Lecture 7 More On The Bayes' Theorem

BIO210 Biostatistics

Xi Chen

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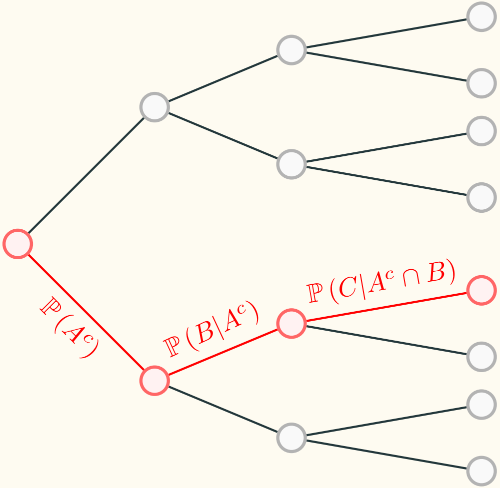
School of Life Sciences
Southern University of Science and Technology

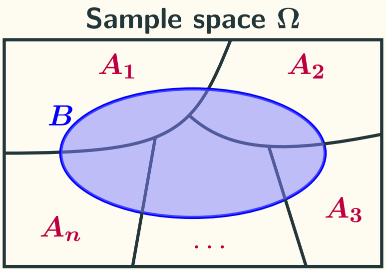
南方科技大学生命科学学院
SUSTech · SCHOOL OF
LIFE SCIENCES

The Multiplication Rule

$$
\begin{aligned}
\mathbb{P}\left(\cap_{i=1}^{n} A_i\right) = & \ \mathbb{P}\left(A_1\right) \cdot \\
& \ \mathbb{P}\left(A_2 | A_1\right) \cdot \\
& \ \mathbb{P}\left(A_3 | A_1 \cap A_2\right) \cdot \\
& \ \mathbb{P}\left(A_4 | A_1 \cap A_2 \cap A_3\right) \cdot \\
& \ \ldots \\
& \ \mathbb{P}\left(A_n | \cap_{i=1}^{n-1} A_i\right)
\end{aligned}
$$

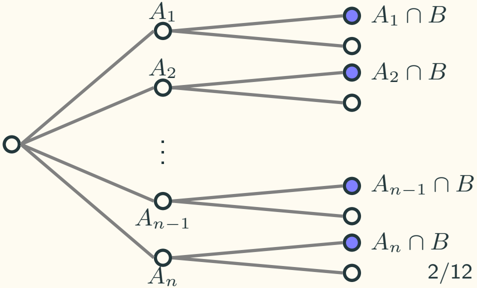


Conditional Probability



The Total Probability Rule

$$\begin{aligned}\mathbb{P}(B) &= \mathbb{P}[(A_1 \cap B) \cup (A_2 \cap B) \cup \ldots \cup (A_n \cap B)] \\ &= \mathbb{P}(A_1 \cap B) + \mathbb{P}(A_2 \cap B) + \ldots + \mathbb{P}(A_n \cap B) \\ &= \sum_{i=1}^{n} \mathbb{P}(A_i) \cdot \mathbb{P}(B|A_i)\end{aligned}$$



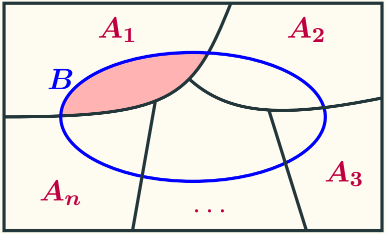
Conditional Probability

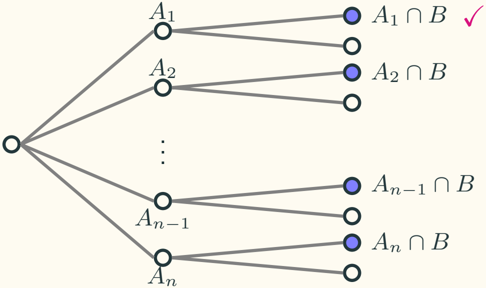
Bayes' Theorem

$$\mathbb{P}(A_i|B) = \frac{\mathbb{P}(A_i)\cdot\mathbb{P}(B|A_i)}{\mathbb{P}(B)}$$

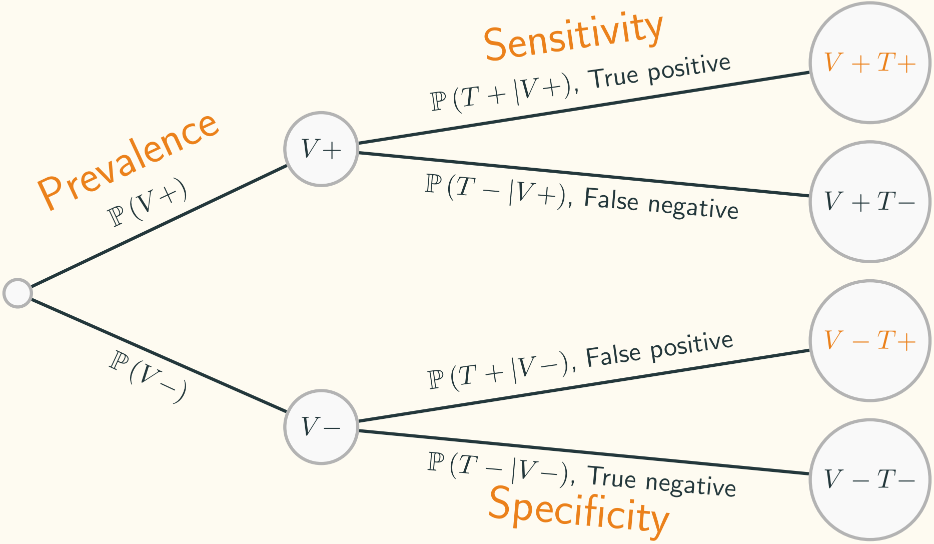
$$= \frac{\mathbb{P}(A_i)\cdot\mathbb{P}(B|A_i)}{\sum_{i=1}^{n}\mathbb{P}(A_i)\cdot\mathbb{P}(B|A_i)}$$

Sample space Ω





Virus Detection



Who Is Steve

ARTICLES

Judgment under Uncertainty: Heuristics and Biases

Amos Tversky[1], Daniel Kahneman[1]

[1]Hebrew University, Jerusalem, Israel

- Hide authors and affiliations

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Article | Info & Metrics | eLetters | PDF

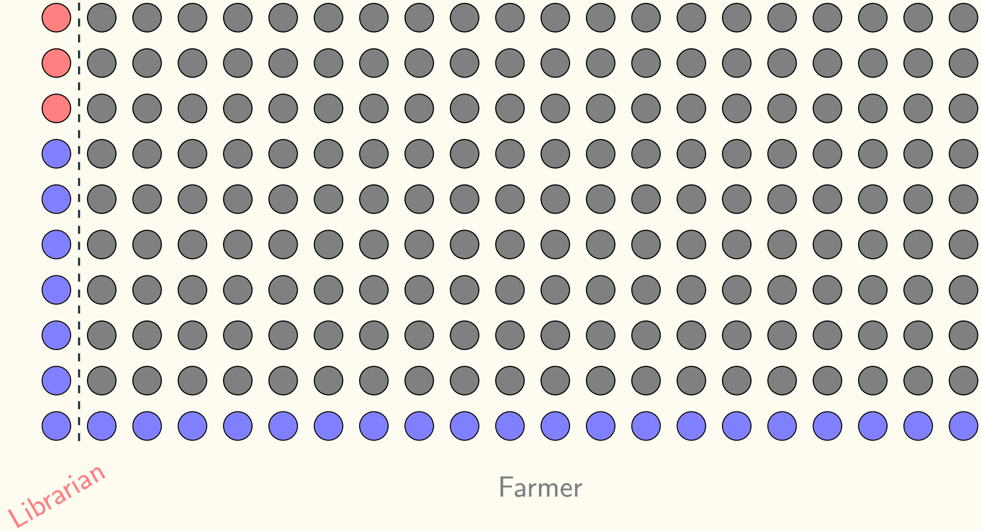
Abstract

This article described three heuristics that are employed in making judgements under uncertainty: (i) representativeness, which is usually employed when people are asked to judge the probability that an object or event A belongs to class or process B; (ii) availability of instances or scenarios, which is often employed when people are asked to assess the frequency of a class or the plausibility of a particular development; and (iii) adjustment from an anchor, which is usually employed in numerical prediction when a relevant value is available. These heuristics are highly economical and usually effective, but they lead to systematic and predictable errors. A better understanding of these heuristics and of the biases to which they lead could improve judgements and decisions in situations of uncertainty.

Amos Tversky & Daniel Kahneman

"Steve is very shy and withdrawn, invariably helpful, but with little interest in people, or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail. How do people assess the probability that Steve is engaged in a particular occupation from a list of possibilities (for example, farmer, salesman, airline pilot, librarian, or physician)?"

Who Is Steve



When To Use The Bayes' Theorem

You have a hypothesis	You have observed some evidence	You want
The person carries the virus; Steve is a librarian	Test result is positive; Steve's characters	Probability of the hypothesis given the evidence, $\mathbb{P}(H \mid E)$

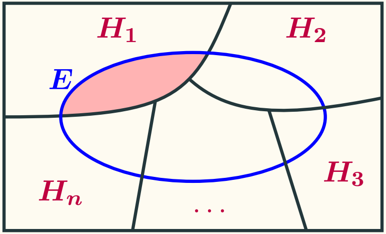
The Alternative Form Of The Bayes' Theorem

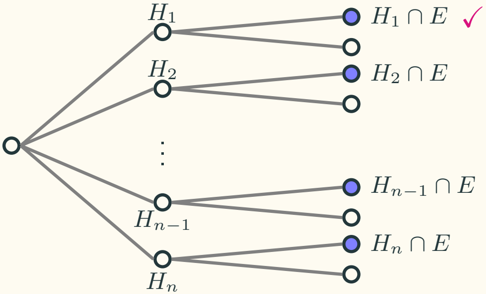
Bayes' Theorem

$$\mathbb{P}(H_i|E) = \frac{\mathbb{P}(H_i) \cdot \mathbb{P}(E|H_i)}{\mathbb{P}(E)}$$

$$= \frac{\mathbb{P}(H_i) \cdot \mathbb{P}(E|H_i)}{\sum_{i=1}^{n} \mathbb{P}(H_i) \cdot \mathbb{P}(E|H_i)}$$

Sample space Ω





The Bayes' Theorem

$$\mathbb{P}\left(H_i \middle| E\right) = \frac{\mathbb{P}\left(E \middle| H_i\right)}{\sum_{i=1}^{n} \mathbb{P}\left(H_i\right) \cdot \mathbb{P}\left(E \middle| H_i\right)} \cdot \mathbb{P}\left(H_i\right)$$

$\mathbb{P}\left(H_i\right)$: **prior probability**

$\mathbb{P}\left(H_i \middle| E\right)$: **posterior probability**

The Bayesian Search

- The 4th H-bomb from American B-52 (1966)
- Air France 447 (2009 - 2011)
- Malaysian Air Flight 370 (2014 -)
- **USS Scorpion (SSN-589)** (1968)

US Navy photo #NH_97214 & 1136658

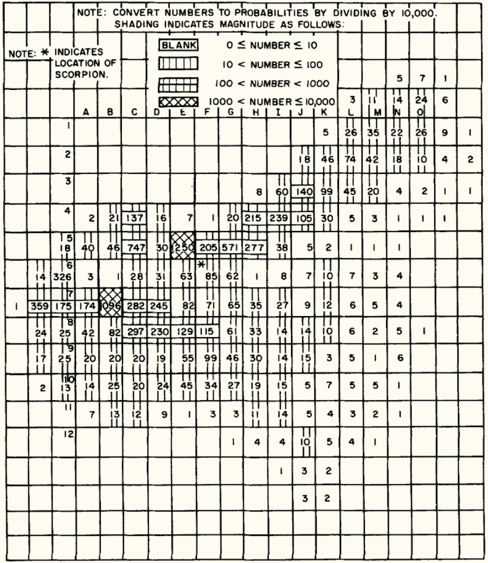
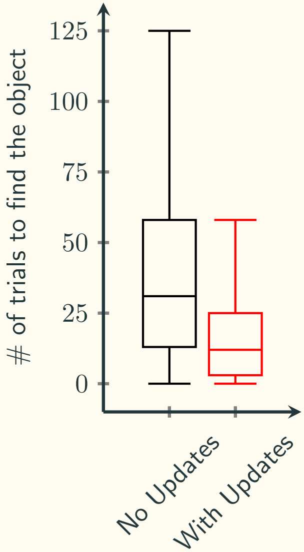


FIGURE 2. Overall A Priori distribution for *Scorpion* search

Richardson & Stone - *Operations analysis during the underwater search for Scorpions* (1971)

Simulation of The Bayesian Search

0.14	0.07	0.11
0.22	0.00	0.03
0.17	0.21	0.04



One Simulation Result

0.14	0.07	0.11
0.22	0.00	0.03
0.17	0.21	0.04

0.15	0.07	0.11
0.19	0.00	0.03
0.18	0.22	0.04

0.15	0.07	0.12
0.19	0.00	0.03
0.187	0.188	0.04

0.15	0.08	0.12
0.16	0.00	0.03
0.194	0.195	0.04

0.16	0.08	0.12
0.16	0.00	0.03
0.20	0.16	0.04

0.16	0.08	0.12
0.169	0.00	0.03
0.168	0.167	0.04