Lecture 9 Counting - basic principle, permutations, combinations & partitions

BIO210 Biostatistics

Xi Chen

Spring, 2024

School of Life Sciences
Southern University of Science and Technology

南方科技大学生命科学学院
SUSTech · SCHOOL OF
LIFE SCIENCES

Counting

Aims

- **Recap from the math class**
- **Be familiar with the notations**

Principles of counting

- Basic principles of counting
- permutations/k-permutations
- combinations
- partitions

Discrete Uniform Law

- **Let all outcomes be equally likely**
- **Then**:

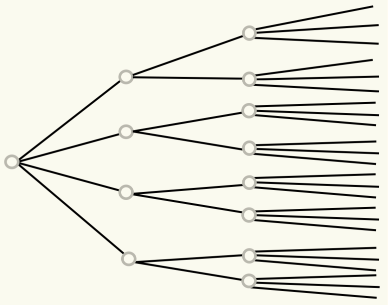
$$\mathbb{P}(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes}} = \frac{|A|}{|\Omega|}$$

$$|A|, |\Omega| : \text{cardinality of the set}$$

- All you need to do is: **counting** !

Basic counting principle

- **Basic scenario**: r stages, n_i choices at stage i

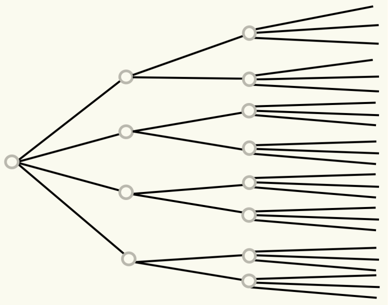


Number of choices is:

$$\prod_{i=1}^{r} n_i = n_1 \cdot n_2 \cdot n_3 \cdots n_r$$

Basic counting principle

- **Basic scenario**: r stages, n_i choices at stage i



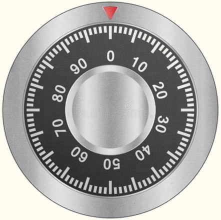
Number of choices is:

$$\prod_{i=1}^{r} n_i = n_1 \cdot n_2 \cdot n_3 \cdots n_r$$

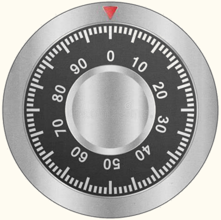
1. ***k*-permutations**: Number of ways of ordering k elements chosen from n elements: $n \cdot (n-1) \cdot (n-2) \cdots (n-k+1) = n!/(n-k)!$
2. **Permutations**: Number of ways of ordering n elements is: $n!$
3. Number of subsets of n elements: 2^n

Richard Feynman, The Safecracker

Richard Feynman, The Safecracker



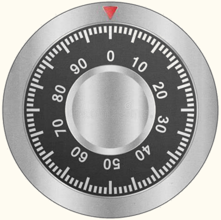
Richard Feynman, The Safecracker



Feynman wanted to brutal force the problem, then

- What is the total number of combinations for a three-number password?

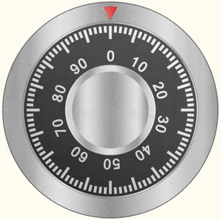
Richard Feynman, The Safecracker



Feynman wanted to brutal force the problem, then

- What is the total number of combinations for a three-number password?
- He found out the dial was not mechanically perfect: ± 2 also works. What is the minimum number of trials to iterate all combinations?

Richard Feynman, The Safecracker



Feynman wanted to brutal force the problem, then

- What is the total number of combinations for a three-number password?
- He found out the dial was not mechanically perfect: ± 2 also works. What is the minimum number of trials to iterate all combinations?
- He noticed people used dates as passwords. Let's assume the year was between 1900-1945. What is the minimum number of trials now?

Example: DNA hexamer

Letter	Base
A	Adenine
C	Cytosine
G	Guanine
T	Thymine
R	A or G
Y	C or T
S	G or C
W	A or T
K	G or T
M	A or C
B	C or G or T
D	A or G or T
H	A or C or T
V	A or C or G
N	any base

- DNA synthesis: how many different sequences can a **random hexamer** represent?
- How many of them satisfy the requirement that two adjacent bases cannot be the same?

The Birthday Problem

- What is chance that in a group of 25 randomly selected people two or more will be found to share the same birthday?

The Birthday Problem

- What is chance that in a group of 25 randomly selected people two or more will be found to share the same birthday?

- $|\Omega| = ?$
- $A = \{$ two or more will be found to share the same birthday $\}$. What is $|A|$?

The Birthday Problem

- What is chance that in a group of 25 randomly selected people two or more will be found to share the same birthday?

- $|\Omega| = ?$
- $A =$ { two or more will be found to share the same birthday }. What is $|A|$?

$$\underbrace{\boxed{}\boxed{}\boxed{}\boxed{\cdots\quad\cdots\quad\cdots}\boxed{}\boxed{}\boxed{}}_{\text{25 people}}$$

The Birthday Problem

- What is chance that in a group of 25 randomly selected people two or more will be found to share the same birthday?

- $|\Omega| = ?$
- $A =$ { two or more will be found to share the same birthday }. What is $|A|$?

$$\underbrace{\boxed{}\,\boxed{}\,\boxed{}\;\cdots\quad\cdots\quad\cdots\;\boxed{}\,\boxed{}\,\boxed{}}_{\text{25 people}}$$

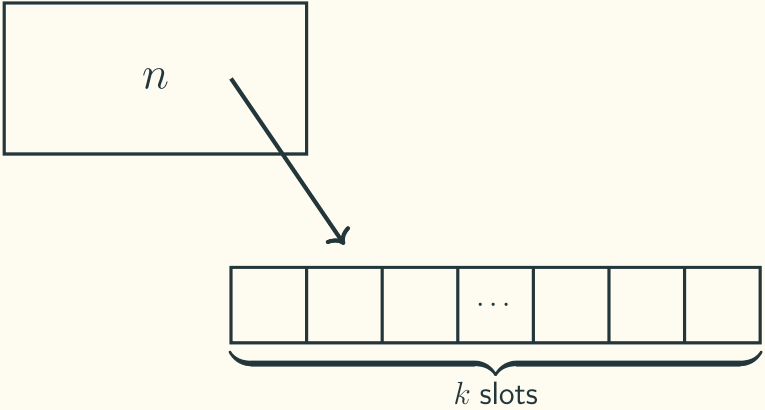
- $A^C =$ { all of them have distinct birthday }. What is $|A^C|$?

Combinations

- Number of k-element subsets of a given n-element set - how many ways are there of choosing k-elements from an n-element set without replacement.

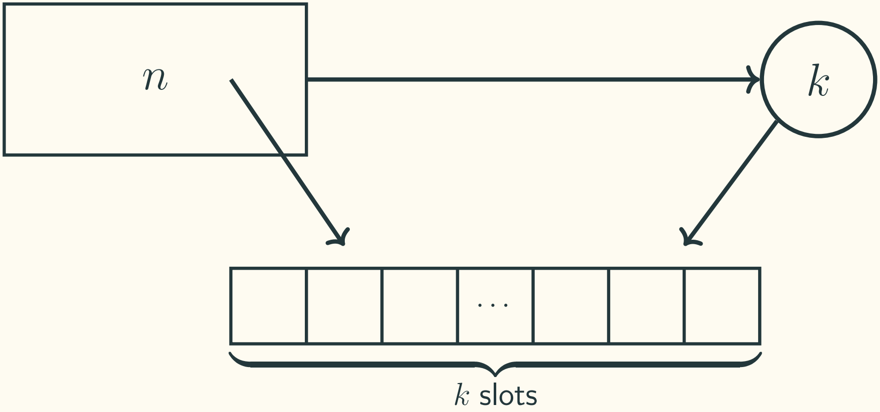
Combinations

- Number of k-element subsets of a given n-element set - how many ways are there of choosing k-elements from an n-element set without replacement.



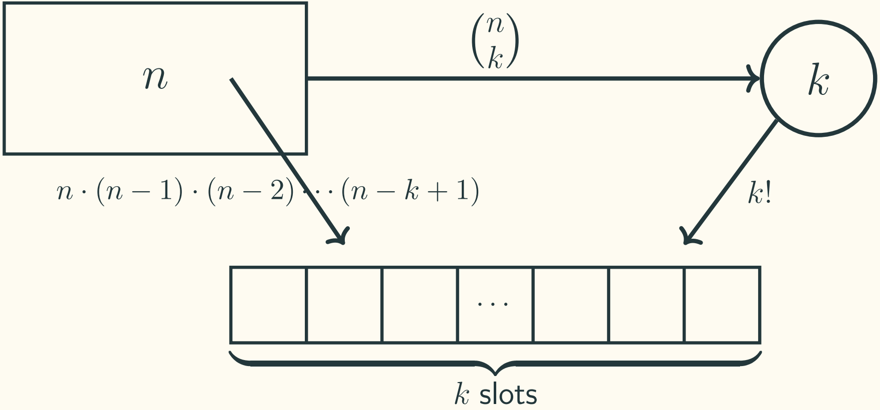
Combinations

- Number of k-element subsets of a given n-element set - how many ways are there of choosing k-elements from an n-element set without replacement.



Combinations

- Number of k-element subsets of a given n-element set - how many ways are there of choosing k-elements from an n-element set without replacement.



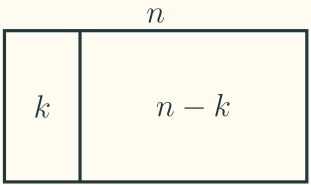
Coin flip example

- Event $A = \{$ 3 out of 10 independent flips were Hs $\}$
- Given that A has occurred, what is the conditional probability that the first two flips were Hs?

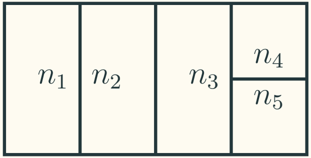
Coin flip example

- Event $A = \{$ 3 out of 10 independent flips were Hs $\}$
- Given that A has occurred, what is the conditional probability that the first two flips were Hs?

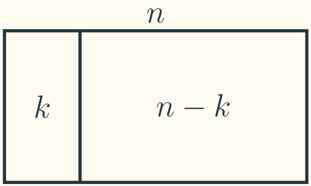
Partitions



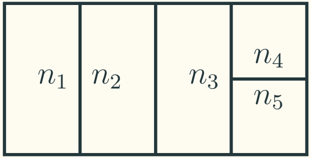
$$n = n_1 + n_2 + n_3 + n_4 + n_5$$



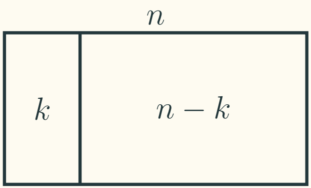
Partitions



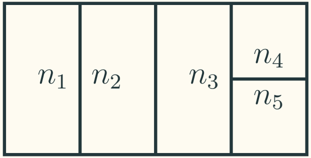
$$n = n_1 + n_2 + n_3 + n_4 + n_5$$



$$|\Omega| = \frac{n!}{n_1!n_2!n_3!n_4!n_5!}$$



$$n = n_1 + n_2 + n_3 + n_4 + n_5$$



$$|\Omega| = \frac{n!}{n_1!n_2!n_3!n_4!n_5!}$$

Example (ABO blood groups): In a room with 100 people, we know that there are 20 people with blood type A, 10 with B, 20 with AB and 50 with O, but we don't have the information of the blood type of each individual person.

1. How many total possible observations are there?
2. If we only know Adam, Bob, Charlie and Dave have four different blood types, how many total possible observations are there for all people in the room?