup_{i=1}^{n} A_i\right) = 1$

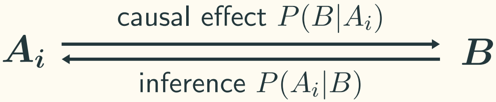
$P(A_i),\ \ P(B|A_i)$ are given

$$
\begin{aligned}
P(A_i|B) &= \frac{P(A_i \cap B)}{P(B)} \\
&= \frac{P(A_i)P(B|A_i)}{P(B)} \\
&= \frac{P(A_i)P(B|A_i)}{\sum_{i=1}^{n} P(A_i)P(B|A_i)}
\end{aligned}
$$

The Bayes Theorem







The Bayes' Theorem

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(A_i)P(B|A_i)}{\sum_{i=1}^{n} P(A_i)P(B|A_i)}$$

$\boldsymbol{P(A_i)}$: **prior probability**

$\boldsymbol{P(A_i|B)}$: **posterior probability**

Carroll's Pillow Problem #5

There is a ball inside a non-transparent bag. The colour of the ball is unknown, but it is equally likely to be either blue or red. Now you put a red ball into the bag, shake the bag, and take a ball without looking inside. The ball you have just taken out is red. What is the probability that the colour of the remaining ball that is still inside the bag is red?

Practice

- It is known that 1% of people in general carries the virus V. A company has a kit to detect this specific virus. It is known that if a person does carry the virus, there are 99% of the chance that the kit will show a positive result; if a person does not carry the virus, there are 98% of the chance that the kit will show a negative result. Based on previous experience, if a person has a stomach ache, the probability of the person carrying the virus is 5%. Now, Li Ming has a stomach ache and goes to the hospital to have the test by the kit. The test shows a positive result. Given those information, what is the probability of Li Ming actually carrying the virus V?