Homework 2

Discrete Mathematics

Please review the **Rules of the Game** from the syllabus. Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. Since mathematical reasoning, problem solving, and critical thinking skills are part of the learning outcomes of this course, all assignments should be prepared by the student. Developing strong competencies in this area will prepare you to be a lifelong learner and give you an edge in a competitive workplace. When it comes to completing assignments for this course, unless explicitly told otherwise, you should *not* look to resources outside the context of this course for help. That is, you should *not* be consulting the web (e.g., Chegg and Course Hero), generative artificial intelligence tools (e.g., ChatGPT), mathematics assistive technologies (e.g., Wolfram Alpha and Photomath), other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. On the other hand, you may use each other, the textbook, me, and your own intuition.You are highly encouraged to seek out assistance by asking questions in our Q&A Discussion Board in Canvas. You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in their own work. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.**

In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments**. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Complete the following problems. Unless explicitly stated otherwise, you are expected to justify your answers. In many problems this means that you should use words to describe what you are doing and why. In other problems, simply providing sufficient arithmetic may be sufficient. If a problem asks you to count something, please $\boxed{\text{box}}$ your final answer.

1. Holly has five different mathematics books, 3 different computer science books, and 2 different history books. She will arrangement on a shelf in an order that keeps the books in the same subject together. In how many ways can this be done?

2. Recall that a **hexadecimal** is a string consisting of the symbols $0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F$. Answer each of the following. No detailed justification required.

   (a) How many hexadecimals of length 5 have at least one repeated symbol?

   (b) How many hexadecimals of length 5 have at least four consecutive 6's?

3. Consider the standard English lowercase alphabet. Recall that a,e,i,o,u are called **vowels** while the remaining letters are called **consonants**.

   (a) How many 4-letter words begin with a two consonants?

   (b) How many 3- or 4-letter words are there that do not contain the letter p?

   (c) How many 4-letter words contain exactly two p's?

   (d) How many 4-letter words contain at least two p's?

4. A **composition** of n with k parts is an ordered list of k positive integers whose sum is n, denoted $\alpha = (\alpha_1, \ldots, \alpha_k)$. We say that α_i is the ith part. How many compositions of n are there? *Hint:* Start by collecting some data and then conjecture a formula. To prove that

your proposed formula is correct, consider using a "sticks and stones" model, where the ith part consists of α_i many stones and each part is separated by a stick. For example, the composition $(1,3,2)$ on $n = 6$ corresponds to $\circ \mid \circ\circ\circ \mid \circ\circ$.

5. King Arthur and Guinevere are hosting the knights Lancelot, Gawain, Galahad, Percival, Tristan, and Iseult to dinner. The eight of them sit at the Round Table, which consists of eight seats. Rotations of the group do not constitute different seating orders.

   (a) How many ways can the eight of them be seated around the Round Table?

   (b) How many ways can the eight of them be seated around the Round Table if Arthur never sits next to Guinevere?