Lecture 24 Hypothesis Testing Terms

BIO210 Biostatistics

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Hypothesis Testing

Logic:

Some expectations:

- parameters of interest
- compare various groups

- 32% of people have blood type A
- 9% of people have blood type AB
- normal body temperature is 37 °C

→ Hypothesis →

Based on the sample you have, you come up with some hypotheses:

- The proportion of people with blood type A is more than 32%
- The proportion of people with blood type AB is not 9%
- The normal body temp. is not 37 °C

Tests:

If the opposite were true, the probability of observing ... is ...

Is the probability small or large?

The Null Hypothesis And The Alternative Hypothesis

- In inferential statistics, the null hypothesis is a general statement or default position that there is no relationship/difference/association between measured phenomena/groups.

- The alternative hypothesis is the opposite to the null hypothesis. They are collectively exhaustive and mutually exclusive.

The Null Hypothesis And The Alternative Hypothesis

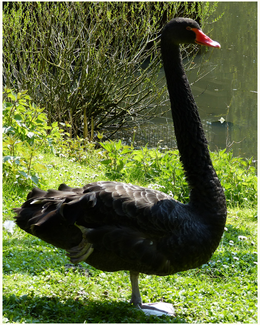
H_0	H_1 or H_a
The proportion of blood type AB in the COVID-19 patients is 0.09. $(H_0 : \pi = 0.09)$	The proportion of blood type AB in the COVID-19 patients is not 0.09. $(H_0 : \pi \neq 0.09)$
The proportion of blood type A in the COVID-19 patients is equal to or lower than 0.32. $(H_0 : \pi \leqslant 0.32)$	The proportion of blood type A in the COVID-19 patients is higher than 0.32. $(H_1 : \pi > 0.32)$
The mean body temperature of normal people is 37 °C. $(H_0 : \mu = 37)$	The mean body temperature of normal people is not 37 °C. $(H_1 : \mu \neq 37)$

The Null Hypothesis And The Alternative Hypothesis

- $\boldsymbol{H_0}$ vs $\boldsymbol{H_1}$: H_1 is the negation of H_0, and vice versa.

- Why test the null hypothesis ?
  - Scientific methods: can be falsified/disproved.
  - Introduce less bias, such as confirmation bias.
  - Practically easier.

All swans are white.



Fisher vs Neyman-Pearson

https://en.wikipedia.org/wiki/Statistical_hypothesis_testing

Hypothesis Testing

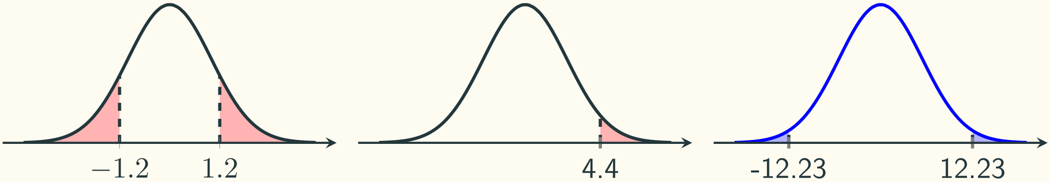
- Given that the null hypothesis is true, the probability of obtaining a measurement as extreme as or more extreme than the observed sample is called the p-value.

$$P(\text{data or more extreme} \mid H_0 \text{ is true})$$

- How do we perform the calculation ?

✓ By Calculating **the test statistic** and use the properties of the sampling distributions.

$$z = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = 1.2 \qquad z = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} \geqslant 4.4 \qquad t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = -12.23$$



Significance Test

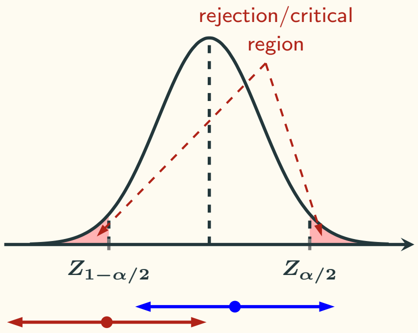
- If the p-value is **is small**, we reject the **null hypothesis**.
- How small ?

\- Ronald Fisher. **Statistical Methods for Research Workers (1925).**

only once in 370 trials, while Table II. shows that to exceed the standard deviation sixfold would need nearly a thousand million trials. The value for which P = ·05, or 1 in 20, is 1·96 or nearly 2 ; it is convenient to take this point as a limit in judging whether a deviation is to be considered significant or not. Deviations exceeding twice the standard deviation are thus formally regarded as significant. Using this criterion, we should be led to follow up a false indication only once in 22 trials, even if the statistics were the only guide available. Small effects will still escape notice *if the data are insufficiently numerous to bring them* out, but no lowering of the standard of significance would meet this difficulty.

Significance Test And Confidence Interval

- Common p-value cutoffs: 0.01, 0.05, 0.10.
- In 1933, Jerzy Neyman and Egon Pearson called those cutoffs as significance levels, denoted by α. A significance level must be decided ahead of time.
  $Z_{\alpha}, Z_{1-\alpha}, Z_{\alpha/2}, Z_{1-\alpha/2}$, are called critical values.
- When the p-value is smaller than α, we reject the null hypothesis, and we say the result is statistically significant.



$p \geqslant \alpha \Leftrightarrow$ the test statistic falls into the middle zone $\Leftrightarrow$ $(1-\alpha) \times 100\%$ covers π_0 or μ_0

$p < \alpha \Leftrightarrow$ the test statistic falls into the rejection/critical region $\Leftrightarrow (1-\alpha) \times 100\%$ does NOT cover π_0 or μ_0

Interpret The Result

- p-value $< \alpha$, reject H_0, "accept" H_1;
- p-value $\geqslant \alpha$, do not reject H_0 .

- Meanings and warnings:

  1. failing to reject H_0 does NOT mean that the null hypothesis is true. The same goes to H_1.

  2. The test is about the data, NOT your theory or hypothesis! Remember the p-value is $P(\text{data or more extreme} \mid H_0 \text{ is true})$.

courtroom trial

Innocent	Guilty
H_0	H_1
Assumed to be true until convincing evidence suggests otherwise	Accepted if H_0 is rejected by convincing evidence

Rationale of Significance Test

We assume that the POPULATION parameter of interest takes a certain value π_0 *or* μ_0

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A random sample is collected

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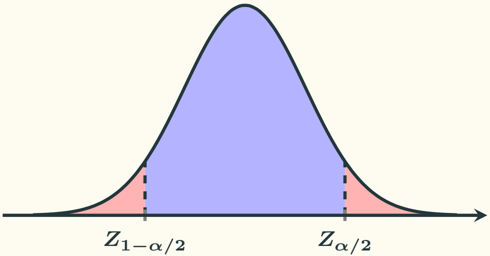
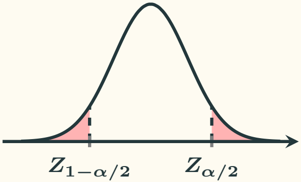
We use the properties of the sampling distribution of the sample proportion/mean to analyse the sample statistic: High chance; Low chance



A step by step hypothesis testing

1. Specify what you are comparing
2. Formulate hypotheses
3. Check assumptions
4. Determine significance level α
5. Compute the test statistic
6. Check significance
7. Make a decision about whether to reject H_0
8. Interpret findings

6a)



6b) Calculate the p-value

6c) Construct $(1-\alpha) \times 100\%$ confidence interval to see if it covers the H_0 value.