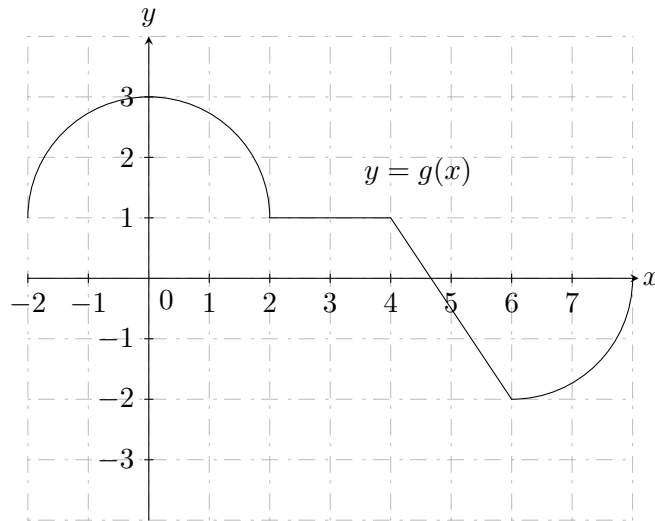


## Supplemental Problems for Final Exam

Here are some problems involving materials that we have discussed since Exam 3.

### Fundamental Theorem of Calculus

1. Explain why the Fundamental Theorem of Calculus cannot be used to evaluate  $\int_{-1}^1 \frac{1}{x^2} dx$ .
2. Consider the graph of the function  $g$  that is given below. Assume that the graph is built from line segments, semi-circles, and quarter-circles.



If  $G$  is the antiderivative of  $g$  satisfying  $G(0) = 42$ , what is  $G(4)$ ?

3. Complete the following.

(a) Let  $A(x) = \int_0^x t^2 - t \, dt$ . Find  $A'(x)$ .

(b) Let  $f(x) = \int_0^x \sqrt[3]{t^2 + 1} \, dt$ . Find  $f'(x)$ .

(c) Let  $G(x) = \int_0^{x^2} t^3 \sin(t) \, dt$ . Find  $G'(x)$ .

(d) Let  $C(x) = \int_x^{x^3} \cos(\cos(t)) \, dt$ . Find  $C'(x)$ .

4. Let  $A(x) = \int_0^x \sin^2(t) \, dt$ . Determine where  $A$  attains its absolute maximum value on  $[0, \pi]$ .

## Indefinite and Definite Integrals

Compute each of the following integrals.

5.  $\int \frac{1}{x^3} dx$

6.  $\int \frac{x+5}{x^2} dx$

7.  $\int \frac{\sin(x)}{\cos^2(x)} dx$

8.  $\int_{-1}^1 x^4 - \frac{1}{2}x^3 + \frac{1}{4}x - 2 dx$

9.  $\int_0^\pi \cos(2x) dx$

10.  $\int_0^{\ln(2)} e^{x/3} dx$

11.  $\int_1^{e^2} \frac{x+1}{x^2} dx$

12.  $\int_1^2 \frac{x^3 - 2\sqrt{x}}{x} dx$

13.  $\int_0^{1/2} \frac{4}{\sqrt{1-x^2}} dx$

14.  $\int (3x-1)^{99} dx$

15.  $\int 5x^2 \sqrt{x^3-2} dx$

16.  $\int_0^2 xe^{x^2} dx$

17.  $\int \sin^2(x) \cos(x) dx$

18.  $\int_0^1 \frac{x}{x^2+1} dx$

19.  $\int x^2 \sec^2(x^3) dx$

20.  $\int \frac{x}{x^4+1} dx$

21.  $\int x\sqrt{x-1} dx$

22.  $\int \frac{e^x}{e^x+1} dx$

23.  $\int \frac{1}{\sqrt{1-9x^2}} dx$

## Miscellaneous

24. Determine whether each of the following statements is true or false. Circle the correct answer.

(a) **True** or **False**:  $\int_a^b f(x)g(x) dx = \int_a^b f(x) dx \cdot \int_a^b g(x) dx$

(b) **True** or **False**:  $\int_a^b f(x) + g(x) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$

(c) **True** or **False**: If  $f(x) \leq g(x)$  on  $[a, b]$ , then  $\int_a^b f(x) dx \leq \int_a^b g(x) dx$ .

(d) **True** or **False**: If  $f'(x) = g'(x)$ , then  $f(x) = g(x)$ .

(e) **True** or **False**: The formula  $\int x^n dx = \frac{x^{n+1}}{n+1} + C$  works for *all* values of  $n$ .

25. Find  $f$  that satisfies  $f'(x) = \sqrt{x}$  and  $f(4) = 0$ .

26. Find  $f$  that satisfies  $f''(x) = x^2 + 4$ ,  $f'(3) = 1$ , and  $f(1) = 6$ .

27. Use basic properties of integrals to evaluate the following.

(a)  $\int_1^{11} f(x) dx$  if  $\int_0^1 f(x) dx = -7$  and  $\int_0^{11} f(x) dx = 29$

(b)  $\int_0^4 5f(x) + \sqrt{x} \, dx$  if  $\int_0^{10} f(x) \, dx = 8$  and  $\int_{10}^4 f(x) \, dx = -3$

28. Find a positive value of  $a$  such that  $\int_a^{2a} \frac{3}{4}x(x^2 - a^2)^2 \, dx = 1$ .
29. A zombie moves in a straight line with velocity  $v(t) = -t + 4$  mph after  $t$  hours of his start. How far is he from his original position after 6 hours?
30. A bungee jumper jumps off a bridge. Her downward velocity in feet per second, after  $t$  seconds of the fall, is  $v(t) = 160(1 - e^{-t/5})$ . This function is good for the first 5 seconds, after  $t = 5$  the bungee cord slows her fall. How far did she fall in those 5 seconds?