The Friendship Paradox: Your friends, on average, have more friends than you do

Math on Tap at Mother Road Brewery

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The Friendship Paradox

The Paradox

Despite your best efforts, you probably have fewer friends than most of your friends.

But you aren't special: the same is true for most of us.

The Friendship Paradox

Study of Facebook (2012)

- 721 million users ($\approx$ 10% world's population) $\leftarrow$ vertices of graph
- 69 billion friendships $\leftarrow$ edges of graph
- Researchers looked at a user's friend count & compared to that user's circle of friends
- 98% of the time: user's friend count $<$ average friend count of their friends
- Users had an average of 190 friends while their friends averaged 635 friends

It might seem counterintuitive that this is possible.

The Friendship Paradox

The findings of the Facebook study are not unusual and have been replicated in other social networks and in real life.

For any network where some people have more "friends" than others, we have the following.

Theorem (The Friendship Paradox, 1991)

The average number of friends of individuals is always less than the average number of friends of friends.

Understanding why this is true hinges on understanding a kind of "weighted average."

The Friendship Paradox

Example 1

Consider a professor that teaches two classes.

- Large class consisting of 90 students
- Small class consisting of 10 students

What is average class size for this professor?

$$\frac{90+10}{2} = 50 \text{ students per class}$$

Not wrong, but perhaps misleading...

The Friendship Paradox

Example 1 (continued)

Let's consider the student point of view: 90/100 find themselves sitting in a big class while 10/100 experience a small class.

Student-weighted average: How big is your class?

- 90 say 90
- 10 say 10

Average class size that students experience:

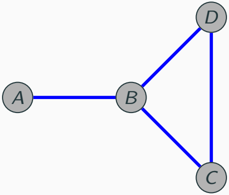
$$\frac{(90 \times 90) + (10 \times 10)}{90 + 10} = \frac{8200}{100} = 82 > 50$$

Observation: two 90s and two 10s appear in the numerator of the student-weighted average above. Two different roles: as a number being averaged and as a weight in front of that number.

The Friendship Paradox

Example 2

Consider the following friendship network.



What is average number of friends of individuals?

$$\frac{1+3+2+2}{4} = \frac{8}{4} = 2$$

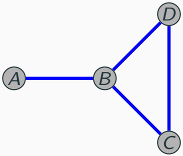
The Friendship Paradox claims that this number is smaller than the average number of friends of friends.

The Friendship Paradox

Example 2 (continued)

What is the average number of friends of friends? Ask each individual how many friends their friends have.

- A: B has a score of 3.
- B: A has a score of 1, C has a score of 2, D has a score of 2.
- C: B has a score of 3, D has a score of 2.
- D: B has a score of 3, C has a score of 2.



Average number of friends of friends:

$$\frac{3 + (1 + 2 + 2) + (3 + 2) + (3 + 2)}{8} = \frac{18}{8} = 2.25 > 2$$

The Friendship Paradox

Example 2 (continued)

Why does this happen? In addition to having a high score, popular friends contribute to other people's scores more frequently. So, popular friends like B contribute disproportionately to the average.

- A mentioned 1 time since score of 1 $\rightarrow$ contributes 1×1
- B mentioned 3 times since score of 3 $\rightarrow$ contributes 3×3
- C mentioned 2 times since score of 2 $\rightarrow$ contributes 2×2
- D mentioned 2 times since score of 2 $\rightarrow$ contributes 2×2

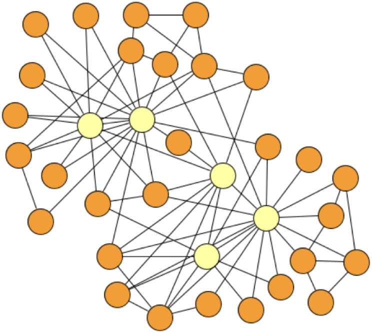
Weighted average of scores $1, 3, 2, 2$ weighted by scores themselves:

$$\frac{(1 \times 1) + (3 \times 3) + (2 \times 2) + (2 \times 2)}{1 + 3 + 2 + 2} = 2.25 > 2 = \frac{1 + 3 + 2 + 2}{1 + 1 + 1 + 1}$$

Big idea: Square terms are large!

Example 3

The following network[1] represents friendship relationships between members of a university karate club. Darker nodes represent members who observe that they are less popular than their friends are on average.



[1] Figure from Strong Friendship Paradox in Social Networks (2024).

Application

Strategy for detecting and combatting disease outbreaks.

- Monitor/immunize friends of random individuals instead of random individuals.
- One study got two-week lead time on H1N1.
- Model: If random individuals are immunized, 80–90% of population must be immunized to attain herd immunity. However, herd immunity achieved when 20–40% of friend population immunized.

There is a stronger variation of the Friendship Paradox that is a statement about the median as opposed to the mean.

Theorem (The Strong Friendship Paradox)

For most individuals, most of their friends have more friends than they do.

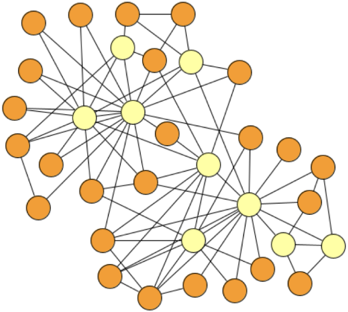
Both variations lead to what is often referred to as the Generalized Friendship Paradox, which states that on average, your friends are not only more popular than you but also richer, better looking, etc.

Example 4

For most of you, most of your lovers have had more lovers than you...

Example 5

Back to karate club network[2]. Darker nodes represent members that observe that they have fewer friends than most of their friends.



[2]Figure from Strong Friendship Paradox in Social Networks (2024).

Majority Illusion

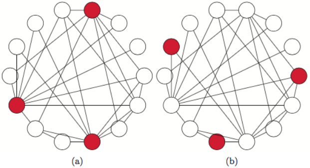
Under the right conditions: When individuals have traits/behaviors, many will observe that most of their friends have more of that trait/behavior than they do. This can lead to the Majority Illusion, in which a rare trait will appear highly prevalent within a network.

See the paper The Majority Illusion in Social Networks (2015) for an explanation of the "right conditions".

The Friendship Paradox

Example 6

Consider the following friendship network[3], where red corresponds to heavy drinker.



(a) Each of the 11 non-heavy drinkers observe that at least half of their friends are heavy drinkers, which leads them to think that heavy drinking is common. Only 3/14 of the group are heavy drinkers!

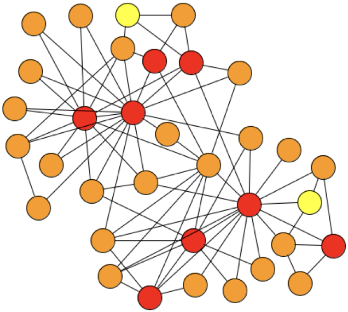
(b) Nobody in group has heavy drinkers as most of their friends.

[3]Figure from The Majority Illusion in Social Networks (2015).

The Friendship Paradox

Example 7

Back to the karate club network[4]. Although only eight of the 34 nodes are red (undesirable trait), all but two of the remaining nodes (yellow) observe that at least half of their friends are red.



[4]Figure from Strong Friendship Paradox in Social Networks (2024).

Questions

- What ramifications do the Generalized Friendship Paradox and Majority Illusion have on how individuals perceive themselves relative to "friends" on social media?
- Which scenario do you think leads to a more drastic (Strong) Friendship Paradox or Majority Illusion?
  (a) Network where all nodes have roughly the same high degree of connections.
  (b) Network where a small number of nodes have a very high degree of connections.

References

- A lot of content of my talk comes directly from Friends You Can Count On by Stephen Strogatz at NY Times Opinator.
- The Wikipedia article on the Friendship Paradox is excellent and contains a sketch of the proof of the paradox.
- The Majority Illusion in Social Networks (2015) by Lerman, Yan, Wu explores some phenomena that are related to the Friendship Paradox.
- As title suggests, Strong Friendship Paradox in Social Networks (2024) by Lerman discusses the stronger version of the Friendship Paradox.
- Social Network Sensors for Early Detection of Contagious Outbreaks (2010) by Christakis and Fowler describes how Friendship Paradox can be used to monitor and detect disease spread.

Next up at Math on Tap!

Join us on Wednesday, December 3 at 6pm for our next Math on Tap.

Spot It! - The Hidden Math Behind a Fast-Paced Card Game

Speaker: Dr. Angie Hodge-Zickerman

Abstract: Think Spot It! is just a quick reflex game? Think again! Beneath the colorful chaos of matching symbols lies some seriously cool math. In this lively, hands-on session, we'll play a few rounds, peek under the hood at how the deck is built and discover how 19th-century math puzzles inspired one of today's most popular games. No formulas required - just bring your curiosity (and maybe your competitive streak).