Homework 7

Combinatorics

Please review the **Rules of the Game** from the syllabus. Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. Since mathematical reasoning, problem solving, and critical thinking skills are part of the learning outcomes of this course, all assignments should be prepared by the student. Developing strong competencies in this area will prepare you to be a lifelong learner and give you an edge in a competitive workplace. When it comes to completing assignments for this course, unless explicitly told otherwise, you should *not* look to resources outside the context of this course for help. That is, you should *not* be consulting the web (e.g., Chegg and Course Hero), generative artificial intelligence tools (e.g., ChatGPT), mathematics assistive technologies (e.g., Wolfram Alpha and Photomath), other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. On the other hand, you may use each other, me, and your own intuition. You are highly encouraged to seek out assistance by asking questions on our Discord server. You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in their own work. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.**

In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments**. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Complete the following problems. Unless explicitly stated otherwise, you are expected to justify your answers.

1. Use Worpitzsky's Identity to find a closed form for $\left\langle {n \atop 3} \right\rangle$.

2. Recall the definition of Stirling numbers given on a previous homework assignment. For $k \geq 1$, let $S_k(t)$ be the exponential generating function for the number of set partitions with k blocks:
$$S_k(t) := \sum_{n \geq k} \left\{ {n \atop k} \right\} \frac{t^n}{n!}.$$
Notice that for each k, we are letting n vary. Prove that $S_k'(t) = S_{k-1}(t) + kS_k(t)$.

3. Recall the definition of the Bell numbers given on a previous homework assignment. Define the exponential generating function for the Bell numbers via
$$B(t) := \sum_{k \geq 0} B_k \frac{t^k}{k!}.$$
   (a) Prove that $B'(t) = e^t B(t)$.
   (b) Notice that the equation in part (a) is a differential equation. Taking for granted that we can formally integrate as expected, solve this differential equation to obtain a closed form for the exponential generating function for the Bell numbers.