

Lecture 7 More On The Bayes' Theorem

BIO210 Biostatistics

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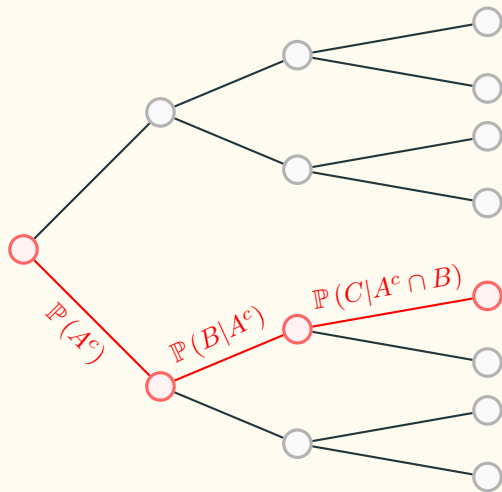


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Conditional Probability

The Multiplication Rule

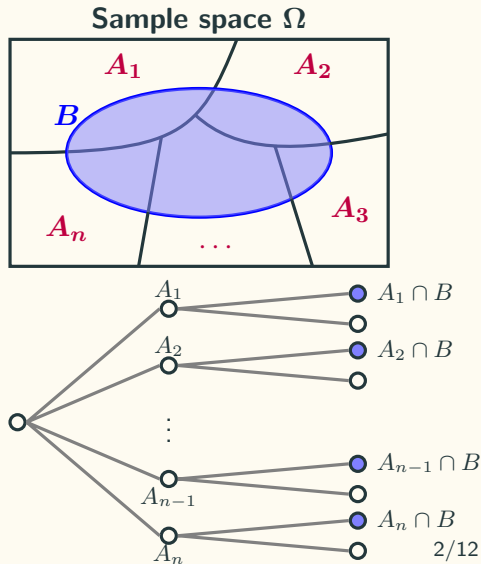
$$\begin{aligned}\mathbb{P}(\cap_{i=1}^n A_i) &= \mathbb{P}(A_1) \cdot \\ &\quad \mathbb{P}(A_2|A_1) \cdot \\ &\quad \mathbb{P}(A_3|A_1 \cap A_2) \cdot \\ &\quad \mathbb{P}(A_4|A_1 \cap A_2 \cap A_3) \cdot \\ &\quad \dots \\ &\quad \mathbb{P}(A_n|\cap_{i=1}^{n-1} A_i)\end{aligned}$$



Conditional Probability

The Total Probability Rule

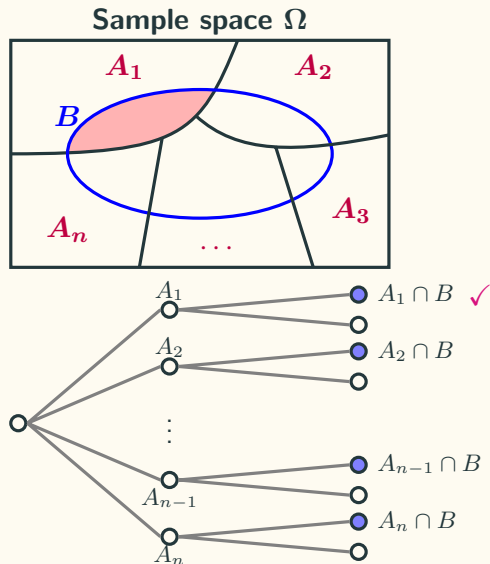
$$\begin{aligned}\mathbb{P}(B) &= \mathbb{P}[(A_1 \cap B) \cup (A_2 \cap B) \cup \dots \cup (A_n \cap B)] \\ &= \mathbb{P}(A_1 \cap B) + \mathbb{P}(A_2 \cap B) + \dots + \mathbb{P}(A_n \cap B) \\ &= \sum_{i=1}^n \mathbb{P}(A_i) \cdot \mathbb{P}(B|A_i)\end{aligned}$$



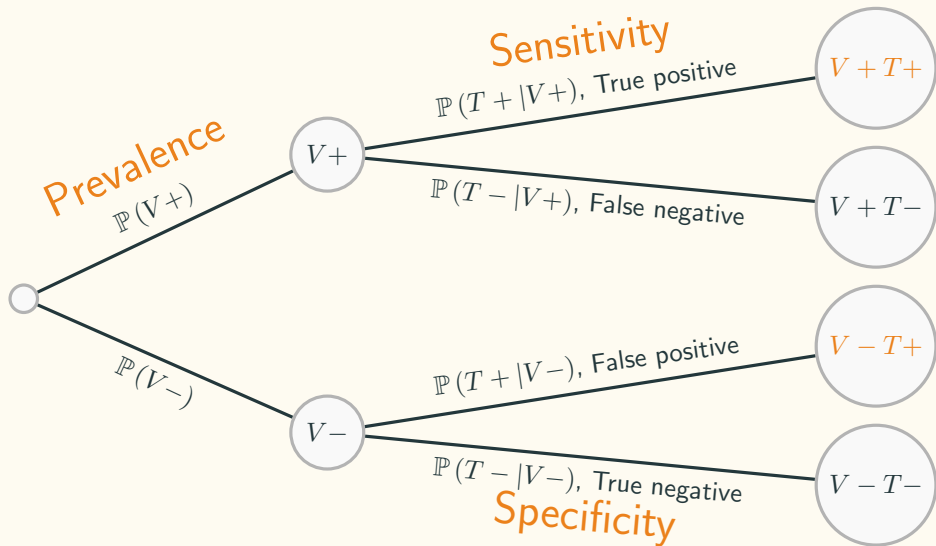
Conditional Probability

Bayes' Theorem

$$\begin{aligned}\mathbb{P}(A_i|B) &= \frac{\mathbb{P}(A_i) \cdot \mathbb{P}(B|A_i)}{\mathbb{P}(B)} \\ &= \frac{\mathbb{P}(A_i) \cdot \mathbb{P}(B|A_i)}{\sum_{i=1}^n \mathbb{P}(A_i) \cdot \mathbb{P}(B|A_i)}\end{aligned}$$



Virus Detection



ARTICLES

Judgment under Uncertainty: Heuristics and Biases

Amos Tversky¹, Daniel Kahneman¹

¹Hebrew University, Jerusalem, Israel

– Hide authors and affiliations

Science 27 Sep 1974;
Vol. 185, Issue 4157, pp. 1124-1131
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Article

Info & Metrics

eLetters

 PDF

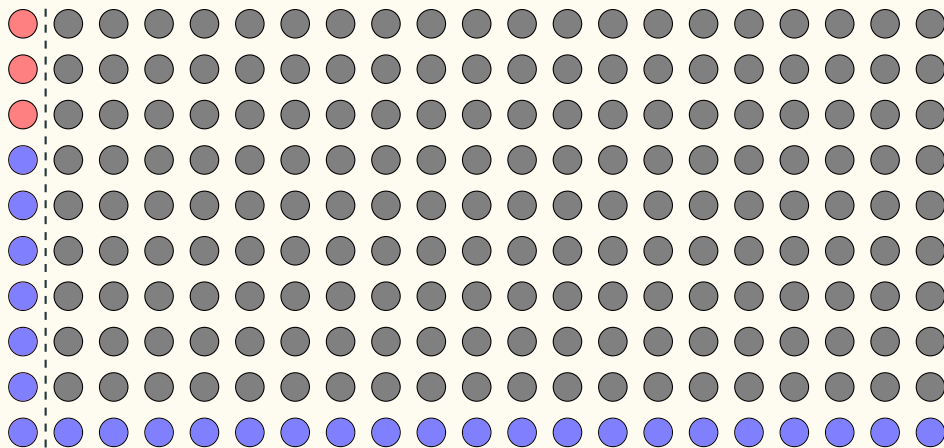
Abstract

This article described three heuristics that are employed in making judgements under uncertainty: (i) representativeness, which is usually employed when people are asked to judge the probability that an object or event A belongs to class or process B; (ii) availability of instances or scenarios, which is often employed when people are asked to assess the frequency of a class or the plausibility of a particular development; and (iii) adjustment from an anchor, which is usually employed in numerical prediction when a relevant value is available. These heuristics are highly economical and usually effective, but they lead to systematic and predictable errors. A better understanding of these heuristics and of the biases to which they lead could improve judgements and decisions in situations of uncertainty.

Amos Tversky & Daniel Kahneman

“Steve is very **shy and withdrawn**, invariably helpful, but with little interest in people, or in the world of reality. A **meek and tidy soul**, he has a need for **order and structure**, and a **passion for detail**. How do people assess the probability that Steve is engaged in a particular occupation from a list of possibilities (for example, farmer, salesman, airline pilot, librarian, or physician)?”

Who Is Steve



Librarian

Farmer

When To Use The Bayes' Theorem

You have a
hypothesis

You have observed some **evidence**

You want

The person
carries the
virus; Steve
is a librarian

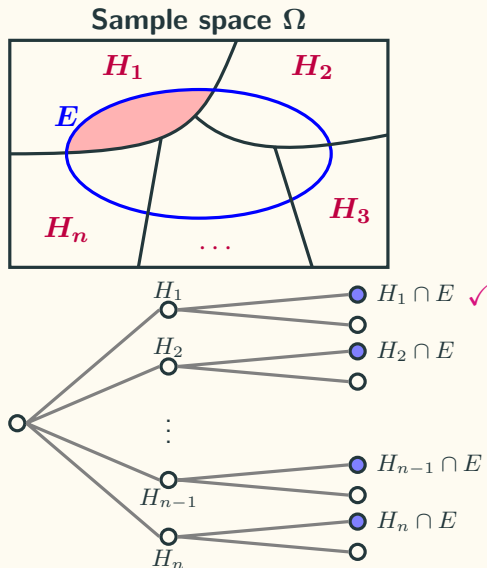
Test result is positive; Steve's characters

Probability of the
hypothesis given
the **evidence**,
 $\mathbb{P}(H|E)$

The Alternative Form Of The Bayes' Theorem

Bayes' Theorem

$$\begin{aligned}\mathbb{P}(H_i|E) &= \frac{\mathbb{P}(H_i) \cdot \mathbb{P}(E|H_i)}{\mathbb{P}(E)} \\ &= \frac{\mathbb{P}(H_i) \cdot \mathbb{P}(E|H_i)}{\sum_{i=1}^n \mathbb{P}(H_i) \cdot \mathbb{P}(E|H_i)}\end{aligned}$$



The Bayes' Theorem

$$\mathbb{P}(H_i|E) = \frac{\mathbb{P}(E|H_i)}{\sum_{i=1}^n \mathbb{P}(H_i) \cdot \mathbb{P}(E|H_i)} \cdot \mathbb{P}(H_i)$$

$\mathbb{P}(H_i)$: prior probability

$\mathbb{P}(H_i|E)$: posterior probability

The Bayesian Search

- The 4th H-bomb from American B-52 (1966)
- Air France 447 (2009 - 2011)
- Malaysian Air Flight 370 (2014 -)
- **USS Scorpion (SSN-589) (1968)**



US Navy photo #NH_97214 & 1136658

The Bayesian Search

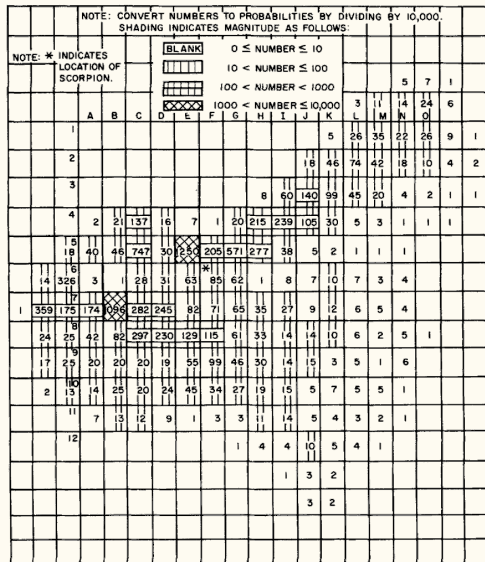
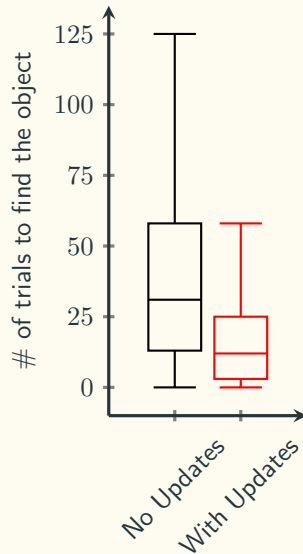


FIGURE 2. Overall A Priori distribution for *Scorpion* search

Richardson & Stone - *Operations analysis during the underwater search for Scorpions* (1971)

Simulation of The Bayesian Search

| | | |
|------|------|------|
| 0.14 | 0.07 | 0.11 |
| 0.22 | 0.00 | 0.03 |
| 0.17 | 0.21 | 0.04 |



One Simulation Result

