Homework 7

Combinatorial Game Theory

Please review the **Rules of the Game** from the syllabus. Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. Since mathematical reasoning, problem solving, and critical thinking skills are part of the learning outcomes of this course, all assignments should be prepared by the student. Developing strong competencies in this area will prepare you to be a lifelong learner and give you an edge in a competitive workplace. When it comes to completing assignments for this course, unless explicitly told otherwise, you should *not* look to resources outside the context of this course for help. That is, you should *not* be consulting the web (e.g., Chegg and Course Hero), generative artificial intelligence tools (e.g., ChatGPT), mathematics assistive technologies (e.g., Wolfram Alpha and Photomath), other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. On the other hand, you may use each other, the textbook, me, and your own intuition. You are highly encouraged to seek out assistance by asking questions on Discord. You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in their own work. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.**

In general, late homework will not be accepted. However, you are allowed to turn in **up to two late homework assignments**. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Complete the following problems. Unless explicitly stated otherwise, you are expected to justify your answers.

Note: I do realize that Problem 1 is fairly brute-force intensive, but I think the total amount of work isn't unreasonable. Please communicate with me if I'm way off here.

1. Find the simplified structure diagram for each of the following and identify the nim-value of the game.
   (a) $\mathsf{DNG}(\mathbb{Z}_8)$
   (b) $\mathsf{GEN}(\mathbb{Z}_8)$
   (c) $\mathsf{DNG}(\mathbb{Z}_{10})$
   (d) $\mathsf{GEN}(\mathbb{Z}_{10})$
   (e) $\mathsf{DNG}(\mathbb{Z}_{12})$
   (f) $\mathsf{GEN}(\mathbb{Z}_{12})$
   (g) $\mathsf{DNG}(\mathbb{Z}_{15})$
   (h) $\mathsf{GEN}(\mathbb{Z}_{15})$
   (i) $\mathsf{DNG}(D_6)$ (where D_6 is the dihedral group for the triangle)
   (j) $\mathsf{GEN}(D_6)$
   (k) $\mathsf{DNG}(D_8)$ (where D_8 is the dihedral group for the square)
   (l) $\mathsf{GEN}(D_8)$
2. Prove that if X_I is a structure class of $\mathsf{DNG}(G)$ such that I is only contained in odd maximal subgroups, then $\text{type}(X_I) = (1,1,0)$.
3. Prove that if G is a nontrivial odd finite group, then $\text{nim}(\mathsf{DNG}(G)) = 1$.