Homework 3

Discrete Mathematics

Please review the *Rules of the Game* from the syllabus. Reviewing material from previous courses and looking up definitions and theorems you may have forgotten is fair game. Since mathematical reasoning, problem solving, and critical thinking skills are part of the learning outcomes of this course, all assignments should be prepared by the student. Developing strong competencies in this area will prepare you to be a lifelong learner and give you an edge in a competitive workplace. When it comes to completing assignments for this course, unless explicitly told otherwise, you should *not* look to resources outside the context of this course for help. That is, you should *not* be consulting the web (e.g., Chegg and Course Hero), generative artificial intelligence tools (e.g., ChatGPT), mathematics assistive technologies (e.g., Wolfram Alpha and Photomath), other texts, other faculty, or students outside of our course in an attempt to find solutions to the problems you are assigned. On the other hand, you may use each other, the textbook, me, and your own intuition.You are highly encouraged to seek out assistance by asking questions in our Q&A Discussion Board in Canvas. You are allowed and encouraged to work together on homework. Yet, each student is expected to turn in their own work. **If you feel you need additional resources, please come talk to me and we will come up with an appropriate plan of action.**

In general, late homework will not be accepted. However, you are allowed to turn in **up to three late homework assignments**. Unless you have made arrangements in advance with me, homework turned in after class will be considered late.

Complete the following problems. Unless explicitly stated otherwise, you are expected to justify your answers. In many problems this means that you should use words to describe what you are doing and why. In other problems, simply providing sufficient arithmetic may be sufficient. If a problem asks you to count something, please $\boxed{\text{box}}$ your final answer.

1. Recall the definition of Sterling numbers from Homework 2. Explain why $\left\{ {n \atop n-1} \right\} = \dfrac{n(n-1)}{2}$.

2. Imagine we have $n \geq 1$ distinct balls labeled $1, 2, \ldots, n$ and $k \geq 1$ distinct buckets labeled $1, 2, \ldots, k$. How any ways can we distribute the n distinct balls into the k distinct buckets?

3. A special word has 4 capital letters and satisfies the rule that if the word has an A, the next letter to the right (if there is one) is also an A. How many special words are there?

4. In how many ways can the letters of the word FLAGSTAFF be arranged?

5. A player's tray in Scrabble© spells the word PIKACHU. The player cannot play and decides to discard 3 tiles and pick again. In how many ways can three letters be discarded?

6. A mathematics professor has 8 marker pens in her bag, and one day puts them along the bottom of the white board. How many ways can this be done if 3 are black, 2 are red, and there are 1 each of green, orange, and purple?

7. King Arthur and Guinevere are hosting the knights Lancelot, Gawain, Galahad, Percival, Tristan, and Iseult to dinner. The eight of them sit at the Round Table, which consists of eight seats. Rotations of the group do not constitute different seating orders.

   (a) How many ways can the eight of them be seated around the Round Table?

(b) How many ways can the eight of them be seated around the Round Table if Arthur never sits next to Guinevere?

8. Consider a group of 10 students, where 5 are wearing blue shirts and 5 are wearing red shirts.

   (a) How many orders are there to seat the 10 students in a row?

   (b) How many ways are there to seat them so that the students with red shirts are together and the students with blue shirt are together?

   (c) How many ways are there to see them so that the colors alternate?

   (d) How many ways are there to see them so that all the students with red shirts are together (and the blue shirts need not be)?