Homework 3

Combinatorics

You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in his or her own work. In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments with no questions asked**. When doing your homework, I encourage you to consult the Elements of Style for Proofs. Unless otherwise indicated, submit each of the following assignments via BbLearn by the due date. You will need to capture your handwritten work digitally and then upload a PDF to BbLearn. There are many free smartphone apps for doing this. I use TurboScan on my iPhone.

Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. However, when it comes to completing assignments for this course, you should *not* look to resources outside the context of this course for help. That is, you should not be consulting the web, other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. This includes Chegg and Course Hero. On the other hand, you may use each other, the textbook, me, and your own intuition. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.** Please read NAU's Academic Integrity Policy.

Complete the following problems.

1. Find all 231-avoiding permutations in S_5 (*Hint:* There are 42) and organize them based on the number of maximal decreasing runs.
2. Find all non-crossing partitions on 5 elements and organize them based on the number of blocks.
3. Pick any five 231-avoiding permutations from S_5 and determine which NC-partitions they map to using the bijection that I outlined in class on January 13.
4. Complete Problem 1.1.
5. Prove that the total number of compositions of n is 2^{n-1} without appealing to Problem 1.1. Try to find a bijective proof. Consider using the stones and bars model described in the solution to Problem 1.1. For example, the composition $(1,3,2)$ on $n=6$ corresponds to $\circ \mid \circ\circ\circ \mid \circ\circ$.
6. Use the previous two problems to explain why $\sum_{k=1}^{n} \binom{n-1}{k-1} = 2^{n-1}$.
7. Complete Problem 1.2 (all three parts).
8. Consider a $1 \times n$ array of the numbers 1 through n. Suppose we have tiles of size 1×1 and 1×2 such that the tiles cover exactly one and two numbers of our array, respectively. In how many ways can we tile our array?
9. Enumerate the compositions of n such that each part is odd and greater than 1.
10. Complete Problem 1.3. Do this in two different ways: Using Pascal's Recurrence vs. using Problem 1.2.