One iteration of 'table2_power.R'

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2022-08-12

Summary

Below is one iteration of the code from "table2_power.R" from Wang et al. (2021). We show that they use a perturbation of the *true* genotype frequencies when constructing their test statistic. Since the true genotype frequencies are never known in reality, this test statistic could never be constructed in reality.

Code

This holds the output of the simulations

```
pvalue <- c()
geno.L <- c()
lr <- c()
kff <- c()
```

This generates `prob1`

```
a <- 0.1
b <- 0.1
c <- 0.15
d <- 0.1
e <- 0.2
f <- 0.1
g <- 0.05
h <- 0.1
i <- 0.1

alpha = 0
prob0 <- c(0.1, 0.1, 0.15, 0.1, 0.2, 0.1, 0.05, 0.1, 0.1)

Q9.1 <- (a^2 * 1 + 2 * a * b * (9/16 + 1/64 * alpha^2 + 3/16 *
    alpha) + 2 * a * c * (225/784 + 9/196 * alpha^2 + 45/196 *
    alpha) + 2 * a * d * (25/196 + 225/3136 * alpha^2 + 75/392 *
    alpha) + 2 * a * e * (9/196 + 4/49 * alpha^2 + 6/49 * alpha) +
    2 * a * f * (9/784 + 225/3136 * alpha^2 + 45/784 * alpha) +
    2 * a * g * (1/784 + 9/196 * alpha^2 + 3/196 * alpha) + 2 *
    a * h * (1/64 * alpha^2) + 0 + b^2 * (81/256 + 1/4096 * alpha^4 +
    3/512 * alpha^3 + 27/512 * alpha^2 + 27/128 * alpha) + 2 *
    b * c * (2025/12544 + 9/12544 * alpha^4 + 153/12544 * alpha^3 +
    3681/50176 * alpha^2 + 2295/12544 * alpha) + 2 * b * d *
    ((225 * alpha^4)/200704 + (825 * alpha^3)/50176 + (3925 *
        alpha^2)/50176 + (825 * alpha)/6272 + 225/3136) + 2 *
    b * e * (alpha^4/784 + (27 * alpha^3)/1568 + (873 * alpha^2)/12544 +
```

```
    (243 * alpha)/3136 + 81/3136) + 2 * b * f * ((225 * alpha^4)/200704 +
    (45 * alpha^3)/3136 + (1287 * alpha^2)/25088 + (27 * alpha)/784 +
    81/12544) + 2 * b * g * ((9 * alpha^4)/12544 + (111 * alpha^3)/12544 +
    (1441 * alpha^2)/50176 + (111 * alpha)/12544 + 9/12544) +
    2 * b * h * (alpha^4/4096 + (3 * alpha^3)/1024 + (9 * alpha^2)/1024) +
    0 + c^2 * ((81 * alpha^4)/38416 + (405 * alpha^3)/19208 +
    (6075 * alpha^2)/76832 + (10125 * alpha)/76832 + 50625/614656) +
    2 * c * d * ((2025 * alpha^4)/614656 + (15525 * alpha^3)/614656 +
        (173025 * alpha^2)/2458624 + (25875 * alpha)/307328 +
        5625/153664) + 2 * c * e * ((9 * alpha^4)/2401 + (117 *
    alpha^3)/4802 + (2061 * alpha^2)/38416 + (1755 * alpha)/38416 +
    2025/153664) + 2 * c * f * ((2025 * alpha^4)/614656 + (11745 *
    alpha^3)/614656 + (84321 * alpha^2)/2458624 + (11745 * alpha)/614656 +
    2025/614656) + 2 * c * g * ((81 * alpha^4)/38416 + (27 *
    alpha^3)/2401 + (1287 * alpha^2)/76832 + (45 * alpha)/9604 +
    225/614656) + 2 * c * h * ((9 * alpha^4)/12544 + (45 * alpha^3)/12544 +
    (225 * alpha^2)/50176) + 0 + d^2 * ((50625 * alpha^4)/9834496 +
    (16875 * alpha^3)/614656 + (16875 * alpha^2)/307328 + (1875 *
    alpha)/38416 + 625/38416) + 2 * d * e * ((225 * alpha^4)/38416 +
    (1875 * alpha^3)/76832 + (22825 * alpha^2)/614656 + (1875 *
    alpha)/76832 + 225/38416) + 2 * d * f * ((50625 * alpha^4)/9834496 +
    (43875 * alpha^3)/2458624 + (51525 * alpha^2)/2458624 + (2925 *
    alpha)/307328 + 225/153664) + 2 * d * g * ((2025 * alpha^4)/614656 +
    (6075 * alpha^3)/614656 + (21825 * alpha^2)/2458624 + (675 *
    alpha)/307328 + 25/153664) + 2 * d * h * ((225 * alpha^4)/200704 +
    (75 * alpha^3)/25088 + (25 * alpha^2)/12544) + 0 + e^2 *
    ((16 * alpha^4)/2401 + (48 * alpha^3)/2401 + (54 * alpha^2)/2401 +
        (27 * alpha)/2401 + 81/38416) + 2 * e * f * ((225 * alpha^4)/38416 +
    (1035 * alpha^3)/76832 + (6921 * alpha^2)/614656 + (621 *
    alpha)/153664 + 81/153664) + 2 * e * g * ((9 * alpha^4)/2401 +
    (33 * alpha^3)/4802 + (157 * alpha^2)/38416 + (33 * alpha)/38416 +
    9/153664) + 2 * e * h * (alpha^4/784 + (3 * alpha^3)/1568 +
    (9 * alpha^2)/12544) + 0 + f^2 * ((50625 * alpha^4)/9834496 +
    (10125 * alpha^3)/1229312 + (6075 * alpha^2)/1229312 + (405 *
    alpha)/307328 + 81/614656) + 2 * f * g * ((2025 * alpha^4)/614656 +
    (2295 * alpha^3)/614656 + (3681 * alpha^2)/2458624 + (153 *
    alpha)/614656 + 9/614656) + 2 * f * h * ((225 * alpha^4)/200704 +
    (45 * alpha^3)/50176 + (9 * alpha^2)/50176) + 0 + g^2 * ((81 *
    alpha^4)/38416 + (27 * alpha^3)/19208 + (27 * alpha^2)/76832 +
    (3 * alpha)/76832 + 1/614656) + 2 * g * h * (9/12544 * alpha^4 +
    3/12544 * alpha^3 + 1/50176 * alpha^2) + 0 + h^2 * (1/4096 *
    alpha^4) + 0 + i^2 * 0)


Q8.1 <- (a^2 * 0 + 2 * a * b * (3/8 - 1/16 * alpha^2 - 5/16 *
    alpha) + 2 * a * c * (45/98 - 9/49 * alpha^2 - 27/98 * alpha) +
    2 * a * d * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) +
    2 * a * e * (12/49 - 16/49 * alpha^2 + 4/49 * alpha) + 2 *
    a * f * (45/392 - 225/784 * alpha^2 + 135/784 * alpha) +
    2 * a * g * (3/98 - 9/49 * alpha^2 + 15/98 * alpha) + 2 *
    a * h * (1/16 * alpha - 1/16 * alpha^2) + 0 + b^2 * (27/64 -
    1/512 * alpha^4 - 17/512 * alpha^3 - 45/256 * alpha^2 - 27/128 *
    alpha) + 2 * b * c * (2295/6272 - 9/1568 * alpha^4 - 423/6272 *
```

```
alpha^3 - 2763/12544 * alpha^2 - 909/12544 * alpha) + 2 *
b * d * ((-(225 * alpha^4)/25088) - (1125 * alpha^3)/12544 -
(5375 * alpha^2)/25088 + (625 * alpha)/12544 + 825/3136) +
2 * b * e * ((-alpha^4/98) - (73 * alpha^3)/784 - (549 *
    alpha^2)/3136 + (387 * alpha)/3136 + 243/1568) + 2 *
b * f * ((-(225 * alpha^4)/25088) - (1935 * alpha^3)/25088 -
(747 * alpha^2)/6272 + (855 * alpha)/6272 + 27/392) + 2 *
b * g * ((-(9 * alpha^4)/1568) - (297 * alpha^3)/6272 - (775 *
alpha^2)/12544 + (1219 * alpha)/12544 + 111/6272) + 2 * b *
h * ((-alpha^4/512) - alpha^3/64 - (9 * alpha^2)/512 + (9 *
alpha)/256) + 0 + c^2 * ((-(81 * alpha^4)/4802) - (1053 *
alpha^3)/9604 - (3645 * alpha^2)/19208 + (2025 * alpha)/38416 +
10125/38416) + 2 * c * d * ((-(2025 * alpha^4)/76832) - (38475 *
alpha^3)/307328 - (79875 * alpha^2)/614656 + (69525 * alpha)/614656 +
25875/153664) + 2 * c * e * ((-(72 * alpha^4)/2401) - (279 *
alpha^3)/2401 - (657 * alpha^2)/9604 + (2367 * alpha)/19208 +
1755/19208) + 2 * c * f * ((-(2025 * alpha^4)/76832) - (27135 *
alpha^3)/307328 - (13851 * alpha^2)/614656 + (60831 * alpha)/614656 +
11745/307328) + 2 * c * g * ((-(81 * alpha^4)/4802) - (243 *
alpha^3)/4802 + (9 * alpha^2)/19208 + (1107 * alpha)/19208 +
45/4802) + 2 * c * h * ((-(9 * alpha^4)/1568) - (99 * alpha^3)/6272 +
(45 * alpha^2)/12544 + (225 * alpha)/12544) + 0 + d^2 * ((-(50625 *
alpha^4)/1229312) - (151875 * alpha^3)/1229312 - (16875 *
alpha^2)/307328 + (9375 * alpha)/76832 + 1875/19208) + 2 *
d * e * ((-(225 * alpha^4)/4802) - (3825 * alpha^3)/38416 -
(325 * alpha^2)/153664 + (15325 * alpha)/153664 + 1875/38416) +
2 * d * f * ((-(50625 * alpha^4)/1229312) - (10125 * alpha^3)/153664 +
    (28575 * alpha^2)/1229312 + (39825 * alpha)/614656 +
    2925/153664) + 2 * d * g * ((-(2025 * alpha^4)/76832) -
(10125 * alpha^3)/307328 + (14625 * alpha^2)/614656 + (19125 *
alpha)/614656 + 675/153664) + 2 * d * h * ((-(225 * alpha^4)/25088) -
(225 * alpha^3)/25088 + (125 * alpha^2)/12544 + (25 * alpha)/3136) +
0 + e^2 * ((-(128 * alpha^4)/2401) - (160 * alpha^3)/2401 +
(72 * alpha^2)/2401 + (162 * alpha)/2401 + 54/2401) + 2 *
e * f * ((-(225 * alpha^4)/4802) - (1305 * alpha^3)/38416 +
(5499 * alpha^2)/153664 + (5679 * alpha)/153664 + 621/76832) +
2 * e * g * ((-(72 * alpha^4)/2401) - (27 * alpha^3)/2401 +
    (239 * alpha^2)/9604 + (281 * alpha)/19208 + 33/19208) +
2 * e * h * ((-alpha^4/98) - alpha^3/784 + (27 * alpha^2)/3136 +
    (9 * alpha)/3136) + 0 + f^2 * ((-(50625 * alpha^4)/1229312) -
(10125 * alpha^3)/1229312 + (18225 * alpha^2)/614656 + (5265 *
alpha)/307328 + 405/153664) + 2 * f * g * ((-(2025 * alpha^4)/76832) +
(1215 * alpha^3)/307328 + (10089 * alpha^2)/614656 + (3375 *
alpha)/614656 + 153/307328) + 2 * f * h * ((-(225 * alpha^4)/25088) +
(45 * alpha^3)/12544 + (117 * alpha^2)/25088 + (9 * alpha)/12544) +
0 + g^2 * ((-(81 * alpha^4)/4802) + (81 * alpha^3)/9604 +
(135 * alpha^2)/19208 + (51 * alpha)/38416 + 3/38416) + 2 *
g * h * (-9/1568 * alpha^4 + 27/6272 * alpha^3 + 17/12544 *
alpha^2 + 1/12544 * alpha) + 0 + h^2 * (-1/512 * alpha^4 +
1/512 * alpha^3) + 0 + i^2 * 0)


Q7.1 <- (a^2 * 0 + 2 * a * b * (1/16 + 3/32 * alpha^2 + 1/16 *
```

3

alpha) + 2 * a * c * (87/392 + 27/98 * alpha^2 - 6/49 * alpha) +
2 * a * d * (285/784 + 675/1568 * alpha^2 - 255/784 * alpha) +
2 * a * e * (41/98 + 24/49 * alpha^2 - 20/49 * alpha) + 2 *
a * f * (285/784 + 675/1568 * alpha^2 - 255/784 * alpha) +
2 * a * g * (87/392 + 27/98 * alpha^2 - 6/49 * alpha) + 2 *
a * h * (1/16 + 3/32 * alpha^2 + 1/16 * alpha) + 0 + b^2 *
(27/128 + 7/1024 * alpha^4 + 39/512 * alpha^3 + 93/512 *
    alpha^2 - 9/64 * alpha) + 2 * b * c * (3951/12544 + 9/448 *
alpha^4 + 933/6272 * alpha^3 + 2109/12544 * alpha^2 - 3033/12544 *
alpha) + 2 * b * d * ((225 * alpha^4)/7168 + (1215 * alpha^3)/6272 +
(1425 * alpha^2)/12544 - (785 * alpha)/3136 + 4465/12544) +
2 * b * e * (alpha^4/28 + (39 * alpha^3)/196 + (93 * alpha^2)/1568 -
    (585 * alpha)/3136 + 1035/3136) + 2 * b * f * ((225 *
alpha^4)/7168 + (4125 * alpha^3)/25088 + (687 * alpha^2)/25088 -
(513 * alpha)/6272 + 1557/6272) + 2 * b * g * ((9 * alpha^4)/448 +
(639 * alpha^3)/6272 + (261 * alpha^2)/12544 + (271 * alpha)/12544 +
1711/12544) + 2 * b * h * ((7 * alpha^4)/1024 + (9 * alpha^3)/256 +
(3 * alpha^2)/128 + (9 * alpha)/128 + 9/256) + 0 + c^2 *
((81 * alpha^4)/1372 + (297 * alpha^3)/1372 + (81 * alpha^2)/2744 -
    (1215 * alpha)/5488 + 7425/21952) + 2 * c * d * ((2025 *
alpha^4)/21952 + (9855 * alpha^3)/43904 - (1035 * alpha^2)/12544 -
(11625 * alpha)/87808 + 27075/87808) + 2 * c * e * ((36 *
alpha^4)/343 + (66 * alpha^3)/343 - (93 * alpha^2)/686 -
(81 * alpha)/2744 + 333/1372) + 2 * c * f * ((2025 * alpha^4)/21952 +
(6075 * alpha^3)/43904 - (1539 * alpha^2)/12544 + (4293 *
alpha)/87808 + 14013/87808) + 2 * c * g * ((81 * alpha^4)/1372 +
(27 * alpha^3)/343 - (171 * alpha^2)/2744 + (111 * alpha)/1372 +
1713/21952) + 2 * c * h * ((9 * alpha^4)/448 + (177 * alpha^3)/6272 +
(3 * alpha^2)/1792 + (765 * alpha)/12544 + 225/12544) + 0 +
d^2 * ((50625 * alpha^4)/351232 + (30375 * alpha^3)/175616 -
    (30375 * alpha^2)/175616 - (375 * alpha)/21952 + 375/1568) +
2 * d * e * ((225 * alpha^4)/1372 + (135 * alpha^3)/1372 -
    (1965 * alpha^2)/10976 + (205 * alpha)/3136 + 3595/21952) +
2 * d * f * ((50625 * alpha^4)/351232 + (3375 * alpha^3)/87808 -
    (5625 * alpha^2)/43904 + (4395 * alpha)/43904 + 1185/12544) +
2 * d * g * ((2025 * alpha^4)/21952 + (405 * alpha^3)/43904 -
    (675 * alpha^2)/12544 + (7695 * alpha)/87808 + 3555/87808) +
2 * d * h * ((225 * alpha^4)/7168 + (135 * alpha^3)/25088 +
    (15 * alpha^2)/3584 + (275 * alpha)/6272 + 25/3136) +
0 + e^2 * ((64 * alpha^4)/343 - (48 * alpha^2)/343 + (36 *
alpha)/343 + 135/1372) + 2 * e * f * ((225 * alpha^4)/1372 -
(75 * alpha^3)/1372 - (789 * alpha^2)/10976 + (45 * alpha)/448 +
1089/21952) + 2 * e * g * ((36 * alpha^4)/343 - (18 * alpha^3)/343 -
(9 * alpha^2)/686 + (185 * alpha)/2744 + 25/1372) + 2 * e *
h * (alpha^4/28 - (3 * alpha^3)/196 + (3 * alpha^2)/224 +
(81 * alpha)/3136 + 9/3136) + 0 + f^2 * ((50625 * alpha^4)/351232 -
(16875 * alpha^3)/175616 - (2025 * alpha^2)/175616 + (405 *
alpha)/5488 + 135/6272) + 2 * f * g * ((2025 * alpha^4)/21952 -
(3375 * alpha^3)/43904 + (261 * alpha^2)/12544 + (3453 *
alpha)/87808 + 573/87808) + 2 * f * h * ((225 * alpha^4)/7168 -
(75 * alpha^3)/3136 + (33 * alpha^2)/1792 + (9 * alpha)/784 +
9/12544) + 0 + g^2 * ((81 * alpha^4)/1372 - (81 * alpha^3)/1372 +
(81 * alpha^2)/2744 + (87 * alpha)/5488 + 33/21952) + 2 *

```
    g * h * (1/12544 + 9/448 * alpha^4 - 117/6272 * alpha^3 +
    27/1792 * alpha^2 + 37/12544 * alpha) + 0 + h^2 * (7/1024 *
    alpha^4 - 3/512 * alpha^3 + 3/512 * alpha^2) + 0 + i^2 *
    0)


Q6.1 <- (a^2 * 0 + 2 * a * b * (1/16 * alpha - 1/16 * alpha^2) +
    2 * a * c * (3/98 - 9/49 * alpha^2 + 15/98 * alpha) + 2 *
    a * d * (45/392 - 225/784 * alpha^2 + 135/784 * alpha) +
    2 * a * e * (12/49 - 16/49 * alpha^2 + 4/49 * alpha) + 2 *
    a * f * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) + 2 *
    a * g * (45/98 - 9/49 * alpha^2 - 27/98 * alpha) + 2 * a *
    h * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) + 0 + b^2 * (3/64 -
    7/512 * alpha^4 - 45/512 * alpha^3 - 3/128 * alpha^2 + 5/64 *
    alpha) + 2 * b * c * (405/3136 - 9/224 * alpha^4 - 999/6272 *
    alpha^3 + 807/12544 * alpha^2 + 75/12544 * alpha) + 2 * b *
    d * ((-(225 * alpha^4)/3584) - (2505 * alpha^3)/12544 + (3235 *
    alpha^2)/25088 - (1145 * alpha)/12544 + 705/3136) + 2 * b *
    e * ((-alpha^4/14) - (159 * alpha^3)/784 + (417 * alpha^2)/3136 -
    (529 * alpha)/3136 + 243/784) + 2 * b * f * ((-(225 * alpha^4)/3584) -
    (4275 * alpha^3)/25088 + (921 * alpha^2)/12544 - (39 * alpha)/196 +
    1125/3136) + 2 * b * g * ((-(9 * alpha^4)/224) - (705 * alpha^3)/6272 -
    (229 * alpha^2)/12544 - (2165 * alpha)/12544 + 1077/3136) +
    2 * b * h * ((-(7 * alpha^4)/512) - (3 * alpha^3)/64 - (39 *
        alpha^2)/512 - (25 * alpha)/256 + 15/64) + 0 + c^2 *
    ((-(81 * alpha^4)/686) - (243 * alpha^3)/1372 + (81 * alpha^2)/392 -
        (729 * alpha)/5488 + 1215/5488) + 2 * c * d * ((-(2025 *
    alpha^4)/10976) - (5805 * alpha^3)/43904 + (21465 * alpha^2)/87808 -
    (2715 * alpha)/12544 + 12675/43904) + 2 * c * e * ((-(72 *
    alpha^4)/343) - (27 * alpha^3)/343 + (267 * alpha^2)/1372 -
    (615 * alpha)/2744 + 873/2744) + 2 * c * f * ((-(2025 * alpha^4)/10976) -
    (2025 * alpha^3)/43904 + (7857 * alpha^2)/87808 - (2025 *
    alpha)/12544 + 3321/10976) + 2 * c * g * ((-(81 * alpha^4)/686) -
    (27 * alpha^3)/686 - (9 * alpha^2)/392 - (165 * alpha)/2744 +
    165/686) + 2 * c * h * ((-(9 * alpha^4)/224) - (243 * alpha^3)/6272 -
    (993 * alpha^2)/12544 + (39 * alpha)/1792 + 855/6272) + 0 +
    d^2 * ((-(50625 * alpha^4)/175616) + (3375 * alpha^3)/175616 +
        (16875 * alpha^2)/87808 - (10125 * alpha)/43904 + 3375/10976) +
    2 * d * e * ((-(225 * alpha^4)/686) + (15 * alpha^3)/112 +
        (1655 * alpha^2)/21952 - (3665 * alpha)/21952 + 3135/10976) +
    2 * d * f * ((-(50625 * alpha^4)/175616) + (3375 * alpha^3)/21952 -
        (5625 * alpha^2)/175616 - (5655 * alpha)/87808 + 2535/10976) +
    2 * d * g * ((-(2025 * alpha^4)/10976) + (3645 * alpha^3)/43904 -
        (7515 * alpha^2)/87808 + (405 * alpha)/12544 + 6795/43904) +
    2 * d * h * ((-(225 * alpha^4)/3584) - (285 * alpha^3)/25088 -
        (55 * alpha^2)/784 + (65 * alpha)/896 + 225/3136) + 0 +
    e^2 * ((-(128 * alpha^4)/343) + (96 * alpha^3)/343 - (24 *
        alpha^2)/343 - (22 * alpha)/343 + 78/343) + 2 * e * f *
    ((-(225 * alpha^4)/686) + (225 * alpha^3)/784 - (3273 * alpha^2)/21952 +
        (717 * alpha)/21952 + 54/343) + 2 * e * g * ((-(72 *
    alpha^4)/343) + (57 * alpha^3)/343 - (181 * alpha^2)/1372 +
    (239 * alpha)/2744 + 243/2744) + 2 * e * h * ((-alpha^4/14) +
    (9 * alpha^3)/784 - (159 * alpha^2)/3136 + (5 * alpha)/64 +
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```
    51/1568) + 0 + f^2 * ((-(50625 * alpha^4)/175616) + (50625 *
    alpha^3)/175616 - (2025 * alpha^2)/10976 + (2025 * alpha)/21952 +
    2025/21952) + 2 * f * g * ((-(2025 * alpha^4)/10976) + (7425 *
    alpha^3)/43904 - (11043 * alpha^2)/87808 + (177 * alpha)/1792 +
    465/10976) + 2 * f * h * ((-(225 * alpha^4)/3584) + (225 *
    alpha^3)/12544 - (633 * alpha^2)/25088 + (15 * alpha)/256 +
    9/784) + 0 + g^2 * ((-(81 * alpha^4)/686) + (135 * alpha^3)/1372 -
    (27 * alpha^2)/392 + (405 * alpha)/5488 + 81/5488) + 2 *
    g * h * (15/6272 - 9/224 * alpha^4 + 51/6272 * alpha^3 -
    13/12544 * alpha^2 + 55/1792 * alpha) + 0 + h^2 * (-7/512 *
    alpha^4 - 3/512 * alpha^3 + 3/256 * alpha^2 + 1/128 * alpha) +
    0 + i^2 * 0)


Q5.1 <- (a^2 * 0 + 2 * a * b * (1/64 * alpha^2) + 2 * a * c *
    (1/784 + 9/196 * alpha^2 + 3/196 * alpha) + 2 * a * d * (9/784 +
    225/3136 * alpha^2 + 45/784 * alpha) + 2 * a * e * (9/196 +
    4/49 * alpha^2 + 6/49 * alpha) + 2 * a * f * (25/196 + 225/3136 *
    alpha^2 + 75/392 * alpha) + 2 * a * g * (225/784 + 9/196 *
    alpha^2 + 45/196 * alpha) + 2 * a * h * (9/16 + 1/64 * alpha^2 +
    3/16 * alpha) + 2 * a * i * 1 + b^2 * (1/256 + 35/2048 *
    alpha^4 + 25/512 * alpha^3 - 27/512 * alpha^2 + 7/128 * alpha) +
    2 * b * c * (327/12544 + 45/896 * alpha^4 + 225/3136 * alpha^3 -
        2533/25088 * alpha^2 + 1149/12544 * alpha) + 2 * b *
    d * ((1125 * alpha^4)/14336 + (2025 * alpha^3)/25088 - (2623 *
    alpha^2)/25088 + (561 * alpha)/6272 + 453/6272) + 2 * b *
    e * ((5 * alpha^4)/56 + (65 * alpha^3)/784 - (369 * alpha^2)/6272 +
    (149 * alpha)/3136 + 451/3136) + 2 * b * f * ((1125 * alpha^4)/14336 +
    (2025 * alpha^3)/25088 + (527 * alpha^2)/25088 - (45 * alpha)/3136 +
    2985/12544) + 2 * b * g * ((45 * alpha^4)/896 + (225 * alpha^3)/3136 +
    (2507 * alpha^2)/25088 - (699 * alpha)/12544 + 4359/12544) +
    2 * b * h * ((35 * alpha^4)/2048 + (25 * alpha^3)/512 + (63 *
        alpha^2)/512 - alpha/64 + 59/128) + 2 * b * i * (9/16 +
    1/64 * alpha^2 + 3/16 * alpha) + c^2 * ((405 * alpha^4)/2744 -
    (513 * alpha^2)/5488 + (27 * alpha)/343 + 3429/43904) + 2 *
    c * d * ((10125 * alpha^4)/43904 - (2025 * alpha^3)/21952 -
    (2691 * alpha^2)/175616 + (675 * alpha)/87808 + 13045/87808) +
    2 * c * e * ((90 * alpha^4)/343 - (45 * alpha^3)/343 + (197 *
        alpha^2)/2744 - (117 * alpha)/1372 + 2481/10976) + 2 *
    c * f * ((10125 * alpha^4)/43904 - (2025 * alpha^3)/21952 +
    (22509 * alpha^2)/175616 - (13941 * alpha)/87808 + 26037/87808) +
    2 * c * g * ((405 * alpha^4)/2744 + (747 * alpha^2)/5488 -
        (57 * alpha)/343 + 15077/43904) + 2 * c * h * ((45 *
    alpha^4)/896 + (225 * alpha^3)/3136 + (2507 * alpha^2)/25088 -
    (699 * alpha)/12544 + 4359/12544) + 2 * c * i * (225/784 +
    9/196 * alpha^2 + 45/196 * alpha) + d^2 * ((253125 * alpha^4)/702464 -
    (50625 * alpha^3)/175616 + (24975 * alpha^2)/175616 - (675 *
    alpha)/6272 + 19575/87808) + 2 * d * e * ((1125 * alpha^4)/2744 -
    (2025 * alpha^3)/5488 + (10649 * alpha^2)/43904 - (4317 *
    alpha)/21952 + 879/3136) + 2 * d * f * ((253125 * alpha^4)/702464 -
    (50625 * alpha^3)/175616 + (40725 * alpha^2)/175616 - (345 *
    alpha)/1568 + 13529/43904) + 2 * d * g * ((10125 * alpha^4)/43904 -
    (2025 * alpha^3)/21952 + (22509 * alpha^2)/175616 - (13941 *
```

```
    alpha)/87808 + 26037/87808) + 2 * d * h * ((1125 * alpha^4)/14336 +
    (2025 * alpha^3)/25088 + (527 * alpha^2)/25088 - (45 * alpha)/3136 +
    2985/12544) + 2 * d * i * (25/196 + 225/3136 * alpha^2 +
    75/392 * alpha) + e^2 * ((160 * alpha^4)/343 - (160 * alpha^3)/343 +
    (108 * alpha^2)/343 - (82 * alpha)/343 + 821/2744) + 2 *
    e * f * ((1125 * alpha^4)/2744 - (2025 * alpha^3)/5488 +
    (10649 * alpha^2)/43904 - (4317 * alpha)/21952 + 879/3136) +
    2 * e * g * ((90 * alpha^4)/343 - (45 * alpha^3)/343 + (197 *
        alpha^2)/2744 - (117 * alpha)/1372 + 2481/10976) + 2 *
    e * h * ((5 * alpha^4)/56 + (65 * alpha^3)/784 - (369 * alpha^2)/6272 +
    (149 * alpha)/3136 + 451/3136) + 2 * e * i * (9/196 + 4/49 *
    alpha^2 + 6/49 * alpha) + f^2 * ((253125 * alpha^4)/702464 -
    (50625 * alpha^3)/175616 + (24975 * alpha^2)/175616 - (675 *
    alpha)/6272 + 19575/87808) + 2 * f * g * ((10125 * alpha^4)/43904 -
    (2025 * alpha^3)/21952 - (2691 * alpha^2)/175616 + (675 *
    alpha)/87808 + 13045/87808) + 2 * f * h * ((1125 * alpha^4)/14336 +
    (2025 * alpha^3)/25088 - (2623 * alpha^2)/25088 + (561 *
    alpha)/6272 + 453/6272) + 2 * f * i * (9/784 + 225/3136 *
    alpha^2 + 45/784 * alpha) + g^2 * ((405 * alpha^4)/2744 -
    (513 * alpha^2)/5488 + (27 * alpha)/343 + 3429/43904) + 2 *
    g * h * (327/12544 + 45/896 * alpha^4 + 225/3136 * alpha^3 -
    2533/25088 * alpha^2 + 1149/12544 * alpha) + 2 * g * i *
    (1/784 + 9/196 * alpha^2 + 3/196 * alpha) + h^2 * (1/256 +
    35/2048 * alpha^4 + 25/512 * alpha^3 - 27/512 * alpha^2 +
    7/128 * alpha) + 2 * h * i * (1/64 * alpha^2) + i^2 * 0)


Q4.1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
    0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
    h * 0 + 2 * a * i * 0 + b^2 * (-7/512 * alpha^4 - 3/512 *
    alpha^3 + 3/256 * alpha^2 + 1/128 * alpha) + 2 * b * c *
    (15/6272 - 9/224 * alpha^4 + 51/6272 * alpha^3 - 13/12544 *
        alpha^2 + 55/1792 * alpha) + 2 * b * d * ((-(225 * alpha^4)/3584) +
    (225 * alpha^3)/12544 - (633 * alpha^2)/25088 + (15 * alpha)/256 +
    9/784) + 2 * b * e * ((-alpha^4/14) + (9 * alpha^3)/784 -
    (159 * alpha^2)/3136 + (5 * alpha)/64 + 51/1568) + 2 * b *
    f * ((-(225 * alpha^4)/3584) - (285 * alpha^3)/25088 - (55 *
    alpha^2)/784 + (65 * alpha)/896 + 225/3136) + 2 * b * g *
    ((-(9 * alpha^4)/224) - (243 * alpha^3)/6272 - (993 * alpha^2)/12544 +
        (39 * alpha)/1792 + 855/6272) + 2 * b * h * ((-(7 * alpha^4)/512) -
    (3 * alpha^3)/64 - (39 * alpha^2)/512 - (25 * alpha)/256 +
    15/64) + 2 * b * i * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) +
    c^2 * ((-(81 * alpha^4)/686) + (135 * alpha^3)/1372 - (27 *
        alpha^2)/392 + (405 * alpha)/5488 + 81/5488) + 2 * c *
    d * ((-(2025 * alpha^4)/10976) + (7425 * alpha^3)/43904 -
    (11043 * alpha^2)/87808 + (177 * alpha)/1792 + 465/10976) +
    2 * c * e * ((-(72 * alpha^4)/343) + (57 * alpha^3)/343 -
        (181 * alpha^2)/1372 + (239 * alpha)/2744 + 243/2744) +
    2 * c * f * ((-(2025 * alpha^4)/10976) + (3645 * alpha^3)/43904 -
        (7515 * alpha^2)/87808 + (405 * alpha)/12544 + 6795/43904) +
    2 * c * g * ((-(81 * alpha^4)/686) - (27 * alpha^3)/686 -
        (9 * alpha^2)/392 - (165 * alpha)/2744 + 165/686) + 2 *
    c * h * ((-(9 * alpha^4)/224) - (705 * alpha^3)/6272 - (229 *
```

```
    alpha^2)/12544 - (2165 * alpha)/12544 + 1077/3136) + 2 *
    c * i * (45/98 - 9/49 * alpha^2 - 27/98 * alpha) + d^2 *
    ((-(50625 * alpha^4)/175616) + (50625 * alpha^3)/175616 -
        (2025 * alpha^2)/10976 + (2025 * alpha)/21952 + 2025/21952) +
    2 * d * e * ((-(225 * alpha^4)/686) + (225 * alpha^3)/784 -
        (3273 * alpha^2)/21952 + (717 * alpha)/21952 + 54/343) +
    2 * d * f * ((-(50625 * alpha^4)/175616) + (3375 * alpha^3)/21952 -
        (5625 * alpha^2)/175616 - (5655 * alpha)/87808 + 2535/10976) +
    2 * d * g * ((-(2025 * alpha^4)/10976) - (2025 * alpha^3)/43904 +
        (7857 * alpha^2)/87808 - (2025 * alpha)/12544 + 3321/10976) +
    2 * d * h * ((-(225 * alpha^4)/3584) - (4275 * alpha^3)/25088 +
        (921 * alpha^2)/12544 - (39 * alpha)/196 + 1125/3136) +
    2 * d * i * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) +
    e^2 * ((-(128 * alpha^4)/343) + (96 * alpha^3)/343 - (24 *
        alpha^2)/343 - (22 * alpha)/343 + 78/343) + 2 * e * f *
    ((-(225 * alpha^4)/686) + (15 * alpha^3)/112 + (1655 * alpha^2)/21952 -
        (3665 * alpha)/21952 + 3135/10976) + 2 * e * g * ((-(72 *
    alpha^4)/343) - (27 * alpha^3)/343 + (267 * alpha^2)/1372 -
    (615 * alpha)/2744 + 873/2744) + 2 * e * h * ((-alpha^4/14) -
    (159 * alpha^3)/784 + (417 * alpha^2)/3136 - (529 * alpha)/3136 +
    243/784) + 2 * e * i * (12/49 - 16/49 * alpha^2 + 4/49 *
    alpha) + f^2 * ((-(50625 * alpha^4)/175616) + (3375 * alpha^3)/175616 +
    (16875 * alpha^2)/87808 - (10125 * alpha)/43904 + 3375/10976) +
    2 * f * g * ((-(2025 * alpha^4)/10976) - (5805 * alpha^3)/43904 +
        (21465 * alpha^2)/87808 - (2715 * alpha)/12544 + 12675/43904) +
    2 * f * h * ((-(225 * alpha^4)/3584) - (2505 * alpha^3)/12544 +
        (3235 * alpha^2)/25088 - (1145 * alpha)/12544 + 705/3136) +
    2 * f * i * (45/392 - 225/784 * alpha^2 + 135/784 * alpha) +
    g^2 * ((-(81 * alpha^4)/686) - (243 * alpha^3)/1372 + (81 *
        alpha^2)/392 - (729 * alpha)/5488 + 1215/5488) + 2 *
    g * h * (405/3136 - 9/224 * alpha^4 - 999/6272 * alpha^3 +
    807/12544 * alpha^2 + 75/12544 * alpha) + 2 * g * i * (3/98 -
    9/49 * alpha^2 + 15/98 * alpha) + h^2 * (3/64 - 7/512 * alpha^4 -
    45/512 * alpha^3 - 3/128 * alpha^2 + 5/64 * alpha) + 2 *
    h * i * (1/16 * alpha - 1/16 * alpha^2) + i^2 * 0)


Q3.1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
    0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
    h * 0 + 2 * a * i * 0 + b^2 * (7/1024 * alpha^4 - 3/512 *
    alpha^3 + 3/512 * alpha^2) + 2 * b * c * (1/12544 + 9/448 *
    alpha^4 - 117/6272 * alpha^3 + 27/1792 * alpha^2 + 37/12544 *
    alpha) + 2 * b * d * ((225 * alpha^4)/7168 - (75 * alpha^3)/3136 +
    (33 * alpha^2)/1792 + (9 * alpha)/784 + 9/12544) + 2 * b *
    e * (alpha^4/28 - (3 * alpha^3)/196 + (3 * alpha^2)/224 +
    (81 * alpha)/3136 + 9/3136) + 2 * b * f * ((225 * alpha^4)/7168 +
    (135 * alpha^3)/25088 + (15 * alpha^2)/3584 + (275 * alpha)/6272 +
    25/3136) + 2 * b * g * ((9 * alpha^4)/448 + (177 * alpha^3)/6272 +
    (3 * alpha^2)/1792 + (765 * alpha)/12544 + 225/12544) + 2 *
    b * h * ((7 * alpha^4)/1024 + (9 * alpha^3)/256 + (3 * alpha^2)/128 +
    (9 * alpha)/128 + 9/256) + 2 * b * i * (1/16 + 3/32 * alpha^2 +
    1/16 * alpha) + c^2 * ((81 * alpha^4)/1372 - (81 * alpha^3)/1372 +
    (81 * alpha^2)/2744 + (87 * alpha)/5488 + 33/21952) + 2 *
```

```
    c * d * ((2025 * alpha^4)/21952 - (3375 * alpha^3)/43904 +
    (261 * alpha^2)/12544 + (3453 * alpha)/87808 + 573/87808) +
    2 * c * e * ((36 * alpha^4)/343 - (18 * alpha^3)/343 - (9 *
        alpha^2)/686 + (185 * alpha)/2744 + 25/1372) + 2 * c *
    f * ((2025 * alpha^4)/21952 + (405 * alpha^3)/43904 - (675 *
    alpha^2)/12544 + (7695 * alpha)/87808 + 3555/87808) + 2 *
    c * g * ((81 * alpha^4)/1372 + (27 * alpha^3)/343 - (171 *
    alpha^2)/2744 + (111 * alpha)/1372 + 1713/21952) + 2 * c *
    h * ((9 * alpha^4)/448 + (639 * alpha^3)/6272 + (261 * alpha^2)/12544 +
    (271 * alpha)/12544 + 1711/12544) + 2 * c * i * (87/392 +
    27/98 * alpha^2 - 6/49 * alpha) + d^2 * ((50625 * alpha^4)/351232 -
    (16875 * alpha^3)/175616 - (2025 * alpha^2)/175616 + (405 *
    alpha)/5488 + 135/6272) + 2 * d * e * ((225 * alpha^4)/1372 -
    (75 * alpha^3)/1372 - (789 * alpha^2)/10976 + (45 * alpha)/448 +
    1089/21952) + 2 * d * f * ((50625 * alpha^4)/351232 + (3375 *
    alpha^3)/87808 - (5625 * alpha^2)/43904 + (4395 * alpha)/43904 +
    1185/12544) + 2 * d * g * ((2025 * alpha^4)/21952 + (6075 *
    alpha^3)/43904 - (1539 * alpha^2)/12544 + (4293 * alpha)/87808 +
    14013/87808) + 2 * d * h * ((225 * alpha^4)/7168 + (4125 *
    alpha^3)/25088 + (687 * alpha^2)/25088 - (513 * alpha)/6272 +
    1557/6272) + 2 * d * i * (285/784 + 675/1568 * alpha^2 -
    255/784 * alpha) + e^2 * ((64 * alpha^4)/343 - (48 * alpha^2)/343 +
    (36 * alpha)/343 + 135/1372) + 2 * e * f * ((225 * alpha^4)/1372 +
    (135 * alpha^3)/1372 - (1965 * alpha^2)/10976 + (205 * alpha)/3136 +
    3595/21952) + 2 * e * g * ((36 * alpha^4)/343 + (66 * alpha^3)/343 -
    (93 * alpha^2)/686 - (81 * alpha)/2744 + 333/1372) + 2 *
    e * h * (alpha^4/28 + (39 * alpha^3)/196 + (93 * alpha^2)/1568 -
    (585 * alpha)/3136 + 1035/3136) + 2 * e * i * (41/98 + 24/49 *
    alpha^2 - 20/49 * alpha) + f^2 * ((50625 * alpha^4)/351232 +
    (30375 * alpha^3)/175616 - (30375 * alpha^2)/175616 - (375 *
    alpha)/21952 + 375/1568) + 2 * f * g * ((2025 * alpha^4)/21952 +
    (9855 * alpha^3)/43904 - (1035 * alpha^2)/12544 - (11625 *
    alpha)/87808 + 27075/87808) + 2 * f * h * ((225 * alpha^4)/7168 +
    (1215 * alpha^3)/6272 + (1425 * alpha^2)/12544 - (785 * alpha)/3136 +
    4465/12544) + 2 * f * i * (285/784 + 675/1568 * alpha^2 -
    255/784 * alpha) + g^2 * ((81 * alpha^4)/1372 + (297 * alpha^3)/1372 +
    (81 * alpha^2)/2744 - (1215 * alpha)/5488 + 7425/21952) +
    2 * g * h * (3951/12544 + 9/448 * alpha^4 + 933/6272 * alpha^3 +
        2109/12544 * alpha^2 - 3033/12544 * alpha) + 2 * g *
    i * (87/392 + 27/98 * alpha^2 - 6/49 * alpha) + h^2 * (27/128 +
    7/1024 * alpha^4 + 39/512 * alpha^3 + 93/512 * alpha^2 -
    9/64 * alpha) + 2 * h * i * (1/16 + 3/32 * alpha^2 + 1/16 *
    alpha) + i^2 * 0)


Q2.1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
    0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
    h * 0 + 2 * a * i * 0 + b^2 * (-1/512 * alpha^4 + 1/512 *
    alpha^3) + 2 * b * c * (-9/1568 * alpha^4 + 27/6272 * alpha^3 +
    17/12544 * alpha^2 + 1/12544 * alpha) + 2 * b * d * ((-(225 *
    alpha^4)/25088) + (45 * alpha^3)/12544 + (117 * alpha^2)/25088 +
    (9 * alpha)/12544) + 2 * b * e * ((-alpha^4/98) - alpha^3/784 +
    (27 * alpha^2)/3136 + (9 * alpha)/3136) + 2 * b * f * ((-(225 *
```

```
alpha^4)/25088) - (225 * alpha^3)/25088 + (125 * alpha^2)/12544 +
(25 * alpha)/3136) + 2 * b * g * ((-(9 * alpha^4)/1568) -
(99 * alpha^3)/6272 + (45 * alpha^2)/12544 + (225 * alpha)/12544) +
2 * b * h * ((-alpha^4/512) - alpha^3/64 - (9 * alpha^2)/512 +
    (9 * alpha)/256) + 2 * b * i * (1/16 * alpha - 1/16 *
alpha^2) + c^2 * ((-(81 * alpha^4)/4802) + (81 * alpha^3)/9604 +
(135 * alpha^2)/19208 + (51 * alpha)/38416 + 3/38416) + 2 *
c * d * ((-(2025 * alpha^4)/76832) + (1215 * alpha^3)/307328 +
(10089 * alpha^2)/614656 + (3375 * alpha)/614656 + 153/307328) +
2 * c * e * ((-(72 * alpha^4)/2401) - (27 * alpha^3)/2401 +
    (239 * alpha^2)/9604 + (281 * alpha)/19208 + 33/19208) +
2 * c * f * ((-(2025 * alpha^4)/76832) - (10125 * alpha^3)/307328 +
    (14625 * alpha^2)/614656 + (19125 * alpha)/614656 + 675/153664) +
2 * c * g * ((-(81 * alpha^4)/4802) - (243 * alpha^3)/4802 +
    (9 * alpha^2)/19208 + (1107 * alpha)/19208 + 45/4802) +
2 * c * h * ((-(9 * alpha^4)/1568) - (297 * alpha^3)/6272 -
    (775 * alpha^2)/12544 + (1219 * alpha)/12544 + 111/6272) +
2 * c * i * (3/98 - 9/49 * alpha^2 + 15/98 * alpha) + d^2 *
((-(50625 * alpha^4)/1229312) - (10125 * alpha^3)/1229312 +
    (18225 * alpha^2)/614656 + (5265 * alpha)/307328 + 405/153664) +
2 * d * e * ((-(225 * alpha^4)/4802) - (1305 * alpha^3)/38416 +
    (5499 * alpha^2)/153664 + (5679 * alpha)/153664 + 621/76832) +
2 * d * f * ((-(50625 * alpha^4)/1229312) - (10125 * alpha^3)/153664 +
    (28575 * alpha^2)/1229312 + (39825 * alpha)/614656 +
    2925/153664) + 2 * d * g * ((-(2025 * alpha^4)/76832) -
(27135 * alpha^3)/307328 - (13851 * alpha^2)/614656 + (60831 *
alpha)/614656 + 11745/307328) + 2 * d * h * ((-(225 * alpha^4)/25088) -
(1935 * alpha^3)/25088 - (747 * alpha^2)/6272 + (855 * alpha)/6272 +
27/392) + 2 * d * i * (45/392 - 225/784 * alpha^2 + 135/784 *
alpha) + e^2 * ((-(128 * alpha^4)/2401) - (160 * alpha^3)/2401 +
(72 * alpha^2)/2401 + (162 * alpha)/2401 + 54/2401) + 2 *
e * f * ((-(225 * alpha^4)/4802) - (3825 * alpha^3)/38416 -
(325 * alpha^2)/153664 + (15325 * alpha)/153664 + 1875/38416) +
2 * e * g * ((-(72 * alpha^4)/2401) - (279 * alpha^3)/2401 -
    (657 * alpha^2)/9604 + (2367 * alpha)/19208 + 1755/19208) +
2 * e * h * ((-alpha^4/98) - (73 * alpha^3)/784 - (549 *
    alpha^2)/3136 + (387 * alpha)/3136 + 243/1568) + 2 *
e * i * (12/49 - 16/49 * alpha^2 + 4/49 * alpha) + f^2 *
((-(50625 * alpha^4)/1229312) - (151875 * alpha^3)/1229312 -
    (16875 * alpha^2)/307328 + (9375 * alpha)/76832 + 1875/19208) +
2 * f * g * ((-(2025 * alpha^4)/76832) - (38475 * alpha^3)/307328 -
    (79875 * alpha^2)/614656 + (69525 * alpha)/614656 + 25875/153664) +
2 * f * h * ((-(225 * alpha^4)/25088) - (1125 * alpha^3)/12544 -
    (5375 * alpha^2)/25088 + (625 * alpha)/12544 + 825/3136) +
2 * f * i * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) +
g^2 * ((-(81 * alpha^4)/4802) - (1053 * alpha^3)/9604 - (3645 *
    alpha^2)/19208 + (2025 * alpha)/38416 + 10125/38416) +
2 * g * h * (2295/6272 - 9/1568 * alpha^4 - 423/6272 * alpha^3 -
    2763/12544 * alpha^2 - 909/12544 * alpha) + 2 * g * i *
(45/98 - 9/49 * alpha^2 - 27/98 * alpha) + h^2 * (27/64 -
1/512 * alpha^4 - 17/512 * alpha^3 - 45/256 * alpha^2 - 27/128 *
alpha) + 2 * h * i * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) +
i^2 * 0)
```

```
Q1.1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
    0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
    h * 0 + 2 * a * i * 0 + b^2 * (1/4096 * alpha^4) + 2 * b *
    c * (9/12544 * alpha^4 + 3/12544 * alpha^3 + 1/50176 * alpha^2) +
    2 * b * d * ((225 * alpha^4)/200704 + (45 * alpha^3)/50176 +
        (9 * alpha^2)/50176) + 2 * b * e * (alpha^4/784 + (3 *
    alpha^3)/1568 + (9 * alpha^2)/12544) + 2 * b * f * ((225 *
    alpha^4)/200704 + (75 * alpha^3)/25088 + (25 * alpha^2)/12544) +
    2 * b * g * ((9 * alpha^4)/12544 + (45 * alpha^3)/12544 +
        (225 * alpha^2)/50176) + 2 * b * h * (alpha^4/4096 +
    (3 * alpha^3)/1024 + (9 * alpha^2)/1024) + 2 * b * i * (1/64 *
    alpha^2) + c^2 * ((81 * alpha^4)/38416 + (27 * alpha^3)/19208 +
    (27 * alpha^2)/76832 + (3 * alpha)/76832 + 1/614656) + 2 *
    c * d * ((2025 * alpha^4)/614656 + (2295 * alpha^3)/614656 +
    (3681 * alpha^2)/2458624 + (153 * alpha)/614656 + 9/614656) +
    2 * c * e * ((9 * alpha^4)/2401 + (33 * alpha^3)/4802 + (157 *
        alpha^2)/38416 + (33 * alpha)/38416 + 9/153664) + 2 *
    c * f * ((2025 * alpha^4)/614656 + (6075 * alpha^3)/614656 +
    (21825 * alpha^2)/2458624 + (675 * alpha)/307328 + 25/153664) +
    2 * c * g * ((81 * alpha^4)/38416 + (27 * alpha^3)/2401 +
        (1287 * alpha^2)/76832 + (45 * alpha)/9604 + 225/614656) +
    2 * c * h * ((9 * alpha^4)/12544 + (111 * alpha^3)/12544 +
        (1441 * alpha^2)/50176 + (111 * alpha)/12544 + 9/12544) +
    2 * c * i * (1/784 + 9/196 * alpha^2 + 3/196 * alpha) + d^2 *
    ((50625 * alpha^4)/9834496 + (10125 * alpha^3)/1229312 +
        (6075 * alpha^2)/1229312 + (405 * alpha)/307328 + 81/614656) +
    2 * d * e * ((225 * alpha^4)/38416 + (1035 * alpha^3)/76832 +
        (6921 * alpha^2)/614656 + (621 * alpha)/153664 + 81/153664) +
    2 * d * f * ((50625 * alpha^4)/9834496 + (43875 * alpha^3)/2458624 +
        (51525 * alpha^2)/2458624 + (2925 * alpha)/307328 + 225/153664) +
    2 * d * g * ((2025 * alpha^4)/614656 + (11745 * alpha^3)/614656 +
        (84321 * alpha^2)/2458624 + (11745 * alpha)/614656 +
        2025/614656) + 2 * d * h * ((225 * alpha^4)/200704 +
    (45 * alpha^3)/3136 + (1287 * alpha^2)/25088 + (27 * alpha)/784 +
    81/12544) + 2 * d * i * (9/784 + 225/3136 * alpha^2 + 45/784 *
    alpha) + e^2 * ((16 * alpha^4)/2401 + (48 * alpha^3)/2401 +
    (54 * alpha^2)/2401 + (27 * alpha)/2401 + 81/38416) + 2 *
    e * f * ((225 * alpha^4)/38416 + (1875 * alpha^3)/76832 +
    (22825 * alpha^2)/614656 + (1875 * alpha)/76832 + 225/38416) +
    2 * e * g * ((9 * alpha^4)/2401 + (117 * alpha^3)/4802 +
        (2061 * alpha^2)/38416 + (1755 * alpha)/38416 + 2025/153664) +
    2 * e * h * (alpha^4/784 + (27 * alpha^3)/1568 + (873 * alpha^2)/12544 +
        (243 * alpha)/3136 + 81/3136) + 2 * e * i * (9/196 +
    4/49 * alpha^2 + 6/49 * alpha) + f^2 * ((50625 * alpha^4)/9834496 +
    (16875 * alpha^3)/614656 + (16875 * alpha^2)/307328 + (1875 *
    alpha)/38416 + 625/38416) + 2 * f * g * ((2025 * alpha^4)/614656 +
    (15525 * alpha^3)/614656 + (173025 * alpha^2)/2458624 + (25875 *
    alpha)/307328 + 5625/153664) + 2 * f * h * ((225 * alpha^4)/200704 +
    (825 * alpha^3)/50176 + (3925 * alpha^2)/50176 + (825 * alpha)/6272 +
    225/3136) + 2 * f * i * (25/196 + 225/3136 * alpha^2 + 75/392 *
    alpha) + g^2 * ((81 * alpha^4)/38416 + (405 * alpha^3)/19208 +
    (6075 * alpha^2)/76832 + (10125 * alpha)/76832 + 50625/614656) +
```

```
    2 * g * h * (2025/12544 + 9/12544 * alpha^4 + 153/12544 *
        alpha^3 + 3681/50176 * alpha^2 + 2295/12544 * alpha) +
    2 * g * i * (225/784 + 9/196 * alpha^2 + 45/196 * alpha) +
    h^2 * (81/256 + 1/4096 * alpha^4 + 3/512 * alpha^3 + 27/512 *
        alpha^2 + 27/128 * alpha) + 2 * h * i * (9/16 + 1/64 *
    alpha^2 + 3/16 * alpha) + i^2 * 1)
prob1 <- c(Q9.1, Q8.1, Q7.1, Q6.1, Q5.1, Q4.1, Q3.1, Q2.1, Q1.1)
```

This generates prob

```
alpha = 0

Q9 <- (a^2 * 1 + 2 * a * b * (9/16 + 1/64 * alpha^2 + 3/16 *
    alpha) + 2 * a * c * (225/784 + 9/196 * alpha^2 + 45/196 *
    alpha) + 2 * a * d * (25/196 + 225/3136 * alpha^2 + 75/392 *
    alpha) + 2 * a * e * (9/196 + 4/49 * alpha^2 + 6/49 * alpha) +
    2 * a * f * (9/784 + 225/3136 * alpha^2 + 45/784 * alpha) +
    2 * a * g * (1/784 + 9/196 * alpha^2 + 3/196 * alpha) + 2 *
    a * h * (1/64 * alpha^2) + 0 + b^2 * (81/256 + 1/4096 * alpha^4 +
    3/512 * alpha^3 + 27/512 * alpha^2 + 27/128 * alpha) + 2 *
    b * c * (2025/12544 + 9/12544 * alpha^4 + 153/12544 * alpha^3 +
    3681/50176 * alpha^2 + 2295/12544 * alpha) + 2 * b * d *
    ((225 * alpha^4)/200704 + (825 * alpha^3)/50176 + (3925 *
        alpha^2)/50176 + (825 * alpha)/6272 + 225/3136) + 2 *
    b * e * (alpha^4/784 + (27 * alpha^3)/1568 + (873 * alpha^2)/12544 +
    (243 * alpha)/3136 + 81/3136) + 2 * b * f * ((225 * alpha^4)/200704 +
    (45 * alpha^3)/3136 + (1287 * alpha^2)/25088 + (27 * alpha)/784 +
    81/12544) + 2 * b * g * ((9 * alpha^4)/12544 + (111 * alpha^3)/12544 +
    (1441 * alpha^2)/50176 + (111 * alpha)/12544 + 9/12544) +
    2 * b * h * (alpha^4/4096 + (3 * alpha^3)/1024 + (9 * alpha^2)/1024) +
    0 + c^2 * ((81 * alpha^4)/38416 + (405 * alpha^3)/19208 +
    (6075 * alpha^2)/76832 + (10125 * alpha)/76832 + 50625/614656) +
    2 * c * d * ((2025 * alpha^4)/614656 + (15525 * alpha^3)/614656 +
        (173025 * alpha^2)/2458624 + (25875 * alpha)/307328 +
        5625/153664) + 2 * c * e * ((9 * alpha^4)/2401 + (117 *
    alpha^3)/4802 + (2061 * alpha^2)/38416 + (1755 * alpha)/38416 +
    2025/153664) + 2 * c * f * ((2025 * alpha^4)/614656 + (11745 *
    alpha^3)/614656 + (84321 * alpha^2)/2458624 + (11745 * alpha)/614656 +
    2025/614656) + 2 * c * g * ((81 * alpha^4)/38416 + (27 *
    alpha^3)/2401 + (1287 * alpha^2)/76832 + (45 * alpha)/9604 +
    225/614656) + 2 * c * h * ((9 * alpha^4)/12544 + (45 * alpha^3)/12544 +
    (225 * alpha^2)/50176) + 0 + d^2 * ((50625 * alpha^4)/9834496 +
    (16875 * alpha^3)/614656 + (16875 * alpha^2)/307328 + (1875 *
    alpha)/38416 + 625/38416) + 2 * d * e * ((225 * alpha^4)/38416 +
    (1875 * alpha^3)/76832 + (22825 * alpha^2)/614656 + (1875 *
    alpha)/76832 + 225/38416) + 2 * d * f * ((50625 * alpha^4)/9834496 +
    (43875 * alpha^3)/2458624 + (51525 * alpha^2)/2458624 + (2925 *
    alpha)/307328 + 225/153664) + 2 * d * g * ((2025 * alpha^4)/614656 +
    (6075 * alpha^3)/614656 + (21825 * alpha^2)/2458624 + (675 *
    alpha)/307328 + 25/153664) + 2 * d * h * ((225 * alpha^4)/200704 +
    (75 * alpha^3)/25088 + (25 * alpha^2)/12544) + 0 + e^2 *
    ((16 * alpha^4)/2401 + (48 * alpha^3)/2401 + (54 * alpha^2)/2401 +
        (27 * alpha)/2401 + 81/38416) + 2 * e * f * ((225 * alpha^4)/38416 +
    (1035 * alpha^3)/76832 + (6921 * alpha^2)/614656 + (621 *
```

```
alpha)/153664 + 81/153664) + 2 * e * g * ((9 * alpha^4)/2401 +
(33 * alpha^3)/4802 + (157 * alpha^2)/38416 + (33 * alpha)/38416 +
9/153664) + 2 * e * h * (alpha^4/784 + (3 * alpha^3)/1568 +
(9 * alpha^2)/12544) + 0 + f^2 * ((50625 * alpha^4)/9834496 +
(10125 * alpha^3)/1229312 + (6075 * alpha^2)/1229312 + (405 *
alpha)/307328 + 81/614656) + 2 * f * g * ((2025 * alpha^4)/614656 +
(2295 * alpha^3)/614656 + (3681 * alpha^2)/2458624 + (153 *
alpha)/614656 + 9/614656) + 2 * f * h * ((225 * alpha^4)/200704 +
(45 * alpha^3)/50176 + (9 * alpha^2)/50176) + 0 + g^2 * ((81 *
alpha^4)/38416 + (27 * alpha^3)/19208 + (27 * alpha^2)/76832 +
(3 * alpha)/76832 + 1/614656) + 2 * g * h * (9/12544 * alpha^4 +
3/12544 * alpha^3 + 1/50176 * alpha^2) + 0 + h^2 * (1/4096 *
alpha^4) + 0 + i^2 * 0)


Q8 <- (a^2 * 0 + 2 * a * b * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) +
2 * a * c * (45/98 - 9/49 * alpha^2 - 27/98 * alpha) + 2 *
a * d * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) + 2 *
a * e * (12/49 - 16/49 * alpha^2 + 4/49 * alpha) + 2 * a *
f * (45/392 - 225/784 * alpha^2 + 135/784 * alpha) + 2 *
a * g * (3/98 - 9/49 * alpha^2 + 15/98 * alpha) + 2 * a *
h * (1/16 * alpha - 1/16 * alpha^2) + 0 + b^2 * (27/64 -
1/512 * alpha^4 - 17/512 * alpha^3 - 45/256 * alpha^2 - 27/128 *
alpha) + 2 * b * c * (2295/6272 - 9/1568 * alpha^4 - 423/6272 *
alpha^3 - 2763/12544 * alpha^2 - 909/12544 * alpha) + 2 *
b * d * ((-(225 * alpha^4)/25088) - (1125 * alpha^3)/12544 -
(5375 * alpha^2)/25088 + (625 * alpha)/12544 + 825/3136) +
2 * b * e * ((-alpha^4/98) - (73 * alpha^3)/784 - (549 *
    alpha^2)/3136 + (387 * alpha)/3136 + 243/1568) + 2 *
b * f * ((-(225 * alpha^4)/25088) - (1935 * alpha^3)/25088 -
(747 * alpha^2)/6272 + (855 * alpha)/6272 + 27/392) + 2 *
b * g * ((-(9 * alpha^4)/1568) - (297 * alpha^3)/6272 - (775 *
alpha^2)/12544 + (1219 * alpha)/12544 + 111/6272) + 2 * b *
h * ((-alpha^4/512) - alpha^3/64 - (9 * alpha^2)/512 + (9 *
alpha)/256) + 0 + c^2 * ((-(81 * alpha^4)/4802) - (1053 *
alpha^3)/9604 - (3645 * alpha^2)/19208 + (2025 * alpha)/38416 +
10125/38416) + 2 * c * d * ((-(2025 * alpha^4)/76832) - (38475 *
alpha^3)/307328 - (79875 * alpha^2)/614656 + (69525 * alpha)/614656 +
25875/153664) + 2 * c * e * ((-(72 * alpha^4)/2401) - (279 *
alpha^3)/2401 - (657 * alpha^2)/9604 + (2367 * alpha)/19208 +
1755/19208) + 2 * c * f * ((-(2025 * alpha^4)/76832) - (27135 *
alpha^3)/307328 - (13851 * alpha^2)/614656 + (60831 * alpha)/614656 +
11745/307328) + 2 * c * g * ((-(81 * alpha^4)/4802) - (243 *
alpha^3)/4802 + (9 * alpha^2)/19208 + (1107 * alpha)/19208 +
45/4802) + 2 * c * h * ((-(9 * alpha^4)/1568) - (99 * alpha^3)/6272 +
(45 * alpha^2)/12544 + (225 * alpha)/12544) + 0 + d^2 * ((-(50625 *
alpha^4)/1229312) - (151875 * alpha^3)/1229312 - (16875 *
alpha^2)/307328 + (9375 * alpha)/76832 + 1875/19208) + 2 *
d * e * ((-(225 * alpha^4)/4802) - (3825 * alpha^3)/38416 -
(325 * alpha^2)/153664 + (15325 * alpha)/153664 + 1875/38416) +
2 * d * f * ((-(50625 * alpha^4)/1229312) - (10125 * alpha^3)/153664 +
    (28575 * alpha^2)/1229312 + (39825 * alpha)/614656 +
    2925/153664) + 2 * d * g * ((-(2025 * alpha^4)/76832) -
```

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```
          (10125 * alpha^3)/307328 + (14625 * alpha^2)/614656 + (19125 *
          alpha)/614656 + 675/153664) + 2 * d * h * ((-(225 * alpha^4)/25088) -
          (225 * alpha^3)/25088 + (125 * alpha^2)/12544 + (25 * alpha)/3136) +
          0 + e^2 * ((-(128 * alpha^4)/2401) - (160 * alpha^3)/2401 +
          (72 * alpha^2)/2401 + (162 * alpha)/2401 + 54/2401) + 2 *
          e * f * ((-(225 * alpha^4)/4802) - (1305 * alpha^3)/38416 +
          (5499 * alpha^2)/153664 + (5679 * alpha)/153664 + 621/76832) +
          2 * e * g * ((-(72 * alpha^4)/2401) - (27 * alpha^3)/2401 +
              (239 * alpha^2)/9604 + (281 * alpha)/19208 + 33/19208) +
          2 * e * h * ((-alpha^4/98) - alpha^3/784 + (27 * alpha^2)/3136 +
              (9 * alpha)/3136) + 0 + f^2 * ((-(50625 * alpha^4)/1229312) -
          (10125 * alpha^3)/1229312 + (18225 * alpha^2)/614656 + (5265 *
          alpha)/307328 + 405/153664) + 2 * f * g * ((-(2025 * alpha^4)/76832) +
          (1215 * alpha^3)/307328 + (10089 * alpha^2)/614656 + (3375 *
          alpha)/614656 + 153/307328) + 2 * f * h * ((-(225 * alpha^4)/25088) +
          (45 * alpha^3)/12544 + (117 * alpha^2)/25088 + (9 * alpha)/12544) +
          0 + g^2 * ((-(81 * alpha^4)/4802) + (81 * alpha^3)/9604 +
          (135 * alpha^2)/19208 + (51 * alpha)/38416 + 3/38416) + 2 *
          g * h * (-9/1568 * alpha^4 + 27/6272 * alpha^3 + 17/12544 *
          alpha^2 + 1/12544 * alpha) + 0 + h^2 * (-1/512 * alpha^4 +
          1/512 * alpha^3) + 0 + i^2 * 0)


Q7 <- (a^2 * 0 + 2 * a * b * (1/16 + 3/32 * alpha^2 + 1/16 *
      alpha) + 2 * a * c * (87/392 + 27/98 * alpha^2 - 6/49 * alpha) +
      2 * a * d * (285/784 + 675/1568 * alpha^2 - 255/784 * alpha) +
      2 * a * e * (41/98 + 24/49 * alpha^2 - 20/49 * alpha) + 2 *
      a * f * (285/784 + 675/1568 * alpha^2 - 255/784 * alpha) +
      2 * a * g * (87/392 + 27/98 * alpha^2 - 6/49 * alpha) + 2 *
      a * h * (1/16 + 3/32 * alpha^2 + 1/16 * alpha) + 0 + b^2 *
      (27/128 + 7/1024 * alpha^4 + 39/512 * alpha^3 + 93/512 *
          alpha^2 - 9/64 * alpha) + 2 * b * c * (3951/12544 + 9/448 *
      alpha^4 + 933/6272 * alpha^3 + 2109/12544 * alpha^2 - 3033/12544 *
      alpha) + 2 * b * d * ((225 * alpha^4)/7168 + (1215 * alpha^3)/6272 +
      (1425 * alpha^2)/12544 - (785 * alpha)/3136 + 4465/12544) +
      2 * b * e * (alpha^4/28 + (39 * alpha^3)/196 + (93 * alpha^2)/1568 -
          (585 * alpha)/3136 + 1035/3136) + 2 * b * f * ((225 *
      alpha^4)/7168 + (4125 * alpha^3)/25088 + (687 * alpha^2)/25088 -
      (513 * alpha)/6272 + 1557/6272) + 2 * b * g * ((9 * alpha^4)/448 +
      (639 * alpha^3)/6272 + (261 * alpha^2)/12544 + (271 * alpha)/12544 +
      1711/12544) + 2 * b * h * ((7 * alpha^4)/1024 + (9 * alpha^3)/256 +
      (3 * alpha^2)/128 + (9 * alpha)/128 + 9/256) + 0 + c^2 *
      ((81 * alpha^4)/1372 + (297 * alpha^3)/1372 + (81 * alpha^2)/2744 -
          (1215 * alpha)/5488 + 7425/21952) + 2 * c * d * ((2025 *
      alpha^4)/21952 + (9855 * alpha^3)/43904 - (1035 * alpha^2)/12544 -
      (11625 * alpha)/87808 + 27075/87808) + 2 * c * e * ((36 *
      alpha^4)/343 + (66 * alpha^3)/343 - (93 * alpha^2)/686 -
      (81 * alpha)/2744 + 333/1372) + 2 * c * f * ((2025 * alpha^4)/21952 +
      (6075 * alpha^3)/43904 - (1539 * alpha^2)/12544 + (4293 *
      alpha)/87808 + 14013/87808) + 2 * c * g * ((81 * alpha^4)/1372 +
      (27 * alpha^3)/343 - (171 * alpha^2)/2744 + (111 * alpha)/1372 +
      1713/21952) + 2 * c * h * ((9 * alpha^4)/448 + (177 * alpha^3)/6272 +
      (3 * alpha^2)/1792 + (765 * alpha)/12544 + 225/12544) + 0 +
```

```
    d^2 * ((50625 * alpha^4)/351232 + (30375 * alpha^3)/175616 -
        (30375 * alpha^2)/175616 - (375 * alpha)/21952 + 375/1568) +
    2 * d * e * ((225 * alpha^4)/1372 + (135 * alpha^3)/1372 -
        (1965 * alpha^2)/10976 + (205 * alpha)/3136 + 3595/21952) +
    2 * d * f * ((50625 * alpha^4)/351232 + (3375 * alpha^3)/87808 -
        (5625 * alpha^2)/43904 + (4395 * alpha)/43904 + 1185/12544) +
    2 * d * g * ((2025 * alpha^4)/21952 + (405 * alpha^3)/43904 -
        (675 * alpha^2)/12544 + (7695 * alpha)/87808 + 3555/87808) +
    2 * d * h * ((225 * alpha^4)/7168 + (135 * alpha^3)/25088 +
        (15 * alpha^2)/3584 + (275 * alpha)/6272 + 25/3136) +
    0 + e^2 * ((64 * alpha^4)/343 - (48 * alpha^2)/343 + (36 *
    alpha)/343 + 135/1372) + 2 * e * f * ((225 * alpha^4)/1372 -
    (75 * alpha^3)/1372 - (789 * alpha^2)/10976 + (45 * alpha)/448 +
    1089/21952) + 2 * e * g * ((36 * alpha^4)/343 - (18 * alpha^3)/343 -
    (9 * alpha^2)/686 + (185 * alpha)/2744 + 25/1372) + 2 * e *
    h * (alpha^4/28 - (3 * alpha^3)/196 + (3 * alpha^2)/224 +
    (81 * alpha)/3136 + 9/3136) + 0 + f^2 * ((50625 * alpha^4)/351232 -
    (16875 * alpha^3)/175616 - (2025 * alpha^2)/175616 + (405 *
    alpha)/5488 + 135/6272) + 2 * f * g * ((2025 * alpha^4)/21952 -
    (3375 * alpha^3)/43904 + (261 * alpha^2)/12544 + (3453 *
    alpha)/87808 + 573/87808) + 2 * f * h * ((225 * alpha^4)/7168 -
    (75 * alpha^3)/3136 + (33 * alpha^2)/1792 + (9 * alpha)/784 +
    9/12544) + 0 + g^2 * ((81 * alpha^4)/1372 - (81 * alpha^3)/1372 +
    (81 * alpha^2)/2744 + (87 * alpha)/5488 + 33/21952) + 2 *
    g * h * (1/12544 + 9/448 * alpha^4 - 117/6272 * alpha^3 +
    27/1792 * alpha^2 + 37/12544 * alpha) + 0 + h^2 * (7/1024 *
    alpha^4 - 3/512 * alpha^3 + 3/512 * alpha^2) + 0 + i^2 *
    0)


Q6 <- (a^2 * 0 + 2 * a * b * (1/16 * alpha - 1/16 * alpha^2) +
    2 * a * c * (3/98 - 9/49 * alpha^2 + 15/98 * alpha) + 2 *
    a * d * (45/392 - 225/784 * alpha^2 + 135/784 * alpha) +
    2 * a * e * (12/49 - 16/49 * alpha^2 + 4/49 * alpha) + 2 *
    a * f * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) + 2 *
    a * g * (45/98 - 9/49 * alpha^2 - 27/98 * alpha) + 2 * a *
    h * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) + 0 + b^2 * (3/64 -
    7/512 * alpha^4 - 45/512 * alpha^3 - 3/128 * alpha^2 + 5/64 *
    alpha) + 2 * b * c * (405/3136 - 9/224 * alpha^4 - 999/6272 *
    alpha^3 + 807/12544 * alpha^2 + 75/12544 * alpha) + 2 * b *
    d * ((-(225 * alpha^4)/3584) - (2505 * alpha^3)/12544 + (3235 *
    alpha^2)/25088 - (1145 * alpha)/12544 + 705/3136) + 2 * b *
    e * ((-alpha^4/14) - (159 * alpha^3)/784 + (417 * alpha^2)/3136 -
    (529 * alpha)/3136 + 243/784) + 2 * b * f * ((-(225 * alpha^4)/3584) -
    (4275 * alpha^3)/25088 + (921 * alpha^2)/12544 - (39 * alpha)/196 +
    1125/3136) + 2 * b * g * ((-(9 * alpha^4)/224) - (705 * alpha^3)/6272 -
    (229 * alpha^2)/12544 - (2165 * alpha)/12544 + 1077/3136) +
    2 * b * h * ((-(7 * alpha^4)/512) - (3 * alpha^3)/64 - (39 *
        alpha^2)/512 - (25 * alpha)/256 + 15/64) + 0 + c^2 *
    ((-(81 * alpha^4)/686) - (243 * alpha^3)/1372 + (81 * alpha^2)/392 -
        (729 * alpha)/5488 + 1215/5488) + 2 * c * d * ((-(2025 *
    alpha^4)/10976) - (5805 * alpha^3)/43904 + (21465 * alpha^2)/87808 -
    (2715 * alpha)/12544 + 12675/43904) + 2 * c * e * ((-(72 *
```

```
        alpha^4)/343) - (27 * alpha^3)/343 + (267 * alpha^2)/1372 -
        (615 * alpha)/2744 + 873/2744) + 2 * c * f * ((-(2025 * alpha^4)/10976) -
        (2025 * alpha^3)/43904 + (7857 * alpha^2)/87808 - (2025 *
        alpha)/12544 + 3321/10976) + 2 * c * g * ((-(81 * alpha^4)/686) -
        (27 * alpha^3)/686 - (9 * alpha^2)/392 - (165 * alpha)/2744 +
        165/686) + 2 * c * h * ((-(9 * alpha^4)/224) - (243 * alpha^3)/6272 -
        (993 * alpha^2)/12544 + (39 * alpha)/1792 + 855/6272) + 0 +
        d^2 * ((-(50625 * alpha^4)/175616) + (3375 * alpha^3)/175616 +
            (16875 * alpha^2)/87808 - (10125 * alpha)/43904 + 3375/10976) +
        2 * d * e * ((-(225 * alpha^4)/686) + (15 * alpha^3)/112 +
            (1655 * alpha^2)/21952 - (3665 * alpha)/21952 + 3135/10976) +
        2 * d * f * ((-(50625 * alpha^4)/175616) + (3375 * alpha^3)/21952 -
            (5625 * alpha^2)/175616 - (5655 * alpha)/87808 + 2535/10976) +
        2 * d * g * ((-(2025 * alpha^4)/10976) + (3645 * alpha^3)/43904 -
            (7515 * alpha^2)/87808 + (405 * alpha)/12544 + 6795/43904) +
        2 * d * h * ((-(225 * alpha^4)/3584) - (285 * alpha^3)/25088 -
            (55 * alpha^2)/784 + (65 * alpha)/896 + 225/3136) + 0 +
        e^2 * ((-(128 * alpha^4)/343) + (96 * alpha^3)/343 - (24 *
            alpha^2)/343 - (22 * alpha)/343 + 78/343) + 2 * e * f *
        ((-(225 * alpha^4)/686) + (225 * alpha^3)/784 - (3273 * alpha^2)/21952 +
            (717 * alpha)/21952 + 54/343) + 2 * e * g * ((-(72 *
        alpha^4)/343) + (57 * alpha^3)/343 - (181 * alpha^2)/1372 +
        (239 * alpha)/2744 + 243/2744) + 2 * e * h * ((-alpha^4/14) +
        (9 * alpha^3)/784 - (159 * alpha^2)/3136 + (5 * alpha)/64 +
        51/1568) + 0 + f^2 * ((-(50625 * alpha^4)/175616) + (50625 *
        alpha^3)/175616 - (2025 * alpha^2)/10976 + (2025 * alpha)/21952 +
        2025/21952) + 2 * f * g * ((-(2025 * alpha^4)/10976) + (7425 *
        alpha^3)/43904 - (11043 * alpha^2)/87808 + (177 * alpha)/1792 +
        465/10976) + 2 * f * h * ((-(225 * alpha^4)/3584) + (225 *
        alpha^3)/12544 - (633 * alpha^2)/25088 + (15 * alpha)/256 +
        9/784) + 0 + g^2 * ((-(81 * alpha^4)/686) + (135 * alpha^3)/1372 -
        (27 * alpha^2)/392 + (405 * alpha)/5488 + 81/5488) + 2 *
        g * h * (15/6272 - 9/224 * alpha^4 + 51/6272 * alpha^3 -
        13/12544 * alpha^2 + 55/1792 * alpha) + 0 + h^2 * (-7/512 *
        alpha^4 - 3/512 * alpha^3 + 3/256 * alpha^2 + 1/128 * alpha) +
        0 + i^2 * 0)


Q5 <- (a^2 * 0 + 2 * a * b * (1/64 * alpha^2) + 2 * a * c * (1/784 +
    9/196 * alpha^2 + 3/196 * alpha) + 2 * a * d * (9/784 + 225/3136 *
    alpha^2 + 45/784 * alpha) + 2 * a * e * (9/196 + 4/49 * alpha^2 +
    6/49 * alpha) + 2 * a * f * (25/196 + 225/3136 * alpha^2 +
    75/392 * alpha) + 2 * a * g * (225/784 + 9/196 * alpha^2 +
    45/196 * alpha) + 2 * a * h * (9/16 + 1/64 * alpha^2 + 3/16 *
    alpha) + 2 * a * i * 1 + b^2 * (1/256 + 35/2048 * alpha^4 +
    25/512 * alpha^3 - 27/512 * alpha^2 + 7/128 * alpha) + 2 *
    b * c * (327/12544 + 45/896 * alpha^4 + 225/3136 * alpha^3 -
    2533/25088 * alpha^2 + 1149/12544 * alpha) + 2 * b * d *
    ((1125 * alpha^4)/14336 + (2025 * alpha^3)/25088 - (2623 *
        alpha^2)/25088 + (561 * alpha)/6272 + 453/6272) + 2 *
    b * e * ((5 * alpha^4)/56 + (65 * alpha^3)/784 - (369 * alpha^2)/6272 +
    (149 * alpha)/3136 + 451/3136) + 2 * b * f * ((1125 * alpha^4)/14336 +
    (2025 * alpha^3)/25088 + (527 * alpha^2)/25088 - (45 * alpha)/3136 +
```

```
2985/12544) + 2 * b * g * ((45 * alpha^4)/896 + (225 * alpha^3)/3136 +
(2507 * alpha^2)/25088 - (699 * alpha)/12544 + 4359/12544) +
2 * b * h * ((35 * alpha^4)/2048 + (25 * alpha^3)/512 + (63 *
    alpha^2)/512 - alpha/64 + 59/128) + 2 * b * i * (9/16 +
1/64 * alpha^2 + 3/16 * alpha) + c^2 * ((405 * alpha^4)/2744 -
(513 * alpha^2)/5488 + (27 * alpha)/343 + 3429/43904) + 2 *
c * d * ((10125 * alpha^4)/43904 - (2025 * alpha^3)/21952 -
(2691 * alpha^2)/175616 + (675 * alpha)/87808 + 13045/87808) +
2 * c * e * ((90 * alpha^4)/343 - (45 * alpha^3)/343 + (197 *
    alpha^2)/2744 - (117 * alpha)/1372 + 2481/10976) + 2 *
c * f * ((10125 * alpha^4)/43904 - (2025 * alpha^3)/21952 +
(22509 * alpha^2)/175616 - (13941 * alpha)/87808 + 26037/87808) +
2 * c * g * ((405 * alpha^4)/2744 + (747 * alpha^2)/5488 -
    (57 * alpha)/343 + 15077/43904) + 2 * c * h * ((45 *
alpha^4)/896 + (225 * alpha^3)/3136 + (2507 * alpha^2)/25088 -
(699 * alpha)/12544 + 4359/12544) + 2 * c * i * (225/784 +
9/196 * alpha^2 + 45/196 * alpha) + d^2 * ((253125 * alpha^4)/702464 -
(50625 * alpha^3)/175616 + (24975 * alpha^2)/175616 - (675 *
alpha)/6272 + 19575/87808) + 2 * d * e * ((1125 * alpha^4)/2744 -
(2025 * alpha^3)/5488 + (10649 * alpha^2)/43904 - (4317 *
alpha)/21952 + 879/3136) + 2 * d * f * ((253125 * alpha^4)/702464 -
(50625 * alpha^3)/175616 + (40725 * alpha^2)/175616 - (345 *
alpha)/1568 + 13529/43904) + 2 * d * g * ((10125 * alpha^4)/43904 -
(2025 * alpha^3)/21952 + (22509 * alpha^2)/175616 - (13941 *
alpha)/87808 + 26037/87808) + 2 * d * h * ((1125 * alpha^4)/14336 +
(2025 * alpha^3)/25088 + (527 * alpha^2)/25088 - (45 * alpha)/3136 +
2985/12544) + 2 * d * i * (25/196 + 225/3136 * alpha^2 +
75/392 * alpha) + e^2 * ((160 * alpha^4)/343 - (160 * alpha^3)/343 +
(108 * alpha^2)/343 - (82 * alpha)/343 + 821/2744) + 2 *
e * f * ((1125 * alpha^4)/2744 - (2025 * alpha^3)/5488 +
(10649 * alpha^2)/43904 - (4317 * alpha)/21952 + 879/3136) +
2 * e * g * ((90 * alpha^4)/343 - (45 * alpha^3)/343 + (197 *
    alpha^2)/2744 - (117 * alpha)/1372 + 2481/10976) + 2 *
e * h * ((5 * alpha^4)/56 + (65 * alpha^3)/784 - (369 * alpha^2)/6272 +
(149 * alpha)/3136 + 451/3136) + 2 * e * i * (9/196 + 4/49 *
alpha^2 + 6/49 * alpha) + f^2 * ((253125 * alpha^4)/702464 -
(50625 * alpha^3)/175616 + (24975 * alpha^2)/175616 - (675 *
alpha)/6272 + 19575/87808) + 2 * f * g * ((10125 * alpha^4)/43904 -
(2025 * alpha^3)/21952 - (2691 * alpha^2)/175616 + (675 *
alpha)/87808 + 13045/87808) + 2 * f * h * ((1125 * alpha^4)/14336 +
(2025 * alpha^3)/25088 - (2623 * alpha^2)/25088 + (561 *
alpha)/6272 + 453/6272) + 2 * f * i * (9/784 + 225/3136 *
alpha^2 + 45/784 * alpha) + g^2 * ((405 * alpha^4)/2744 -
(513 * alpha^2)/5488 + (27 * alpha)/343 + 3429/43904) + 2 *
g * h * (327/12544 + 45/896 * alpha^4 + 225/3136 * alpha^3 -
2533/25088 * alpha^2 + 1149/12544 * alpha) + 2 * g * i *
(1/784 + 9/196 * alpha^2 + 3/196 * alpha) + h^2 * (1/256 +
35/2048 * alpha^4 + 25/512 * alpha^3 - 27/512 * alpha^2 +
7/128 * alpha) + 2 * h * i * (1/64 * alpha^2) + i^2 * 0)


Q4 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
    0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
```

```
h * 0 + 2 * a * i * 0 + b^2 * (-7/512 * alpha^4 - 3/512 *
alpha^3 + 3/256 * alpha^2 + 1/128 * alpha) + 2 * b * c *
(15/6272 - 9/224 * alpha^4 + 51/6272 * alpha^3 - 13/12544 *
    alpha^2 + 55/1792 * alpha) + 2 * b * d * ((-(225 * alpha^4)/3584) +
(225 * alpha^3)/12544 - (633 * alpha^2)/25088 + (15 * alpha)/256 +
9/784) + 2 * b * e * ((-alpha^4/14) + (9 * alpha^3)/784 -
(159 * alpha^2)/3136 + (5 * alpha)/64 + 51/1568) + 2 * b *
f * ((-(225 * alpha^4)/3584) - (285 * alpha^3)/25088 - (55 *
alpha^2)/784 + (65 * alpha)/896 + 225/3136) + 2 * b * g *
((-(9 * alpha^4)/224) - (243 * alpha^3)/6272 - (993 * alpha^2)/12544 +
    (39 * alpha)/1792 + 855/6272) + 2 * b * h * ((-(7 * alpha^4)/512) -
(3 * alpha^3)/64 - (39 * alpha^2)/512 - (25 * alpha)/256 +
15/64) + 2 * b * i * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) +
c^2 * ((-(81 * alpha^4)/686) + (135 * alpha^3)/1372 - (27 *
    alpha^2)/392 + (405 * alpha)/5488 + 81/5488) + 2 * c *
d * ((-(2025 * alpha^4)/10976) + (7425 * alpha^3)/43904 -
(11043 * alpha^2)/87808 + (177 * alpha)/1792 + 465/10976) +
2 * c * e * ((-(72 * alpha^4)/343) + (57 * alpha^3)/343 -
    (181 * alpha^2)/1372 + (239 * alpha)/2744 + 243/2744) +
2 * c * f * ((-(2025 * alpha^4)/10976) + (3645 * alpha^3)/43904 -
    (7515 * alpha^2)/87808 + (405 * alpha)/12544 + 6795/43904) +
2 * c * g * ((-(81 * alpha^4)/686) - (27 * alpha^3)/686 -
    (9 * alpha^2)/392 - (165 * alpha)/2744 + 165/686) + 2 *
c * h * ((-(9 * alpha^4)/224) - (705 * alpha^3)/6272 - (229 *
alpha^2)/12544 - (2165 * alpha)/12544 + 1077/3136) + 2 *
c * i * (45/98 - 9/49 * alpha^2 - 27/98 * alpha) + d^2 *
((-(50625 * alpha^4)/175616) + (50625 * alpha^3)/175616 -
    (2025 * alpha^2)/10976 + (2025 * alpha)/21952 + 2025/21952) +
2 * d * e * ((-(225 * alpha^4)/686) + (225 * alpha^3)/784 -
    (3273 * alpha^2)/21952 + (717 * alpha)/21952 + 54/343) +
2 * d * f * ((-(50625 * alpha^4)/175616) + (3375 * alpha^3)/21952 -
    (5625 * alpha^2)/175616 - (5655 * alpha)/87808 + 2535/10976) +
2 * d * g * ((-(2025 * alpha^4)/10976) - (2025 * alpha^3)/43904 +
    (7857 * alpha^2)/87808 - (2025 * alpha)/12544 + 3321/10976) +
2 * d * h * ((-(225 * alpha^4)/3584) - (4275 * alpha^3)/25088 +
    (921 * alpha^2)/12544 - (39 * alpha)/196 + 1125/3136) +
2 * d * i * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) +
e^2 * ((-(128 * alpha^4)/343) + (96 * alpha^3)/343 - (24 *
    alpha^2)/343 - (22 * alpha)/343 + 78/343) + 2 * e * f *
((-(225 * alpha^4)/686) + (15 * alpha^3)/112 + (1655 * alpha^2)/21952 -
    (3665 * alpha)/21952 + 3135/10976) + 2 * e * g * ((-(72 *
alpha^4)/343) - (27 * alpha^3)/343 + (267 * alpha^2)/1372 -
(615 * alpha)/2744 + 873/2744) + 2 * e * h * ((-alpha^4/14) -
(159 * alpha^3)/784 + (417 * alpha^2)/3136 - (529 * alpha)/3136 +
243/784) + 2 * e * i * (12/49 - 16/49 * alpha^2 + 4/49 *
alpha) + f^2 * ((-(50625 * alpha^4)/175616) + (3375 * alpha^3)/175616 +
(16875 * alpha^2)/87808 - (10125 * alpha)/43904 + 3375/10976) +
2 * f * g * ((-(2025 * alpha^4)/10976) - (5805 * alpha^3)/43904 +
    (21465 * alpha^2)/87808 - (2715 * alpha)/12544 + 12675/43904) +
2 * f * h * ((-(225 * alpha^4)/3584) - (2505 * alpha^3)/12544 +
    (3235 * alpha^2)/25088 - (1145 * alpha)/12544 + 705/3136) +
2 * f * i * (45/392 - 225/784 * alpha^2 + 135/784 * alpha) +
g^2 * ((-(81 * alpha^4)/686) - (243 * alpha^3)/1372 + (81 *
```

```
        alpha^2)/392 - (729 * alpha)/5488 + 1215/5488) + 2 *
    g * h * (405/3136 - 9/224 * alpha^4 - 999/6272 * alpha^3 +
    807/12544 * alpha^2 + 75/12544 * alpha) + 2 * g * i * (3/98 -
    9/49 * alpha^2 + 15/98 * alpha) + h^2 * (3/64 - 7/512 * alpha^4 -
    45/512 * alpha^3 - 3/128 * alpha^2 + 5/64 * alpha) + 2 *
    h * i * (1/16 * alpha - 1/16 * alpha^2) + i^2 * 0)


Q3 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
    0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
    h * 0 + 2 * a * i * 0 + b^2 * (7/1024 * alpha^4 - 3/512 *
    alpha^3 + 3/512 * alpha^2) + 2 * b * c * (1/12544 + 9/448 *
    alpha^4 - 117/6272 * alpha^3 + 27/1792 * alpha^2 + 37/12544 *
    alpha) + 2 * b * d * ((225 * alpha^4)/7168 - (75 * alpha^3)/3136 +
    (33 * alpha^2)/1792 + (9 * alpha)/784 + 9/12544) + 2 * b *
    e * (alpha^4/28 - (3 * alpha^3)/196 + (3 * alpha^2)/224 +
    (81 * alpha)/3136 + 9/3136) + 2 * b * f * ((225 * alpha^4)/7168 +
    (135 * alpha^3)/25088 + (15 * alpha^2)/3584 + (275 * alpha)/6272 +
    25/3136) + 2 * b * g * ((9 * alpha^4)/448 + (177 * alpha^3)/6272 +
    (3 * alpha^2)/1792 + (765 * alpha)/12544 + 225/12544) + 2 *
    b * h * ((7 * alpha^4)/1024 + (9 * alpha^3)/256 + (3 * alpha^2)/128 +
    (9 * alpha)/128 + 9/256) + 2 * b * i * (1/16 + 3/32 * alpha^2 +
    1/16 * alpha) + c^2 * ((81 * alpha^4)/1372 - (81 * alpha^3)/1372 +
    (81 * alpha^2)/2744 + (87 * alpha)/5488 + 33/21952) + 2 *
    c * d * ((2025 * alpha^4)/21952 - (3375 * alpha^3)/43904 +
    (261 * alpha^2)/12544 + (3453 * alpha)/87808 + 573/87808) +
    2 * c * e * ((36 * alpha^4)/343 - (18 * alpha^3)/343 - (9 *
        alpha^2)/686 + (185 * alpha)/2744 + 25/1372) + 2 * c *
    f * ((2025 * alpha^4)/21952 + (405 * alpha^3)/43904 - (675 *
    alpha^2)/12544 + (7695 * alpha)/87808 + 3555/87808) + 2 *
    c * g * ((81 * alpha^4)/1372 + (27 * alpha^3)/343 - (171 *
    alpha^2)/2744 + (111 * alpha)/1372 + 1713/21952) + 2 * c *
    h * ((9 * alpha^4)/448 + (639 * alpha^3)/6272 + (261 * alpha^2)/12544 +
    (271 * alpha)/12544 + 1711/12544) + 2 * c * i * (87/392 +
    27/98 * alpha^2 - 6/49 * alpha) + d^2 * ((50625 * alpha^4)/351232 -
    (16875 * alpha^3)/175616 - (2025 * alpha^2)/175616 + (405 *
    alpha)/5488 + 135/6272) + 2 * d * e * ((225 * alpha^4)/1372 -
    (75 * alpha^3)/1372 - (789 * alpha^2)/10976 + (45 * alpha)/448 +
    1089/21952) + 2 * d * f * ((50625 * alpha^4)/351232 + (3375 *
    alpha^3)/87808 - (5625 * alpha^2)/43904 + (4395 * alpha)/43904 +
    1185/12544) + 2 * d * g * ((2025 * alpha^4)/21952 + (6075 *
    alpha^3)/43904 - (1539 * alpha^2)/12544 + (4293 * alpha)/87808 +
    14013/87808) + 2 * d * h * ((225 * alpha^4)/7168 + (4125 *
    alpha^3)/25088 + (687 * alpha^2)/25088 - (513 * alpha)/6272 +
    1557/6272) + 2 * d * i * (285/784 + 675/1568 * alpha^2 -
    255/784 * alpha) + e^2 * ((64 * alpha^4)/343 - (48 * alpha^2)/343 +
    (36 * alpha)/343 + 135/1372) + 2 * e * f * ((225 * alpha^4)/1372 +
    (135 * alpha^3)/1372 - (1965 * alpha^2)/10976 + (205 * alpha)/3136 +
    3595/21952) + 2 * e * g * ((36 * alpha^4)/343 + (66 * alpha^3)/343 -
    (93 * alpha^2)/686 - (81 * alpha)/2744 + 333/1372) + 2 *
    e * h * (alpha^4/28 + (39 * alpha^3)/196 + (93 * alpha^2)/1568 -
    (585 * alpha)/3136 + 1035/3136) + 2 * e * i * (41/98 + 24/49 *
    alpha^2 - 20/49 * alpha) + f^2 * ((50625 * alpha^4)/351232 +
```

```
       (30375 * alpha^3)/175616 - (30375 * alpha^2)/175616 - (375 *
       alpha)/21952 + 375/1568) + 2 * f * g * ((2025 * alpha^4)/21952 +
       (9855 * alpha^3)/43904 - (1035 * alpha^2)/12544 - (11625 *
       alpha)/87808 + 27075/87808) + 2 * f * h * ((225 * alpha^4)/7168 +
       (1215 * alpha^3)/6272 + (1425 * alpha^2)/12544 - (785 * alpha)/3136 +
       4465/12544) + 2 * f * i * (285/784 + 675/1568 * alpha^2 -
       255/784 * alpha) + g^2 * ((81 * alpha^4)/1372 + (297 * alpha^3)/1372 +
       (81 * alpha^2)/2744 - (1215 * alpha)/5488 + 7425/21952) +
       2 * g * h * (3951/12544 + 9/448 * alpha^4 + 933/6272 * alpha^3 +
          2109/12544 * alpha^2 - 3033/12544 * alpha) + 2 * g *
       i * (87/392 + 27/98 * alpha^2 - 6/49 * alpha) + h^2 * (27/128 +
       7/1024 * alpha^4 + 39/512 * alpha^3 + 93/512 * alpha^2 -
       9/64 * alpha) + 2 * h * i * (1/16 + 3/32 * alpha^2 + 1/16 *
       alpha) + i^2 * 0)


Q2 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
       0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
       h * 0 + 2 * a * i * 0 + b^2 * (-1/512 * alpha^4 + 1/512 *
       alpha^3) + 2 * b * c * (-9/1568 * alpha^4 + 27/6272 * alpha^3 +
       17/12544 * alpha^2 + 1/12544 * alpha) + 2 * b * d * ((-(225 *
       alpha^4)/25088) + (45 * alpha^3)/12544 + (117 * alpha^2)/25088 +
       (9 * alpha)/12544) + 2 * b * e * ((-alpha^4/98) - alpha^3/784 +
       (27 * alpha^2)/3136 + (9 * alpha)/3136) + 2 * b * f * ((-(225 *
       alpha^4)/25088) - (225 * alpha^3)/25088 + (125 * alpha^2)/12544 +
       (25 * alpha)/3136) + 2 * b * g * ((-(9 * alpha^4)/1568) -
       (99 * alpha^3)/6272 + (45 * alpha^2)/12544 + (225 * alpha)/12544) +
       2 * b * h * ((-alpha^4/512) - alpha^3/64 - (9 * alpha^2)/512 +
          (9 * alpha)/256) + 2 * b * i * (1/16 * alpha - 1/16 *
       alpha^2) + c^2 * ((-(81 * alpha^4)/4802) + (81 * alpha^3)/9604 +
       (135 * alpha^2)/19208 + (51 * alpha)/38416 + 3/38416) + 2 *
       c * d * ((-(2025 * alpha^4)/76832) + (1215 * alpha^3)/307328 +
       (10089 * alpha^2)/614656 + (3375 * alpha)/614656 + 153/307328) +
       2 * c * e * ((-(72 * alpha^4)/2401) - (27 * alpha^3)/2401 +
          (239 * alpha^2)/9604 + (281 * alpha)/19208 + 33/19208) +
       2 * c * f * ((-(2025 * alpha^4)/76832) - (10125 * alpha^3)/307328 +
          (14625 * alpha^2)/614656 + (19125 * alpha)/614656 + 675/153664) +
       2 * c * g * ((-(81 * alpha^4)/4802) - (243 * alpha^3)/4802 +
          (9 * alpha^2)/19208 + (1107 * alpha)/19208 + 45/4802) +
       2 * c * h * ((-(9 * alpha^4)/1568) - (297 * alpha^3)/6272 -
          (775 * alpha^2)/12544 + (1219 * alpha)/12544 + 111/6272) +
       2 * c * i * (3/98 - 9/49 * alpha^2 + 15/98 * alpha) + d^2 *
       ((-(50625 * alpha^4)/1229312) - (10125 * alpha^3)/1229312 +
          (18225 * alpha^2)/614656 + (5265 * alpha)/307328 + 405/153664) +
       2 * d * e * ((-(225 * alpha^4)/4802) - (1305 * alpha^3)/38416 +
          (5499 * alpha^2)/153664 + (5679 * alpha)/153664 + 621/76832) +
       2 * d * f * ((-(50625 * alpha^4)/1229312) - (10125 * alpha^3)/153664 +
          (28575 * alpha^2)/1229312 + (39825 * alpha)/614656 +
          2925/153664) + 2 * d * g * ((-(2025 * alpha^4)/76832) -
       (27135 * alpha^3)/307328 - (13851 * alpha^2)/614656 + (60831 *
       alpha)/614656 + 11745/307328) + 2 * d * h * ((-(225 * alpha^4)/25088) -
       (1935 * alpha^3)/25088 - (747 * alpha^2)/6272 + (855 * alpha)/6272 +
       27/392) + 2 * d * i * (45/392 - 225/784 * alpha^2 + 135/784 *
```

```
    alpha) + e^2 * ((-(128 * alpha^4)/2401) - (160 * alpha^3)/2401 +
    (72 * alpha^2)/2401 + (162 * alpha)/2401 + 54/2401) + 2 *
    e * f * ((-(225 * alpha^4)/4802) - (3825 * alpha^3)/38416 -
    (325 * alpha^2)/153664 + (15325 * alpha)/153664 + 1875/38416) +
    2 * e * g * ((-(72 * alpha^4)/2401) - (279 * alpha^3)/2401 -
        (657 * alpha^2)/9604 + (2367 * alpha)/19208 + 1755/19208) +
    2 * e * h * ((-alpha^4/98) - (73 * alpha^3)/784 - (549 *
        alpha^2)/3136 + (387 * alpha)/3136 + 243/1568) + 2 *
    e * i * (12/49 - 16/49 * alpha^2 + 4/49 * alpha) + f^2 *
    ((-(50625 * alpha^4)/1229312) - (151875 * alpha^3)/1229312 -
        (16875 * alpha^2)/307328 + (9375 * alpha)/76832 + 1875/19208) +
    2 * f * g * ((-(2025 * alpha^4)/76832) - (38475 * alpha^3)/307328 -
        (79875 * alpha^2)/614656 + (69525 * alpha)/614656 + 25875/153664) +
    2 * f * h * ((-(225 * alpha^4)/25088) - (1125 * alpha^3)/12544 -
        (5375 * alpha^2)/25088 + (625 * alpha)/12544 + 825/3136) +
    2 * f * i * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) +
    g^2 * ((-(81 * alpha^4)/4802) - (1053 * alpha^3)/9604 - (3645 *
        alpha^2)/19208 + (2025 * alpha)/38416 + 10125/38416) +
    2 * g * h * (2295/6272 - 9/1568 * alpha^4 - 423/6272 * alpha^3 -
        2763/12544 * alpha^2 - 909/12544 * alpha) + 2 * g * i *
    (45/98 - 9/49 * alpha^2 - 27/98 * alpha) + h^2 * (27/64 -
    1/512 * alpha^4 - 17/512 * alpha^3 - 45/256 * alpha^2 - 27/128 *
    alpha) + 2 * h * i * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) +
    i^2 * 0)


Q1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
    0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
    h * 0 + 2 * a * i * 0 + b^2 * (1/4096 * alpha^4) + 2 * b *
    c * (9/12544 * alpha^4 + 3/12544 * alpha^3 + 1/50176 * alpha^2) +
    2 * b * d * ((225 * alpha^4)/200704 + (45 * alpha^3)/50176 +
        (9 * alpha^2)/50176) + 2 * b * e * (alpha^4/784 + (3 *
    alpha^3)/1568 + (9 * alpha^2)/12544) + 2 * b * f * ((225 *
    alpha^4)/200704 + (75 * alpha^3)/25088 + (25 * alpha^2)/12544) +
    2 * b * g * ((9 * alpha^4)/12544 + (45 * alpha^3)/12544 +
        (225 * alpha^2)/50176) + 2 * b * h * (alpha^4/4096 +
    (3 * alpha^3)/1024 + (9 * alpha^2)/1024) + 2 * b * i * (1/64 *
    alpha^2) + c^2 * ((81 * alpha^4)/38416 + (27 * alpha^3)/19208 +
    (27 * alpha^2)/76832 + (3 * alpha)/76832 + 1/614656) + 2 *
    c * d * ((2025 * alpha^4)/614656 + (2295 * alpha^3)/614656 +
    (3681 * alpha^2)/2458624 + (153 * alpha)/614656 + 9/614656) +
    2 * c * e * ((9 * alpha^4)/2401 + (33 * alpha^3)/4802 + (157 *
        alpha^2)/38416 + (33 * alpha)/38416 + 9/153664) + 2 *
    c * f * ((2025 * alpha^4)/614656 + (6075 * alpha^3)/614656 +
    (21825 * alpha^2)/2458624 + (675 * alpha)/307328 + 25/153664) +
    2 * c * g * ((81 * alpha^4)/38416 + (27 * alpha^3)/2401 +
        (1287 * alpha^2)/76832 + (45 * alpha)/9604 + 225/614656) +
    2 * c * h * ((9 * alpha^4)/12544 + (111 * alpha^3)/12544 +
        (1441 * alpha^2)/50176 + (111 * alpha)/12544 + 9/12544) +
    2 * c * i * (1/784 + 9/196 * alpha^2 + 3/196 * alpha) + d^2 *
    ((50625 * alpha^4)/9834496 + (10125 * alpha^3)/1229312 +
        (6075 * alpha^2)/1229312 + (405 * alpha)/307328 + 81/614656) +
    2 * d * e * ((225 * alpha^4)/38416 + (1035 * alpha^3)/76832 +
```

```
      (6921 * alpha^2)/614656 + (621 * alpha)/153664 + 81/153664) +
   2 * d * f * ((50625 * alpha^4)/9834496 + (43875 * alpha^3)/2458624 +
      (51525 * alpha^2)/2458624 + (2925 * alpha)/307328 + 225/153664) +
   2 * d * g * ((2025 * alpha^4)/614656 + (11745 * alpha^3)/614656 +
      (84321 * alpha^2)/2458624 + (11745 * alpha)/614656 +
      2025/614656) + 2 * d * h * ((225 * alpha^4)/200704 +
   (45 * alpha^3)/3136 + (1287 * alpha^2)/25088 + (27 * alpha)/784 +
   81/12544) + 2 * d * i * (9/784 + 225/3136 * alpha^2 + 45/784 *
   alpha) + e^2 * ((16 * alpha^4)/2401 + (48 * alpha^3)/2401 +
   (54 * alpha^2)/2401 + (27 * alpha)/2401 + 81/38416) + 2 *
   e * f * ((225 * alpha^4)/38416 + (1875 * alpha^3)/76832 +
   (22825 * alpha^2)/614656 + (1875 * alpha)/76832 + 225/38416) +
   2 * e * g * ((9 * alpha^4)/2401 + (117 * alpha^3)/4802 +
      (2061 * alpha^2)/38416 + (1755 * alpha)/38416 + 2025/153664) +
   2 * e * h * (alpha^4/784 + (27 * alpha^3)/1568 + (873 * alpha^2)/12544 +
      (243 * alpha)/3136 + 81/3136) + 2 * e * i * (9/196 +
   4/49 * alpha^2 + 6/49 * alpha) + f^2 * ((50625 * alpha^4)/9834496 +
   (16875 * alpha^3)/614656 + (16875 * alpha^2)/307328 + (1875 *
   alpha)/38416 + 625/38416) + 2 * f * g * ((2025 * alpha^4)/614656 +
   (15525 * alpha^3)/614656 + (173025 * alpha^2)/2458624 + (25875 *
   alpha)/307328 + 5625/153664) + 2 * f * h * ((225 * alpha^4)/200704 +
   (825 * alpha^3)/50176 + (3925 * alpha^2)/50176 + (825 * alpha)/6272 +
   225/3136) + 2 * f * i * (25/196 + 225/3136 * alpha^2 + 75/392 *
   alpha) + g^2 * ((81 * alpha^4)/38416 + (405 * alpha^3)/19208 +
   (6075 * alpha^2)/76832 + (10125 * alpha)/76832 + 50625/614656) +
   2 * g * h * (2025/12544 + 9/12544 * alpha^4 + 153/12544 *
      alpha^3 + 3681/50176 * alpha^2 + 2295/12544 * alpha) +
   2 * g * i * (225/784 + 9/196 * alpha^2 + 45/196 * alpha) +
   h^2 * (81/256 + 1/4096 * alpha^4 + 3/512 * alpha^3 + 27/512 *
      alpha^2 + 27/128 * alpha) + 2 * h * i * (9/16 + 1/64 *
   alpha^2 + 3/16 * alpha) + i^2 * 1)

prob <- c(Q9, Q8, Q7, Q6, Q5, Q4, Q3, Q2, Q1)
```

prob and prob1 are equivalent

```
prob
```

```
## [1] 0.04948 0.09193 0.13758 0.16100 0.19942 0.14998 0.10952 0.06350 0.03759
```

```
prob1
```

```
## [1] 0.04948 0.09193 0.13758 0.16100 0.19942 0.14998 0.10952 0.06350 0.03759
```

This perturbs prob (a function of Q1,…,Q9) to get prob_new

```
d1 = (Q9 + Q1) * 0.05
d2 = (Q8 + Q2) * 0.05
d3 = (Q7 + Q3) * 0.05
d4 = (Q6 + Q4) * 0.05

P_Q9 = Q9 + d1
P_Q1 = Q1 - d1
P_Q8 = Q8 + d2
P_Q2 = Q2 - d2
P_Q7 = Q7 + d3
```

```
P_Q3 = Q3 - d3
P_Q6 = Q6 + d4
P_Q4 = Q4 - d4
P_Q5 = Q5

prob_new <- c(P_Q9, P_Q8, P_Q7, P_Q6, P_Q5, P_Q4, P_Q3, P_Q2,
    P_Q1)
```

Minor perturbations:

```
prob_new
```

```
## [1] 0.05384 0.09970 0.14993 0.17655 0.19942 0.13443 0.09716 0.05573 0.03323
```

```
prob
```

```
## [1] 0.04948 0.09193 0.13758 0.16100 0.19942 0.14998 0.10952 0.06350 0.03759
```

This simulates data from `prob_new`

```
index <- sample(c(1, 2, 3, 4, 5, 6, 7, 8, 9), 50, replace = T,
    prob = prob_new)

NN <- rep(0, 9)
st <- table(index)
NNn <- as.numeric(names(st))
NN[NNn] <- as.numeric(st)
n9 <- NN[1]
n8 <- NN[2]
n7 <- NN[3]
n6 <- NN[4]
n5 <- NN[5]
n4 <- NN[6]
n3 <- NN[7]
n2 <- NN[8]
n1 <- NN[9]
N = n9 + n8 + n7 + n6 + n5 + n4 + n3 + n2 + n1
```

This runs some iterative algorithm to estimate the genotype frequencies

```
geno.p <- c()
for (j in 1:500) {

    R9 <- (a^2 * 1 + 2 * a * b * (9/16 + 1/64 * 0^2 + 3/16 *
        0) + 2 * a * c * (225/784 + 9/196 * 0^2 + 45/196 * 0) +
        2 * a * d * (25/196 + 225/3136 * 0^2 + 75/392 * 0) +
        2 * a * e * (9/196 + 4/49 * 0^2 + 6/49 * 0) + 2 * a *
        f * (9/784 + 225/3136 * 0^2 + 45/784 * 0) + 2 * a * g *
        (1/784 + 9/196 * 0^2 + 3/196 * 0) + 2 * a * h * (1/64 *
        0^2) + 0 + b^2 * (81/256 + 1/4096 * 0^4 + 3/512 * 0^3 +
        27/512 * 0^2 + 27/128 * 0) + 2 * b * c * (2025/12544 +
        9/12544 * 0^4 + 153/12544 * 0^3 + 3681/50176 * 0^2 +
        2295/12544 * 0) + 2 * b * d * ((225 * 0^4)/200704 + (825 *
        0^3)/50176 + (3925 * 0^2)/50176 + (825 * 0)/6272 + 225/3136) +
        2 * b * e * (0^4/784 + (27 * 0^3)/1568 + (873 * 0^2)/12544 +
            (243 * 0)/3136 + 81/3136) + 2 * b * f * ((225 * 0^4)/200704 +
        (45 * 0^3)/3136 + (1287 * 0^2)/25088 + (27 * 0)/784 +
```

```
81/12544) + 2 * b * g * ((9 * 0^4)/12544 + (111 * 0^3)/12544 +
(1441 * 0^2)/50176 + (111 * 0)/12544 + 9/12544) + 2 *
b * h * (0^4/4096 + (3 * 0^3)/1024 + (9 * 0^2)/1024) +
0 + c^2 * ((81 * 0^4)/38416 + (405 * 0^3)/19208 + (6075 *
0^2)/76832 + (10125 * 0)/76832 + 50625/614656) + 2 *
c * d * ((2025 * 0^4)/614656 + (15525 * 0^3)/614656 +
(173025 * 0^2)/2458624 + (25875 * 0)/307328 + 5625/153664) +
2 * c * e * ((9 * 0^4)/2401 + (117 * 0^3)/4802 + (2061 *
    0^2)/38416 + (1755 * 0)/38416 + 2025/153664) + 2 *
c * f * ((2025 * 0^4)/614656 + (11745 * 0^3)/614656 +
(84321 * 0^2)/2458624 + (11745 * 0)/614656 + 2025/614656) +
2 * c * g * ((81 * 0^4)/38416 + (27 * 0^3)/2401 + (1287 *
    0^2)/76832 + (45 * 0)/9604 + 225/614656) + 2 * c *
h * ((9 * 0^4)/12544 + (45 * 0^3)/12544 + (225 * 0^2)/50176) +
0 + d^2 * ((50625 * 0^4)/9834496 + (16875 * 0^3)/614656 +
(16875 * 0^2)/307328 + (1875 * 0)/38416 + 625/38416) +
2 * d * e * ((225 * 0^4)/38416 + (1875 * 0^3)/76832 +
    (22825 * 0^2)/614656 + (1875 * 0)/76832 + 225/38416) +
2 * d * f * ((50625 * 0^4)/9834496 + (43875 * 0^3)/2458624 +
    (51525 * 0^2)/2458624 + (2925 * 0)/307328 + 225/153664) +
2 * d * g * ((2025 * 0^4)/614656 + (6075 * 0^3)/614656 +
    (21825 * 0^2)/2458624 + (675 * 0)/307328 + 25/153664) +
2 * d * h * ((225 * 0^4)/200704 + (75 * 0^3)/25088 +
    (25 * 0^2)/12544) + 0 + e^2 * ((16 * 0^4)/2401 +
(48 * 0^3)/2401 + (54 * 0^2)/2401 + (27 * 0)/2401 + 81/38416) +
2 * e * f * ((225 * 0^4)/38416 + (1035 * 0^3)/76832 +
    (6921 * 0^2)/614656 + (621 * 0)/153664 + 81/153664) +
2 * e * g * ((9 * 0^4)/2401 + (33 * 0^3)/4802 + (157 *
    0^2)/38416 + (33 * 0)/38416 + 9/153664) + 2 * e *
h * (0^4/784 + (3 * 0^3)/1568 + (9 * 0^2)/12544) + 0 +
f^2 * ((50625 * 0^4)/9834496 + (10125 * 0^3)/1229312 +
    (6075 * 0^2)/1229312 + (405 * 0)/307328 + 81/614656) +
2 * f * g * ((2025 * 0^4)/614656 + (2295 * 0^3)/614656 +
    (3681 * 0^2)/2458624 + (153 * 0)/614656 + 9/614656) +
2 * f * h * ((225 * 0^4)/200704 + (45 * 0^3)/50176 +
    (9 * 0^2)/50176) + 0 + g^2 * ((81 * 0^4)/38416 +
(27 * 0^3)/19208 + (27 * 0^2)/76832 + (3 * 0)/76832 +
1/614656) + 2 * g * h * (9/12544 * 0^4 + 3/12544 * 0^3 +
1/50176 * 0^2) + 0 + h^2 * (1/4096 * 0^4) + 0 + i^2 *
0)

R8 <- (a^2 * 0 + 2 * a * b * (3/8 - 1/16 * 0^2 - 5/16 * 0) +
    2 * a * c * (45/98 - 9/49 * 0^2 - 27/98 * 0) + 2 * a *
    d * (75/196 - 225/784 * 0^2 - 75/784 * 0) + 2 * a * e *
    (12/49 - 16/49 * 0^2 + 4/49 * 0) + 2 * a * f * (45/392 -
    225/784 * 0^2 + 135/784 * 0) + 2 * a * g * (3/98 - 9/49 *
    0^2 + 15/98 * 0) + 2 * a * h * (1/16 * 0 - 1/16 * 0^2) +
    0 + b^2 * (27/64 - 1/512 * 0^4 - 17/512 * 0^3 - 45/256 *
    0^2 - 27/128 * 0) + 2 * b * c * (2295/6272 - 9/1568 *
    0^4 - 423/6272 * 0^3 - 2763/12544 * 0^2 - 909/12544 *
    0) + 2 * b * d * ((-(225 * 0^4)/25088) - (1125 * 0^3)/12544 -
    (5375 * 0^2)/25088 + (625 * 0)/12544 + 825/3136) + 2 *
    b * e * ((-0^4/98) - (73 * 0^3)/784 - (549 * 0^2)/3136 +
```

```
      (387 * 0)/3136 + 243/1568) + 2 * b * f * ((-(225 * 0^4)/25088) -
      (1935 * 0^3)/25088 - (747 * 0^2)/6272 + (855 * 0)/6272 +
      27/392) + 2 * b * g * ((-(9 * 0^4)/1568) - (297 * 0^3)/6272 -
      (775 * 0^2)/12544 + (1219 * 0)/12544 + 111/6272) + 2 *
      b * h * ((-0^4/512) - 0^3/64 - (9 * 0^2)/512 + (9 * 0)/256) +
      0 + c^2 * ((-(81 * 0^4)/4802) - (1053 * 0^3)/9604 - (3645 *
      0^2)/19208 + (2025 * 0)/38416 + 10125/38416) + 2 * c *
      d * ((-(2025 * 0^4)/76832) - (38475 * 0^3)/307328 - (79875 *
      0^2)/614656 + (69525 * 0)/614656 + 25875/153664) + 2 *
      c * e * ((-(72 * 0^4)/2401) - (279 * 0^3)/2401 - (657 *
      0^2)/9604 + (2367 * 0)/19208 + 1755/19208) + 2 * c *
      f * ((-(2025 * 0^4)/76832) - (27135 * 0^3)/307328 - (13851 *
      0^2)/614656 + (60831 * 0)/614656 + 11745/307328) + 2 *
      c * g * ((-(81 * 0^4)/4802) - (243 * 0^3)/4802 + (9 *
      0^2)/19208 + (1107 * 0)/19208 + 45/4802) + 2 * c * h *
      ((-(9 * 0^4)/1568) - (99 * 0^3)/6272 + (45 * 0^2)/12544 +
          (225 * 0)/12544) + 0 + d^2 * ((-(50625 * 0^4)/1229312) -
      (151875 * 0^3)/1229312 - (16875 * 0^2)/307328 + (9375 *
      0)/76832 + 1875/19208) + 2 * d * e * ((-(225 * 0^4)/4802) -
      (3825 * 0^3)/38416 - (325 * 0^2)/153664 + (15325 * 0)/153664 +
      1875/38416) + 2 * d * f * ((-(50625 * 0^4)/1229312) -
      (10125 * 0^3)/153664 + (28575 * 0^2)/1229312 + (39825 *
      0)/614656 + 2925/153664) + 2 * d * g * ((-(2025 * 0^4)/76832) -
      (10125 * 0^3)/307328 + (14625 * 0^2)/614656 + (19125 *
      0)/614656 + 675/153664) + 2 * d * h * ((-(225 * 0^4)/25088) -
      (225 * 0^3)/25088 + (125 * 0^2)/12544 + (25 * 0)/3136) +
      0 + e^2 * ((-(128 * 0^4)/2401) - (160 * 0^3)/2401 + (72 *
      0^2)/2401 + (162 * 0)/2401 + 54/2401) + 2 * e * f * ((-(225 *
      0^4)/4802) - (1305 * 0^3)/38416 + (5499 * 0^2)/153664 +
      (5679 * 0)/153664 + 621/76832) + 2 * e * g * ((-(72 *
      0^4)/2401) - (27 * 0^3)/2401 + (239 * 0^2)/9604 + (281 *
      0)/19208 + 33/19208) + 2 * e * h * ((-0^4/98) - 0^3/784 +
      (27 * 0^2)/3136 + (9 * 0)/3136) + 0 + f^2 * ((-(50625 *
      0^4)/1229312) - (10125 * 0^3)/1229312 + (18225 * 0^2)/614656 +
      (5265 * 0)/307328 + 405/153664) + 2 * f * g * ((-(2025 *
      0^4)/76832) + (1215 * 0^3)/307328 + (10089 * 0^2)/614656 +
      (3375 * 0)/614656 + 153/307328) + 2 * f * h * ((-(225 *
      0^4)/25088) + (45 * 0^3)/12544 + (117 * 0^2)/25088 +
      (9 * 0)/12544) + 0 + g^2 * ((-(81 * 0^4)/4802) + (81 *
      0^3)/9604 + (135 * 0^2)/19208 + (51 * 0)/38416 + 3/38416) +
      2 * g * h * (-9/1568 * 0^4 + 27/6272 * 0^3 + 17/12544 *
          0^2 + 1/12544 * 0) + 0 + h^2 * (-1/512 * 0^4 + 1/512 *
      0^3) + 0 + i^2 * 0)

  R7 <- (a^2 * 0 + 2 * a * b * (1/16 + 3/32 * 0^2 + 1/16 *
      0) + 2 * a * c * (87/392 + 27/98 * 0^2 - 6/49 * 0) +
      2 * a * d * (285/784 + 675/1568 * 0^2 - 255/784 * 0) +
      2 * a * e * (41/98 + 24/49 * 0^2 - 20/49 * 0) + 2 * a *
      f * (285/784 + 675/1568 * 0^2 - 255/784 * 0) + 2 * a *
      g * (87/392 + 27/98 * 0^2 - 6/49 * 0) + 2 * a * h * (1/16 +
      3/32 * 0^2 + 1/16 * 0) + 0 + b^2 * (27/128 + 7/1024 *
      0^4 + 39/512 * 0^3 + 93/512 * 0^2 - 9/64 * 0) + 2 * b *
      c * (3951/12544 + 9/448 * 0^4 + 933/6272 * 0^3 + 2109/12544 *
```

```
      0^2 - 3033/12544 * 0) + 2 * b * d * ((225 * 0^4)/7168 +
      (1215 * 0^3)/6272 + (1425 * 0^2)/12544 - (785 * 0)/3136 +
      4465/12544) + 2 * b * e * (0^4/28 + (39 * 0^3)/196 +
      (93 * 0^2)/1568 - (585 * 0)/3136 + 1035/3136) + 2 * b *
      f * ((225 * 0^4)/7168 + (4125 * 0^3)/25088 + (687 * 0^2)/25088 -
      (513 * 0)/6272 + 1557/6272) + 2 * b * g * ((9 * 0^4)/448 +
      (639 * 0^3)/6272 + (261 * 0^2)/12544 + (271 * 0)/12544 +
      1711/12544) + 2 * b * h * ((7 * 0^4)/1024 + (9 * 0^3)/256 +
      (3 * 0^2)/128 + (9 * 0)/128 + 9/256) + 0 + c^2 * ((81 *
      0^4)/1372 + (297 * 0^3)/1372 + (81 * 0^2)/2744 - (1215 *
      0)/5488 + 7425/21952) + 2 * c * d * ((2025 * 0^4)/21952 +
      (9855 * 0^3)/43904 - (1035 * 0^2)/12544 - (11625 * 0)/87808 +
      27075/87808) + 2 * c * e * ((36 * 0^4)/343 + (66 * 0^3)/343 -
      (93 * 0^2)/686 - (81 * 0)/2744 + 333/1372) + 2 * c *
      f * ((2025 * 0^4)/21952 + (6075 * 0^3)/43904 - (1539 *
      0^2)/12544 + (4293 * 0)/87808 + 14013/87808) + 2 * c *
      g * ((81 * 0^4)/1372 + (27 * 0^3)/343 - (171 * 0^2)/2744 +
      (111 * 0)/1372 + 1713/21952) + 2 * c * h * ((9 * 0^4)/448 +
      (177 * 0^3)/6272 + (3 * 0^2)/1792 + (765 * 0)/12544 +
      225/12544) + 0 + d^2 * ((50625 * 0^4)/351232 + (30375 *
      0^3)/175616 - (30375 * 0^2)/175616 - (375 * 0)/21952 +
      375/1568) + 2 * d * e * ((225 * 0^4)/1372 + (135 * 0^3)/1372 -
      (1965 * 0^2)/10976 + (205 * 0)/3136 + 3595/21952) + 2 *
      d * f * ((50625 * 0^4)/351232 + (3375 * 0^3)/87808 -
      (5625 * 0^2)/43904 + (4395 * 0)/43904 + 1185/12544) +
      2 * d * g * ((2025 * 0^4)/21952 + (405 * 0^3)/43904 -
         (675 * 0^2)/12544 + (7695 * 0)/87808 + 3555/87808) +
      2 * d * h * ((225 * 0^4)/7168 + (135 * 0^3)/25088 + (15 *
         0^2)/3584 + (275 * 0)/6272 + 25/3136) + 0 + e^2 *
      ((64 * 0^4)/343 - (48 * 0^2)/343 + (36 * 0)/343 + 135/1372) +
      2 * e * f * ((225 * 0^4)/1372 - (75 * 0^3)/1372 - (789 *
         0^2)/10976 + (45 * 0)/448 + 1089/21952) + 2 * e *
      g * ((36 * 0^4)/343 - (18 * 0^3)/343 - (9 * 0^2)/686 +
      (185 * 0)/2744 + 25/1372) + 2 * e * h * (0^4/28 - (3 *
      0^3)/196 + (3 * 0^2)/224 + (81 * 0)/3136 + 9/3136) +
      0 + f^2 * ((50625 * 0^4)/351232 - (16875 * 0^3)/175616 -
      (2025 * 0^2)/175616 + (405 * 0)/5488 + 135/6272) + 2 *
      f * g * ((2025 * 0^4)/21952 - (3375 * 0^3)/43904 + (261 *
      0^2)/12544 + (3453 * 0)/87808 + 573/87808) + 2 * f *
      h * ((225 * 0^4)/7168 - (75 * 0^3)/3136 + (33 * 0^2)/1792 +
      (9 * 0)/784 + 9/12544) + 0 + g^2 * ((81 * 0^4)/1372 -
      (81 * 0^3)/1372 + (81 * 0^2)/2744 + (87 * 0)/5488 + 33/21952) +
      2 * g * h * (1/12544 + 9/448 * 0^4 - 117/6272 * 0^3 +
         27/1792 * 0^2 + 37/12544 * 0) + 0 + h^2 * (7/1024 *
      0^4 - 3/512 * 0^3 + 3/512 * 0^2) + 0 + i^2 * 0)

R6 <- (a^2 * 0 + 2 * a * b * (1/16 * 0 - 1/16 * 0^2) + 2 *
      a * c * (3/98 - 9/49 * 0^2 + 15/98 * 0) + 2 * a * d *
      (45/392 - 225/784 * 0^2 + 135/784 * 0) + 2 * a * e *
      (12/49 - 16/49 * 0^2 + 4/49 * 0) + 2 * a * f * (75/196 -
      225/784 * 0^2 - 75/784 * 0) + 2 * a * g * (45/98 - 9/49 *
      0^2 - 27/98 * 0) + 2 * a * h * (3/8 - 1/16 * 0^2 - 5/16 *
      0) + 0 + b^2 * (3/64 - 7/512 * 0^4 - 45/512 * 0^3 - 3/128 *
```

```
0^2 + 5/64 * 0) + 2 * b * c * (405/3136 - 9/224 * 0^4 -
999/6272 * 0^3 + 807/12544 * 0^2 + 75/12544 * 0) + 2 *
b * d * ((-(225 * 0^4)/3584) - (2505 * 0^3)/12544 + (3235 *
0^2)/25088 - (1145 * 0)/12544 + 705/3136) + 2 * b * e *
((-0^4/14) - (159 * 0^3)/784 + (417 * 0^2)/3136 - (529 *
    0)/3136 + 243/784) + 2 * b * f * ((-(225 * 0^4)/3584) -
(4275 * 0^3)/25088 + (921 * 0^2)/12544 - (39 * 0)/196 +
1125/3136) + 2 * b * g * ((-(9 * 0^4)/224) - (705 * 0^3)/6272 -
(229 * 0^2)/12544 - (2165 * 0)/12544 + 1077/3136) + 2 *
b * h * ((-(7 * 0^4)/512) - (3 * 0^3)/64 - (39 * 0^2)/512 -
(25 * 0)/256 + 15/64) + 0 + c^2 * ((-(81 * 0^4)/686) -
(243 * 0^3)/1372 + (81 * 0^2)/392 - (729 * 0)/5488 +
1215/5488) + 2 * c * d * ((-(2025 * 0^4)/10976) - (5805 *
0^3)/43904 + (21465 * 0^2)/87808 - (2715 * 0)/12544 +
12675/43904) + 2 * c * e * ((-(72 * 0^4)/343) - (27 *
0^3)/343 + (267 * 0^2)/1372 - (615 * 0)/2744 + 873/2744) +
2 * c * f * ((-(2025 * 0^4)/10976) - (2025 * 0^3)/43904 +
    (7857 * 0^2)/87808 - (2025 * 0)/12544 + 3321/10976) +
2 * c * g * ((-(81 * 0^4)/686) - (27 * 0^3)/686 - (9 *
    0^2)/392 - (165 * 0)/2744 + 165/686) + 2 * c * h *
((-(9 * 0^4)/224) - (243 * 0^3)/6272 - (993 * 0^2)/12544 +
    (39 * 0)/1792 + 855/6272) + 0 + d^2 * ((-(50625 *
0^4)/175616) + (3375 * 0^3)/175616 + (16875 * 0^2)/87808 -
(10125 * 0)/43904 + 3375/10976) + 2 * d * e * ((-(225 *
0^4)/686) + (15 * 0^3)/112 + (1655 * 0^2)/21952 - (3665 *
0)/21952 + 3135/10976) + 2 * d * f * ((-(50625 * 0^4)/175616) +
(3375 * 0^3)/21952 - (5625 * 0^2)/175616 - (5655 * 0)/87808 +
2535/10976) + 2 * d * g * ((-(2025 * 0^4)/10976) + (3645 *
0^3)/43904 - (7515 * 0^2)/87808 + (405 * 0)/12544 + 6795/43904) +
2 * d * h * ((-(225 * 0^4)/3584) - (285 * 0^3)/25088 -
    (55 * 0^2)/784 + (65 * 0)/896 + 225/3136) + 0 + e^2 *
((-(128 * 0^4)/343) + (96 * 0^3)/343 - (24 * 0^2)/343 -
    (22 * 0)/343 + 78/343) + 2 * e * f * ((-(225 * 0^4)/686) +
(225 * 0^3)/784 - (3273 * 0^2)/21952 + (717 * 0)/21952 +
54/343) + 2 * e * g * ((-(72 * 0^4)/343) + (57 * 0^3)/343 -
(181 * 0^2)/1372 + (239 * 0)/2744 + 243/2744) + 2 * e *
h * ((-0^4/14) + (9 * 0^3)/784 - (159 * 0^2)/3136 + (5 *
0)/64 + 51/1568) + 0 + f^2 * ((-(50625 * 0^4)/175616) +
(50625 * 0^3)/175616 - (2025 * 0^2)/10976 + (2025 * 0)/21952 +
2025/21952) + 2 * f * g * ((-(2025 * 0^4)/10976) + (7425 *
0^3)/43904 - (11043 * 0^2)/87808 + (177 * 0)/1792 + 465/10976) +
2 * f * h * ((-(225 * 0^4)/3584) + (225 * 0^3)/12544 -
    (633 * 0^2)/25088 + (15 * 0)/256 + 9/784) + 0 + g^2 *
((-(81 * 0^4)/686) + (135 * 0^3)/1372 - (27 * 0^2)/392 +
    (405 * 0)/5488 + 81/5488) + 2 * g * h * (15/6272 -
9/224 * 0^4 + 51/6272 * 0^3 - 13/12544 * 0^2 + 55/1792 *
0) + 0 + h^2 * (-7/512 * 0^4 - 3/512 * 0^3 + 3/256 *
0^2 + 1/128 * 0) + 0 + i^2 * 0)

R5 <- (a^2 * 0 + 2 * a * b * (1/64 * 0^2) + 2 * a * c * (1/784 +
9/196 * 0^2 + 3/196 * 0) + 2 * a * d * (9/784 + 225/3136 *
0^2 + 45/784 * 0) + 2 * a * e * (9/196 + 4/49 * 0^2 +
6/49 * 0) + 2 * a * f * (25/196 + 225/3136 * 0^2 + 75/392 *
```

0) + 2 * a * g * (225/784 + 9/196 * 0^2 + 45/196 * 0) +
2 * a * h * (9/16 + 1/64 * 0^2 + 3/16 * 0) + 2 * a *
i * 1 + b^2 * (1/256 + 35/2048 * 0^4 + 25/512 * 0^3 -
27/512 * 0^2 + 7/128 * 0) + 2 * b * c * (327/12544 +
45/896 * 0^4 + 225/3136 * 0^3 - 2533/25088 * 0^2 + 1149/12544 *
0) + 2 * b * d * ((1125 * 0^4)/14336 + (2025 * 0^3)/25088 -
(2623 * 0^2)/25088 + (561 * 0)/6272 + 453/6272) + 2 *
b * e * ((5 * 0^4)/56 + (65 * 0^3)/784 - (369 * 0^2)/6272 +
(149 * 0)/3136 + 451/3136) + 2 * b * f * ((1125 * 0^4)/14336 +
(2025 * 0^3)/25088 + (527 * 0^2)/25088 - (45 * 0)/3136 +
2985/12544) + 2 * b * g * ((45 * 0^4)/896 + (225 * 0^3)/3136 +
(2507 * 0^2)/25088 - (699 * 0)/12544 + 4359/12544) +
2 * b * h * ((35 * 0^4)/2048 + (25 * 0^3)/512 + (63 *
    0^2)/512 - 0/64 + 59/128) + 2 * b * i * (9/16 + 1/64 *
0^2 + 3/16 * 0) + c^2 * ((405 * 0^4)/2744 - (513 * 0^2)/5488 +
(27 * 0)/343 + 3429/43904) + 2 * c * d * ((10125 * 0^4)/43904 -
(2025 * 0^3)/21952 - (2691 * 0^2)/175616 + (675 * 0)/87808 +
13045/87808) + 2 * c * e * ((90 * 0^4)/343 - (45 * 0^3)/343 +
(197 * 0^2)/2744 - (117 * 0)/1372 + 2481/10976) + 2 *
c * f * ((10125 * 0^4)/43904 - (2025 * 0^3)/21952 + (22509 *
0^2)/175616 - (13941 * 0)/87808 + 26037/87808) + 2 *
c * g * ((405 * 0^4)/2744 + (747 * 0^2)/5488 - (57 *
0)/343 + 15077/43904) + 2 * c * h * ((45 * 0^4)/896 +
(225 * 0^3)/3136 + (2507 * 0^2)/25088 - (699 * 0)/12544 +
4359/12544) + 2 * c * i * (225/784 + 9/196 * 0^2 + 45/196 *
0) + d^2 * ((253125 * 0^4)/702464 - (50625 * 0^3)/175616 +
(24975 * 0^2)/175616 - (675 * 0)/6272 + 19575/87808) +
2 * d * e * ((1125 * 0^4)/2744 - (2025 * 0^3)/5488 +
    (10649 * 0^2)/43904 - (4317 * 0)/21952 + 879/3136) +
2 * d * f * ((253125 * 0^4)/702464 - (50625 * 0^3)/175616 +
    (40725 * 0^2)/175616 - (345 * 0)/1568 + 13529/43904) +
2 * d * g * ((10125 * 0^4)/43904 - (2025 * 0^3)/21952 +
    (22509 * 0^2)/175616 - (13941 * 0)/87808 + 26037/87808) +
2 * d * h * ((1125 * 0^4)/14336 + (2025 * 0^3)/25088 +
    (527 * 0^2)/25088 - (45 * 0)/3136 + 2985/12544) +
2 * d * i * (25/196 + 225/3136 * 0^2 + 75/392 * 0) +
e^2 * ((160 * 0^4)/343 - (160 * 0^3)/343 + (108 * 0^2)/343 -
    (82 * 0)/343 + 821/2744) + 2 * e * f * ((1125 * 0^4)/2744 -
(2025 * 0^3)/5488 + (10649 * 0^2)/43904 - (4317 * 0)/21952 +
879/3136) + 2 * e * g * ((90 * 0^4)/343 - (45 * 0^3)/343 +
(197 * 0^2)/2744 - (117 * 0)/1372 + 2481/10976) + 2 *
e * h * ((5 * 0^4)/56 + (65 * 0^3)/784 - (369 * 0^2)/6272 +
(149 * 0)/3136 + 451/3136) + 2 * e * i * (9/196 + 4/49 *
0^2 + 6/49 * 0) + f^2 * ((253125 * 0^4)/702464 - (50625 *
0^3)/175616 + (24975 * 0^2)/175616 - (675 * 0)/6272 +
19575/87808) + 2 * f * g * ((10125 * 0^4)/43904 - (2025 *
0^3)/21952 - (2691 * 0^2)/175616 + (675 * 0)/87808 +
13045/87808) + 2 * f * h * ((1125 * 0^4)/14336 + (2025 *
0^3)/25088 - (2623 * 0^2)/25088 + (561 * 0)/6272 + 453/6272) +
2 * f * i * (9/784 + 225/3136 * 0^2 + 45/784 * 0) + g^2 *
((405 * 0^4)/2744 - (513 * 0^2)/5488 + (27 * 0)/343 +
    3429/43904) + 2 * g * h * (327/12544 + 45/896 * 0^4 +
225/3136 * 0^3 - 2533/25088 * 0^2 + 1149/12544 * 0) +

```
      2 * g * i * (1/784 + 9/196 * 0^2 + 3/196 * 0) + h^2 *
      (1/256 + 35/2048 * 0^4 + 25/512 * 0^3 - 27/512 * 0^2 +
          7/128 * 0) + 2 * h * i * (1/64 * 0^2) + i^2 * 0)

  R4 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a *
      d * 0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 +
      2 * a * h * 0 + 2 * a * i * 0 + b^2 * (-7/512 * 0^4 -
      3/512 * 0^3 + 3/256 * 0^2 + 1/128 * 0) + 2 * b * c *
      (15/6272 - 9/224 * 0^4 + 51/6272 * 0^3 - 13/12544 * 0^2 +
          55/1792 * 0) + 2 * b * d * ((-(225 * 0^4)/3584) +
      (225 * 0^3)/12544 - (633 * 0^2)/25088 + (15 * 0)/256 +
      9/784) + 2 * b * e * ((-0^4/14) + (9 * 0^3)/784 - (159 *
      0^2)/3136 + (5 * 0)/64 + 51/1568) + 2 * b * f * ((-(225 *
      0^4)/3584) - (285 * 0^3)/25088 - (55 * 0^2)/784 + (65 *
      0)/896 + 225/3136) + 2 * b * g * ((-(9 * 0^4)/224) -
      (243 * 0^3)/6272 - (993 * 0^2)/12544 + (39 * 0)/1792 +
      855/6272) + 2 * b * h * ((-(7 * 0^4)/512) - (3 * 0^3)/64 -
      (39 * 0^2)/512 - (25 * 0)/256 + 15/64) + 2 * b * i *
      (3/8 - 1/16 * 0^2 - 5/16 * 0) + c^2 * ((-(81 * 0^4)/686) +
      (135 * 0^3)/1372 - (27 * 0^2)/392 + (405 * 0)/5488 +
      81/5488) + 2 * c * d * ((-(2025 * 0^4)/10976) + (7425 *
      0^3)/43904 - (11043 * 0^2)/87808 + (177 * 0)/1792 + 465/10976) +
      2 * c * e * ((-(72 * 0^4)/343) + (57 * 0^3)/343 - (181 *
          0^2)/1372 + (239 * 0)/2744 + 243/2744) + 2 * c *
      f * ((-(2025 * 0^4)/10976) + (3645 * 0^3)/43904 - (7515 *
      0^2)/87808 + (405 * 0)/12544 + 6795/43904) + 2 * c *
      g * ((-(81 * 0^4)/686) - (27 * 0^3)/686 - (9 * 0^2)/392 -
      (165 * 0)/2744 + 165/686) + 2 * c * h * ((-(9 * 0^4)/224) -
      (705 * 0^3)/6272 - (229 * 0^2)/12544 - (2165 * 0)/12544 +
      1077/3136) + 2 * c * i * (45/98 - 9/49 * 0^2 - 27/98 *
      0) + d^2 * ((-(50625 * 0^4)/175616) + (50625 * 0^3)/175616 -
      (2025 * 0^2)/10976 + (2025 * 0)/21952 + 2025/21952) +
      2 * d * e * ((-(225 * 0^4)/686) + (225 * 0^3)/784 - (3273 *
          0^2)/21952 + (717 * 0)/21952 + 54/343) + 2 * d *
      f * ((-(50625 * 0^4)/175616) + (3375 * 0^3)/21952 - (5625 *
      0^2)/175616 - (5655 * 0)/87808 + 2535/10976) + 2 * d *
      g * ((-(2025 * 0^4)/10976) - (2025 * 0^3)/43904 + (7857 *
      0^2)/87808 - (2025 * 0)/12544 + 3321/10976) + 2 * d *
      h * ((-(225 * 0^4)/3584) - (4275 * 0^3)/25088 + (921 *
      0^2)/12544 - (39 * 0)/196 + 1125/3136) + 2 * d * i *
      (75/196 - 225/784 * 0^2 - 75/784 * 0) + e^2 * ((-(128 *
      0^4)/343) + (96 * 0^3)/343 - (24 * 0^2)/343 - (22 * 0)/343 +
      78/343) + 2 * e * f * ((-(225 * 0^4)/686) + (15 * 0^3)/112 +
      (1655 * 0^2)/21952 - (3665 * 0)/21952 + 3135/10976) +
      2 * e * g * ((-(72 * 0^4)/343) - (27 * 0^3)/343 + (267 *
          0^2)/1372 - (615 * 0)/2744 + 873/2744) + 2 * e *
      h * ((-0^4/14) - (159 * 0^3)/784 + (417 * 0^2)/3136 -
      (529 * 0)/3136 + 243/784) + 2 * e * i * (12/49 - 16/49 *
      0^2 + 4/49 * 0) + f^2 * ((-(50625 * 0^4)/175616) + (3375 *
      0^3)/175616 + (16875 * 0^2)/87808 - (10125 * 0)/43904 +
      3375/10976) + 2 * f * g * ((-(2025 * 0^4)/10976) - (5805 *
      0^3)/43904 + (21465 * 0^2)/87808 - (2715 * 0)/12544 +
      12675/43904) + 2 * f * h * ((-(225 * 0^4)/3584) - (2505 *
```

```
0^3)/12544 + (3235 * 0^2)/25088 - (1145 * 0)/12544 +
705/3136) + 2 * f * i * (45/392 - 225/784 * 0^2 + 135/784 *
0) + g^2 * ((-(81 * 0^4)/686) - (243 * 0^3)/1372 + (81 *
0^2)/392 - (729 * 0)/5488 + 1215/5488) + 2 * g * h *
(405/3136 - 9/224 * 0^4 - 999/6272 * 0^3 + 807/12544 *
    0^2 + 75/12544 * 0) + 2 * g * i * (3/98 - 9/49 *
0^2 + 15/98 * 0) + h^2 * (3/64 - 7/512 * 0^4 - 45/512 *
0^3 - 3/128 * 0^2 + 5/64 * 0) + 2 * h * i * (1/16 * 0 -
1/16 * 0^2) + i^2 * 0)

R3 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a *
    d * 0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 +
    2 * a * h * 0 + 2 * a * i * 0 + b^2 * (7/1024 * 0^4 -
    3/512 * 0^3 + 3/512 * 0^2) + 2 * b * c * (1/12544 + 9/448 *
    0^4 - 117/6272 * 0^3 + 27/1792 * 0^2 + 37/12544 * 0) +
    2 * b * d * ((225 * 0^4)/7168 - (75 * 0^3)/3136 + (33 *
        0^2)/1792 + (9 * 0)/784 + 9/12544) + 2 * b * e *
    (0^4/28 - (3 * 0^3)/196 + (3 * 0^2)/224 + (81 * 0)/3136 +
        9/3136) + 2 * b * f * ((225 * 0^4)/7168 + (135 *
    0^3)/25088 + (15 * 0^2)/3584 + (275 * 0)/6272 + 25/3136) +
    2 * b * g * ((9 * 0^4)/448 + (177 * 0^3)/6272 + (3 *
        0^2)/1792 + (765 * 0)/12544 + 225/12544) + 2 * b *
    h * ((7 * 0^4)/1024 + (9 * 0^3)/256 + (3 * 0^2)/128 +
    (9 * 0)/128 + 9/256) + 2 * b * i * (1/16 + 3/32 * 0^2 +
    1/16 * 0) + c^2 * ((81 * 0^4)/1372 - (81 * 0^3)/1372 +
    (81 * 0^2)/2744 + (87 * 0)/5488 + 33/21952) + 2 * c *
    d * ((2025 * 0^4)/21952 - (3375 * 0^3)/43904 + (261 *
    0^2)/12544 + (3453 * 0)/87808 + 573/87808) + 2 * c *
    e * ((36 * 0^4)/343 - (18 * 0^3)/343 - (9 * 0^2)/686 +
    (185 * 0)/2744 + 25/1372) + 2 * c * f * ((2025 * 0^4)/21952 +
    (405 * 0^3)/43904 - (675 * 0^2)/12544 + (7695 * 0)/87808 +
    3555/87808) + 2 * c * g * ((81 * 0^4)/1372 + (27 * 0^3)/343 -
    (171 * 0^2)/2744 + (111 * 0)/1372 + 1713/21952) + 2 *
    c * h * ((9 * 0^4)/448 + (639 * 0^3)/6272 + (261 * 0^2)/12544 +
    (271 * 0)/12544 + 1711/12544) + 2 * c * i * (87/392 +
    27/98 * 0^2 - 6/49 * 0) + d^2 * ((50625 * 0^4)/351232 -
    (16875 * 0^3)/175616 - (2025 * 0^2)/175616 + (405 * 0)/5488 +
    135/6272) + 2 * d * e * ((225 * 0^4)/1372 - (75 * 0^3)/1372 -
    (789 * 0^2)/10976 + (45 * 0)/448 + 1089/21952) + 2 *
    d * f * ((50625 * 0^4)/351232 + (3375 * 0^3)/87808 -
    (5625 * 0^2)/43904 + (4395 * 0)/43904 + 1185/12544) +
    2 * d * g * ((2025 * 0^4)/21952 + (6075 * 0^3)/43904 -
        (1539 * 0^2)/12544 + (4293 * 0)/87808 + 14013/87808) +
    2 * d * h * ((225 * 0^4)/7168 + (4125 * 0^3)/25088 +
        (687 * 0^2)/25088 - (513 * 0)/6272 + 1557/6272) +
    2 * d * i * (285/784 + 675/1568 * 0^2 - 255/784 * 0) +
    e^2 * ((64 * 0^4)/343 - (48 * 0^2)/343 + (36 * 0)/343 +
        135/1372) + 2 * e * f * ((225 * 0^4)/1372 + (135 *
    0^3)/1372 - (1965 * 0^2)/10976 + (205 * 0)/3136 + 3595/21952) +
    2 * e * g * ((36 * 0^4)/343 + (66 * 0^3)/343 - (93 *
        0^2)/686 - (81 * 0)/2744 + 333/1372) + 2 * e * h *
    (0^4/28 + (39 * 0^3)/196 + (93 * 0^2)/1568 - (585 * 0)/3136 +
        1035/3136) + 2 * e * i * (41/98 + 24/49 * 0^2 - 20/49 *
```

```
0) + f^2 * ((50625 * 0^4)/351232 + (30375 * 0^3)/175616 -
(30375 * 0^2)/175616 - (375 * 0)/21952 + 375/1568) +
2 * f * g * ((2025 * 0^4)/21952 + (9855 * 0^3)/43904 -
    (1035 * 0^2)/12544 - (11625 * 0)/87808 + 27075/87808) +
2 * f * h * ((225 * 0^4)/7168 + (1215 * 0^3)/6272 + (1425 *
    0^2)/12544 - (785 * 0)/3136 + 4465/12544) + 2 * f *
i * (285/784 + 675/1568 * 0^2 - 255/784 * 0) + g^2 *
((81 * 0^4)/1372 + (297 * 0^3)/1372 + (81 * 0^2)/2744 -
    (1215 * 0)/5488 + 7425/21952) + 2 * g * h * (3951/12544 +
9/448 * 0^4 + 933/6272 * 0^3 + 2109/12544 * 0^2 - 3033/12544 *
0) + 2 * g * i * (87/392 + 27/98 * 0^2 - 6/49 * 0) +
h^2 * (27/128 + 7/1024 * 0^4 + 39/512 * 0^3 + 93/512 *
    0^2 - 9/64 * 0) + 2 * h * i * (1/16 + 3/32 * 0^2 +
1/16 * 0) + i^2 * 0)

R2 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a *
    d * 0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 +
    2 * a * h * 0 + 2 * a * i * 0 + b^2 * (-1/512 * 0^4 +
    1/512 * 0^3) + 2 * b * c * (-9/1568 * 0^4 + 27/6272 *
    0^3 + 17/12544 * 0^2 + 1/12544 * 0) + 2 * b * d * ((-(225 *
    0^4)/25088) + (45 * 0^3)/12544 + (117 * 0^2)/25088 +
    (9 * 0)/12544) + 2 * b * e * ((-0^4/98) - 0^3/784 + (27 *
    0^2)/3136 + (9 * 0)/3136) + 2 * b * f * ((-(225 * 0^4)/25088) -
    (225 * 0^3)/25088 + (125 * 0^2)/12544 + (25 * 0)/3136) +
    2 * b * g * ((-(9 * 0^4)/1568) - (99 * 0^3)/6272 + (45 *
        0^2)/12544 + (225 * 0)/12544) + 2 * b * h * ((-0^4/512) -
    0^3/64 - (9 * 0^2)/512 + (9 * 0)/256) + 2 * b * i * (1/16 *
    0 - 1/16 * 0^2) + c^2 * ((-(81 * 0^4)/4802) + (81 * 0^3)/9604 +
    (135 * 0^2)/19208 + (51 * 0)/38416 + 3/38416) + 2 * c *
    d * ((-(2025 * 0^4)/76832) + (1215 * 0^3)/307328 + (10089 *
    0^2)/614656 + (3375 * 0)/614656 + 153/307328) + 2 * c *
    e * ((-(72 * 0^4)/2401) - (27 * 0^3)/2401 + (239 * 0^2)/9604 +
    (281 * 0)/19208 + 33/19208) + 2 * c * f * ((-(2025 *
    0^4)/76832) - (10125 * 0^3)/307328 + (14625 * 0^2)/614656 +
    (19125 * 0)/614656 + 675/153664) + 2 * c * g * ((-(81 *
    0^4)/4802) - (243 * 0^3)/4802 + (9 * 0^2)/19208 + (1107 *
    0)/19208 + 45/4802) + 2 * c * h * ((-(9 * 0^4)/1568) -
    (297 * 0^3)/6272 - (775 * 0^2)/12544 + (1219 * 0)/12544 +
    111/6272) + 2 * c * i * (3/98 - 9/49 * 0^2 + 15/98 *
    0) + d^2 * ((-(50625 * 0^4)/1229312) - (10125 * 0^3)/1229312 +
    (18225 * 0^2)/614656 + (5265 * 0)/307328 + 405/153664) +
    2 * d * e * ((-(225 * 0^4)/4802) - (1305 * 0^3)/38416 +
        (5499 * 0^2)/153664 + (5679 * 0)/153664 + 621/76832) +
    2 * d * f * ((-(50625 * 0^4)/1229312) - (10125 * 0^3)/153664 +
        (28575 * 0^2)/1229312 + (39825 * 0)/614656 + 2925/153664) +
    2 * d * g * ((-(2025 * 0^4)/76832) - (27135 * 0^3)/307328 -
        (13851 * 0^2)/614656 + (60831 * 0)/614656 + 11745/307328) +
    2 * d * h * ((-(225 * 0^4)/25088) - (1935 * 0^3)/25088 -
        (747 * 0^2)/6272 + (855 * 0)/6272 + 27/392) + 2 *
    d * i * (45/392 - 225/784 * 0^2 + 135/784 * 0) + e^2 *
    ((-(128 * 0^4)/2401) - (160 * 0^3)/2401 + (72 * 0^2)/2401 +
        (162 * 0)/2401 + 54/2401) + 2 * e * f * ((-(225 *
    0^4)/4802) - (3825 * 0^3)/38416 - (325 * 0^2)/153664 +
```

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        (15325 * 0)/153664 + 1875/38416) + 2 * e * g * ((-(72 *
        0^4)/2401) - (279 * 0^3)/2401 - (657 * 0^2)/9604 + (2367 *
        0)/19208 + 1755/19208) + 2 * e * h * ((-0^4/98) - (73 *
        0^3)/784 - (549 * 0^2)/3136 + (387 * 0)/3136 + 243/1568) +
        2 * e * i * (12/49 - 16/49 * 0^2 + 4/49 * 0) + f^2 *
        ((-(50625 * 0^4)/1229312) - (151875 * 0^3)/1229312 -
            (16875 * 0^2)/307328 + (9375 * 0)/76832 + 1875/19208) +
        2 * f * g * ((-(2025 * 0^4)/76832) - (38475 * 0^3)/307328 -
            (79875 * 0^2)/614656 + (69525 * 0)/614656 + 25875/153664) +
        2 * f * h * ((-(225 * 0^4)/25088) - (1125 * 0^3)/12544 -
            (5375 * 0^2)/25088 + (625 * 0)/12544 + 825/3136) +
        2 * f * i * (75/196 - 225/784 * 0^2 - 75/784 * 0) + g^2 *
        ((-(81 * 0^4)/4802) - (1053 * 0^3)/9604 - (3645 * 0^2)/19208 +
            (2025 * 0)/38416 + 10125/38416) + 2 * g * h * (2295/6272 -
        9/1568 * 0^4 - 423/6272 * 0^3 - 2763/12544 * 0^2 - 909/12544 *
        0) + 2 * g * i * (45/98 - 9/49 * 0^2 - 27/98 * 0) + h^2 *
        (27/64 - 1/512 * 0^4 - 17/512 * 0^3 - 45/256 * 0^2 -
            27/128 * 0) + 2 * h * i * (3/8 - 1/16 * 0^2 - 5/16 *
        0) + i^2 * 0)

R1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a *
    d * 0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 +
    2 * a * h * 0 + 2 * a * i * 0 + b^2 * (1/4096 * 0^4) +
    2 * b * c * (9/12544 * 0^4 + 3/12544 * 0^3 + 1/50176 *
        0^2) + 2 * b * d * ((225 * 0^4)/200704 + (45 * 0^3)/50176 +
    (9 * 0^2)/50176) + 2 * b * e * (0^4/784 + (3 * 0^3)/1568 +
    (9 * 0^2)/12544) + 2 * b * f * ((225 * 0^4)/200704 +
    (75 * 0^3)/25088 + (25 * 0^2)/12544) + 2 * b * g * ((9 *
    0^4)/12544 + (45 * 0^3)/12544 + (225 * 0^2)/50176) +
    2 * b * h * (0^4/4096 + (3 * 0^3)/1024 + (9 * 0^2)/1024) +
    2 * b * i * (1/64 * 0^2) + c^2 * ((81 * 0^4)/38416 +
    (27 * 0^3)/19208 + (27 * 0^2)/76832 + (3 * 0)/76832 +
    1/614656) + 2 * c * d * ((2025 * 0^4)/614656 + (2295 *
    0^3)/614656 + (3681 * 0^2)/2458624 + (153 * 0)/614656 +
    9/614656) + 2 * c * e * ((9 * 0^4)/2401 + (33 * 0^3)/4802 +
    (157 * 0^2)/38416 + (33 * 0)/38416 + 9/153664) + 2 *
    c * f * ((2025 * 0^4)/614656 + (6075 * 0^3)/614656 +
    (21825 * 0^2)/2458624 + (675 * 0)/307328 + 25/153664) +
    2 * c * g * ((81 * 0^4)/38416 + (27 * 0^3)/2401 + (1287 *
        0^2)/76832 + (45 * 0)/9604 + 225/614656) + 2 * c *
    h * ((9 * 0^4)/12544 + (111 * 0^3)/12544 + (1441 * 0^2)/50176 +
    (111 * 0)/12544 + 9/12544) + 2 * c * i * (1/784 + 9/196 *
    0^2 + 3/196 * 0) + d^2 * ((50625 * 0^4)/9834496 + (10125 *
    0^3)/1229312 + (6075 * 0^2)/1229312 + (405 * 0)/307328 +
    81/614656) + 2 * d * e * ((225 * 0^4)/38416 + (1035 *
    0^3)/76832 + (6921 * 0^2)/614656 + (621 * 0)/153664 +
    81/153664) + 2 * d * f * ((50625 * 0^4)/9834496 + (43875 *
    0^3)/2458624 + (51525 * 0^2)/2458624 + (2925 * 0)/307328 +
    225/153664) + 2 * d * g * ((2025 * 0^4)/614656 + (11745 *
    0^3)/614656 + (84321 * 0^2)/2458624 + (11745 * 0)/614656 +
    2025/614656) + 2 * d * h * ((225 * 0^4)/200704 + (45 *
    0^3)/3136 + (1287 * 0^2)/25088 + (27 * 0)/784 + 81/12544) +
    2 * d * i * (9/784 + 225/3136 * 0^2 + 45/784 * 0) + e^2 *
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      ((16 * 0^4)/2401 + (48 * 0^3)/2401 + (54 * 0^2)/2401 +
          (27 * 0)/2401 + 81/38416) + 2 * e * f * ((225 * 0^4)/38416 +
      (1875 * 0^3)/76832 + (22825 * 0^2)/614656 + (1875 * 0)/76832 +
      225/38416) + 2 * e * g * ((9 * 0^4)/2401 + (117 * 0^3)/4802 +
      (2061 * 0^2)/38416 + (1755 * 0)/38416 + 2025/153664) +
      2 * e * h * (0^4/784 + (27 * 0^3)/1568 + (873 * 0^2)/12544 +
          (243 * 0)/3136 + 81/3136) + 2 * e * i * (9/196 +
      4/49 * 0^2 + 6/49 * 0) + f^2 * ((50625 * 0^4)/9834496 +
      (16875 * 0^3)/614656 + (16875 * 0^2)/307328 + (1875 *
      0)/38416 + 625/38416) + 2 * f * g * ((2025 * 0^4)/614656 +
      (15525 * 0^3)/614656 + (173025 * 0^2)/2458624 + (25875 *
      0)/307328 + 5625/153664) + 2 * f * h * ((225 * 0^4)/200704 +
      (825 * 0^3)/50176 + (3925 * 0^2)/50176 + (825 * 0)/6272 +
      225/3136) + 2 * f * i * (25/196 + 225/3136 * 0^2 + 75/392 *
      0) + g^2 * ((81 * 0^4)/38416 + (405 * 0^3)/19208 + (6075 *
      0^2)/76832 + (10125 * 0)/76832 + 50625/614656) + 2 *
      g * h * (2025/12544 + 9/12544 * 0^4 + 153/12544 * 0^3 +
      3681/50176 * 0^2 + 2295/12544 * 0) + 2 * g * i * (225/784 +
      9/196 * 0^2 + 45/196 * 0) + h^2 * (81/256 + 1/4096 *
      0^4 + 3/512 * 0^3 + 27/512 * 0^2 + 27/128 * 0) + 2 *
      h * i * (9/16 + 1/64 * 0^2 + 3/16 * 0) + i^2 * 1)

fai9.9 = (2 * a^2 * 1 + 2 * a * b * (9/16) + 2 * a * c *
    (225/784) + 2 * a * d * (25/196) + 2 * a * e * (9/196) +
    2 * a * f * (9/784) + 2 * a * g * (1/784))/R9
fai9.8 = (2 * a^2 * 0 + 2 * a * b * (3/8) + 2 * a * c * (45/98) +
    2 * a * d * (75/196) + 2 * a * e * (12/49) + 2 * a *
    f * (45/392) + 2 * a * g * (3/98))/R8
fai9.7 = (2 * a^2 * 0 + 2 * a * b * (1/16) + 2 * a * c *
    (87/392) + 2 * a * d * (285/784) + 2 * a * e * (41/98) +
    2 * a * f * (285/784) + 2 * a * g * (87/392) + 2 * a *
    h * (1/16))/R7
fai9.6 = (2 * a^2 * 0 + 2 * a * b * 0 + 2 * a * c * (3/98) +
    2 * a * d * (45/392) + 2 * a * e * (12/49) + 2 * a *
    f * (75/196) + 2 * a * g * (45/98) + 2 * a * h * (3/8))/R6
fai9.5 = (2 * a^2 * 0 + 2 * a * b * 0 + 2 * a * c * (1/784) +
    2 * a * d * (9/784) + 2 * a * e * (9/196) + 2 * a * f *
    (25/196) + 2 * a * g * (225/784) + 2 * a * h * (9/16) +
    2 * a * i * 1)/R5

fai8.9 = (2 * a * b * (9/16) + 2 * b^2 * (81/256) + 2 * b *
    c * (2025/12544) + 2 * b * d * (225/3136) + 2 * b * e *
    (81/3136) + 2 * b * f * (81/12544) + 2 * b * g * (9/12544))/R9
fai8.8 = (2 * a * b * (3/8) + 2 * b^2 * (27/64) + 2 * b *
    c * (2295/6272) + 2 * b * d * (825/3136) + 2 * b * e *
    (243/1568) + 2 * b * f * (27/392) + 2 * b * g * (111/6272))/R8
fai8.7 = (2 * a * b * (1/16) + 2 * b^2 * (27/128) + 2 * b *
    c * (3951/12544) + 2 * b * d * (4465/12544) + 2 * b *
    e * (1035/3136) + 2 * b * f * (1557/6272) + 2 * b * g *
    (1711/12544) + 2 * b * h * (9/256))/R7
fai8.6 = (2 * a * b * 0 + 2 * b^2 * (3/64) + 2 * b * c *
    (405/3136) + 2 * b * d * (705/3136) + 2 * b * e * (243/784) +
    2 * b * f * (1125/3136) + 2 * b * g * (1077/3136) + 2 *
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        b * h * (15/64))/R6
fai8.5 = (2 * a * b * 0 + 2 * b^2 * (1/256) + 2 * b * c *
    (327/12544) + 2 * b * d * (453/6272) + 2 * b * e * (451/3136) +
    2 * b * f * (2985/12544) + 2 * b * g * (4359/12544) +
    2 * b * h * (59/128))/R5
fai8.4 = (2 * a * b * 0 + 2 * b^2 * 0 + 2 * b * c * (15/6272) +
    2 * b * d * (9/784) + 2 * b * e * (51/1568) + 2 * b *
    f * (225/3136) + 2 * b * g * (855/6272) + 2 * b * h *
    (15/64) + 2 * b * i * (3/8))/R4
fai8.3 = (2 * a * b * 0 + 2 * b^2 * 0 + 2 * b * c * (1/12544) +
    2 * b * d * (9/12544) + 2 * b * e * (9/3136) + 2 * b *
    f * (25/3136) + 2 * b * g * (225/12544) + 2 * b * h *
    (9/256) + 2 * b * i * (1/16))/R3

fai7.9 = (2 * a * c * (225/784) + 2 * b * c * (2025/12544) +
    2 * c^2 * (50625/614656) + 2 * c * d * (5625/153664) +
    2 * c * e * (2025/153664) + 2 * c * f * (2025/614656) +
    2 * c * g * (225/614656))/R9
fai7.8 = (2 * a * c * (45/98) + 2 * b * c * (2295/6272) +
    2 * c^2 * (10125/38416) + 2 * c * d * (25875/153664) +
    2 * c * e * (1755/19208) + 2 * c * f * (11745/307328) +
    2 * c * g * (45/4802))/R8
fai7.7 = (2 * a * c * (87/392) + 2 * b * c * (3951/12544) +
    2 * c^2 * (7425/21952) + 2 * c * d * (27075/87808) +
    2 * c * e * (333/1372) + 2 * c * f * (14013/87808) +
    2 * c * g * (1713/21952) + 2 * c * h * (225/12544))/R7
fai7.6 = (2 * a * c * (3/98) + 2 * b * c * (405/3136) + 2 *
    c^2 * (1215/5488) + 2 * c * d * (12675/43904) + 2 * c *
    e * (873/2744) + 2 * c * f * (3321/10976) + 2 * c * g *
    (165/686) + 2 * c * h * (855/6272))/R6
fai7.5 = (2 * a * c * (1/784) + 2 * b * c * (327/12544) +
    2 * c^2 * (3429/43904) + 2 * c * d * (13045/87808) +
    2 * c * e * (2481/10976) + 2 * c * f * (26037/87808) +
    2 * c * g * (15077/43904) + 2 * c * h * (4359/12544) +
    2 * c * i * (225/784))/R5
fai7.4 = (2 * a * c * 0 + 2 * b * c * (15/6272) + 2 * c^2 *
    (81/5488) + 2 * c * d * (465/10976) + 2 * c * e * (243/2744) +
    2 * c * f * (6795/43904) + 2 * c * g * (165/686) + 2 *
    c * h * (1077/3136) + 2 * c * i * (45/98))/R4
fai7.3 = (2 * a * c * 0 + 2 * b * c * (1/12544) + 2 * c^2 *
    (33/21952) + 2 * c * d * (573/87808) + 2 * c * e * (25/1372) +
    2 * c * f * (3555/87808) + 2 * c * g * (1713/21952) +
    2 * c * h * (1711/12544) + 2 * c * i * (87/392))/R3
fai7.2 = (2 * a * c * 0 + 2 * b * c * 0 + 2 * c^2 * (3/38416) +
    2 * c * d * (153/307328) + 2 * c * e * (33/19208) + 2 *
    c * f * (675/153664) + 2 * c * g * (45/4802) + 2 * c *
    h * (111/6272) + 2 * c * i * (3/98))/R2
fai7.1 = (2 * a * c * 0 + 2 * b * c * 0 + 2 * c^2 * (1/614656) +
    2 * c * d * (9/614656) + 2 * c * e * (9/153664) + 2 *
    c * f * (25/153664) + 2 * c * g * (225/614656) + 2 *
    c * h * (9/12544) + 2 * c * i * (1/784))/R1

fai6.9 = (2 * a * d * (25/196) + 2 * b * d * (225/3136) +
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        2 * c * d * (5625/153664) + 2 * d^2 * (625/38416) + 2 *
        d * e * (225/38416) + 2 * d * f * (225/153664) + 2 *
        d * g * (25/153664) + 2 * d * h * (0) + 2 * d * i * (0))/R9
fai6.8 = (2 * a * d * (75/196) + 2 * b * d * (825/3136) +
        2 * c * d * (25875/153664) + 2 * d^2 * (1875/19208) +
        2 * d * e * (1875/38416) + 2 * d * f * (2925/153664) +
        2 * d * g * (675/153664) + 2 * d * h * (0) + 2 * d *
        i * (0))/R8
fai6.7 = (2 * a * d * (285/784) + 2 * b * d * (4465/12544) +
        2 * c * d * (27075/87808) + 2 * d^2 * (375/1568) + 2 *
        d * e * (3595/21952) + 2 * d * f * (1185/12544) + 2 *
        d * g * (3555/87808) + 2 * d * h * (25/3136) + 2 * d *
        i * (0))/R7
fai6.6 = (2 * a * d * (45/392) + 2 * b * d * (705/3136) +
        2 * c * d * (12675/43904) + 2 * d^2 * (3375/10976) +
        2 * d * e * (3135/10976) + 2 * d * f * (2535/10976) +
        2 * d * g * (6795/43904) + 2 * d * h * (225/3136) + 2 *
        d * i * (0))/R6
fai6.5 = (2 * a * d * (9/784) + 2 * b * d * (453/6272) +
        2 * c * d * (13045/87808) + 2 * d^2 * (19575/87808) +
        2 * d * e * (879/3136) + 2 * d * f * (13529/43904) +
        2 * d * g * (26037/87808) + 2 * d * h * (2985/12544) +
        2 * d * i * (25/196))/R5
fai6.4 = (2 * a * d * (0) + 2 * b * d * (9/784) + 2 * c *
        d * (465/10976) + 2 * d^2 * (2025/21952) + 2 * d * e *
        (54/343) + 2 * d * f * (2535/10976) + 2 * d * g * (3321/10976) +
        2 * d * h * (1125/3136) + 2 * d * i * (75/196))/R4
fai6.3 = (2 * a * d * (0) + 2 * b * d * (9/12544) + 2 * c *
        d * (573/87808) + 2 * d^2 * (135/6272) + 2 * d * e *
        (1089/21952) + 2 * d * f * (1185/12544) + 2 * d * g *
        (14013/87808) + 2 * d * h * (1557/6272) + 2 * d * i *
        (285/784))/R3
fai6.2 = (2 * a * d * (0) + 2 * b * d * (0) + 2 * c * d *
        (153/307328) + 2 * d^2 * (405/153664) + 2 * d * e * (621/76832) +
        2 * d * f * (2925/153664) + 2 * d * g * (11745/307328) +
        2 * d * h * (27/392) + 2 * d * i * (45/392))/R2
fai6.1 = (2 * a * d * (0) + 2 * b * d * (0) + 2 * c * d *
        (9/614656) + 2 * d^2 * (81/614656) + 2 * d * e * (81/153664) +
        2 * d * f * (225/153664) + 2 * d * g * (2025/614656) +
        2 * d * h * (81/12544) + 2 * d * i * (9/784))/R1

fai5.9 = (2 * a * e * (9/196) + 2 * b * e * (81/3136) + 2 *
        c * e * (2025/153664) + 2 * d * e * (225/38416) + 2 *
        e^2 * (81/38416) + 2 * e * f * (81/153664) + 2 * e *
        g * (9/153664) + 2 * e * h * (0) + 2 * e * i * (0))/R9
fai5.8 = (2 * a * e * (12/49) + 2 * b * e * (243/1568) +
        2 * c * e * (1755/19208) + 2 * d * e * (1875/38416) +
        2 * e^2 * (54/2401) + 2 * e * f * (621/76832) + 2 * e *
        g * (33/19208) + 2 * e * h * (0) + 2 * e * i * (0))/R8
fai5.7 = (2 * a * e * (41/98) + 2 * b * e * (1035/3136) +
        2 * c * e * (333/1372) + 2 * d * e * (3595/21952) + 2 *
        e^2 * (135/1372) + 2 * e * f * (1089/21952) + 2 * e *
        g * (25/1372) + 2 * e * h * (9/3136) + 2 * e * i * (0))/R7
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fai5.6 = (2 * a * e * (12/49) + 2 * b * e * (243/784) + 2 *
    c * e * (873/2744) + 2 * d * e * (3135/10976) + 2 * e^2 *
    (78/343) + 2 * e * f * (54/343) + 2 * e * g * (243/2744) +
    2 * e * h * (51/1568) + 2 * e * i * (0))/R6
fai5.5 = (2 * a * e * (9/196) + 2 * b * e * (451/3136) +
    2 * c * e * (2481/10976) + 2 * d * e * (879/3136) + 2 *
    e^2 * (821/2744) + 2 * e * f * (879/3136) + 2 * e * g *
    (2481/10976) + 2 * e * h * (451/3136) + 2 * e * i * (9/196))/R5
fai5.4 = (2 * a * e * (0) + 2 * b * e * (51/1568) + 2 * c *
    e * (243/2744) + 2 * d * e * (54/343) + 2 * e^2 * (78/343) +
    2 * e * f * (3135/10976) + 2 * e * g * (873/2744) + 2 *
    e * h * (243/784) + 2 * e * i * (12/49))/R4
fai5.3 = (2 * a * e * (0) + 2 * b * e * (9/3136) + 2 * c *
    e * (25/1372) + 2 * d * e * (1089/21952) + 2 * e^2 *
    (135/1372) + 2 * e * f * (3595/21952) + 2 * e * g * (333/1372) +
    2 * e * h * (1035/3136) + 2 * e * i * (41/98))/R3
fai5.2 = (2 * a * e * (0) + 2 * b * e * (0) + 2 * c * e *
    (33/19208) + 2 * d * e * (621/76832) + 2 * e^2 * (54/2401) +
    2 * e * f * (1875/38416) + 2 * e * g * (1755/19208) +
    2 * e * h * (243/1568) + 2 * e * i * (12/49))/R2
fai5.1 = (2 * a * e * (0) + 2 * b * e * (0) + 2 * c * e *
    (9/153664) + 2 * d * e * (81/153664) + 2 * e^2 * (81/38416) +
    2 * e * f * (225/38416) + 2 * e * g * (2025/153664) +
    2 * e * h * (81/3136) + 2 * e * i * (9/196))/R1

fai4.9 = (2 * a * f * (9/784) + 2 * b * f * (81/12544) +
    2 * c * f * (2025/614656) + 2 * d * f * (225/153664) +
    2 * e * f * (81/153664) + 2 * f^2 * (81/614656) + 2 *
    f * g * (9/614656) + 2 * f * h * (0) + 2 * f * i * (0))/R9
fai4.8 = (2 * a * f * (45/392) + 2 * b * f * (27/392) + 2 *
    c * f * (11745/307328) + 2 * d * f * (2925/153664) +
    2 * e * f * (621/76832) + 2 * f^2 * (405/153664) + 2 *
    f * g * (153/307328) + 2 * f * h * (0) + 2 * f * i *
    (0))/R8
fai4.7 = (2 * a * f * (285/784) + 2 * b * f * (1557/6272) +
    2 * c * f * (14013/87808) + 2 * d * f * (1185/12544) +
    2 * e * f * (1089/21952) + 2 * f^2 * (135/6272) + 2 *
    f * g * (573/87808) + 2 * f * h * (9/12544) + 2 * f *
    i * (0))/R7
fai4.6 = (2 * a * f * (75/196) + 2 * b * f * (1125/3136) +
    2 * c * f * (3321/10976) + 2 * d * f * (2535/10976) +
    2 * e * f * (54/343) + 2 * f^2 * (2025/21952) + 2 * f *
    g * (465/10976) + 2 * f * h * (9/784) + 2 * f * i * (0))/R6
fai4.5 = (2 * a * f * (25/196) + 2 * b * f * (2985/12544) +
    2 * c * f * (26037/87808) + 2 * d * f * (13529/43904) +
    2 * e * f * (879/3136) + 2 * f^2 * (19575/87808) + 2 *
    f * g * (13045/87808) + 2 * f * h * (453/6272) + 2 *
    f * i * (9/784))/R5
fai4.4 = (2 * a * f * (0) + 2 * b * f * (225/3136) + 2 *
    c * f * (6795/43904) + 2 * d * f * (2535/10976) + 2 *
    e * f * (3135/10976) + 2 * f^2 * (3375/10976) + 2 * f *
    g * (12675/43904) + 2 * f * h * (705/3136) + 2 * f *
    i * (45/392))/R4
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fai4.3 = (2 * a * f * (0) + 2 * b * f * (25/3136) + 2 * c *
    f * (3555/87808) + 2 * d * f * (1185/12544) + 2 * e *
    f * (3595/21952) + 2 * f^2 * (375/1568) + 2 * f * g *
    (27075/87808) + 2 * f * h * (4465/12544) + 2 * f * i *
    (285/784))/R3
fai4.2 = (2 * a * f * (0) + 2 * b * f * (0) + 2 * c * f *
    (675/153664) + 2 * d * f * (2925/153664) + 2 * e * f *
    (1875/38416) + 2 * f^2 * (1875/19208) + 2 * f * g * (25875/153664) +
    2 * f * h * (825/3136) + 2 * f * i * (75/196))/R2
fai4.1 = (2 * a * f * (0) + 2 * b * f * (0) + 2 * c * f *
    (25/153664) + 2 * d * f * (225/153664) + 2 * e * f *
    (225/38416) + 2 * f^2 * (625/38416) + 2 * f * g * (5625/153664) +
    2 * f * h * (225/3136) + 2 * f * i * (25/196))/R1

fai3.9 = (2 * a * g * (1/784) + 2 * b * g * (9/12544) + 2 *
    c * g * (225/614656) + 2 * d * g * (25/153664) + 2 *
    e * g * (9/153664) + 2 * f * g * (9/614656) + 2 * g^2 *
    (1/614656) + 2 * g * h * (0) + 2 * g * i * (0))/R9
fai3.8 = (2 * a * g * (3/98) + 2 * b * g * (111/6272) + 2 *
    c * g * (45/4802) + 2 * d * g * (675/153664) + 2 * e *
    g * (33/19208) + 2 * f * g * (153/307328) + 2 * g^2 *
    (3/38416) + 2 * g * h * (0) + 2 * g * i * (0))/R8
fai3.7 = (2 * a * g * (87/392) + 2 * b * g * (1711/12544) +
    2 * c * g * (1713/21952) + 2 * d * g * (3555/87808) +
    2 * e * g * (25/1372) + 2 * f * g * (573/87808) + 2 *
    g^2 * (33/21952) + 2 * g * h * (1/12544) + 2 * g * i *
    (0))/R7
fai3.6 = (2 * a * g * (45/98) + 2 * b * g * (1077/3136) +
    2 * c * g * (165/686) + 2 * d * g * (6795/43904) + 2 *
    e * g * (243/2744) + 2 * f * g * (465/10976) + 2 * g^2 *
    (81/5488) + 2 * g * h * (15/6272) + 2 * g * i * (0))/R6
fai3.5 = (2 * a * g * (225/784) + 2 * b * g * (4359/12544) +
    2 * c * g * (15077/43904) + 2 * d * g * (26037/87808) +
    2 * e * g * (2481/10976) + 2 * f * g * (13045/87808) +
    2 * g^2 * (3429/43904) + 2 * g * h * (327/12544) + 2 *
    g * i * (1/784))/R5
fai3.4 = (2 * a * g * (0) + 2 * b * g * (855/6272) + 2 *
    c * g * (165/686) + 2 * d * g * (3321/10976) + 2 * e *
    g * (873/2744) + 2 * f * g * (12675/43904) + 2 * g^2 *
    (1215/5488) + 2 * g * h * (405/3136) + 2 * g * i * (3/98))/R4
fai3.3 = (2 * a * g * (0) + 2 * b * g * (225/12544) + 2 *
    c * g * (1713/21952) + 2 * d * g * (14013/87808) + 2 *
    e * g * (333/1372) + 2 * f * g * (27075/87808) + 2 *
    g^2 * (7425/21952) + 2 * g * h * (3951/12544) + 2 * g *
    i * (87/392))/R3
fai3.2 = (2 * a * g * (0) + 2 * b * g * (0) + 2 * c * g *
    (45/4802) + 2 * d * g * (11745/307328) + 2 * e * g *
    (1755/19208) + 2 * f * g * (25875/153664) + 2 * g^2 *
    (10125/38416) + 2 * g * h * (2295/6272) + 2 * g * i *
    (45/98))/R2
fai3.1 = (2 * a * g * (0) + 2 * b * g * (0) + 2 * c * g *
    (225/614656) + 2 * d * g * (2025/614656) + 2 * e * g *
    (2025/153664) + 2 * f * g * (5625/153664) + 2 * g^2 *
```

```
        (50625/614656) + 2 * g * h * (2025/12544) + 2 * g * i *
        (225/784))/R1

fai2.9 = (2 * a * h * (0) + 2 * b * h * (0) + 2 * c * h *
        (0) + 2 * d * h * (0) + 2 * e * h * (0) + 2 * f * h *
        (0) + 2 * g * h * (0) + 2 * h^2 * (0) + 2 * h * i * (0))/R9
fai2.8 = (2 * a * h * (0) + 2 * b * h * (0) + 2 * c * h *
        (0) + 2 * d * h * (0) + 2 * e * h * (0) + 2 * f * h *
        (0) + 2 * g * h * (0) + 2 * h^2 * (0) + 2 * h * i * (0))/R8
fai2.7 = (2 * a * h * (1/16) + 2 * b * h * (9/256) + 2 *
        c * h * (225/12544) + 2 * d * h * (25/3136) + 2 * e *
        h * (9/3136) + 2 * f * h * (9/12544) + 2 * g * h * (1/12544) +
        2 * h^2 * (0) + 2 * h * i * (0))/R7
fai2.6 = (2 * a * h * (3/8) + 2 * b * h * (15/64) + 2 * c *
        h * (855/6272) + 2 * d * h * (225/3136) + 2 * e * h *
        (51/1568) + 2 * f * h * (9/784) + 2 * g * h * (15/6272) +
        2 * h^2 * (0) + 2 * h * i * (0))/R6
fai2.5 = (2 * a * h * (9/16) + 2 * b * h * (59/128) + 2 *
        c * h * (4359/12544) + 2 * d * h * (2985/12544) + 2 *
        e * h * (451/3136) + 2 * f * h * (453/6272) + 2 * g *
        h * (327/12544) + 2 * h^2 * (1/256) + 2 * h * i * (0))/R5
fai2.4 = (2 * a * h * (0) + 2 * b * h * (15/64) + 2 * c *
        h * (1077/3136) + 2 * d * h * (1125/3136) + 2 * e * h *
        (243/784) + 2 * f * h * (705/3136) + 2 * g * h * (405/3136) +
        2 * h^2 * (3/64) + 2 * h * i * (0))/R4
fai2.3 = (2 * a * h * (0) + 2 * b * h * (9/256) + 2 * c *
        h * (1711/12544) + 2 * d * h * (1557/6272) + 2 * e *
        h * (1035/3136) + 2 * f * h * (4465/12544) + 2 * g *
        h * (3951/12544) + 2 * h^2 * (27/128) + 2 * h * i * (1/16))/R3
fai2.2 = (2 * a * h * (0) + 2 * b * h * (0) + 2 * c * h *
        (111/6272) + 2 * d * h * (27/392) + 2 * e * h * (243/1568) +
        2 * f * h * (825/3136) + 2 * g * h * (2295/6272) + 2 *
        h^2 * (27/64) + 2 * h * i * (3/8))/R2
fai2.1 = (2 * a * h * (0) + 2 * b * h * (0) + 2 * c * h *
        (9/12544) + 2 * d * h * (81/12544) + 2 * e * h * (81/3136) +
        2 * f * h * (225/3136) + 2 * g * h * (2025/12544) + 2 *
        h^2 * (81/256) + 2 * h * i * (9/16))/R1

fai1.9 = (2 * a * i * (0) + 2 * b * i * (0) + 2 * c * i *
        (0) + 2 * d * i * (0) + 2 * e * i * (0) + 2 * f * i *
        (0) + 2 * g * i * (0) + 2 * h * i * (0) + 2 * i^2 * (0))/R9
fai1.8 = (2 * a * i * (0) + 2 * b * i * (0) + 2 * c * i *
        (0) + 2 * d * i * (0) + 2 * e * i * (0) + 2 * f * i *
        (0) + 2 * g * i * (0) + 2 * h * i * (0) + 2 * i^2 * (0))/R8
fai1.7 = (2 * a * i * (0) + 2 * b * i * (0) + 2 * c * i *
        (0) + 2 * d * i * (0) + 2 * e * i * (0) + 2 * f * i *
        (0) + 2 * g * i * (0) + 2 * h * i * (0) + 2 * i^2 * (0))/R7
fai1.6 = (2 * a * i * (0) + 2 * b * i * (0) + 2 * c * i *
        (0) + 2 * d * i * (0) + 2 * e * i * (0) + 2 * f * i *
        (0) + 2 * g * i * (0) + 2 * h * i * (0) + 2 * i^2 * (0))/R6
fai1.5 = (2 * a * i * (1) + 2 * b * i * (9/16) + 2 * c *
        i * (225/784) + 2 * d * i * (25/196) + 2 * e * i * (9/196) +
        2 * f * i * (9/784) + 2 * g * i * (1/784) + 2 * h * i *
```

```
    (0) + 2 * i^2 * (0))/R5
fai1.4 = (2 * a * i * (0) + 2 * b * i * (3/8) + 2 * c * i *
    (45/98) + 2 * d * i * (75/196) + 2 * e * i * (12/49) +
    2 * f * i * (45/392) + 2 * g * i * (3/98) + 2 * h * i *
    (0) + 2 * i^2 * (0))/R4
fai1.3 = (2 * a * i * (0) + 2 * b * i * (1/16) + 2 * c *
    i * (87/392) + 2 * d * i * (285/784) + 2 * e * i * (41/98) +
    2 * f * i * (285/784) + 2 * g * i * (87/392) + 2 * h *
    i * (1/16) + 2 * i^2 * (0))/R3
fai1.2 = (2 * a * i * (0) + 2 * b * i * (0) + 2 * c * i *
    (3/98) + 2 * d * i * (45/392) + 2 * e * i * (12/49) +
    2 * f * i * (75/196) + 2 * g * i * (45/98) + 2 * h *
    i * (3/8) + 2 * i^2 * (0))/R2
fai1.1 = (2 * a * i * (0) + 2 * b * i * (0) + 2 * c * i *
    (1/784) + 2 * d * i * (9/784) + 2 * e * i * (9/196) +
    2 * f * i * (25/196) + 2 * g * i * (225/784) + 2 * h *
    i * (9/16) + 2 * i^2 * (1))/R1


P.9 = (fai9.9 * n9 + fai9.8 * n8 + fai9.7 * n7 + fai9.6 *
    n6 + fai9.5 * n5)/(2 * N)
P.8 = (fai8.9 * n9 + fai8.8 * n8 + fai8.7 * n7 + fai8.6 *
    n6 + fai8.5 * n5 + fai8.4 * n4 + fai8.3 * n3)/(2 * N)
P.7 = (fai7.9 * n9 + fai7.8 * n8 + fai7.7 * n7 + fai7.6 *
    n6 + fai7.5 * n5 + fai7.4 * n4 + fai7.3 * n3 + fai7.2 *
    n2 + fai7.1 * n1)/(2 * N)
P.6 = (fai6.9 * n9 + fai6.8 * n8 + fai6.7 * n7 + fai6.6 *
    n6 + fai6.5 * n5 + fai6.4 * n4 + fai6.3 * n3 + fai6.2 *
    n2 + fai6.1 * n1)/(2 * N)
P.5 = (fai5.9 * n9 + fai5.8 * n8 + fai5.7 * n7 + fai5.6 *
    n6 + fai5.5 * n5 + fai5.4 * n4 + fai5.3 * n3 + fai5.2 *
    n2 + fai5.1 * n1)/(2 * N)
P.4 = (fai4.9 * n9 + fai4.8 * n8 + fai4.7 * n7 + fai4.6 *
    n6 + fai4.5 * n5 + fai4.4 * n4 + fai4.3 * n3 + fai4.2 *
    n2 + fai4.1 * n1)/(2 * N)
P.3 = (fai3.9 * n9 + fai3.8 * n8 + fai3.7 * n7 + fai3.6 *
    n6 + fai3.5 * n5 + fai3.4 * n4 + fai3.3 * n3 + fai3.2 *
    n2 + fai3.1 * n1)/(2 * N)
P.2 = (fai2.9 * n9 + fai2.8 * n8 + fai2.7 * n7 + fai2.6 *
    n6 + fai2.5 * n5 + fai2.4 * n4 + fai2.3 * n3 + fai2.2 *
    n2 + fai2.1 * n1)/(2 * N)
P.1 = (fai1.9 * n9 + fai1.8 * n8 + fai1.7 * n7 + fai1.6 *
    n6 + fai1.5 * n5 + fai1.4 * n4 + fai1.3 * n3 + fai1.2 *
    n2 + fai1.1 * n1)/(2 * N)

geno.p <- rbind(geno.p, c(a, b, c, d, e, f, g, h, i))
a <- P.9
b <- P.8
c <- P.7
d <- P.6
e <- P.5
f <- P.4
g <- P.3
h <- P.2
```

```
    i <- P.1
}
a <- geno.p[500, 1]
b <- geno.p[500, 2]
c <- geno.p[500, 3]
d <- geno.p[500, 4]
e <- geno.p[500, 5]
f <- geno.p[500, 6]
g <- geno.p[500, 7]
h <- geno.p[500, 8]
i <- geno.p[500, 9]

geno.L <- rbind(geno.L, c(a, b, c, d, e, f, g, h, i))
```

This obtains the final genotype frequency estimates, which they call `exp`

```
R.9 <- (a^2 * 1 + 2 * a * b * (9/16 + 1/64 * 0^2 + 3/16 * 0) +
    2 * a * c * (225/784 + 9/196 * 0^2 + 45/196 * 0) + 2 * a *
    d * (25/196 + 225/3136 * 0^2 + 75/392 * 0) + 2 * a * e *
    (9/196 + 4/49 * 0^2 + 6/49 * 0) + 2 * a * f * (9/784 + 225/3136 *
    0^2 + 45/784 * 0) + 2 * a * g * (1/784 + 9/196 * 0^2 + 3/196 *
    0) + 2 * a * h * (1/64 * 0^2) + 0 + b^2 * (81/256 + 1/4096 *
    0^4 + 3/512 * 0^3 + 27/512 * 0^2 + 27/128 * 0) + 2 * b *
    c * (2025/12544 + 9/12544 * 0^4 + 153/12544 * 0^3 + 3681/50176 *
    0^2 + 2295/12544 * 0) + 2 * b * d * ((225 * 0^4)/200704 +
    (825 * 0^3)/50176 + (3925 * 0^2)/50176 + (825 * 0)/6272 +
    225/3136) + 2 * b * e * (0^4/784 + (27 * 0^3)/1568 + (873 *
    0^2)/12544 + (243 * 0)/3136 + 81/3136) + 2 * b * f * ((225 *
    0^4)/200704 + (45 * 0^3)/3136 + (1287 * 0^2)/25088 + (27 *
    0)/784 + 81/12544) + 2 * b * g * ((9 * 0^4)/12544 + (111 *
    0^3)/12544 + (1441 * 0^2)/50176 + (111 * 0)/12544 + 9/12544) +
    2 * b * h * (0^4/4096 + (3 * 0^3)/1024 + (9 * 0^2)/1024) +
    0 + c^2 * ((81 * 0^4)/38416 + (405 * 0^3)/19208 + (6075 *
    0^2)/76832 + (10125 * 0)/76832 + 50625/614656) + 2 * c *
    d * ((2025 * 0^4)/614656 + (15525 * 0^3)/614656 + (173025 *
    0^2)/2458624 + (25875 * 0)/307328 + 5625/153664) + 2 * c *
    e * ((9 * 0^4)/2401 + (117 * 0^3)/4802 + (2061 * 0^2)/38416 +
    (1755 * 0)/38416 + 2025/153664) + 2 * c * f * ((2025 * 0^4)/614656 +
    (11745 * 0^3)/614656 + (84321 * 0^2)/2458624 + (11745 * 0)/614656 +
    2025/614656) + 2 * c * g * ((81 * 0^4)/38416 + (27 * 0^3)/2401 +
    (1287 * 0^2)/76832 + (45 * 0)/9604 + 225/614656) + 2 * c *
    h * ((9 * 0^4)/12544 + (45 * 0^3)/12544 + (225 * 0^2)/50176) +
    0 + d^2 * ((50625 * 0^4)/9834496 + (16875 * 0^3)/614656 +
    (16875 * 0^2)/307328 + (1875 * 0)/38416 + 625/38416) + 2 *
    d * e * ((225 * 0^4)/38416 + (1875 * 0^3)/76832 + (22825 *
    0^2)/614656 + (1875 * 0)/76832 + 225/38416) + 2 * d * f *
    ((50625 * 0^4)/9834496 + (43875 * 0^3)/2458624 + (51525 *
        0^2)/2458624 + (2925 * 0)/307328 + 225/153664) + 2 *
    d * g * ((2025 * 0^4)/614656 + (6075 * 0^3)/614656 + (21825 *
    0^2)/2458624 + (675 * 0)/307328 + 25/153664) + 2 * d * h *
    ((225 * 0^4)/200704 + (75 * 0^3)/25088 + (25 * 0^2)/12544) +
    0 + e^2 * ((16 * 0^4)/2401 + (48 * 0^3)/2401 + (54 * 0^2)/2401 +
    (27 * 0)/2401 + 81/38416) + 2 * e * f * ((225 * 0^4)/38416 +
    (1035 * 0^3)/76832 + (6921 * 0^2)/614656 + (621 * 0)/153664 +
```

```
         81/153664) + 2 * e * g * ((9 * 0^4)/2401 + (33 * 0^3)/4802 +
         (157 * 0^2)/38416 + (33 * 0)/38416 + 9/153664) + 2 * e *
         h * (0^4/784 + (3 * 0^3)/1568 + (9 * 0^2)/12544) + 0 + f^2 *
         ((50625 * 0^4)/9834496 + (10125 * 0^3)/1229312 + (6075 *
             0^2)/1229312 + (405 * 0)/307328 + 81/614656) + 2 * f *
         g * ((2025 * 0^4)/614656 + (2295 * 0^3)/614656 + (3681 *
         0^2)/2458624 + (153 * 0)/614656 + 9/614656) + 2 * f * h *
         ((225 * 0^4)/200704 + (45 * 0^3)/50176 + (9 * 0^2)/50176) +
         0 + g^2 * ((81 * 0^4)/38416 + (27 * 0^3)/19208 + (27 * 0^2)/76832 +
         (3 * 0)/76832 + 1/614656) + 2 * g * h * (9/12544 * 0^4 +
         3/12544 * 0^3 + 1/50176 * 0^2) + 0 + h^2 * (1/4096 * 0^4) +
         0 + i^2 * 0)


R.8 <- (a^2 * 0 + 2 * a * b * (3/8 - 1/16 * 0^2 - 5/16 * 0) +
         2 * a * c * (45/98 - 9/49 * 0^2 - 27/98 * 0) + 2 * a * d *
         (75/196 - 225/784 * 0^2 - 75/784 * 0) + 2 * a * e * (12/49 -
         16/49 * 0^2 + 4/49 * 0) + 2 * a * f * (45/392 - 225/784 *
         0^2 + 135/784 * 0) + 2 * a * g * (3/98 - 9/49 * 0^2 + 15/98 *
         0) + 2 * a * h * (1/16 * 0 - 1/16 * 0^2) + 0 + b^2 * (27/64 -
         1/512 * 0^4 - 17/512 * 0^3 - 45/256 * 0^2 - 27/128 * 0) +
         2 * b * c * (2295/6272 - 9/1568 * 0^4 - 423/6272 * 0^3 -
             2763/12544 * 0^2 - 909/12544 * 0) + 2 * b * d * ((-(225 *
         0^4)/25088) - (1125 * 0^3)/12544 - (5375 * 0^2)/25088 + (625 *
         0)/12544 + 825/3136) + 2 * b * e * ((-0^4/98) - (73 * 0^3)/784 -
         (549 * 0^2)/3136 + (387 * 0)/3136 + 243/1568) + 2 * b * f *
         ((-(225 * 0^4)/25088) - (1935 * 0^3)/25088 - (747 * 0^2)/6272 +
             (855 * 0)/6272 + 27/392) + 2 * b * g * ((-(9 * 0^4)/1568) -
         (297 * 0^3)/6272 - (775 * 0^2)/12544 + (1219 * 0)/12544 +
         111/6272) + 2 * b * h * ((-0^4/512) - 0^3/64 - (9 * 0^2)/512 +
         (9 * 0)/256) + 0 + c^2 * ((-(81 * 0^4)/4802) - (1053 * 0^3)/9604 -
         (3645 * 0^2)/19208 + (2025 * 0)/38416 + 10125/38416) + 2 *
         c * d * ((-(2025 * 0^4)/76832) - (38475 * 0^3)/307328 - (79875 *
         0^2)/614656 + (69525 * 0)/614656 + 25875/153664) + 2 * c *
         e * ((-(72 * 0^4)/2401) - (279 * 0^3)/2401 - (657 * 0^2)/9604 +
         (2367 * 0)/19208 + 1755/19208) + 2 * c * f * ((-(2025 * 0^4)/76832) -
         (27135 * 0^3)/307328 - (13851 * 0^2)/614656 + (60831 * 0)/614656 +
         11745/307328) + 2 * c * g * ((-(81 * 0^4)/4802) - (243 *
         0^3)/4802 + (9 * 0^2)/19208 + (1107 * 0)/19208 + 45/4802) +
         2 * c * h * ((-(9 * 0^4)/1568) - (99 * 0^3)/6272 + (45 *
             0^2)/12544 + (225 * 0)/12544) + 0 + d^2 * ((-(50625 *
         0^4)/1229312) - (151875 * 0^3)/1229312 - (16875 * 0^2)/307328 +
         (9375 * 0)/76832 + 1875/19208) + 2 * d * e * ((-(225 * 0^4)/4802) -
         (3825 * 0^3)/38416 - (325 * 0^2)/153664 + (15325 * 0)/153664 +
         1875/38416) + 2 * d * f * ((-(50625 * 0^4)/1229312) - (10125 *
         0^3)/153664 + (28575 * 0^2)/1229312 + (39825 * 0)/614656 +
         2925/153664) + 2 * d * g * ((-(2025 * 0^4)/76832) - (10125 *
         0^3)/307328 + (14625 * 0^2)/614656 + (19125 * 0)/614656 +
         675/153664) + 2 * d * h * ((-(225 * 0^4)/25088) - (225 *
         0^3)/25088 + (125 * 0^2)/12544 + (25 * 0)/3136) + 0 + e^2 *
         ((-(128 * 0^4)/2401) - (160 * 0^3)/2401 + (72 * 0^2)/2401 +
             (162 * 0)/2401 + 54/2401) + 2 * e * f * ((-(225 * 0^4)/4802) -
         (1305 * 0^3)/38416 + (5499 * 0^2)/153664 + (5679 * 0)/153664 +
```

```
       621/76832) + 2 * e * g * ((-(72 * 0^4)/2401) - (27 * 0^3)/2401 +
       (239 * 0^2)/9604 + (281 * 0)/19208 + 33/19208) + 2 * e *
       h * ((-0^4/98) - 0^3/784 + (27 * 0^2)/3136 + (9 * 0)/3136) +
       0 + f^2 * ((-(50625 * 0^4)/1229312) - (10125 * 0^3)/1229312 +
       (18225 * 0^2)/614656 + (5265 * 0)/307328 + 405/153664) +
       2 * f * g * ((-(2025 * 0^4)/76832) + (1215 * 0^3)/307328 +
           (10089 * 0^2)/614656 + (3375 * 0)/614656 + 153/307328) +
       2 * f * h * ((-(225 * 0^4)/25088) + (45 * 0^3)/12544 + (117 *
           0^2)/25088 + (9 * 0)/12544) + 0 + g^2 * ((-(81 * 0^4)/4802) +
       (81 * 0^3)/9604 + (135 * 0^2)/19208 + (51 * 0)/38416 + 3/38416) +
       2 * g * h * (-9/1568 * 0^4 + 27/6272 * 0^3 + 17/12544 * 0^2 +
           1/12544 * 0) + 0 + h^2 * (-1/512 * 0^4 + 1/512 * 0^3) +
       0 + i^2 * 0)


R.7 <- (a^2 * 0 + 2 * a * b * (1/16 + 3/32 * 0^2 + 1/16 * 0) +
       2 * a * c * (87/392 + 27/98 * 0^2 - 6/49 * 0) + 2 * a * d *
       (285/784 + 675/1568 * 0^2 - 255/784 * 0) + 2 * a * e * (41/98 +
       24/49 * 0^2 - 20/49 * 0) + 2 * a * f * (285/784 + 675/1568 *
       0^2 - 255/784 * 0) + 2 * a * g * (87/392 + 27/98 * 0^2 -
       6/49 * 0) + 2 * a * h * (1/16 + 3/32 * 0^2 + 1/16 * 0) +
       0 + b^2 * (27/128 + 7/1024 * 0^4 + 39/512 * 0^3 + 93/512 *
       0^2 - 9/64 * 0) + 2 * b * c * (3951/12544 + 9/448 * 0^4 +
       933/6272 * 0^3 + 2109/12544 * 0^2 - 3033/12544 * 0) + 2 *
       b * d * ((225 * 0^4)/7168 + (1215 * 0^3)/6272 + (1425 * 0^2)/12544 -
       (785 * 0)/3136 + 4465/12544) + 2 * b * e * (0^4/28 + (39 *
       0^3)/196 + (93 * 0^2)/1568 - (585 * 0)/3136 + 1035/3136) +
       2 * b * f * ((225 * 0^4)/7168 + (4125 * 0^3)/25088 + (687 *
           0^2)/25088 - (513 * 0)/6272 + 1557/6272) + 2 * b * g *
       ((9 * 0^4)/448 + (639 * 0^3)/6272 + (261 * 0^2)/12544 + (271 *
           0)/12544 + 1711/12544) + 2 * b * h * ((7 * 0^4)/1024 +
       (9 * 0^3)/256 + (3 * 0^2)/128 + (9 * 0)/128 + 9/256) + 0 +
       c^2 * ((81 * 0^4)/1372 + (297 * 0^3)/1372 + (81 * 0^2)/2744 -
           (1215 * 0)/5488 + 7425/21952) + 2 * c * d * ((2025 *
       0^4)/21952 + (9855 * 0^3)/43904 - (1035 * 0^2)/12544 - (11625 *
       0)/87808 + 27075/87808) + 2 * c * e * ((36 * 0^4)/343 + (66 *
       0^3)/343 - (93 * 0^2)/686 - (81 * 0)/2744 + 333/1372) + 2 *
       c * f * ((2025 * 0^4)/21952 + (6075 * 0^3)/43904 - (1539 *
       0^2)/12544 + (4293 * 0)/87808 + 14013/87808) + 2 * c * g *
       ((81 * 0^4)/1372 + (27 * 0^3)/343 - (171 * 0^2)/2744 + (111 *
           0)/1372 + 1713/21952) + 2 * c * h * ((9 * 0^4)/448 +
       (177 * 0^3)/6272 + (3 * 0^2)/1792 + (765 * 0)/12544 + 225/12544) +
       0 + d^2 * ((50625 * 0^4)/351232 + (30375 * 0^3)/175616 -
       (30375 * 0^2)/175616 - (375 * 0)/21952 + 375/1568) + 2 *
       d * e * ((225 * 0^4)/1372 + (135 * 0^3)/1372 - (1965 * 0^2)/10976 +
       (205 * 0)/3136 + 3595/21952) + 2 * d * f * ((50625 * 0^4)/351232 +
       (3375 * 0^3)/87808 - (5625 * 0^2)/43904 + (4395 * 0)/43904 +
       1185/12544) + 2 * d * g * ((2025 * 0^4)/21952 + (405 * 0^3)/43904 -
       (675 * 0^2)/12544 + (7695 * 0)/87808 + 3555/87808) + 2 *
       d * h * ((225 * 0^4)/7168 + (135 * 0^3)/25088 + (15 * 0^2)/3584 +
       (275 * 0)/6272 + 25/3136) + 0 + e^2 * ((64 * 0^4)/343 - (48 *
       0^2)/343 + (36 * 0)/343 + 135/1372) + 2 * e * f * ((225 *
       0^4)/1372 - (75 * 0^3)/1372 - (789 * 0^2)/10976 + (45 * 0)/448 +
```

```
    1089/21952) + 2 * e * g * ((36 * 0^4)/343 - (18 * 0^3)/343 -
    (9 * 0^2)/686 + (185 * 0)/2744 + 25/1372) + 2 * e * h * (0^4/28 -
    (3 * 0^3)/196 + (3 * 0^2)/224 + (81 * 0)/3136 + 9/3136) +
    0 + f^2 * ((50625 * 0^4)/351232 - (16875 * 0^3)/175616 -
    (2025 * 0^2)/175616 + (405 * 0)/5488 + 135/6272) + 2 * f *
    g * ((2025 * 0^4)/21952 - (3375 * 0^3)/43904 + (261 * 0^2)/12544 +
    (3453 * 0)/87808 + 573/87808) + 2 * f * h * ((225 * 0^4)/7168 -
    (75 * 0^3)/3136 + (33 * 0^2)/1792 + (9 * 0)/784 + 9/12544) +
    0 + g^2 * ((81 * 0^4)/1372 - (81 * 0^3)/1372 + (81 * 0^2)/2744 +
    (87 * 0)/5488 + 33/21952) + 2 * g * h * (1/12544 + 9/448 *
    0^4 - 117/6272 * 0^3 + 27/1792 * 0^2 + 37/12544 * 0) + 0 +
    h^2 * (7/1024 * 0^4 - 3/512 * 0^3 + 3/512 * 0^2) + 0 + i^2 *
    0)


R.6 <- (a^2 * 0 + 2 * a * b * (1/16 * 0 - 1/16 * 0^2) + 2 * a *
    c * (3/98 - 9/49 * 0^2 + 15/98 * 0) + 2 * a * d * (45/392 -
    225/784 * 0^2 + 135/784 * 0) + 2 * a * e * (12/49 - 16/49 *
    0^2 + 4/49 * 0) + 2 * a * f * (75/196 - 225/784 * 0^2 - 75/784 *
    0) + 2 * a * g * (45/98 - 9/49 * 0^2 - 27/98 * 0) + 2 * a *
    h * (3/8 - 1/16 * 0^2 - 5/16 * 0) + 0 + b^2 * (3/64 - 7/512 *
    0^4 - 45/512 * 0^3 - 3/128 * 0^2 + 5/64 * 0) + 2 * b * c *
    (405/3136 - 9/224 * 0^4 - 999/6272 * 0^3 + 807/12544 * 0^2 +
        75/12544 * 0) + 2 * b * d * ((-(225 * 0^4)/3584) - (2505 *
    0^3)/12544 + (3235 * 0^2)/25088 - (1145 * 0)/12544 + 705/3136) +
    2 * b * e * ((-0^4/14) - (159 * 0^3)/784 + (417 * 0^2)/3136 -
        (529 * 0)/3136 + 243/784) + 2 * b * f * ((-(225 * 0^4)/3584) -
    (4275 * 0^3)/25088 + (921 * 0^2)/12544 - (39 * 0)/196 + 1125/3136) +
    2 * b * g * ((-(9 * 0^4)/224) - (705 * 0^3)/6272 - (229 *
        0^2)/12544 - (2165 * 0)/12544 + 1077/3136) + 2 * b *
    h * ((-(7 * 0^4)/512) - (3 * 0^3)/64 - (39 * 0^2)/512 - (25 *
    0)/256 + 15/64) + 0 + c^2 * ((-(81 * 0^4)/686) - (243 * 0^3)/1372 +
    (81 * 0^2)/392 - (729 * 0)/5488 + 1215/5488) + 2 * c * d *
    ((-(2025 * 0^4)/10976) - (5805 * 0^3)/43904 + (21465 * 0^2)/87808 -
        (2715 * 0)/12544 + 12675/43904) + 2 * c * e * ((-(72 *
    0^4)/343) - (27 * 0^3)/343 + (267 * 0^2)/1372 - (615 * 0)/2744 +
    873/2744) + 2 * c * f * ((-(2025 * 0^4)/10976) - (2025 *
    0^3)/43904 + (7857 * 0^2)/87808 - (2025 * 0)/12544 + 3321/10976) +
    2 * c * g * ((-(81 * 0^4)/686) - (27 * 0^3)/686 - (9 * 0^2)/392 -
        (165 * 0)/2744 + 165/686) + 2 * c * h * ((-(9 * 0^4)/224) -
    (243 * 0^3)/6272 - (993 * 0^2)/12544 + (39 * 0)/1792 + 855/6272) +
    0 + d^2 * ((-(50625 * 0^4)/175616) + (3375 * 0^3)/175616 +
    (16875 * 0^2)/87808 - (10125 * 0)/43904 + 3375/10976) + 2 *
    d * e * ((-(225 * 0^4)/686) + (15 * 0^3)/112 + (1655 * 0^2)/21952 -
    (3665 * 0)/21952 + 3135/10976) + 2 * d * f * ((-(50625 *
    0^4)/175616) + (3375 * 0^3)/21952 - (5625 * 0^2)/175616 -
    (5655 * 0)/87808 + 2535/10976) + 2 * d * g * ((-(2025 * 0^4)/10976) +
    (3645 * 0^3)/43904 - (7515 * 0^2)/87808 + (405 * 0)/12544 +
    6795/43904) + 2 * d * h * ((-(225 * 0^4)/3584) - (285 * 0^3)/25088 -
    (55 * 0^2)/784 + (65 * 0)/896 + 225/3136) + 0 + e^2 * ((-(128 *
    0^4)/343) + (96 * 0^3)/343 - (24 * 0^2)/343 - (22 * 0)/343 +
    78/343) + 2 * e * f * ((-(225 * 0^4)/686) + (225 * 0^3)/784 -
    (3273 * 0^2)/21952 + (717 * 0)/21952 + 54/343) + 2 * e *
```

```
g * ((-(72 * 0^4)/343) + (57 * 0^3)/343 - (181 * 0^2)/1372 +
(239 * 0)/2744 + 243/2744) + 2 * e * h * ((-0^4/14) + (9 *
0^3)/784 - (159 * 0^2)/3136 + (5 * 0)/64 + 51/1568) + 0 +
f^2 * ((-(50625 * 0^4)/175616) + (50625 * 0^3)/175616 - (2025 *
    0^2)/10976 + (2025 * 0)/21952 + 2025/21952) + 2 * f *
g * ((-(2025 * 0^4)/10976) + (7425 * 0^3)/43904 - (11043 *
0^2)/87808 + (177 * 0)/1792 + 465/10976) + 2 * f * h * ((-(225 *
0^4)/3584) + (225 * 0^3)/12544 - (633 * 0^2)/25088 + (15 *
0)/256 + 9/784) + 0 + g^2 * ((-(81 * 0^4)/686) + (135 * 0^3)/1372 -
(27 * 0^2)/392 + (405 * 0)/5488 + 81/5488) + 2 * g * h *
(15/6272 - 9/224 * 0^4 + 51/6272 * 0^3 - 13/12544 * 0^2 +
    55/1792 * 0) + 0 + h^2 * (-7/512 * 0^4 - 3/512 * 0^3 +
3/256 * 0^2 + 1/128 * 0) + 0 + i^2 * 0)


R.5 <- (a^2 * 0 + 2 * a * b * (1/64 * 0^2) + 2 * a * c * (1/784 +
9/196 * 0^2 + 3/196 * 0) + 2 * a * d * (9/784 + 225/3136 *
0^2 + 45/784 * 0) + 2 * a * e * (9/196 + 4/49 * 0^2 + 6/49 *
0) + 2 * a * f * (25/196 + 225/3136 * 0^2 + 75/392 * 0) +
2 * a * g * (225/784 + 9/196 * 0^2 + 45/196 * 0) + 2 * a *
h * (9/16 + 1/64 * 0^2 + 3/16 * 0) + 2 * a * i * 1 + b^2 *
(1/256 + 35/2048 * 0^4 + 25/512 * 0^3 - 27/512 * 0^2 + 7/128 *
    0) + 2 * b * c * (327/12544 + 45/896 * 0^4 + 225/3136 *
0^3 - 2533/25088 * 0^2 + 1149/12544 * 0) + 2 * b * d * ((1125 *
0^4)/14336 + (2025 * 0^3)/25088 - (2623 * 0^2)/25088 + (561 *
0)/6272 + 453/6272) + 2 * b * e * ((5 * 0^4)/56 + (65 * 0^3)/784 -
(369 * 0^2)/6272 + (149 * 0)/3136 + 451/3136) + 2 * b * f *
((1125 * 0^4)/14336 + (2025 * 0^3)/25088 + (527 * 0^2)/25088 -
    (45 * 0)/3136 + 2985/12544) + 2 * b * g * ((45 * 0^4)/896 +
(225 * 0^3)/3136 + (2507 * 0^2)/25088 - (699 * 0)/12544 +
4359/12544) + 2 * b * h * ((35 * 0^4)/2048 + (25 * 0^3)/512 +
(63 * 0^2)/512 - 0/64 + 59/128) + 2 * b * i * (9/16 + 1/64 *
0^2 + 3/16 * 0) + c^2 * ((405 * 0^4)/2744 - (513 * 0^2)/5488 +
(27 * 0)/343 + 3429/43904) + 2 * c * d * ((10125 * 0^4)/43904 -
(2025 * 0^3)/21952 - (2691 * 0^2)/175616 + (675 * 0)/87808 +
13045/87808) + 2 * c * e * ((90 * 0^4)/343 - (45 * 0^3)/343 +
(197 * 0^2)/2744 - (117 * 0)/1372 + 2481/10976) + 2 * c *
f * ((10125 * 0^4)/43904 - (2025 * 0^3)/21952 + (22509 *
0^2)/175616 - (13941 * 0)/87808 + 26037/87808) + 2 * c *
g * ((405 * 0^4)/2744 + (747 * 0^2)/5488 - (57 * 0)/343 +
15077/43904) + 2 * c * h * ((45 * 0^4)/896 + (225 * 0^3)/3136 +
(2507 * 0^2)/25088 - (699 * 0)/12544 + 4359/12544) + 2 *
c * i * (225/784 + 9/196 * 0^2 + 45/196 * 0) + d^2 * ((253125 *
0^4)/702464 - (50625 * 0^3)/175616 + (24975 * 0^2)/175616 -
(675 * 0)/6272 + 19575/87808) + 2 * d * e * ((1125 * 0^4)/2744 -
(2025 * 0^3)/5488 + (10649 * 0^2)/43904 - (4317 * 0)/21952 +
879/3136) + 2 * d * f * ((253125 * 0^4)/702464 - (50625 *
0^3)/175616 + (40725 * 0^2)/175616 - (345 * 0)/1568 + 13529/43904) +
2 * d * g * ((10125 * 0^4)/43904 - (2025 * 0^3)/21952 + (22509 *
    0^2)/175616 - (13941 * 0)/87808 + 26037/87808) + 2 *
d * h * ((1125 * 0^4)/14336 + (2025 * 0^3)/25088 + (527 *
0^2)/25088 - (45 * 0)/3136 + 2985/12544) + 2 * d * i * (25/196 +
225/3136 * 0^2 + 75/392 * 0) + e^2 * ((160 * 0^4)/343 - (160 *
```

```
    0^3)/343 + (108 * 0^2)/343 - (82 * 0)/343 + 821/2744) + 2 *
    e * f * ((1125 * 0^4)/2744 - (2025 * 0^3)/5488 + (10649 *
    0^2)/43904 - (4317 * 0)/21952 + 879/3136) + 2 * e * g * ((90 *
    0^4)/343 - (45 * 0^3)/343 + (197 * 0^2)/2744 - (117 * 0)/1372 +
    2481/10976) + 2 * e * h * ((5 * 0^4)/56 + (65 * 0^3)/784 -
    (369 * 0^2)/6272 + (149 * 0)/3136 + 451/3136) + 2 * e * i *
    (9/196 + 4/49 * 0^2 + 6/49 * 0) + f^2 * ((253125 * 0^4)/702464 -
    (50625 * 0^3)/175616 + (24975 * 0^2)/175616 - (675 * 0)/6272 +
    19575/87808) + 2 * f * g * ((10125 * 0^4)/43904 - (2025 *
    0^3)/21952 - (2691 * 0^2)/175616 + (675 * 0)/87808 + 13045/87808) +
    2 * f * h * ((1125 * 0^4)/14336 + (2025 * 0^3)/25088 - (2623 *
        0^2)/25088 + (561 * 0)/6272 + 453/6272) + 2 * f * i *
    (9/784 + 225/3136 * 0^2 + 45/784 * 0) + g^2 * ((405 * 0^4)/2744 -
    (513 * 0^2)/5488 + (27 * 0)/343 + 3429/43904) + 2 * g * h *
    (327/12544 + 45/896 * 0^4 + 225/3136 * 0^3 - 2533/25088 *
        0^2 + 1149/12544 * 0) + 2 * g * i * (1/784 + 9/196 *
    0^2 + 3/196 * 0) + h^2 * (1/256 + 35/2048 * 0^4 + 25/512 *
    0^3 - 27/512 * 0^2 + 7/128 * 0) + 2 * h * i * (1/64 * 0^2) +
    i^2 * 0)


R.4 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
    0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
    h * 0 + 2 * a * i * 0 + b^2 * (-7/512 * 0^4 - 3/512 * 0^3 +
    3/256 * 0^2 + 1/128 * 0) + 2 * b * c * (15/6272 - 9/224 *
    0^4 + 51/6272 * 0^3 - 13/12544 * 0^2 + 55/1792 * 0) + 2 *
    b * d * ((-(225 * 0^4)/3584) + (225 * 0^3)/12544 - (633 *
    0^2)/25088 + (15 * 0)/256 + 9/784) + 2 * b * e * ((-0^4/14) +
    (9 * 0^3)/784 - (159 * 0^2)/3136 + (5 * 0)/64 + 51/1568) +
    2 * b * f * ((-(225 * 0^4)/3584) - (285 * 0^3)/25088 - (55 *
        0^2)/784 + (65 * 0)/896 + 225/3136) + 2 * b * g * ((-(9 *
    0^4)/224) - (243 * 0^3)/6272 - (993 * 0^2)/12544 + (39 *
    0)/1792 + 855/6272) + 2 * b * h * ((-(7 * 0^4)/512) - (3 *
    0^3)/64 - (39 * 0^2)/512 - (25 * 0)/256 + 15/64) + 2 * b *
    i * (3/8 - 1/16 * 0^2 - 5/16 * 0) + c^2 * ((-(81 * 0^4)/686) +
    (135 * 0^3)/1372 - (27 * 0^2)/392 + (405 * 0)/5488 + 81/5488) +
    2 * c * d * ((-(2025 * 0^4)/10976) + (7425 * 0^3)/43904 -
        (11043 * 0^2)/87808 + (177 * 0)/1792 + 465/10976) + 2 *
    c * e * ((-(72 * 0^4)/343) + (57 * 0^3)/343 - (181 * 0^2)/1372 +
    (239 * 0)/2744 + 243/2744) + 2 * c * f * ((-(2025 * 0^4)/10976) +
    (3645 * 0^3)/43904 - (7515 * 0^2)/87808 + (405 * 0)/12544 +
    6795/43904) + 2 * c * g * ((-(81 * 0^4)/686) - (27 * 0^3)/686 -
    (9 * 0^2)/392 - (165 * 0)/2744 + 165/686) + 2 * c * h * ((-(9 *
    0^4)/224) - (705 * 0^3)/6272 - (229 * 0^2)/12544 - (2165 *
    0)/12544 + 1077/3136) + 2 * c * i * (45/98 - 9/49 * 0^2 -
    27/98 * 0) + d^2 * ((-(50625 * 0^4)/175616) + (50625 * 0^3)/175616 -
    (2025 * 0^2)/10976 + (2025 * 0)/21952 + 2025/21952) + 2 *
    d * e * ((-(225 * 0^4)/686) + (225 * 0^3)/784 - (3273 * 0^2)/21952 +
    (717 * 0)/21952 + 54/343) + 2 * d * f * ((-(50625 * 0^4)/175616) +
    (3375 * 0^3)/21952 - (5625 * 0^2)/175616 - (5655 * 0)/87808 +
    2535/10976) + 2 * d * g * ((-(2025 * 0^4)/10976) - (2025 *
    0^3)/43904 + (7857 * 0^2)/87808 - (2025 * 0)/12544 + 3321/10976) +
    2 * d * h * ((-(225 * 0^4)/3584) - (4275 * 0^3)/25088 + (921 *
```

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      0^2)/12544 - (39 * 0)/196 + 1125/3136) + 2 * d * i *
(75/196 - 225/784 * 0^2 - 75/784 * 0) + e^2 * ((-(128 * 0^4)/343) +
(96 * 0^3)/343 - (24 * 0^2)/343 - (22 * 0)/343 + 78/343) +
2 * e * f * ((-(225 * 0^4)/686) + (15 * 0^3)/112 + (1655 *
      0^2)/21952 - (3665 * 0)/21952 + 3135/10976) + 2 * e *
g * ((-(72 * 0^4)/343) - (27 * 0^3)/343 + (267 * 0^2)/1372 -
(615 * 0)/2744 + 873/2744) + 2 * e * h * ((-0^4/14) - (159 *
0^3)/784 + (417 * 0^2)/3136 - (529 * 0)/3136 + 243/784) +
2 * e * i * (12/49 - 16/49 * 0^2 + 4/49 * 0) + f^2 * ((-(50625 *
0^4)/175616) + (3375 * 0^3)/175616 + (16875 * 0^2)/87808 -
(10125 * 0)/43904 + 3375/10976) + 2 * f * g * ((-(2025 *
0^4)/10976) - (5805 * 0^3)/43904 + (21465 * 0^2)/87808 -
(2715 * 0)/12544 + 12675/43904) + 2 * f * h * ((-(225 * 0^4)/3584) -
(2505 * 0^3)/12544 + (3235 * 0^2)/25088 - (1145 * 0)/12544 +
705/3136) + 2 * f * i * (45/392 - 225/784 * 0^2 + 135/784 *
0) + g^2 * ((-(81 * 0^4)/686) - (243 * 0^3)/1372 + (81 *
0^2)/392 - (729 * 0)/5488 + 1215/5488) + 2 * g * h * (405/3136 -
9/224 * 0^4 - 999/6272 * 0^3 + 807/12544 * 0^2 + 75/12544 *
0) + 2 * g * i * (3/98 - 9/49 * 0^2 + 15/98 * 0) + h^2 *
(3/64 - 7/512 * 0^4 - 45/512 * 0^3 - 3/128 * 0^2 + 5/64 *
      0) + 2 * h * i * (1/16 * 0 - 1/16 * 0^2) + i^2 * 0)


R.3 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
    0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
    h * 0 + 2 * a * i * 0 + b^2 * (7/1024 * 0^4 - 3/512 * 0^3 +
    3/512 * 0^2) + 2 * b * c * (1/12544 + 9/448 * 0^4 - 117/6272 *
    0^3 + 27/1792 * 0^2 + 37/12544 * 0) + 2 * b * d * ((225 *
    0^4)/7168 - (75 * 0^3)/3136 + (33 * 0^2)/1792 + (9 * 0)/784 +
    9/12544) + 2 * b * e * (0^4/28 - (3 * 0^3)/196 + (3 * 0^2)/224 +
    (81 * 0)/3136 + 9/3136) + 2 * b * f * ((225 * 0^4)/7168 +
    (135 * 0^3)/25088 + (15 * 0^2)/3584 + (275 * 0)/6272 + 25/3136) +
    2 * b * g * ((9 * 0^4)/448 + (177 * 0^3)/6272 + (3 * 0^2)/1792 +
      (765 * 0)/12544 + 225/12544) + 2 * b * h * ((7 * 0^4)/1024 +
    (9 * 0^3)/256 + (3 * 0^2)/128 + (9 * 0)/128 + 9/256) + 2 *
    b * i * (1/16 + 3/32 * 0^2 + 1/16 * 0) + c^2 * ((81 * 0^4)/1372 -
    (81 * 0^3)/1372 + (81 * 0^2)/2744 + (87 * 0)/5488 + 33/21952) +
    2 * c * d * ((2025 * 0^4)/21952 - (3375 * 0^3)/43904 + (261 *
      0^2)/12544 + (3453 * 0)/87808 + 573/87808) + 2 * c *
    e * ((36 * 0^4)/343 - (18 * 0^3)/343 - (9 * 0^2)/686 + (185 *
    0)/2744 + 25/1372) + 2 * c * f * ((2025 * 0^4)/21952 + (405 *
    0^3)/43904 - (675 * 0^2)/12544 + (7695 * 0)/87808 + 3555/87808) +
    2 * c * g * ((81 * 0^4)/1372 + (27 * 0^3)/343 - (171 * 0^2)/2744 +
      (111 * 0)/1372 + 1713/21952) + 2 * c * h * ((9 * 0^4)/448 +
    (639 * 0^3)/6272 + (261 * 0^2)/12544 + (271 * 0)/12544 +
    1711/12544) + 2 * c * i * (87/392 + 27/98 * 0^2 - 6/49 *
    0) + d^2 * ((50625 * 0^4)/351232 - (16875 * 0^3)/175616 -
    (2025 * 0^2)/175616 + (405 * 0)/5488 + 135/6272) + 2 * d *
    e * ((225 * 0^4)/1372 - (75 * 0^3)/1372 - (789 * 0^2)/10976 +
    (45 * 0)/448 + 1089/21952) + 2 * d * f * ((50625 * 0^4)/351232 +
    (3375 * 0^3)/87808 - (5625 * 0^2)/43904 + (4395 * 0)/43904 +
    1185/12544) + 2 * d * g * ((2025 * 0^4)/21952 + (6075 * 0^3)/43904 -
    (1539 * 0^2)/12544 + (4293 * 0)/87808 + 14013/87808) + 2 *
```

```
d * h * ((225 * 0^4)/7168 + (4125 * 0^3)/25088 + (687 * 0^2)/25088 -
(513 * 0)/6272 + 1557/6272) + 2 * d * i * (285/784 + 675/1568 *
0^2 - 255/784 * 0) + e^2 * ((64 * 0^4)/343 - (48 * 0^2)/343 +
(36 * 0)/343 + 135/1372) + 2 * e * f * ((225 * 0^4)/1372 +
(135 * 0^3)/1372 - (1965 * 0^2)/10976 + (205 * 0)/3136 +
3595/21952) + 2 * e * g * ((36 * 0^4)/343 + (66 * 0^3)/343 -
(93 * 0^2)/686 - (81 * 0)/2744 + 333/1372) + 2 * e * h *
(0^4/28 + (39 * 0^3)/196 + (93 * 0^2)/1568 - (585 * 0)/3136 +
    1035/3136) + 2 * e * i * (41/98 + 24/49 * 0^2 - 20/49 *
0) + f^2 * ((50625 * 0^4)/351232 + (30375 * 0^3)/175616 -
(30375 * 0^2)/175616 - (375 * 0)/21952 + 375/1568) + 2 *
f * g * ((2025 * 0^4)/21952 + (9855 * 0^3)/43904 - (1035 *
0^2)/12544 - (11625 * 0)/87808 + 27075/87808) + 2 * f * h *
((225 * 0^4)/7168 + (1215 * 0^3)/6272 + (1425 * 0^2)/12544 -
    (785 * 0)/3136 + 4465/12544) + 2 * f * i * (285/784 +
675/1568 * 0^2 - 255/784 * 0) + g^2 * ((81 * 0^4)/1372 +
(297 * 0^3)/1372 + (81 * 0^2)/2744 - (1215 * 0)/5488 + 7425/21952) +
2 * g * h * (3951/12544 + 9/448 * 0^4 + 933/6272 * 0^3 +
    2109/12544 * 0^2 - 3033/12544 * 0) + 2 * g * i * (87/392 +
27/98 * 0^2 - 6/49 * 0) + h^2 * (27/128 + 7/1024 * 0^4 +
39/512 * 0^3 + 93/512 * 0^2 - 9/64 * 0) + 2 * h * i * (1/16 +
3/32 * 0^2 + 1/16 * 0) + i^2 * 0)


R.2 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
    0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
    h * 0 + 2 * a * i * 0 + b^2 * (-1/512 * 0^4 + 1/512 * 0^3) +
    2 * b * c * (-9/1568 * 0^4 + 27/6272 * 0^3 + 17/12544 * 0^2 +
        1/12544 * 0) + 2 * b * d * ((-(225 * 0^4)/25088) + (45 *
    0^3)/12544 + (117 * 0^2)/25088 + (9 * 0)/12544) + 2 * b *
    e * ((-0^4/98) - 0^3/784 + (27 * 0^2)/3136 + (9 * 0)/3136) +
    2 * b * f * ((-(225 * 0^4)/25088) - (225 * 0^3)/25088 + (125 *
        0^2)/12544 + (25 * 0)/3136) + 2 * b * g * ((-(9 * 0^4)/1568) -
    (99 * 0^3)/6272 + (45 * 0^2)/12544 + (225 * 0)/12544) + 2 *
    b * h * ((-0^4/512) - 0^3/64 - (9 * 0^2)/512 + (9 * 0)/256) +
    2 * b * i * (1/16 * 0 - 1/16 * 0^2) + c^2 * ((-(81 * 0^4)/4802) +
    (81 * 0^3)/9604 + (135 * 0^2)/19208 + (51 * 0)/38416 + 3/38416) +
    2 * c * d * ((-(2025 * 0^4)/76832) + (1215 * 0^3)/307328 +
        (10089 * 0^2)/614656 + (3375 * 0)/614656 + 153/307328) +
    2 * c * e * ((-(72 * 0^4)/2401) - (27 * 0^3)/2401 + (239 *
        0^2)/9604 + (281 * 0)/19208 + 33/19208) + 2 * c * f *
    ((-(2025 * 0^4)/76832) - (10125 * 0^3)/307328 + (14625 *
        0^2)/614656 + (19125 * 0)/614656 + 675/153664) + 2 *
    c * g * ((-(81 * 0^4)/4802) - (243 * 0^3)/4802 + (9 * 0^2)/19208 +
    (1107 * 0)/19208 + 45/4802) + 2 * c * h * ((-(9 * 0^4)/1568) -
    (297 * 0^3)/6272 - (775 * 0^2)/12544 + (1219 * 0)/12544 +
    111/6272) + 2 * c * i * (3/98 - 9/49 * 0^2 + 15/98 * 0) +
    d^2 * ((-(50625 * 0^4)/1229312) - (10125 * 0^3)/1229312 +
        (18225 * 0^2)/614656 + (5265 * 0)/307328 + 405/153664) +
    2 * d * e * ((-(225 * 0^4)/4802) - (1305 * 0^3)/38416 + (5499 *
        0^2)/153664 + (5679 * 0)/153664 + 621/76832) + 2 * d *
    f * ((-(50625 * 0^4)/1229312) - (10125 * 0^3)/153664 + (28575 *
    0^2)/1229312 + (39825 * 0)/614656 + 2925/153664) + 2 * d *
```

```
g * ((-(2025 * 0^4)/76832) - (27135 * 0^3)/307328 - (13851 *
0^2)/614656 + (60831 * 0)/614656 + 11745/307328) + 2 * d *
h * ((-(225 * 0^4)/25088) - (1935 * 0^3)/25088 - (747 * 0^2)/6272 +
(855 * 0)/6272 + 27/392) + 2 * d * i * (45/392 - 225/784 *
0^2 + 135/784 * 0) + e^2 * ((-(128 * 0^4)/2401) - (160 *
0^3)/2401 + (72 * 0^2)/2401 + (162 * 0)/2401 + 54/2401) +
2 * e * f * ((-(225 * 0^4)/4802) - (3825 * 0^3)/38416 - (325 *
    0^2)/153664 + (15325 * 0)/153664 + 1875/38416) + 2 *
e * g * ((-(72 * 0^4)/2401) - (279 * 0^3)/2401 - (657 * 0^2)/9604 +
(2367 * 0)/19208 + 1755/19208) + 2 * e * h * ((-0^4/98) -
(73 * 0^3)/784 - (549 * 0^2)/3136 + (387 * 0)/3136 + 243/1568) +
2 * e * i * (12/49 - 16/49 * 0^2 + 4/49 * 0) + f^2 * ((-(50625 *
0^4)/1229312) - (151875 * 0^3)/1229312 - (16875 * 0^2)/307328 +
(9375 * 0)/76832 + 1875/19208) + 2 * f * g * ((-(2025 * 0^4)/76832) -
(38475 * 0^3)/307328 - (79875 * 0^2)/614656 + (69525 * 0)/614656 +
25875/153664) + 2 * f * h * ((-(225 * 0^4)/25088) - (1125 *
0^3)/12544 - (5375 * 0^2)/25088 + (625 * 0)/12544 + 825/3136) +
2 * f * i * (75/196 - 225/784 * 0^2 - 75/784 * 0) + g^2 *
((-(81 * 0^4)/4802) - (1053 * 0^3)/9604 - (3645 * 0^2)/19208 +
    (2025 * 0)/38416 + 10125/38416) + 2 * g * h * (2295/6272 -
9/1568 * 0^4 - 423/6272 * 0^3 - 2763/12544 * 0^2 - 909/12544 *
0) + 2 * g * i * (45/98 - 9/49 * 0^2 - 27/98 * 0) + h^2 *
(27/64 - 1/512 * 0^4 - 17/512 * 0^3 - 45/256 * 0^2 - 27/128 *
    0) + 2 * h * i * (3/8 - 1/16 * 0^2 - 5/16 * 0) + i^2 *
0)


R.1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
    0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
    h * 0 + 2 * a * i * 0 + b^2 * (1/4096 * 0^4) + 2 * b * c *
    (9/12544 * 0^4 + 3/12544 * 0^3 + 1/50176 * 0^2) + 2 * b *
    d * ((225 * 0^4)/200704 + (45 * 0^3)/50176 + (9 * 0^2)/50176) +
    2 * b * e * (0^4/784 + (3 * 0^3)/1568 + (9 * 0^2)/12544) +
    2 * b * f * ((225 * 0^4)/200704 + (75 * 0^3)/25088 + (25 *
        0^2)/12544) + 2 * b * g * ((9 * 0^4)/12544 + (45 * 0^3)/12544 +
    (225 * 0^2)/50176) + 2 * b * h * (0^4/4096 + (3 * 0^3)/1024 +
    (9 * 0^2)/1024) + 2 * b * i * (1/64 * 0^2) + c^2 * ((81 *
    0^4)/38416 + (27 * 0^3)/19208 + (27 * 0^2)/76832 + (3 * 0)/76832 +
    1/614656) + 2 * c * d * ((2025 * 0^4)/614656 + (2295 * 0^3)/614656 +
    (3681 * 0^2)/2458624 + (153 * 0)/614656 + 9/614656) + 2 *
    c * e * ((9 * 0^4)/2401 + (33 * 0^3)/4802 + (157 * 0^2)/38416 +
    (33 * 0)/38416 + 9/153664) + 2 * c * f * ((2025 * 0^4)/614656 +
    (6075 * 0^3)/614656 + (21825 * 0^2)/2458624 + (675 * 0)/307328 +
    25/153664) + 2 * c * g * ((81 * 0^4)/38416 + (27 * 0^3)/2401 +
    (1287 * 0^2)/76832 + (45 * 0)/9604 + 225/614656) + 2 * c *
    h * ((9 * 0^4)/12544 + (111 * 0^3)/12544 + (1441 * 0^2)/50176 +
    (111 * 0)/12544 + 9/12544) + 2 * c * i * (1/784 + 9/196 *
    0^2 + 3/196 * 0) + d^2 * ((50625 * 0^4)/9834496 + (10125 *
    0^3)/1229312 + (6075 * 0^2)/1229312 + (405 * 0)/307328 +
    81/614656) + 2 * d * e * ((225 * 0^4)/38416 + (1035 * 0^3)/76832 +
    (6921 * 0^2)/614656 + (621 * 0)/153664 + 81/153664) + 2 *
    d * f * ((50625 * 0^4)/9834496 + (43875 * 0^3)/2458624 +
    (51525 * 0^2)/2458624 + (2925 * 0)/307328 + 225/153664) +
```

```
    2 * d * g * ((2025 * 0^4)/614656 + (11745 * 0^3)/614656 +
        (84321 * 0^2)/2458624 + (11745 * 0)/614656 + 2025/614656) +
    2 * d * h * ((225 * 0^4)/200704 + (45 * 0^3)/3136 + (1287 *
        0^2)/25088 + (27 * 0)/784 + 81/12544) + 2 * d * i * (9/784 +
    225/3136 * 0^2 + 45/784 * 0) + e^2 * ((16 * 0^4)/2401 + (48 *
    0^3)/2401 + (54 * 0^2)/2401 + (27 * 0)/2401 + 81/38416) +
    2 * e * f * ((225 * 0^4)/38416 + (1875 * 0^3)/76832 + (22825 *
        0^2)/614656 + (1875 * 0)/76832 + 225/38416) + 2 * e *
    g * ((9 * 0^4)/2401 + (117 * 0^3)/4802 + (2061 * 0^2)/38416 +
    (1755 * 0)/38416 + 2025/153664) + 2 * e * h * (0^4/784 +
    (27 * 0^3)/1568 + (873 * 0^2)/12544 + (243 * 0)/3136 + 81/3136) +
    2 * e * i * (9/196 + 4/49 * 0^2 + 6/49 * 0) + f^2 * ((50625 *
    0^4)/9834496 + (16875 * 0^3)/614656 + (16875 * 0^2)/307328 +
    (1875 * 0)/38416 + 625/38416) + 2 * f * g * ((2025 * 0^4)/614656 +
    (15525 * 0^3)/614656 + (173025 * 0^2)/2458624 + (25875 *
    0)/307328 + 5625/153664) + 2 * f * h * ((225 * 0^4)/200704 +
    (825 * 0^3)/50176 + (3925 * 0^2)/50176 + (825 * 0)/6272 +
    225/3136) + 2 * f * i * (25/196 + 225/3136 * 0^2 + 75/392 *
    0) + g^2 * ((81 * 0^4)/38416 + (405 * 0^3)/19208 + (6075 *
    0^2)/76832 + (10125 * 0)/76832 + 50625/614656) + 2 * g *
    h * (2025/12544 + 9/12544 * 0^4 + 153/12544 * 0^3 + 3681/50176 *
    0^2 + 2295/12544 * 0) + 2 * g * i * (225/784 + 9/196 * 0^2 +
    45/196 * 0) + h^2 * (81/256 + 1/4096 * 0^4 + 3/512 * 0^3 +
    27/512 * 0^2 + 27/128 * 0) + 2 * h * i * (9/16 + 1/64 * 0^2 +
    3/16 * 0) + i^2 * 1)

exp <- c(R.9, R.8, R.7, R.6, R.5, R.4, R.3, R.2, R.1)
```

This constructs the test statistic. Notice that it uses `prob1`, a perturbation of the true genotype frequencies. This makes no sense.

```
KF <- ((prob1[1] - exp[1])^2/exp[1] + (prob1[2] - exp[2])^2/exp[2] +
    (prob1[3] - exp[3])^2/exp[3] + (prob1[4] - exp[4])^2/exp[4] +
    (prob1[5] - exp[5])^2/exp[5] + (prob1[6] - exp[6])^2/exp[6] +
    (prob1[7] - exp[7])^2/exp[7] + (prob1[8] - exp[8])^2/exp[8] +
    (prob1[9] - exp[9])^2/exp[9]) * N
kff <- c(kff, KF)
```

We can verify that `prob1` is the same as above

```
prob1
```

```
## [1] 0.04948 0.09193 0.13758 0.16100 0.19942 0.14998 0.10952 0.06350 0.03759
```

References

Wang, Jing, Xuemin Lv, Li Feng, Ang Dong, Dan Liang, and Rongling Wu. 2021. "A Tracing Model for the Evolutionary Equilibrium of Octoploids." *Frontiers in Genetics* 12. https://doi.org/10.3389/fgene.2021.794907.