

One iteration of ‘table2_power.R’

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Summary

Below is one iteration of the code from “table2_power.R” from Wang et al. (2021). We show that they use a perturbation of the *true* genotype frequencies when constructing their test statistic. Since the true genotype frequencies are never known in reality, this test statistic could never be constructed in reality.

Code

This holds the output of the simulations

```
pvalue <- c()
geno.L <- c()
lr <- c()
kff <- c()
```

This generates prob1

```
a <- 0.1
b <- 0.1
c <- 0.15
d <- 0.1
e <- 0.2
f <- 0.1
g <- 0.05
h <- 0.1
i <- 0.1

alpha = 0
prob0 <- c(0.1, 0.1, 0.15, 0.1, 0.2, 0.1, 0.05, 0.1, 0.1)

Q9.1 <- (a^2 * 1 + 2 * a * b * (9/16 + 1/64 * alpha^2 + 3/16 *
  alpha) + 2 * a * c * (225/784 + 9/196 * alpha^2 + 45/196 *
  alpha) + 2 * a * d * (25/196 + 225/3136 * alpha^2 + 75/392 *
  alpha) + 2 * a * e * (9/196 + 4/49 * alpha^2 + 6/49 * alpha) +
  2 * a * f * (9/784 + 225/3136 * alpha^2 + 45/784 * alpha) +
  2 * a * g * (1/784 + 9/196 * alpha^2 + 3/196 * alpha) + 2 *
  a * h * (1/64 * alpha^2) + 0 + b^2 * (81/256 + 1/4096 * alpha^4 +
  3/512 * alpha^3 + 27/512 * alpha^2 + 27/128 * alpha) + 2 *
  b * c * (2025/12544 + 9/12544 * alpha^4 + 153/12544 * alpha^3 +
  3681/50176 * alpha^2 + 2295/12544 * alpha) + 2 * b * d *
  ((225 * alpha^4)/200704 + (825 * alpha^3)/50176 + (3925 *
  alpha^2)/50176 + (825 * alpha)/6272 + 225/3136) + 2 *
  b * e * (alpha^4/784 + (27 * alpha^3)/1568 + (873 * alpha^2)/12544 +
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(243 * alpha)/3136 + 81/3136) + 2 * b * f * ((225 * alpha^4)/200704 +
(45 * alpha^3)/3136 + (1287 * alpha^2)/25088 + (27 * alpha)/784 +
81/12544) + 2 * b * g * ((9 * alpha^4)/12544 + (111 * alpha^3)/12544 +
(1441 * alpha^2)/50176 + (111 * alpha)/12544 + 9/12544) +
2 * b * h * (alpha^4/4096 + (3 * alpha^3)/1024 + (9 * alpha^2)/1024) +
0 + c^2 * ((81 * alpha^4)/38416 + (405 * alpha^3)/19208 +
(6075 * alpha^2)/76832 + (10125 * alpha)/76832 + 50625/614656) +
2 * c * d * ((2025 * alpha^4)/614656 + (15525 * alpha^3)/614656 +
(173025 * alpha^2)/2458624 + (25875 * alpha)/307328 +
5625/153664) + 2 * c * e * ((9 * alpha^4)/2401 + (117 *
alpha^3)/4802 + (2061 * alpha^2)/38416 + (1755 * alpha)/38416 +
2025/153664) + 2 * c * f * ((2025 * alpha^4)/614656 + (11745 *
alpha^3)/614656 + (84321 * alpha^2)/2458624 + (11745 * alpha)/614656 +
2025/614656) + 2 * c * g * ((81 * alpha^4)/38416 + (27 *
alpha^3)/2401 + (1287 * alpha^2)/76832 + (45 * alpha)/9604 +
225/614656) + 2 * c * h * ((9 * alpha^4)/12544 + (45 * alpha^3)/12544 +
(225 * alpha^2)/50176) + 0 + d^2 * ((50625 * alpha^4)/9834496 +
(16875 * alpha^3)/614656 + (16875 * alpha^2)/307328 + (1875 *
alpha)/38416 + 625/38416) + 2 * d * e * ((225 * alpha^4)/38416 +
(1875 * alpha^3)/76832 + (22825 * alpha^2)/614656 + (1875 *
alpha)/76832 + 225/38416) + 2 * d * f * ((50625 * alpha^4)/9834496 +
(43875 * alpha^3)/2458624 + (51525 * alpha^2)/2458624 + (2925 *
alpha)/307328 + 225/153664) + 2 * d * g * ((2025 * alpha^4)/614656 +
(6075 * alpha^3)/614656 + (21825 * alpha^2)/2458624 + (675 *
alpha)/307328 + 25/153664) + 2 * d * h * ((225 * alpha^4)/200704 +
(75 * alpha^3)/25088 + (25 * alpha^2)/12544) + 0 + e^2 *
((16 * alpha^4)/2401 + (48 * alpha^3)/2401 + (54 * alpha^2)/2401 +
(27 * alpha)/2401 + 81/38416) + 2 * e * f * ((225 * alpha^4)/38416 +
(1035 * alpha^3)/76832 + (6921 * alpha^2)/614656 + (621 *
alpha)/153664 + 81/153664) + 2 * e * g * ((9 * alpha^4)/2401 +
(33 * alpha^3)/4802 + (157 * alpha^2)/38416 + (33 * alpha)/38416 +
9/153664) + 2 * e * h * (alpha^4/784 + (3 * alpha^3)/1568 +
(9 * alpha^2)/12544) + 0 + f^2 * ((50625 * alpha^4)/9834496 +
(10125 * alpha^3)/1229312 + (6075 * alpha^2)/1229312 + (405 *
alpha)/307328 + 81/614656) + 2 * f * g * ((2025 * alpha^4)/614656 +
(2295 * alpha^3)/614656 + (3681 * alpha^2)/2458624 + (153 *
alpha)/614656 + 9/614656) + 2 * f * h * ((225 * alpha^4)/200704 +
(45 * alpha^3)/50176 + (9 * alpha^2)/50176) + 0 + g^2 * ((81 *
alpha^4)/38416 + (27 * alpha^3)/19208 + (27 * alpha^2)/76832 +
(3 * alpha)/76832 + 1/614656) + 2 * g * h * (9/12544 * alpha^4 +
3/12544 * alpha^3 + 1/50176 * alpha^2) + 0 + h^2 * (1/4096 *
alpha^4) + 0 + i^2 * 0)

```

```

Q8.1 <- (a^2 * 0 + 2 * a * b * (3/8 - 1/16 * alpha^2 - 5/16 *
alpha) + 2 * a * c * (45/98 - 9/49 * alpha^2 - 27/98 * alpha) +
2 * a * d * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) +
2 * a * e * (12/49 - 16/49 * alpha^2 + 4/49 * alpha) + 2 *
a * f * (45/392 - 225/784 * alpha^2 + 135/784 * alpha) +
2 * a * g * (3/98 - 9/49 * alpha^2 + 15/98 * alpha) + 2 *
a * h * (1/16 * alpha - 1/16 * alpha^2) + 0 + b^2 * (27/64 -
1/512 * alpha^4 - 17/512 * alpha^3 - 45/256 * alpha^2 - 27/128 *
alpha) + 2 * b * c * (2295/6272 - 9/1568 * alpha^4 - 423/6272 *

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alpha^3 - 2763/12544 * alpha^2 - 909/12544 * alpha) + 2 *
b * d * ((-(225 * alpha^4)/25088) - (1125 * alpha^3)/12544 -
(5375 * alpha^2)/25088 + (625 * alpha)/12544 + 825/3136) +
2 * b * e * ((-alpha^4/98) - (73 * alpha^3)/784 - (549 *
alpha^2)/3136 + (387 * alpha)/3136 + 243/1568) + 2 *
b * f * ((-(225 * alpha^4)/25088) - (1935 * alpha^3)/25088 -
(747 * alpha^2)/6272 + (855 * alpha)/6272 + 27/392) + 2 *
b * g * ((-(9 * alpha^4)/1568) - (297 * alpha^3)/6272 - (775 *
alpha^2)/12544 + (1219 * alpha)/12544 + 111/6272) + 2 * b *
h * ((-alpha^4/512) - alpha^3/64 - (9 * alpha^2)/512 + (9 *
alpha)/256) + 0 + c^2 * ((-(81 * alpha^4)/4802) - (1053 *
alpha^3)/9604 - (3645 * alpha^2)/19208 + (2025 * alpha)/38416 +
10125/38416) + 2 * c * d * ((-(2025 * alpha^4)/76832) - (38475 *
alpha^3)/307328 - (79875 * alpha^2)/614656 + (69525 * alpha)/614656 +
25875/153664) + 2 * c * e * ((-(72 * alpha^4)/2401) - (279 *
alpha^3)/2401 - (657 * alpha^2)/9604 + (2367 * alpha)/19208 +
1755/19208) + 2 * c * f * ((-(2025 * alpha^4)/76832) - (27135 *
alpha^3)/307328 - (13851 * alpha^2)/614656 + (60831 * alpha)/614656 +
11745/307328) + 2 * c * g * ((-(81 * alpha^4)/4802) - (243 *
alpha^3)/4802 + (9 * alpha^2)/19208 + (1107 * alpha)/19208 +
45/4802) + 2 * c * h * ((-(9 * alpha^4)/1568) - (99 * alpha^3)/6272 +
(45 * alpha^2)/12544 + (225 * alpha)/12544) + 0 + d^2 * ((-(50625 *
alpha^4)/1229312) - (151875 * alpha^3)/1229312 - (16875 *
alpha^2)/307328 + (9375 * alpha)/76832 + 1875/19208) + 2 *
d * e * ((-(225 * alpha^4)/4802) - (3825 * alpha^3)/38416 -
(325 * alpha^2)/153664 + (15325 * alpha)/153664 + 1875/38416) +
2 * d * f * ((-(50625 * alpha^4)/1229312) - (10125 * alpha^3)/153664 +
(28575 * alpha^2)/1229312 + (39825 * alpha)/614656 +
2925/153664) + 2 * d * g * ((-(2025 * alpha^4)/76832) -
(10125 * alpha^3)/307328 + (14625 * alpha^2)/614656 + (19125 *
alpha)/614656 + 675/153664) + 2 * d * h * ((-(225 * alpha^4)/25088) -
(225 * alpha^3)/25088 + (125 * alpha^2)/12544 + (25 * alpha)/3136) +
0 + e^2 * ((-(128 * alpha^4)/2401) - (160 * alpha^3)/2401 +
(72 * alpha^2)/2401 + (162 * alpha)/2401 + 54/2401) + 2 *
e * f * ((-(225 * alpha^4)/4802) - (1305 * alpha^3)/38416 +
(5499 * alpha^2)/153664 + (5679 * alpha)/153664 + 621/76832) +
2 * e * g * ((-(72 * alpha^4)/2401) - (27 * alpha^3)/2401 +
(239 * alpha^2)/9604 + (281 * alpha)/19208 + 33/19208) +
2 * e * h * ((-alpha^4/98) - alpha^3/784 + (27 * alpha^2)/3136 +
(9 * alpha)/3136) + 0 + f^2 * ((-(50625 * alpha^4)/1229312) -
(10125 * alpha^3)/1229312 + (18225 * alpha^2)/614656 + (5265 *
alpha)/307328 + 405/153664) + 2 * f * g * ((-(2025 * alpha^4)/76832) +
(1215 * alpha^3)/307328 + (10089 * alpha^2)/614656 + (3375 *
alpha)/614656 + 153/307328) + 2 * f * h * ((-(225 * alpha^4)/25088) +
(45 * alpha^3)/12544 + (117 * alpha^2)/25088 + (9 * alpha)/12544) +
0 + g^2 * ((-(81 * alpha^4)/4802) + (81 * alpha^3)/9604 +
(135 * alpha^2)/19208 + (51 * alpha)/38416 + 3/38416) + 2 *
g * h * (-9/1568 * alpha^4 + 27/6272 * alpha^3 + 17/12544 *
alpha^2 + 1/12544 * alpha) + 0 + h^2 * (-1/512 * alpha^4 +
1/512 * alpha^3) + 0 + i^2 * 0)

```

```

Q7.1 <- (a^2 * 0 + 2 * a * b * (1/16 + 3/32 * alpha^2 + 1/16 *

```

$$\begin{aligned}
& \alpha) + 2 * a * c * (87/392 + 27/98 * \alpha^2 - 6/49 * \alpha) + \\
& 2 * a * d * (285/784 + 675/1568 * \alpha^2 - 255/784 * \alpha) + \\
& 2 * a * e * (41/98 + 24/49 * \alpha^2 - 20/49 * \alpha) + 2 * \\
& a * f * (285/784 + 675/1568 * \alpha^2 - 255/784 * \alpha) + \\
& 2 * a * g * (87/392 + 27/98 * \alpha^2 - 6/49 * \alpha) + 2 * \\
& a * h * (1/16 + 3/32 * \alpha^2 + 1/16 * \alpha) + 0 + b^2 * \\
& (27/128 + 7/1024 * \alpha^4 + 39/512 * \alpha^3 + 93/512 * \\
& \alpha^2 - 9/64 * \alpha) + 2 * b * c * (3951/12544 + 9/448 * \\
& \alpha^4 + 933/6272 * \alpha^3 + 2109/12544 * \alpha^2 - 3033/12544 * \\
& \alpha) + 2 * b * d * ((225 * \alpha^4)/7168 + (1215 * \alpha^3)/6272 + \\
& (1425 * \alpha^2)/12544 - (785 * \alpha)/3136 + 4465/12544) + \\
& 2 * b * e * (\alpha^4/28 + (39 * \alpha^3)/196 + (93 * \alpha^2)/1568 - \\
& (585 * \alpha)/3136 + 1035/3136) + 2 * b * f * ((225 * \\
& \alpha^4)/7168 + (4125 * \alpha^3)/25088 + (687 * \alpha^2)/25088 - \\
& (513 * \alpha)/6272 + 1557/6272) + 2 * b * g * ((9 * \alpha^4)/448 + \\
& (639 * \alpha^3)/6272 + (261 * \alpha^2)/12544 + (271 * \alpha)/12544 + \\
& 1711/12544) + 2 * b * h * ((7 * \alpha^4)/1024 + (9 * \alpha^3)/256 + \\
& (3 * \alpha^2)/128 + (9 * \alpha)/128 + 9/256) + 0 + c^2 * \\
& ((81 * \alpha^4)/1372 + (297 * \alpha^3)/1372 + (81 * \alpha^2)/2744 - \\
& (1215 * \alpha)/5488 + 7425/21952) + 2 * c * d * ((2025 * \\
& \alpha^4)/21952 + (9855 * \alpha^3)/43904 - (1035 * \alpha^2)/12544 - \\
& (11625 * \alpha)/87808 + 27075/87808) + 2 * c * e * ((36 * \\
& \alpha^4)/343 + (66 * \alpha^3)/343 - (93 * \alpha^2)/686 - \\
& (81 * \alpha)/2744 + 333/1372) + 2 * c * f * ((2025 * \alpha^4)/21952 + \\
& (6075 * \alpha^3)/43904 - (1539 * \alpha^2)/12544 + (4293 * \\
& \alpha)/87808 + 14013/87808) + 2 * c * g * ((81 * \alpha^4)/1372 + \\
& (27 * \alpha^3)/343 - (171 * \alpha^2)/2744 + (111 * \alpha)/1372 + \\
& 1713/21952) + 2 * c * h * ((9 * \alpha^4)/448 + (177 * \alpha^3)/6272 + \\
& (3 * \alpha^2)/1792 + (765 * \alpha)/12544 + 225/12544) + 0 + \\
& d^2 * ((50625 * \alpha^4)/351232 + (30375 * \alpha^3)/175616 - \\
& (30375 * \alpha^2)/175616 - (375 * \alpha)/21952 + 375/1568) + \\
& 2 * d * e * ((225 * \alpha^4)/1372 + (135 * \alpha^3)/1372 - \\
& (1965 * \alpha^2)/10976 + (205 * \alpha)/3136 + 3595/21952) + \\
& 2 * d * f * ((50625 * \alpha^4)/351232 + (3375 * \alpha^3)/87808 - \\
& (5625 * \alpha^2)/43904 + (4395 * \alpha)/43904 + 1185/12544) + \\
& 2 * d * g * ((2025 * \alpha^4)/21952 + (405 * \alpha^3)/43904 - \\
& (675 * \alpha^2)/12544 + (7695 * \alpha)/87808 + 3555/87808) + \\
& 2 * d * h * ((225 * \alpha^4)/7168 + (135 * \alpha^3)/25088 + \\
& (15 * \alpha^2)/3584 + (275 * \alpha)/6272 + 25/3136) + \\
& 0 + e^2 * ((64 * \alpha^4)/343 - (48 * \alpha^2)/343 + (36 * \\
& \alpha)/343 + 135/1372) + 2 * e * f * ((225 * \alpha^4)/1372 - \\
& (75 * \alpha^3)/1372 - (789 * \alpha^2)/10976 + (45 * \alpha)/448 + \\
& 1089/21952) + 2 * e * g * ((36 * \alpha^4)/343 - (18 * \alpha^3)/343 - \\
& (9 * \alpha^2)/686 + (185 * \alpha)/2744 + 25/1372) + 2 * e * \\
& h * (\alpha^4/28 - (3 * \alpha^3)/196 + (3 * \alpha^2)/224 + \\
& (81 * \alpha)/3136 + 9/3136) + 0 + f^2 * ((50625 * \alpha^4)/351232 - \\
& (16875 * \alpha^3)/175616 - (2025 * \alpha^2)/175616 + (405 * \\
& \alpha)/5488 + 135/6272) + 2 * f * g * ((2025 * \alpha^4)/21952 - \\
& (3375 * \alpha^3)/43904 + (261 * \alpha^2)/12544 + (3453 * \\
& \alpha)/87808 + 573/87808) + 2 * f * h * ((225 * \alpha^4)/7168 - \\
& (75 * \alpha^3)/3136 + (33 * \alpha^2)/1792 + (9 * \alpha)/784 + \\
& 9/12544) + 0 + g^2 * ((81 * \alpha^4)/1372 - (81 * \alpha^3)/1372 + \\
& (81 * \alpha^2)/2744 + (87 * \alpha)/5488 + 33/21952) + 2 *
\end{aligned}$$

```

g * h * (1/12544 + 9/448 * alpha^4 - 117/6272 * alpha^3 +
27/1792 * alpha^2 + 37/12544 * alpha) + 0 + h^2 * (7/1024 *
alpha^4 - 3/512 * alpha^3 + 3/512 * alpha^2) + 0 + i^2 *
0)

```

```

Q6.1 <- (a^2 * 0 + 2 * a * b * (1/16 * alpha - 1/16 * alpha^2) +
2 * a * c * (3/98 - 9/49 * alpha^2 + 15/98 * alpha) + 2 *
a * d * (45/392 - 225/784 * alpha^2 + 135/784 * alpha) +
2 * a * e * (12/49 - 16/49 * alpha^2 + 4/49 * alpha) + 2 *
a * f * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) + 2 *
a * g * (45/98 - 9/49 * alpha^2 - 27/98 * alpha) + 2 * a *
h * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) + 0 + b^2 * (3/64 -
7/512 * alpha^4 - 45/512 * alpha^3 - 3/128 * alpha^2 + 5/64 *
alpha) + 2 * b * c * (405/3136 - 9/224 * alpha^4 - 999/6272 *
alpha^3 + 807/12544 * alpha^2 + 75/12544 * alpha) + 2 * b *
d * ((- (225 * alpha^4)/3584) - (2505 * alpha^3)/12544 + (3235 *
alpha^2)/25088 - (1145 * alpha)/12544 + 705/3136) + 2 * b *
e * ((-alpha^4/14) - (159 * alpha^3)/784 + (417 * alpha^2)/3136 -
(529 * alpha)/3136 + 243/784) + 2 * b * f * ((- (225 * alpha^4)/3584) -
(4275 * alpha^3)/25088 + (921 * alpha^2)/12544 - (39 * alpha)/196 +
1125/3136) + 2 * b * g * ((- (9 * alpha^4)/224) - (705 * alpha^3)/6272 -
(229 * alpha^2)/12544 - (2165 * alpha)/12544 + 1077/3136) +
2 * b * h * ((- (7 * alpha^4)/512) - (3 * alpha^3)/64 - (39 *
alpha^2)/512 - (25 * alpha)/256 + 15/64) + 0 + c^2 *
((- (81 * alpha^4)/686) - (243 * alpha^3)/1372 + (81 * alpha^2)/392 -
(729 * alpha)/5488 + 1215/5488) + 2 * c * d * ((- (2025 *
alpha^4)/10976) - (5805 * alpha^3)/43904 + (21465 * alpha^2)/87808 -
(2715 * alpha)/12544 + 12675/43904) + 2 * c * e * ((- (72 *
alpha^4)/343) - (27 * alpha^3)/343 + (267 * alpha^2)/1372 -
(615 * alpha)/2744 + 873/2744) + 2 * c * f * ((- (2025 * alpha^4)/10976) -
(2025 * alpha^3)/43904 + (7857 * alpha^2)/87808 - (2025 *
alpha)/12544 + 3321/10976) + 2 * c * g * ((- (81 * alpha^4)/686) -
(27 * alpha^3)/686 - (9 * alpha^2)/392 - (165 * alpha)/2744 +
165/686) + 2 * c * h * ((- (9 * alpha^4)/224) - (243 * alpha^3)/6272 -
(993 * alpha^2)/12544 + (39 * alpha)/1792 + 855/6272) + 0 +
d^2 * ((- (50625 * alpha^4)/175616) + (3375 * alpha^3)/175616 +
(16875 * alpha^2)/87808 - (10125 * alpha)/43904 + 3375/10976) +
2 * d * e * ((- (225 * alpha^4)/686) + (15 * alpha^3)/112 +
(1655 * alpha^2)/21952 - (3665 * alpha)/21952 + 3135/10976) +
2 * d * f * ((- (50625 * alpha^4)/175616) + (3375 * alpha^3)/21952 -
(5625 * alpha^2)/175616 - (5655 * alpha)/87808 + 2535/10976) +
2 * d * g * ((- (2025 * alpha^4)/10976) + (3645 * alpha^3)/43904 -
(7515 * alpha^2)/87808 + (405 * alpha)/12544 + 6795/43904) +
2 * d * h * ((- (225 * alpha^4)/3584) - (285 * alpha^3)/25088 -
(55 * alpha^2)/784 + (65 * alpha)/896 + 225/3136) + 0 +
e^2 * ((- (128 * alpha^4)/343) + (96 * alpha^3)/343 - (24 *
alpha^2)/343 - (22 * alpha)/343 + 78/343) + 2 * e * f *
((- (225 * alpha^4)/686) + (225 * alpha^3)/784 - (3273 * alpha^2)/21952 +
(717 * alpha)/21952 + 54/343) + 2 * e * g * ((- (72 *
alpha^4)/343) + (57 * alpha^3)/343 - (181 * alpha^2)/1372 +
(239 * alpha)/2744 + 243/2744) + 2 * e * h * ((-alpha^4/14) +
(9 * alpha^3)/784 - (159 * alpha^2)/3136 + (5 * alpha)/64 +

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```

51/1568) + 0 + f^2 * ((-(50625 * alpha^4)/175616) + (50625 *
alpha^3)/175616 - (2025 * alpha^2)/10976 + (2025 * alpha)/21952 +
2025/21952) + 2 * f * g * ((-(2025 * alpha^4)/10976) + (7425 *
alpha^3)/43904 - (11043 * alpha^2)/87808 + (177 * alpha)/1792 +
465/10976) + 2 * f * h * ((-(225 * alpha^4)/3584) + (225 *
alpha^3)/12544 - (633 * alpha^2)/25088 + (15 * alpha)/256 +
9/784) + 0 + g^2 * ((-(81 * alpha^4)/686) + (135 * alpha^3)/1372 -
(27 * alpha^2)/392 + (405 * alpha)/5488 + 81/5488) + 2 *
g * h * (15/6272 - 9/224 * alpha^4 + 51/6272 * alpha^3 -
13/12544 * alpha^2 + 55/1792 * alpha) + 0 + h^2 * (-7/512 *
alpha^4 - 3/512 * alpha^3 + 3/256 * alpha^2 + 1/128 * alpha) +
0 + i^2 * 0)

```

```

Q5.1 <- (a^2 * 0 + 2 * a * b * (1/64 * alpha^2) + 2 * a * c *
(1/784 + 9/196 * alpha^2 + 3/196 * alpha) + 2 * a * d * (9/784 +
225/3136 * alpha^2 + 45/784 * alpha) + 2 * a * e * (9/196 +
4/49 * alpha^2 + 6/49 * alpha) + 2 * a * f * (25/196 + 225/3136 *
alpha^2 + 75/392 * alpha) + 2 * a * g * (225/784 + 9/196 *
alpha^2 + 45/196 * alpha) + 2 * a * h * (9/16 + 1/64 * alpha^2 +
3/16 * alpha) + 2 * a * i * 1 + b^2 * (1/256 + 35/2048 *
alpha^4 + 25/512 * alpha^3 - 27/512 * alpha^2 + 7/128 * alpha) +
2 * b * c * (327/12544 + 45/896 * alpha^4 + 225/3136 * alpha^3 -
2533/25088 * alpha^2 + 1149/12544 * alpha) + 2 * b *
d * ((1125 * alpha^4)/14336 + (2025 * alpha^3)/25088 - (2623 *
alpha^2)/25088 + (561 * alpha)/6272 + 453/6272) + 2 * b *
e * ((5 * alpha^4)/56 + (65 * alpha^3)/784 - (369 * alpha^2)/6272 +
(149 * alpha)/3136 + 451/3136) + 2 * b * f * ((1125 * alpha^4)/14336 +
(2025 * alpha^3)/25088 + (527 * alpha^2)/25088 - (45 * alpha)/3136 +
2985/12544) + 2 * b * g * ((45 * alpha^4)/896 + (225 * alpha^3)/3136 +
(2507 * alpha^2)/25088 - (699 * alpha)/12544 + 4359/12544) +
2 * b * h * ((35 * alpha^4)/2048 + (25 * alpha^3)/512 + (63 *
alpha^2)/512 - alpha/64 + 59/128) + 2 * b * i * (9/16 +
1/64 * alpha^2 + 3/16 * alpha) + c^2 * ((405 * alpha^4)/2744 -
(513 * alpha^2)/5488 + (27 * alpha)/343 + 3429/43904) + 2 *
c * d * ((10125 * alpha^4)/43904 - (2025 * alpha^3)/21952 -
(2691 * alpha^2)/175616 + (675 * alpha)/87808 + 13045/87808) +
2 * c * e * ((90 * alpha^4)/343 - (45 * alpha^3)/343 + (197 *
alpha^2)/2744 - (117 * alpha)/1372 + 2481/10976) + 2 *
c * f * ((10125 * alpha^4)/43904 - (2025 * alpha^3)/21952 +
(22509 * alpha^2)/175616 - (13941 * alpha)/87808 + 26037/87808) +
2 * c * g * ((405 * alpha^4)/2744 + (747 * alpha^2)/5488 -
(57 * alpha)/343 + 15077/43904) + 2 * c * h * ((45 *
alpha^4)/896 + (225 * alpha^3)/3136 + (2507 * alpha^2)/25088 -
(699 * alpha)/12544 + 4359/12544) + 2 * c * i * (225/784 +
9/196 * alpha^2 + 45/196 * alpha) + d^2 * ((253125 * alpha^4)/702464 -
(50625 * alpha^3)/175616 + (24975 * alpha^2)/175616 - (675 *
alpha)/6272 + 19575/87808) + 2 * d * e * ((1125 * alpha^4)/2744 -
(2025 * alpha^3)/5488 + (10649 * alpha^2)/43904 - (4317 *
alpha)/21952 + 879/3136) + 2 * d * f * ((253125 * alpha^4)/702464 -
(50625 * alpha^3)/175616 + (40725 * alpha^2)/175616 - (345 *
alpha)/1568 + 13529/43904) + 2 * d * g * ((10125 * alpha^4)/43904 -
(2025 * alpha^3)/21952 + (22509 * alpha^2)/175616 - (13941 *

```


$$\begin{aligned}
& \alpha)/87808 + 26037/87808) + 2 * d * h * ((1125 * \alpha^4)/14336 + \\
& (2025 * \alpha^3)/25088 + (527 * \alpha^2)/25088 - (45 * \alpha)/3136 + \\
& 2985/12544) + 2 * d * i * (25/196 + 225/3136 * \alpha^2 + \\
& 75/392 * \alpha) + e^2 * ((160 * \alpha^4)/343 - (160 * \alpha^3)/343 + \\
& (108 * \alpha^2)/343 - (82 * \alpha)/343 + 821/2744) + 2 * \\
& e * f * ((1125 * \alpha^4)/2744 - (2025 * \alpha^3)/5488 + \\
& (10649 * \alpha^2)/43904 - (4317 * \alpha)/21952 + 879/3136) + \\
& 2 * e * g * ((90 * \alpha^4)/343 - (45 * \alpha^3)/343 + (197 * \\
& \alpha^2)/2744 - (117 * \alpha)/1372 + 2481/10976) + 2 * \\
& e * h * ((5 * \alpha^4)/56 + (65 * \alpha^3)/784 - (369 * \alpha^2)/6272 + \\
& (149 * \alpha)/3136 + 451/3136) + 2 * e * i * (9/196 + 4/49 * \\
& \alpha^2 + 6/49 * \alpha) + f^2 * ((253125 * \alpha^4)/702464 - \\
& (50625 * \alpha^3)/175616 + (24975 * \alpha^2)/175616 - (675 * \\
& \alpha)/6272 + 19575/87808) + 2 * f * g * ((10125 * \alpha^4)/43904 - \\
& (2025 * \alpha^3)/21952 - (2691 * \alpha^2)/175616 + (675 * \\
& \alpha)/87808 + 13045/87808) + 2 * f * h * ((1125 * \alpha^4)/14336 + \\
& (2025 * \alpha^3)/25088 - (2623 * \alpha^2)/25088 + (561 * \\
& \alpha)/6272 + 453/6272) + 2 * f * i * (9/784 + 225/3136 * \\
& \alpha^2 + 45/784 * \alpha) + g^2 * ((405 * \alpha^4)/2744 - \\
& (513 * \alpha^2)/5488 + (27 * \alpha)/343 + 3429/43904) + 2 * \\
& g * h * (327/12544 + 45/896 * \alpha^4 + 225/3136 * \alpha^3 - \\
& 2533/25088 * \alpha^2 + 1149/12544 * \alpha) + 2 * g * i * \\
& (1/784 + 9/196 * \alpha^2 + 3/196 * \alpha) + h^2 * (1/256 + \\
& 35/2048 * \alpha^4 + 25/512 * \alpha^3 - 27/512 * \alpha^2 + \\
& 7/128 * \alpha) + 2 * h * i * (1/64 * \alpha^2) + i^2 * 0)
\end{aligned}$$

```

Q4.1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
h * 0 + 2 * a * i * 0 + b^2 * (-7/512 * alpha^4 - 3/512 *
alpha^3 + 3/256 * alpha^2 + 1/128 * alpha) + 2 * b * c *
(15/6272 - 9/224 * alpha^4 + 51/6272 * alpha^3 - 13/12544 *
alpha^2 + 55/1792 * alpha) + 2 * b * d * ((-225 * alpha^4)/3584) +
(225 * alpha^3)/12544 - (633 * alpha^2)/25088 + (15 * alpha)/256 +
9/784) + 2 * b * e * ((-alpha^4/14) + (9 * alpha^3)/784 -
(159 * alpha^2)/3136 + (5 * alpha)/64 + 51/1568) + 2 * b *
f * ((-225 * alpha^4)/3584) - (285 * alpha^3)/25088 - (55 *
alpha^2)/784 + (65 * alpha)/896 + 225/3136) + 2 * b * g *
((-9 * alpha^4)/224) - (243 * alpha^3)/6272 - (993 * alpha^2)/12544 +
(39 * alpha)/1792 + 855/6272) + 2 * b * h * ((-7 * alpha^4)/512) -
(3 * alpha^3)/64 - (39 * alpha^2)/512 - (25 * alpha)/256 +
15/64) + 2 * b * i * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) +
c^2 * ((-81 * alpha^4)/686) + (135 * alpha^3)/1372 - (27 *
alpha^2)/392 + (405 * alpha)/5488 + 81/5488) + 2 * c *
d * ((-2025 * alpha^4)/10976) + (7425 * alpha^3)/43904 -
(11043 * alpha^2)/87808 + (177 * alpha)/1792 + 465/10976) +
2 * c * e * ((-72 * alpha^4)/343) + (57 * alpha^3)/343 -
(181 * alpha^2)/1372 + (239 * alpha)/2744 + 243/2744) +
2 * c * f * ((-2025 * alpha^4)/10976) + (3645 * alpha^3)/43904 -
(7515 * alpha^2)/87808 + (405 * alpha)/12544 + 6795/43904) +
2 * c * g * ((-81 * alpha^4)/686) - (27 * alpha^3)/686 -
(9 * alpha^2)/392 - (165 * alpha)/2744 + 165/686) + 2 *
c * h * ((-9 * alpha^4)/224) - (705 * alpha^3)/6272 - (229 *

```

```

alpha^2)/12544 - (2165 * alpha)/12544 + 1077/3136) + 2 *
c * i * (45/98 - 9/49 * alpha^2 - 27/98 * alpha) + d^2 *
((-50625 * alpha^4)/175616) + (50625 * alpha^3)/175616 -
(2025 * alpha^2)/10976 + (2025 * alpha)/21952 + 2025/21952) +
2 * d * e * ((-225 * alpha^4)/686) + (225 * alpha^3)/784 -
(3273 * alpha^2)/21952 + (717 * alpha)/21952 + 54/343) +
2 * d * f * ((-50625 * alpha^4)/175616) + (3375 * alpha^3)/21952 -
(5625 * alpha^2)/175616 - (5655 * alpha)/87808 + 2535/10976) +
2 * d * g * ((-2025 * alpha^4)/10976) - (2025 * alpha^3)/43904 +
(7857 * alpha^2)/87808 - (2025 * alpha)/12544 + 3321/10976) +
2 * d * h * ((-225 * alpha^4)/3584) - (4275 * alpha^3)/25088 +
(921 * alpha^2)/12544 - (39 * alpha)/196 + 1125/3136) +
2 * d * i * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) +
e^2 * ((-128 * alpha^4)/343) + (96 * alpha^3)/343 - (24 *
alpha^2)/343 - (22 * alpha)/343 + 78/343) + 2 * e * f *
((-225 * alpha^4)/686) + (15 * alpha^3)/112 + (1655 * alpha^2)/21952 -
(3665 * alpha)/21952 + 3135/10976) + 2 * e * g * ((-72 *
alpha^4)/343) - (27 * alpha^3)/343 + (267 * alpha^2)/1372 -
(615 * alpha)/2744 + 873/2744) + 2 * e * h * ((-alpha^4)/14) -
(159 * alpha^3)/784 + (417 * alpha^2)/3136 - (529 * alpha)/3136 +
243/784) + 2 * e * i * (12/49 - 16/49 * alpha^2 + 4/49 *
alpha) + f^2 * ((-50625 * alpha^4)/175616) + (3375 * alpha^3)/175616 +
(16875 * alpha^2)/87808 - (10125 * alpha)/43904 + 3375/10976) +
2 * f * g * ((-2025 * alpha^4)/10976) - (5805 * alpha^3)/43904 +
(21465 * alpha^2)/87808 - (2715 * alpha)/12544 + 12675/43904) +
2 * f * h * ((-225 * alpha^4)/3584) - (2505 * alpha^3)/12544 +
(3235 * alpha^2)/25088 - (1145 * alpha)/12544 + 705/3136) +
2 * f * i * (45/392 - 225/784 * alpha^2 + 135/784 * alpha) +
g^2 * ((-81 * alpha^4)/686) - (243 * alpha^3)/1372 + (81 *
alpha^2)/392 - (729 * alpha)/5488 + 1215/5488) + 2 *
g * h * (405/3136 - 9/224 * alpha^4 - 999/6272 * alpha^3 +
807/12544 * alpha^2 + 75/12544 * alpha) + 2 * g * i * (3/98 -
9/49 * alpha^2 + 15/98 * alpha) + h^2 * (3/64 - 7/512 * alpha^4 -
45/512 * alpha^3 - 3/128 * alpha^2 + 5/64 * alpha) + 2 *
h * i * (1/16 * alpha - 1/16 * alpha^2) + i^2 * 0)

```

```

Q3.1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
h * 0 + 2 * a * i * 0 + b^2 * (7/1024 * alpha^4 - 3/512 *
alpha^3 + 3/512 * alpha^2) + 2 * b * c * (1/12544 + 9/448 *
alpha^4 - 117/6272 * alpha^3 + 27/1792 * alpha^2 + 37/12544 *
alpha) + 2 * b * d * ((225 * alpha^4)/7168 - (75 * alpha^3)/3136 +
(33 * alpha^2)/1792 + (9 * alpha)/784 + 9/12544) + 2 * b *
e * (alpha^4/28 - (3 * alpha^3)/196 + (3 * alpha^2)/224 +
(81 * alpha)/3136 + 9/3136) + 2 * b * f * ((225 * alpha^4)/7168 +
(135 * alpha^3)/25088 + (15 * alpha^2)/3584 + (275 * alpha)/6272 +
25/3136) + 2 * b * g * ((9 * alpha^4)/448 + (177 * alpha^3)/6272 +
(3 * alpha^2)/1792 + (765 * alpha)/12544 + 225/12544) + 2 *
b * h * ((7 * alpha^4)/1024 + (9 * alpha^3)/256 + (3 * alpha^2)/128 +
(9 * alpha)/128 + 9/256) + 2 * b * i * (1/16 + 3/32 * alpha^2 +
1/16 * alpha) + c^2 * ((81 * alpha^4)/1372 - (81 * alpha^3)/1372 +
(81 * alpha^2)/2744 + (87 * alpha)/5488 + 33/21952) + 2 *

```



```

c * d * ((2025 * alpha^4)/21952 - (3375 * alpha^3)/43904 +
(261 * alpha^2)/12544 + (3453 * alpha)/87808 + 573/87808) +
2 * c * e * ((36 * alpha^4)/343 - (18 * alpha^3)/343 - (9 *
alpha^2)/686 + (185 * alpha)/2744 + 25/1372) + 2 * c *
f * ((2025 * alpha^4)/21952 + (405 * alpha^3)/43904 - (675 *
alpha^2)/12544 + (7695 * alpha)/87808 + 3555/87808) + 2 *
c * g * ((81 * alpha^4)/1372 + (27 * alpha^3)/343 - (171 *
alpha^2)/2744 + (111 * alpha)/1372 + 1713/21952) + 2 * c *
h * ((9 * alpha^4)/448 + (639 * alpha^3)/6272 + (261 * alpha^2)/12544 +
(271 * alpha)/12544 + 1711/12544) + 2 * c * i * (87/392 +
27/98 * alpha^2 - 6/49 * alpha) + d^2 * ((50625 * alpha^4)/351232 -
(16875 * alpha^3)/175616 - (2025 * alpha^2)/175616 + (405 *
alpha)/5488 + 135/6272) + 2 * d * e * ((225 * alpha^4)/1372 -
(75 * alpha^3)/1372 - (789 * alpha^2)/10976 + (45 * alpha)/448 +
1089/21952) + 2 * d * f * ((50625 * alpha^4)/351232 + (3375 *
alpha^3)/87808 - (5625 * alpha^2)/43904 + (4395 * alpha)/43904 +
1185/12544) + 2 * d * g * ((2025 * alpha^4)/21952 + (6075 *
alpha^3)/43904 - (1539 * alpha^2)/12544 + (4293 * alpha)/87808 +
14013/87808) + 2 * d * h * ((225 * alpha^4)/7168 + (4125 *
alpha^3)/25088 + (687 * alpha^2)/25088 - (513 * alpha)/6272 +
1557/6272) + 2 * d * i * (285/784 + 675/1568 * alpha^2 -
255/784 * alpha) + e^2 * ((64 * alpha^4)/343 - (48 * alpha^2)/343 +
(36 * alpha)/343 + 135/1372) + 2 * e * f * ((225 * alpha^4)/1372 +
(135 * alpha^3)/1372 - (1965 * alpha^2)/10976 + (205 * alpha)/3136 +
3595/21952) + 2 * e * g * ((36 * alpha^4)/343 + (66 * alpha^3)/343 -
(93 * alpha^2)/686 - (81 * alpha)/2744 + 333/1372) + 2 *
e * h * (alpha^4/28 + (39 * alpha^3)/196 + (93 * alpha^2)/1568 -
(585 * alpha)/3136 + 1035/3136) + 2 * e * i * (41/98 + 24/49 *
alpha^2 - 20/49 * alpha) + f^2 * ((50625 * alpha^4)/351232 +
(30375 * alpha^3)/175616 - (30375 * alpha^2)/175616 - (375 *
alpha)/21952 + 375/1568) + 2 * f * g * ((2025 * alpha^4)/21952 +
(9855 * alpha^3)/43904 - (1035 * alpha^2)/12544 - (11625 *
alpha)/87808 + 27075/87808) + 2 * f * h * ((225 * alpha^4)/7168 +
(1215 * alpha^3)/6272 + (1425 * alpha^2)/12544 - (785 * alpha)/3136 +
4465/12544) + 2 * f * i * (285/784 + 675/1568 * alpha^2 -
255/784 * alpha) + g^2 * ((81 * alpha^4)/1372 + (297 * alpha^3)/1372 +
(81 * alpha^2)/2744 - (1215 * alpha)/5488 + 7425/21952) +
2 * g * h * (3951/12544 + 9/448 * alpha^4 + 933/6272 * alpha^3 +
2109/12544 * alpha^2 - 3033/12544 * alpha) + 2 * g *
i * (87/392 + 27/98 * alpha^2 - 6/49 * alpha) + h^2 * (27/128 +
7/1024 * alpha^4 + 39/512 * alpha^3 + 93/512 * alpha^2 -
9/64 * alpha) + 2 * h * i * (1/16 + 3/32 * alpha^2 + 1/16 *
alpha) + i^2 * 0)

```

```

Q2.1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
h * 0 + 2 * a * i * 0 + b^2 * (-1/512 * alpha^4 + 1/512 *
alpha^3) + 2 * b * c * (-9/1568 * alpha^4 + 27/6272 * alpha^3 +
17/12544 * alpha^2 + 1/12544 * alpha) + 2 * b * d * ((-225 *
alpha^4)/25088) + (45 * alpha^3)/12544 + (117 * alpha^2)/25088 +
(9 * alpha)/12544) + 2 * b * e * ((-alpha^4/98) - alpha^3/784 +
(27 * alpha^2)/3136 + (9 * alpha)/3136) + 2 * b * f * ((-225 *

```

$$\begin{aligned}
& \alpha^4/25088) - (225 * \alpha^3)/25088 + (125 * \alpha^2)/12544 + \\
& (25 * \alpha)/3136) + 2 * b * g * ((-9 * \alpha^4)/1568) - \\
& (99 * \alpha^3)/6272 + (45 * \alpha^2)/12544 + (225 * \alpha)/12544) + \\
& 2 * b * h * ((-\alpha^4/512) - \alpha^3/64 - (9 * \alpha^2)/512 + \\
& (9 * \alpha)/256) + 2 * b * i * (1/16 * \alpha - 1/16 * \\
& \alpha^2) + c^2 * ((-81 * \alpha^4)/4802) + (81 * \alpha^3)/9604 + \\
& (135 * \alpha^2)/19208 + (51 * \alpha)/38416 + 3/38416) + 2 * \\
& c * d * ((-(2025 * \alpha^4)/76832) + (1215 * \alpha^3)/307328 + \\
& (10089 * \alpha^2)/614656 + (3375 * \alpha)/614656 + 153/307328) + \\
& 2 * c * e * ((-(72 * \alpha^4)/2401) - (27 * \alpha^3)/2401 + \\
& (239 * \alpha^2)/9604 + (281 * \alpha)/19208 + 33/19208) + \\
& 2 * c * f * ((-(2025 * \alpha^4)/76832) - (10125 * \alpha^3)/307328 + \\
& (14625 * \alpha^2)/614656 + (19125 * \alpha)/614656 + 675/153664) + \\
& 2 * c * g * ((-81 * \alpha^4)/4802) - (243 * \alpha^3)/4802 + \\
& (9 * \alpha^2)/19208 + (1107 * \alpha)/19208 + 45/4802) + \\
& 2 * c * h * ((-9 * \alpha^4)/1568) - (297 * \alpha^3)/6272 - \\
& (775 * \alpha^2)/12544 + (1219 * \alpha)/12544 + 111/6272) + \\
& 2 * c * i * (3/98 - 9/49 * \alpha^2 + 15/98 * \alpha) + d^2 * \\
& ((-(50625 * \alpha^4)/1229312) - (10125 * \alpha^3)/1229312 + \\
& (18225 * \alpha^2)/614656 + (5265 * \alpha)/307328 + 405/153664) + \\
& 2 * d * e * ((-(225 * \alpha^4)/4802) - (1305 * \alpha^3)/38416 + \\
& (5499 * \alpha^2)/153664 + (5679 * \alpha)/153664 + 621/76832) + \\
& 2 * d * f * ((-(50625 * \alpha^4)/1229312) - (10125 * \alpha^3)/153664 + \\
& (28575 * \alpha^2)/1229312 + (39825 * \alpha)/614656 + \\
& 2925/153664) + 2 * d * g * ((-(2025 * \alpha^4)/76832) - \\
& (27135 * \alpha^3)/307328 - (13851 * \alpha^2)/614656 + (60831 * \\
& \alpha)/614656 + 11745/307328) + 2 * d * h * ((-(225 * \alpha^4)/25088) - \\
& (1935 * \alpha^3)/25088 - (747 * \alpha^2)/6272 + (855 * \alpha)/6272 + \\
& 27/392) + 2 * d * i * (45/392 - 225/784 * \alpha^2 + 135/784 * \\
& \alpha) + e^2 * ((-(128 * \alpha^4)/2401) - (160 * \alpha^3)/2401 + \\
& (72 * \alpha^2)/2401 + (162 * \alpha)/2401 + 54/2401) + 2 * \\
& e * f * ((-(225 * \alpha^4)/4802) - (3825 * \alpha^3)/38416 - \\
& (325 * \alpha^2)/153664 + (15325 * \alpha)/153664 + 1875/38416) + \\
& 2 * e * g * ((-(72 * \alpha^4)/2401) - (279 * \alpha^3)/2401 - \\
& (657 * \alpha^2)/9604 + (2367 * \alpha)/19208 + 1755/19208) + \\
& 2 * e * h * ((-\alpha^4/98) - (73 * \alpha^3)/784 - (549 * \\
& \alpha^2)/3136 + (387 * \alpha)/3136 + 243/1568) + 2 * \\
& e * i * (12/49 - 16/49 * \alpha^2 + 4/49 * \alpha) + f^2 * \\
& ((-(50625 * \alpha^4)/1229312) - (151875 * \alpha^3)/1229312 - \\
& (16875 * \alpha^2)/307328 + (9375 * \alpha)/76832 + 1875/19208) + \\
& 2 * f * g * ((-(2025 * \alpha^4)/76832) - (38475 * \alpha^3)/307328 - \\
& (79875 * \alpha^2)/614656 + (69525 * \alpha)/614656 + 25875/153664) + \\
& 2 * f * h * ((-(225 * \alpha^4)/25088) - (1125 * \alpha^3)/12544 - \\
& (5375 * \alpha^2)/25088 + (625 * \alpha)/12544 + 825/3136) + \\
& 2 * f * i * (75/196 - 225/784 * \alpha^2 - 75/784 * \alpha) + \\
& g^2 * ((-81 * \alpha^4)/4802) - (1053 * \alpha^3)/9604 - (3645 * \\
& \alpha^2)/19208 + (2025 * \alpha)/38416 + 10125/38416) + \\
& 2 * g * h * (2295/6272 - 9/1568 * \alpha^4 - 423/6272 * \alpha^3 - \\
& 2763/12544 * \alpha^2 - 909/12544 * \alpha) + 2 * g * i * \\
& (45/98 - 9/49 * \alpha^2 - 27/98 * \alpha) + h^2 * (27/64 - \\
& 1/512 * \alpha^4 - 17/512 * \alpha^3 - 45/256 * \alpha^2 - 27/128 * \\
& \alpha) + 2 * h * i * (3/8 - 1/16 * \alpha^2 - 5/16 * \alpha) + \\
& i^2 * 0)
\end{aligned}$$

```

Q1.1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
h * 0 + 2 * a * i * 0 + b^2 * (1/4096 * alpha^4) + 2 * b *
c * (9/12544 * alpha^4 + 3/12544 * alpha^3 + 1/50176 * alpha^2) +
2 * b * d * ((225 * alpha^4)/200704 + (45 * alpha^3)/50176 +
(9 * alpha^2)/50176) + 2 * b * e * (alpha^4/784 + (3 *
alpha^3)/1568 + (9 * alpha^2)/12544) + 2 * b * f * ((225 *
alpha^4)/200704 + (75 * alpha^3)/25088 + (25 * alpha^2)/12544) +
2 * b * g * ((9 * alpha^4)/12544 + (45 * alpha^3)/12544 +
(225 * alpha^2)/50176) + 2 * b * h * (alpha^4/4096 +
(3 * alpha^3)/1024 + (9 * alpha^2)/1024) + 2 * b * i * (1/64 *
alpha^2) + c^2 * ((81 * alpha^4)/38416 + (27 * alpha^3)/19208 +
(27 * alpha^2)/76832 + (3 * alpha)/76832 + 1/614656) + 2 *
c * d * ((2025 * alpha^4)/614656 + (2295 * alpha^3)/614656 +
(3681 * alpha^2)/2458624 + (153 * alpha)/614656 + 9/614656) +
2 * c * e * ((9 * alpha^4)/2401 + (33 * alpha^3)/4802 + (157 *
alpha^2)/38416 + (33 * alpha)/38416 + 9/153664) + 2 *
c * f * ((2025 * alpha^4)/614656 + (6075 * alpha^3)/614656 +
(21825 * alpha^2)/2458624 + (675 * alpha)/307328 + 25/153664) +
2 * c * g * ((81 * alpha^4)/38416 + (27 * alpha^3)/2401 +
(1287 * alpha^2)/76832 + (45 * alpha)/9604 + 225/614656) +
2 * c * h * ((9 * alpha^4)/12544 + (111 * alpha^3)/12544 +
(1441 * alpha^2)/50176 + (111 * alpha)/12544 + 9/12544) +
2 * c * i * (1/784 + 9/196 * alpha^2 + 3/196 * alpha) + d^2 *
((50625 * alpha^4)/9834496 + (10125 * alpha^3)/1229312 +
(6075 * alpha^2)/1229312 + (405 * alpha)/307328 + 81/614656) +
2 * d * e * ((225 * alpha^4)/38416 + (1035 * alpha^3)/76832 +
(6921 * alpha^2)/614656 + (621 * alpha)/153664 + 81/153664) +
2 * d * f * ((50625 * alpha^4)/9834496 + (43875 * alpha^3)/2458624 +
(51525 * alpha^2)/2458624 + (2925 * alpha)/307328 + 225/153664) +
2 * d * g * ((2025 * alpha^4)/614656 + (11745 * alpha^3)/614656 +
(84321 * alpha^2)/2458624 + (11745 * alpha)/614656 +
2025/614656) + 2 * d * h * ((225 * alpha^4)/200704 +
(45 * alpha^3)/3136 + (1287 * alpha^2)/25088 + (27 * alpha)/784 +
81/12544) + 2 * d * i * (9/784 + 225/3136 * alpha^2 + 45/784 *
alpha) + e^2 * ((16 * alpha^4)/2401 + (48 * alpha^3)/2401 +
(54 * alpha^2)/2401 + (27 * alpha)/2401 + 81/38416) + 2 *
e * f * ((225 * alpha^4)/38416 + (1875 * alpha^3)/76832 +
(22825 * alpha^2)/614656 + (1875 * alpha)/76832 + 225/38416) +
2 * e * g * ((9 * alpha^4)/2401 + (117 * alpha^3)/4802 +
(2061 * alpha^2)/38416 + (1755 * alpha)/38416 + 2025/153664) +
2 * e * h * (alpha^4/784 + (27 * alpha^3)/1568 + (873 * alpha^2)/12544 +
(243 * alpha)/3136 + 81/3136) + 2 * e * i * (9/196 +
4/49 * alpha^2 + 6/49 * alpha) + f^2 * ((50625 * alpha^4)/9834496 +
(16875 * alpha^3)/614656 + (16875 * alpha^2)/307328 + (1875 *
alpha)/38416 + 625/38416) + 2 * f * g * ((2025 * alpha^4)/614656 +
(15525 * alpha^3)/614656 + (173025 * alpha^2)/2458624 + (25875 *
alpha)/307328 + 5625/153664) + 2 * f * h * ((225 * alpha^4)/200704 +
(825 * alpha^3)/50176 + (3925 * alpha^2)/50176 + (825 * alpha)/6272 +
225/3136) + 2 * f * i * (25/196 + 225/3136 * alpha^2 + 75/392 *
alpha) + g^2 * ((81 * alpha^4)/38416 + (405 * alpha^3)/19208 +
(6075 * alpha^2)/76832 + (10125 * alpha)/76832 + 50625/614656) +

```

```

2 * g * h * (2025/12544 + 9/12544 * alpha^4 + 153/12544 *
  alpha^3 + 3681/50176 * alpha^2 + 2295/12544 * alpha) +
2 * g * i * (225/784 + 9/196 * alpha^2 + 45/196 * alpha) +
h^2 * (81/256 + 1/4096 * alpha^4 + 3/512 * alpha^3 + 27/512 *
  alpha^2 + 27/128 * alpha) + 2 * h * i * (9/16 + 1/64 *
  alpha^2 + 3/16 * alpha) + i^2 * 1)
probl <- c(Q9.1, Q8.1, Q7.1, Q6.1, Q5.1, Q4.1, Q3.1, Q2.1, Q1.1)

```

This generates prob

```
alpha = 0
```

```

Q9 <- (a^2 * 1 + 2 * a * b * (9/16 + 1/64 * alpha^2 + 3/16 *
  alpha) + 2 * a * c * (225/784 + 9/196 * alpha^2 + 45/196 *
  alpha) + 2 * a * d * (25/196 + 225/3136 * alpha^2 + 75/392 *
  alpha) + 2 * a * e * (9/196 + 4/49 * alpha^2 + 6/49 * alpha) +
  2 * a * f * (9/784 + 225/3136 * alpha^2 + 45/784 * alpha) +
  2 * a * g * (1/784 + 9/196 * alpha^2 + 3/196 * alpha) + 2 *
  a * h * (1/64 * alpha^2) + 0 + b^2 * (81/256 + 1/4096 * alpha^4 +
  3/512 * alpha^3 + 27/512 * alpha^2 + 27/128 * alpha) + 2 *
  b * c * (2025/12544 + 9/12544 * alpha^4 + 153/12544 * alpha^3 +
  3681/50176 * alpha^2 + 2295/12544 * alpha) + 2 * b * d *
  ((225 * alpha^4)/200704 + (825 * alpha^3)/50176 + (3925 *
  alpha^2)/50176 + (825 * alpha)/6272 + 225/3136) + 2 *
  b * e * (alpha^4/784 + (27 * alpha^3)/1568 + (873 * alpha^2)/12544 +
  (243 * alpha)/3136 + 81/3136) + 2 * b * f * ((225 * alpha^4)/200704 +
  (45 * alpha^3)/3136 + (1287 * alpha^2)/25088 + (27 * alpha)/784 +
  81/12544) + 2 * b * g * ((9 * alpha^4)/12544 + (111 * alpha^3)/12544 +
  (1441 * alpha^2)/50176 + (111 * alpha)/12544 + 9/12544) +
  2 * b * h * (alpha^4/4096 + (3 * alpha^3)/1024 + (9 * alpha^2)/1024) +
  0 + c^2 * ((81 * alpha^4)/38416 + (405 * alpha^3)/19208 +
  (6075 * alpha^2)/76832 + (10125 * alpha)/76832 + 50625/614656) +
  2 * c * d * ((2025 * alpha^4)/614656 + (15525 * alpha^3)/614656 +
  (173025 * alpha^2)/2458624 + (25875 * alpha)/307328 +
  5625/153664) + 2 * c * e * ((9 * alpha^4)/2401 + (117 *
  alpha^3)/4802 + (2061 * alpha^2)/38416 + (1755 * alpha)/38416 +
  2025/153664) + 2 * c * f * ((2025 * alpha^4)/614656 + (11745 *
  alpha^3)/614656 + (84321 * alpha^2)/2458624 + (11745 * alpha)/614656 +
  2025/614656) + 2 * c * g * ((81 * alpha^4)/38416 + (27 *
  alpha^3)/2401 + (1287 * alpha^2)/76832 + (45 * alpha)/9604 +
  225/614656) + 2 * c * h * ((9 * alpha^4)/12544 + (45 * alpha^3)/12544 +
  (225 * alpha^2)/50176) + 0 + d^2 * ((50625 * alpha^4)/9834496 +
  (16875 * alpha^3)/614656 + (16875 * alpha^2)/307328 + (1875 *
  alpha)/38416 + 625/38416) + 2 * d * e * ((225 * alpha^4)/38416 +
  (1875 * alpha^3)/76832 + (22825 * alpha^2)/614656 + (1875 *
  alpha)/76832 + 225/38416) + 2 * d * f * ((50625 * alpha^4)/9834496 +
  (43875 * alpha^3)/2458624 + (51525 * alpha^2)/2458624 + (2925 *
  alpha)/307328 + 225/153664) + 2 * d * g * ((2025 * alpha^4)/614656 +
  (6075 * alpha^3)/614656 + (21825 * alpha^2)/2458624 + (675 *
  alpha)/307328 + 25/153664) + 2 * d * h * ((225 * alpha^4)/200704 +
  (75 * alpha^3)/25088 + (25 * alpha^2)/12544) + 0 + e^2 *
  ((16 * alpha^4)/2401 + (48 * alpha^3)/2401 + (54 * alpha^2)/2401 +
  (27 * alpha)/2401 + 81/38416) + 2 * e * f * ((225 * alpha^4)/38416 +
  (1035 * alpha^3)/76832 + (6921 * alpha^2)/614656 + (621 *

```

```

alpha)/153664 + 81/153664) + 2 * e * g * ((9 * alpha^4)/2401 +
(33 * alpha^3)/4802 + (157 * alpha^2)/38416 + (33 * alpha)/38416 +
9/153664) + 2 * e * h * (alpha^4/784 + (3 * alpha^3)/1568 +
(9 * alpha^2)/12544) + 0 + f^2 * ((50625 * alpha^4)/9834496 +
(10125 * alpha^3)/1229312 + (6075 * alpha^2)/1229312 + (405 *
alpha)/307328 + 81/614656) + 2 * f * g * ((2025 * alpha^4)/614656 +
(2295 * alpha^3)/614656 + (3681 * alpha^2)/2458624 + (153 *
alpha)/614656 + 9/614656) + 2 * f * h * ((225 * alpha^4)/200704 +
(45 * alpha^3)/50176 + (9 * alpha^2)/50176) + 0 + g^2 * ((81 *
alpha^4)/38416 + (27 * alpha^3)/19208 + (27 * alpha^2)/76832 +
(3 * alpha)/76832 + 1/614656) + 2 * g * h * (9/12544 * alpha^4 +
3/12544 * alpha^3 + 1/50176 * alpha^2) + 0 + h^2 * (1/4096 *
alpha^4) + 0 + i^2 * 0)

```

```

Q8 <- (a^2 * 0 + 2 * a * b * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) +
2 * a * c * (45/98 - 9/49 * alpha^2 - 27/98 * alpha) + 2 *
a * d * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) + 2 *
a * e * (12/49 - 16/49 * alpha^2 + 4/49 * alpha) + 2 * a *
f * (45/392 - 225/784 * alpha^2 + 135/784 * alpha) + 2 *
a * g * (3/98 - 9/49 * alpha^2 + 15/98 * alpha) + 2 * a *
h * (1/16 * alpha - 1/16 * alpha^2) + 0 + b^2 * (27/64 -
1/512 * alpha^4 - 17/512 * alpha^3 - 45/256 * alpha^2 - 27/128 *
alpha) + 2 * b * c * (2295/6272 - 9/1568 * alpha^4 - 423/6272 *
alpha^3 - 2763/12544 * alpha^2 - 909/12544 * alpha) + 2 *
b * d * ((-(225 * alpha^4)/25088) - (1125 * alpha^3)/12544 -
(5375 * alpha^2)/25088 + (625 * alpha)/12544 + 825/3136) +
2 * b * e * ((-alpha^4/98) - (73 * alpha^3)/784 - (549 *
alpha^2)/3136 + (387 * alpha)/3136 + 243/1568) + 2 *
b * f * ((-(225 * alpha^4)/25088) - (1935 * alpha^3)/25088 -
(747 * alpha^2)/6272 + (855 * alpha)/6272 + 27/392) + 2 *
b * g * ((-(9 * alpha^4)/1568) - (297 * alpha^3)/6272 - (775 *
alpha^2)/12544 + (1219 * alpha)/12544 + 111/6272) + 2 * b *
h * ((-alpha^4/512) - alpha^3/64 - (9 * alpha^2)/512 + (9 *
alpha)/256) + 0 + c^2 * ((-(81 * alpha^4)/4802) - (1053 *
alpha^3)/9604 - (3645 * alpha^2)/19208 + (2025 * alpha)/38416 +
10125/38416) + 2 * c * d * ((-(2025 * alpha^4)/76832) - (38475 *
alpha^3)/307328 - (79875 * alpha^2)/614656 + (69525 * alpha)/614656 +
25875/153664) + 2 * c * e * ((-(72 * alpha^4)/2401) - (279 *
alpha^3)/2401 - (657 * alpha^2)/9604 + (2367 * alpha)/19208 +
1755/19208) + 2 * c * f * ((-(2025 * alpha^4)/76832) - (27135 *
alpha^3)/307328 - (13851 * alpha^2)/614656 + (60831 * alpha)/614656 +
11745/307328) + 2 * c * g * ((-(81 * alpha^4)/4802) - (243 *
alpha^3)/4802 + (9 * alpha^2)/19208 + (1107 * alpha)/19208 +
45/4802) + 2 * c * h * ((-(9 * alpha^4)/1568) - (99 * alpha^3)/6272 +
(45 * alpha^2)/12544 + (225 * alpha)/12544) + 0 + d^2 * ((-(50625 *
alpha^4)/1229312) - (151875 * alpha^3)/1229312 - (16875 *
alpha^2)/307328 + (9375 * alpha)/76832 + 1875/19208) + 2 *
d * e * ((-(225 * alpha^4)/4802) - (3825 * alpha^3)/38416 -
(325 * alpha^2)/153664 + (15325 * alpha)/153664 + 1875/38416) +
2 * d * f * ((-(50625 * alpha^4)/1229312) - (10125 * alpha^3)/153664 +
(28575 * alpha^2)/1229312 + (39825 * alpha)/614656 +
2925/153664) + 2 * d * g * ((-(2025 * alpha^4)/76832) -

```

```

(10125 * alpha^3)/307328 + (14625 * alpha^2)/614656 + (19125 *
alpha)/614656 + 675/153664) + 2 * d * h * ((-(225 * alpha^4)/25088) -
(225 * alpha^3)/25088 + (125 * alpha^2)/12544 + (25 * alpha)/3136) +
0 + e^2 * ((-(128 * alpha^4)/2401) - (160 * alpha^3)/2401 +
(72 * alpha^2)/2401 + (162 * alpha)/2401 + 54/2401) + 2 *
e * f * ((-(225 * alpha^4)/4802) - (1305 * alpha^3)/38416 +
(5499 * alpha^2)/153664 + (5679 * alpha)/153664 + 621/76832) +
2 * e * g * ((-(72 * alpha^4)/2401) - (27 * alpha^3)/2401 +
(239 * alpha^2)/9604 + (281 * alpha)/19208 + 33/19208) +
2 * e * h * ((-alpha^4/98) - alpha^3/784 + (27 * alpha^2)/3136 +
(9 * alpha)/3136) + 0 + f^2 * ((-(50625 * alpha^4)/1229312) -
(10125 * alpha^3)/1229312 + (18225 * alpha^2)/614656 + (5265 *
alpha)/307328 + 405/153664) + 2 * f * g * ((-(2025 * alpha^4)/76832) +
(1215 * alpha^3)/307328 + (10089 * alpha^2)/614656 + (3375 *
alpha)/614656 + 153/307328) + 2 * f * h * ((-(225 * alpha^4)/25088) +
(45 * alpha^3)/12544 + (117 * alpha^2)/25088 + (9 * alpha)/12544) +
0 + g^2 * ((-(81 * alpha^4)/4802) + (81 * alpha^3)/9604 +
(135 * alpha^2)/19208 + (51 * alpha)/38416 + 3/38416) + 2 *
g * h * (-9/1568 * alpha^4 + 27/6272 * alpha^3 + 17/12544 *
alpha^2 + 1/12544 * alpha) + 0 + h^2 * (-1/512 * alpha^4 +
1/512 * alpha^3) + 0 + i^2 * 0)

```

```

Q7 <- (a^2 * 0 + 2 * a * b * (1/16 + 3/32 * alpha^2 + 1/16 *
alpha) + 2 * a * c * (87/392 + 27/98 * alpha^2 - 6/49 * alpha) +
2 * a * d * (285/784 + 675/1568 * alpha^2 - 255/784 * alpha) +
2 * a * e * (41/98 + 24/49 * alpha^2 - 20/49 * alpha) + 2 *
a * f * (285/784 + 675/1568 * alpha^2 - 255/784 * alpha) +
2 * a * g * (87/392 + 27/98 * alpha^2 - 6/49 * alpha) + 2 *
a * h * (1/16 + 3/32 * alpha^2 + 1/16 * alpha) + 0 + b^2 *
(27/128 + 7/1024 * alpha^4 + 39/512 * alpha^3 + 93/512 *
alpha^2 - 9/64 * alpha) + 2 * b * c * (3951/12544 + 9/448 *
alpha^4 + 933/6272 * alpha^3 + 2109/12544 * alpha^2 - 3033/12544 *
alpha) + 2 * b * d * ((225 * alpha^4)/7168 + (1215 * alpha^3)/6272 +
(1425 * alpha^2)/12544 - (785 * alpha)/3136 + 4465/12544) +
2 * b * e * (alpha^4/28 + (39 * alpha^3)/196 + (93 * alpha^2)/1568 -
(585 * alpha)/3136 + 1035/3136) + 2 * b * f * ((225 *
alpha^4)/7168 + (4125 * alpha^3)/25088 + (687 * alpha^2)/25088 -
(513 * alpha)/6272 + 1557/6272) + 2 * b * g * ((9 * alpha^4)/448 +
(639 * alpha^3)/6272 + (261 * alpha^2)/12544 + (271 * alpha)/12544 +
1711/12544) + 2 * b * h * ((7 * alpha^4)/1024 + (9 * alpha^3)/256 +
(3 * alpha^2)/128 + (9 * alpha)/128 + 9/256) + 0 + c^2 *
((81 * alpha^4)/1372 + (297 * alpha^3)/1372 + (81 * alpha^2)/2744 -
(1215 * alpha)/5488 + 7425/21952) + 2 * c * d * ((2025 *
alpha^4)/21952 + (9855 * alpha^3)/43904 - (1035 * alpha^2)/12544 -
(11625 * alpha)/87808 + 27075/87808) + 2 * c * e * ((36 *
alpha^4)/343 + (66 * alpha^3)/343 - (93 * alpha^2)/686 -
(81 * alpha)/2744 + 333/1372) + 2 * c * f * ((2025 * alpha^4)/21952 +
(6075 * alpha^3)/43904 - (1539 * alpha^2)/12544 + (4293 *
alpha)/87808 + 14013/87808) + 2 * c * g * ((81 * alpha^4)/1372 +
(27 * alpha^3)/343 - (171 * alpha^2)/2744 + (111 * alpha)/1372 +
1713/21952) + 2 * c * h * ((9 * alpha^4)/448 + (177 * alpha^3)/6272 +
(3 * alpha^2)/1792 + (765 * alpha)/12544 + 225/12544) + 0 +

```



```

d^2 * ((50625 * alpha^4)/351232 + (30375 * alpha^3)/175616 -
(30375 * alpha^2)/175616 - (375 * alpha)/21952 + 375/1568) +
2 * d * e * ((225 * alpha^4)/1372 + (135 * alpha^3)/1372 -
(1965 * alpha^2)/10976 + (205 * alpha)/3136 + 3595/21952) +
2 * d * f * ((50625 * alpha^4)/351232 + (3375 * alpha^3)/87808 -
(5625 * alpha^2)/43904 + (4395 * alpha)/43904 + 1185/12544) +
2 * d * g * ((2025 * alpha^4)/21952 + (405 * alpha^3)/43904 -
(675 * alpha^2)/12544 + (7695 * alpha)/87808 + 3555/87808) +
2 * d * h * ((225 * alpha^4)/7168 + (135 * alpha^3)/25088 +
(15 * alpha^2)/3584 + (275 * alpha)/6272 + 25/3136) +
0 + e^2 * ((64 * alpha^4)/343 - (48 * alpha^2)/343 + (36 *
alpha)/343 + 135/1372) + 2 * e * f * ((225 * alpha^4)/1372 -
(75 * alpha^3)/1372 - (789 * alpha^2)/10976 + (45 * alpha)/448 +
1089/21952) + 2 * e * g * ((36 * alpha^4)/343 - (18 * alpha^3)/343 -
(9 * alpha^2)/686 + (185 * alpha)/2744 + 25/1372) + 2 * e *
h * (alpha^4/28 - (3 * alpha^3)/196 + (3 * alpha^2)/224 +
(81 * alpha)/3136 + 9/3136) + 0 + f^2 * ((50625 * alpha^4)/351232 -
(16875 * alpha^3)/175616 - (2025 * alpha^2)/175616 + (405 *
alpha)/5488 + 135/6272) + 2 * f * g * ((2025 * alpha^4)/21952 -
(3375 * alpha^3)/43904 + (261 * alpha^2)/12544 + (3453 *
alpha)/87808 + 573/87808) + 2 * f * h * ((225 * alpha^4)/7168 -
(75 * alpha^3)/3136 + (33 * alpha^2)/1792 + (9 * alpha)/784 +
9/12544) + 0 + g^2 * ((81 * alpha^4)/1372 - (81 * alpha^3)/1372 +
(81 * alpha^2)/2744 + (87 * alpha)/5488 + 33/21952) + 2 *
g * h * (1/12544 + 9/448 * alpha^4 - 117/6272 * alpha^3 +
27/1792 * alpha^2 + 37/12544 * alpha) + 0 + h^2 * (7/1024 *
alpha^4 - 3/512 * alpha^3 + 3/512 * alpha^2) + 0 + i^2 *
0)

```

```

Q6 <- (a^2 * 0 + 2 * a * b * (1/16 * alpha - 1/16 * alpha^2) +
2 * a * c * (3/98 - 9/49 * alpha^2 + 15/98 * alpha) + 2 *
a * d * (45/392 - 225/784 * alpha^2 + 135/784 * alpha) +
2 * a * e * (12/49 - 16/49 * alpha^2 + 4/49 * alpha) + 2 *
a * f * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) + 2 *
a * g * (45/98 - 9/49 * alpha^2 - 27/98 * alpha) + 2 * a *
h * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) + 0 + b^2 * (3/64 -
7/512 * alpha^4 - 45/512 * alpha^3 - 3/128 * alpha^2 + 5/64 *
alpha) + 2 * b * c * (405/3136 - 9/224 * alpha^4 - 999/6272 *
alpha^3 + 807/12544 * alpha^2 + 75/12544 * alpha) + 2 * b *
d * ((- (225 * alpha^4)/3584) - (2505 * alpha^3)/12544 + (3235 *
alpha^2)/25088 - (1145 * alpha)/12544 + 705/3136) + 2 * b *
e * ((-alpha^4/14) - (159 * alpha^3)/784 + (417 * alpha^2)/3136 -
(529 * alpha)/3136 + 243/784) + 2 * b * f * ((- (225 * alpha^4)/3584) -
(4275 * alpha^3)/25088 + (921 * alpha^2)/12544 - (39 * alpha)/196 +
1125/3136) + 2 * b * g * ((- (9 * alpha^4)/224) - (705 * alpha^3)/6272 -
(229 * alpha^2)/12544 - (2165 * alpha)/12544 + 1077/3136) +
2 * b * h * ((- (7 * alpha^4)/512) - (3 * alpha^3)/64 - (39 *
alpha^2)/512 - (25 * alpha)/256 + 15/64) + 0 + c^2 *
((- (81 * alpha^4)/686) - (243 * alpha^3)/1372 + (81 * alpha^2)/392 -
(729 * alpha)/5488 + 1215/5488) + 2 * c * d * ((- (2025 *
alpha^4)/10976) - (5805 * alpha^3)/43904 + (21465 * alpha^2)/87808 -
(2715 * alpha)/12544 + 12675/43904) + 2 * c * e * ((- (72 *

```

$$\begin{aligned}
& \alpha^4/343) - (27 * \alpha^3)/343 + (267 * \alpha^2)/1372 - \\
& (615 * \alpha)/2744 + 873/2744) + 2 * c * f * ((-(2025 * \alpha^4)/10976) - \\
& (2025 * \alpha^3)/43904 + (7857 * \alpha^2)/87808 - (2025 * \\
& \alpha)/12544 + 3321/10976) + 2 * c * g * ((-(81 * \alpha^4)/686) - \\
& (27 * \alpha^3)/686 - (9 * \alpha^2)/392 - (165 * \alpha)/2744 + \\
& 165/686) + 2 * c * h * ((-(9 * \alpha^4)/224) - (243 * \alpha^3)/6272 - \\
& (993 * \alpha^2)/12544 + (39 * \alpha)/1792 + 855/6272) + 0 + \\
& d^2 * ((-(50625 * \alpha^4)/175616) + (3375 * \alpha^3)/175616 + \\
& (16875 * \alpha^2)/87808 - (10125 * \alpha)/43904 + 3375/10976) + \\
& 2 * d * e * ((-(225 * \alpha^4)/686) + (15 * \alpha^3)/112 + \\
& (1655 * \alpha^2)/21952 - (3665 * \alpha)/21952 + 3135/10976) + \\
& 2 * d * f * ((-(50625 * \alpha^4)/175616) + (3375 * \alpha^3)/21952 - \\
& (5625 * \alpha^2)/175616 - (5655 * \alpha)/87808 + 2535/10976) + \\
& 2 * d * g * ((-(2025 * \alpha^4)/10976) + (3645 * \alpha^3)/43904 - \\
& (7515 * \alpha^2)/87808 + (405 * \alpha)/12544 + 6795/43904) + \\
& 2 * d * h * ((-(225 * \alpha^4)/3584) - (285 * \alpha^3)/25088 - \\
& (55 * \alpha^2)/784 + (65 * \alpha)/896 + 225/3136) + 0 + \\
& e^2 * ((-(128 * \alpha^4)/343) + (96 * \alpha^3)/343 - (24 * \\
& \alpha^2)/343 - (22 * \alpha)/343 + 78/343) + 2 * e * f * \\
& ((-(225 * \alpha^4)/686) + (225 * \alpha^3)/784 - (3273 * \alpha^2)/21952 + \\
& (717 * \alpha)/21952 + 54/343) + 2 * e * g * ((-(72 * \\
& \alpha^4)/343) + (57 * \alpha^3)/343 - (181 * \alpha^2)/1372 + \\
& (239 * \alpha)/2744 + 243/2744) + 2 * e * h * ((-\alpha^4/14) + \\
& (9 * \alpha^3)/784 - (159 * \alpha^2)/3136 + (5 * \alpha)/64 + \\
& 51/1568) + 0 + f^2 * ((-(50625 * \alpha^4)/175616) + (50625 * \\
& \alpha^3)/175616 - (2025 * \alpha^2)/10976 + (2025 * \alpha)/21952 + \\
& 2025/21952) + 2 * f * g * ((-(2025 * \alpha^4)/10976) + (7425 * \\
& \alpha^3)/43904 - (11043 * \alpha^2)/87808 + (177 * \alpha)/1792 + \\
& 465/10976) + 2 * f * h * ((-(225 * \alpha^4)/3584) + (225 * \\
& \alpha^3)/12544 - (633 * \alpha^2)/25088 + (15 * \alpha)/256 + \\
& 9/784) + 0 + g^2 * ((-(81 * \alpha^4)/686) + (135 * \alpha^3)/1372 - \\
& (27 * \alpha^2)/392 + (405 * \alpha)/5488 + 81/5488) + 2 * \\
& g * h * (15/6272 - 9/224 * \alpha^4 + 51/6272 * \alpha^3 - \\
& 13/12544 * \alpha^2 + 55/1792 * \alpha) + 0 + h^2 * (-7/512 * \\
& \alpha^4 - 3/512 * \alpha^3 + 3/256 * \alpha^2 + 1/128 * \alpha) + \\
& 0 + i^2 * 0)
\end{aligned}$$

Q5 <- (a^2 * 0 + 2 * a * b * (1/64 * alpha^2) + 2 * a * c * (1/784 + 9/196 * alpha^2 + 3/196 * alpha) + 2 * a * d * (9/784 + 225/3136 * alpha^2 + 45/784 * alpha) + 2 * a * e * (9/196 + 4/49 * alpha^2 + 6/49 * alpha) + 2 * a * f * (25/196 + 225/3136 * alpha^2 + 75/392 * alpha) + 2 * a * g * (225/784 + 9/196 * alpha^2 + 45/196 * alpha) + 2 * a * h * (9/16 + 1/64 * alpha^2 + 3/16 * alpha) + 2 * a * i * 1 + b^2 * (1/256 + 35/2048 * alpha^4 + 25/512 * alpha^3 - 27/512 * alpha^2 + 7/128 * alpha) + 2 * b * c * (327/12544 + 45/896 * alpha^4 + 225/3136 * alpha^3 - 2533/25088 * alpha^2 + 1149/12544 * alpha) + 2 * b * d * ((1125 * alpha^4)/14336 + (2025 * alpha^3)/25088 - (2623 * alpha^2)/25088 + (561 * alpha)/6272 + 453/6272) + 2 * b * e * ((5 * alpha^4)/56 + (65 * alpha^3)/784 - (369 * alpha^2)/6272 + (149 * alpha)/3136 + 451/3136) + 2 * b * f * ((1125 * alpha^4)/14336 + (2025 * alpha^3)/25088 + (527 * alpha^2)/25088 - (45 * alpha)/3136 +

```

2985/12544) + 2 * b * g * ((45 * alpha^4)/896 + (225 * alpha^3)/3136 +
(2507 * alpha^2)/25088 - (699 * alpha)/12544 + 4359/12544) +
2 * b * h * ((35 * alpha^4)/2048 + (25 * alpha^3)/512 + (63 *
alpha^2)/512 - alpha/64 + 59/128) + 2 * b * i * (9/16 +
1/64 * alpha^2 + 3/16 * alpha) + c^2 * ((405 * alpha^4)/2744 -
(513 * alpha^2)/5488 + (27 * alpha)/343 + 3429/43904) + 2 *
c * d * ((10125 * alpha^4)/43904 - (2025 * alpha^3)/21952 -
(2691 * alpha^2)/175616 + (675 * alpha)/87808 + 13045/87808) +
2 * c * e * ((90 * alpha^4)/343 - (45 * alpha^3)/343 + (197 *
alpha^2)/2744 - (117 * alpha)/1372 + 2481/10976) + 2 *
c * f * ((10125 * alpha^4)/43904 - (2025 * alpha^3)/21952 +
(22509 * alpha^2)/175616 - (13941 * alpha)/87808 + 26037/87808) +
2 * c * g * ((405 * alpha^4)/2744 + (747 * alpha^2)/5488 -
(57 * alpha)/343 + 15077/43904) + 2 * c * h * ((45 *
alpha^4)/896 + (225 * alpha^3)/3136 + (2507 * alpha^2)/25088 -
(699 * alpha)/12544 + 4359/12544) + 2 * c * i * (225/784 +
9/196 * alpha^2 + 45/196 * alpha) + d^2 * ((253125 * alpha^4)/702464 -
(50625 * alpha^3)/175616 + (24975 * alpha^2)/175616 - (675 *
alpha)/6272 + 19575/87808) + 2 * d * e * ((1125 * alpha^4)/2744 -
(2025 * alpha^3)/5488 + (10649 * alpha^2)/43904 - (4317 *
alpha)/21952 + 879/3136) + 2 * d * f * ((253125 * alpha^4)/702464 -
(50625 * alpha^3)/175616 + (40725 * alpha^2)/175616 - (345 *
alpha)/1568 + 13529/43904) + 2 * d * g * ((10125 * alpha^4)/43904 -
(2025 * alpha^3)/21952 + (22509 * alpha^2)/175616 - (13941 *
alpha)/87808 + 26037/87808) + 2 * d * h * ((1125 * alpha^4)/14336 +
(2025 * alpha^3)/25088 + (527 * alpha^2)/25088 - (45 * alpha)/3136 +
2985/12544) + 2 * d * i * (25/196 + 225/3136 * alpha^2 +
75/392 * alpha) + e^2 * ((160 * alpha^4)/343 - (160 * alpha^3)/343 +
(108 * alpha^2)/343 - (82 * alpha)/343 + 821/2744) + 2 *
e * f * ((1125 * alpha^4)/2744 - (2025 * alpha^3)/5488 +
(10649 * alpha^2)/43904 - (4317 * alpha)/21952 + 879/3136) +
2 * e * g * ((90 * alpha^4)/343 - (45 * alpha^3)/343 + (197 *
alpha^2)/2744 - (117 * alpha)/1372 + 2481/10976) + 2 *
e * h * ((5 * alpha^4)/56 + (65 * alpha^3)/784 - (369 * alpha^2)/6272 +
(149 * alpha)/3136 + 451/3136) + 2 * e * i * (9/196 + 4/49 *
alpha^2 + 6/49 * alpha) + f^2 * ((253125 * alpha^4)/702464 -
(50625 * alpha^3)/175616 + (24975 * alpha^2)/175616 - (675 *
alpha)/6272 + 19575/87808) + 2 * f * g * ((10125 * alpha^4)/43904 -
(2025 * alpha^3)/21952 - (2691 * alpha^2)/175616 + (675 *
alpha)/87808 + 13045/87808) + 2 * f * h * ((1125 * alpha^4)/14336 +
(2025 * alpha^3)/25088 - (2623 * alpha^2)/25088 + (561 *
alpha)/6272 + 453/6272) + 2 * f * i * (9/784 + 225/3136 *
alpha^2 + 45/784 * alpha) + g^2 * ((405 * alpha^4)/2744 -
(513 * alpha^2)/5488 + (27 * alpha)/343 + 3429/43904) + 2 *
g * h * (327/12544 + 45/896 * alpha^4 + 225/3136 * alpha^3 -
2533/25088 * alpha^2 + 1149/12544 * alpha) + 2 * g * i *
(1/784 + 9/196 * alpha^2 + 3/196 * alpha) + h^2 * (1/256 +
35/2048 * alpha^4 + 25/512 * alpha^3 - 27/512 * alpha^2 +
7/128 * alpha) + 2 * h * i * (1/64 * alpha^2) + i^2 * 0)

```

```

Q4 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *

```

```

h * 0 + 2 * a * i * 0 + b^2 * (-7/512 * alpha^4 - 3/512 *
alpha^3 + 3/256 * alpha^2 + 1/128 * alpha) + 2 * b * c *
(15/6272 - 9/224 * alpha^4 + 51/6272 * alpha^3 - 13/12544 *
alpha^2 + 55/1792 * alpha) + 2 * b * d * ((-225 * alpha^4)/3584) +
(225 * alpha^3)/12544 - (633 * alpha^2)/25088 + (15 * alpha)/256 +
9/784) + 2 * b * e * ((-alpha^4/14) + (9 * alpha^3)/784 -
(159 * alpha^2)/3136 + (5 * alpha)/64 + 51/1568) + 2 * b *
f * ((-225 * alpha^4)/3584) - (285 * alpha^3)/25088 - (55 *
alpha^2)/784 + (65 * alpha)/896 + 225/3136) + 2 * b * g *
((-9 * alpha^4)/224) - (243 * alpha^3)/6272 - (993 * alpha^2)/12544 +
(39 * alpha)/1792 + 855/6272) + 2 * b * h * ((-7 * alpha^4)/512) -
(3 * alpha^3)/64 - (39 * alpha^2)/512 - (25 * alpha)/256 +
15/64) + 2 * b * i * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) +
c^2 * ((-81 * alpha^4)/686) + (135 * alpha^3)/1372 - (27 *
alpha^2)/392 + (405 * alpha)/5488 + 81/5488) + 2 * c *
d * ((-2025 * alpha^4)/10976) + (7425 * alpha^3)/43904 -
(11043 * alpha^2)/87808 + (177 * alpha)/1792 + 465/10976) +
2 * c * e * ((-72 * alpha^4)/343) + (57 * alpha^3)/343 -
(181 * alpha^2)/1372 + (239 * alpha)/2744 + 243/2744) +
2 * c * f * ((-2025 * alpha^4)/10976) + (3645 * alpha^3)/43904 -
(7515 * alpha^2)/87808 + (405 * alpha)/12544 + 6795/43904) +
2 * c * g * ((-81 * alpha^4)/686) - (27 * alpha^3)/686 -
(9 * alpha^2)/392 - (165 * alpha)/2744 + 165/686) + 2 *
c * h * ((-9 * alpha^4)/224) - (705 * alpha^3)/6272 - (229 *
alpha^2)/12544 - (2165 * alpha)/12544 + 1077/3136) + 2 *
c * i * (45/98 - 9/49 * alpha^2 - 27/98 * alpha) + d^2 *
((-50625 * alpha^4)/175616) + (50625 * alpha^3)/175616 -
(2025 * alpha^2)/10976 + (2025 * alpha)/21952 + 2025/21952) +
2 * d * e * ((-225 * alpha^4)/686) + (225 * alpha^3)/784 -
(3273 * alpha^2)/21952 + (717 * alpha)/21952 + 54/343) +
2 * d * f * ((-50625 * alpha^4)/175616) + (3375 * alpha^3)/21952 -
(5625 * alpha^2)/175616 - (5655 * alpha)/87808 + 2535/10976) +
2 * d * g * ((-2025 * alpha^4)/10976) - (2025 * alpha^3)/43904 +
(7857 * alpha^2)/87808 - (2025 * alpha)/12544 + 3321/10976) +
2 * d * h * ((-225 * alpha^4)/3584) - (4275 * alpha^3)/25088 +
(921 * alpha^2)/12544 - (39 * alpha)/196 + 1125/3136) +
2 * d * i * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) +
e^2 * ((-128 * alpha^4)/343) + (96 * alpha^3)/343 - (24 *
alpha^2)/343 - (22 * alpha)/343 + 78/343) + 2 * e * f *
((-225 * alpha^4)/686) + (15 * alpha^3)/112 + (1655 * alpha^2)/21952 -
(3665 * alpha)/21952 + 3135/10976) + 2 * e * g * ((-72 *
alpha^4)/343) - (27 * alpha^3)/343 + (267 * alpha^2)/1372 -
(615 * alpha)/2744 + 873/2744) + 2 * e * h * ((-alpha^4/14) -
(159 * alpha^3)/784 + (417 * alpha^2)/3136 - (529 * alpha)/3136 +
243/784) + 2 * e * i * (12/49 - 16/49 * alpha^2 + 4/49 *
alpha) + f^2 * ((-50625 * alpha^4)/175616) + (3375 * alpha^3)/175616 +
(16875 * alpha^2)/87808 - (10125 * alpha)/43904 + 3375/10976) +
2 * f * g * ((-2025 * alpha^4)/10976) - (5805 * alpha^3)/43904 +
(21465 * alpha^2)/87808 - (2715 * alpha)/12544 + 12675/43904) +
2 * f * h * ((-225 * alpha^4)/3584) - (2505 * alpha^3)/12544 +
(3235 * alpha^2)/25088 - (1145 * alpha)/12544 + 705/3136) +
2 * f * i * (45/392 - 225/784 * alpha^2 + 135/784 * alpha) +
g^2 * ((-81 * alpha^4)/686) - (243 * alpha^3)/1372 + (81 *

```

```

alpha^2)/392 - (729 * alpha)/5488 + 1215/5488) + 2 *
g * h * (405/3136 - 9/224 * alpha^4 - 999/6272 * alpha^3 +
807/12544 * alpha^2 + 75/12544 * alpha) + 2 * g * i * (3/98 -
9/49 * alpha^2 + 15/98 * alpha) + h^2 * (3/64 - 7/512 * alpha^4 -
45/512 * alpha^3 - 3/128 * alpha^2 + 5/64 * alpha) + 2 *
h * i * (1/16 * alpha - 1/16 * alpha^2) + i^2 * 0)

```

```

Q3 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
h * 0 + 2 * a * i * 0 + b^2 * (7/1024 * alpha^4 - 3/512 *
alpha^3 + 3/512 * alpha^2) + 2 * b * c * (1/12544 + 9/448 *
alpha^4 - 117/6272 * alpha^3 + 27/1792 * alpha^2 + 37/12544 *
alpha) + 2 * b * d * ((225 * alpha^4)/7168 - (75 * alpha^3)/3136 +
(33 * alpha^2)/1792 + (9 * alpha)/784 + 9/12544) + 2 * b *
e * (alpha^4/28 - (3 * alpha^3)/196 + (3 * alpha^2)/224 +
(81 * alpha)/3136 + 9/3136) + 2 * b * f * ((225 * alpha^4)/7168 +
(135 * alpha^3)/25088 + (15 * alpha^2)/3584 + (275 * alpha)/6272 +
25/3136) + 2 * b * g * ((9 * alpha^4)/448 + (177 * alpha^3)/6272 +
(3 * alpha^2)/1792 + (765 * alpha)/12544 + 225/12544) + 2 *
b * h * ((7 * alpha^4)/1024 + (9 * alpha^3)/256 + (3 * alpha^2)/128 +
(9 * alpha)/128 + 9/256) + 2 * b * i * (1/16 + 3/32 * alpha^2 +
1/16 * alpha) + c^2 * ((81 * alpha^4)/1372 - (81 * alpha^3)/1372 +
(81 * alpha^2)/2744 + (87 * alpha)/5488 + 33/21952) + 2 *
c * d * ((2025 * alpha^4)/21952 - (3375 * alpha^3)/43904 +
(261 * alpha^2)/12544 + (3453 * alpha)/87808 + 573/87808) +
2 * c * e * ((36 * alpha^4)/343 - (18 * alpha^3)/343 - (9 *
alpha^2)/686 + (185 * alpha)/2744 + 25/1372) + 2 * c *
f * ((2025 * alpha^4)/21952 + (405 * alpha^3)/43904 - (675 *
alpha^2)/12544 + (7695 * alpha)/87808 + 3555/87808) + 2 *
c * g * ((81 * alpha^4)/1372 + (27 * alpha^3)/343 - (171 *
alpha^2)/2744 + (111 * alpha)/1372 + 1713/21952) + 2 * c *
h * ((9 * alpha^4)/448 + (639 * alpha^3)/6272 + (261 * alpha^2)/12544 +
(271 * alpha)/12544 + 1711/12544) + 2 * c * i * (87/392 +
27/98 * alpha^2 - 6/49 * alpha) + d^2 * ((50625 * alpha^4)/351232 -
(16875 * alpha^3)/175616 - (2025 * alpha^2)/175616 + (405 *
alpha)/5488 + 135/6272) + 2 * d * e * ((225 * alpha^4)/1372 -
(75 * alpha^3)/1372 - (789 * alpha^2)/10976 + (45 * alpha)/448 +
1089/21952) + 2 * d * f * ((50625 * alpha^4)/351232 + (3375 *
alpha^3)/87808 - (5625 * alpha^2)/43904 + (4395 * alpha)/43904 +
1185/12544) + 2 * d * g * ((2025 * alpha^4)/21952 + (6075 *
alpha^3)/43904 - (1539 * alpha^2)/12544 + (4293 * alpha)/87808 +
14013/87808) + 2 * d * h * ((225 * alpha^4)/7168 + (4125 *
alpha^3)/25088 + (687 * alpha^2)/25088 - (513 * alpha)/6272 +
1557/6272) + 2 * d * i * (285/784 + 675/1568 * alpha^2 -
255/784 * alpha) + e^2 * ((64 * alpha^4)/343 - (48 * alpha^2)/343 +
(36 * alpha)/343 + 135/1372) + 2 * e * f * ((225 * alpha^4)/1372 +
(135 * alpha^3)/1372 - (1965 * alpha^2)/10976 + (205 * alpha)/3136 +
3595/21952) + 2 * e * g * ((36 * alpha^4)/343 + (66 * alpha^3)/343 -
(93 * alpha^2)/686 - (81 * alpha)/2744 + 333/1372) + 2 *
e * h * (alpha^4/28 + (39 * alpha^3)/196 + (93 * alpha^2)/1568 -
(585 * alpha)/3136 + 1035/3136) + 2 * e * i * (41/98 + 24/49 *
alpha^2 - 20/49 * alpha) + f^2 * ((50625 * alpha^4)/351232 +

```

```

(30375 * alpha^3)/175616 - (30375 * alpha^2)/175616 - (375 *
alpha)/21952 + 375/1568) + 2 * f * g * ((2025 * alpha^4)/21952 +
(9855 * alpha^3)/43904 - (1035 * alpha^2)/12544 - (11625 *
alpha)/87808 + 27075/87808) + 2 * f * h * ((225 * alpha^4)/7168 +
(1215 * alpha^3)/6272 + (1425 * alpha^2)/12544 - (785 * alpha)/3136 +
4465/12544) + 2 * f * i * (285/784 + 675/1568 * alpha^2 -
255/784 * alpha) + g^2 * ((81 * alpha^4)/1372 + (297 * alpha^3)/1372 +
(81 * alpha^2)/2744 - (1215 * alpha)/5488 + 7425/21952) +
2 * g * h * (3951/12544 + 9/448 * alpha^4 + 933/6272 * alpha^3 +
2109/12544 * alpha^2 - 3033/12544 * alpha) + 2 * g *
i * (87/392 + 27/98 * alpha^2 - 6/49 * alpha) + h^2 * (27/128 +
7/1024 * alpha^4 + 39/512 * alpha^3 + 93/512 * alpha^2 -
9/64 * alpha) + 2 * h * i * (1/16 + 3/32 * alpha^2 + 1/16 *
alpha) + i^2 * 0)

```

```

Q2 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
h * 0 + 2 * a * i * 0 + b^2 * (-1/512 * alpha^4 + 1/512 *
alpha^3) + 2 * b * c * (-9/1568 * alpha^4 + 27/6272 * alpha^3 +
17/12544 * alpha^2 + 1/12544 * alpha) + 2 * b * d * ((-225 *
alpha^4)/25088) + (45 * alpha^3)/12544 + (117 * alpha^2)/25088 +
(9 * alpha)/12544) + 2 * b * e * ((-alpha^4/98) - alpha^3/784 +
(27 * alpha^2)/3136 + (9 * alpha)/3136) + 2 * b * f * ((-225 *
alpha^4)/25088) - (225 * alpha^3)/25088 + (125 * alpha^2)/12544 +
(25 * alpha)/3136) + 2 * b * g * ((-9 * alpha^4)/1568) -
(99 * alpha^3)/6272 + (45 * alpha^2)/12544 + (225 * alpha)/12544) +
2 * b * h * ((-alpha^4/512) - alpha^3/64 - (9 * alpha^2)/512 +
(9 * alpha)/256) + 2 * b * i * (1/16 * alpha - 1/16 *
alpha^2) + c^2 * ((-81 * alpha^4)/4802) + (81 * alpha^3)/9604 +
(135 * alpha^2)/19208 + (51 * alpha)/38416 + 3/38416) + 2 *
c * d * ((-2025 * alpha^4)/76832) + (1215 * alpha^3)/307328 +
(10089 * alpha^2)/614656 + (3375 * alpha)/614656 + 153/307328) +
2 * c * e * ((-72 * alpha^4)/2401) - (27 * alpha^3)/2401 +
(239 * alpha^2)/9604 + (281 * alpha)/19208 + 33/19208) +
2 * c * f * ((-2025 * alpha^4)/76832) - (10125 * alpha^3)/307328 +
(14625 * alpha^2)/614656 + (19125 * alpha)/614656 + 675/153664) +
2 * c * g * ((-81 * alpha^4)/4802) - (243 * alpha^3)/4802 +
(9 * alpha^2)/19208 + (1107 * alpha)/19208 + 45/4802) +
2 * c * h * ((-9 * alpha^4)/1568) - (297 * alpha^3)/6272 -
(775 * alpha^2)/12544 + (1219 * alpha)/12544 + 111/6272) +
2 * c * i * (3/98 - 9/49 * alpha^2 + 15/98 * alpha) + d^2 *
((-50625 * alpha^4)/1229312) - (10125 * alpha^3)/1229312 +
(18225 * alpha^2)/614656 + (5265 * alpha)/307328 + 405/153664) +
2 * d * e * ((-225 * alpha^4)/4802) - (1305 * alpha^3)/38416 +
(5499 * alpha^2)/153664 + (5679 * alpha)/153664 + 621/76832) +
2 * d * f * ((-50625 * alpha^4)/1229312) - (10125 * alpha^3)/153664 +
(28575 * alpha^2)/1229312 + (39825 * alpha)/614656 +
2925/153664) + 2 * d * g * ((-2025 * alpha^4)/76832) -
(27135 * alpha^3)/307328 - (13851 * alpha^2)/614656 + (60831 *
alpha)/614656 + 11745/307328) + 2 * d * h * ((-225 * alpha^4)/25088) -
(1935 * alpha^3)/25088 - (747 * alpha^2)/6272 + (855 * alpha)/6272 +
27/392) + 2 * d * i * (45/392 - 225/784 * alpha^2 + 135/784 *

```



```

alpha) + e^2 * ((-(128 * alpha^4)/2401) - (160 * alpha^3)/2401 +
(72 * alpha^2)/2401 + (162 * alpha)/2401 + 54/2401) + 2 *
e * f * ((-(225 * alpha^4)/4802) - (3825 * alpha^3)/38416 -
(325 * alpha^2)/153664 + (15325 * alpha)/153664 + 1875/38416) +
2 * e * g * ((-(72 * alpha^4)/2401) - (279 * alpha^3)/2401 -
(657 * alpha^2)/9604 + (2367 * alpha)/19208 + 1755/19208) +
2 * e * h * ((-alpha^4/98) - (73 * alpha^3)/784 - (549 *
alpha^2)/3136 + (387 * alpha)/3136 + 243/1568) + 2 *
e * i * (12/49 - 16/49 * alpha^2 + 4/49 * alpha) + f^2 *
((-(50625 * alpha^4)/1229312) - (151875 * alpha^3)/1229312 -
(16875 * alpha^2)/307328 + (9375 * alpha)/76832 + 1875/19208) +
2 * f * g * ((-(2025 * alpha^4)/76832) - (38475 * alpha^3)/307328 -
(79875 * alpha^2)/614656 + (69525 * alpha)/614656 + 25875/153664) +
2 * f * h * ((-(225 * alpha^4)/25088) - (1125 * alpha^3)/12544 -
(5375 * alpha^2)/25088 + (625 * alpha)/12544 + 825/3136) +
2 * f * i * (75/196 - 225/784 * alpha^2 - 75/784 * alpha) +
g^2 * ((-(81 * alpha^4)/4802) - (1053 * alpha^3)/9604 - (3645 *
alpha^2)/19208 + (2025 * alpha)/38416 + 10125/38416) +
2 * g * h * (2295/6272 - 9/1568 * alpha^4 - 423/6272 * alpha^3 -
2763/12544 * alpha^2 - 909/12544 * alpha) + 2 * g * i *
(45/98 - 9/49 * alpha^2 - 27/98 * alpha) + h^2 * (27/64 -
1/512 * alpha^4 - 17/512 * alpha^3 - 45/256 * alpha^2 - 27/128 *
alpha) + 2 * h * i * (3/8 - 1/16 * alpha^2 - 5/16 * alpha) +
i^2 * 0)

```

```

Q1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
h * 0 + 2 * a * i * 0 + b^2 * (1/4096 * alpha^4) + 2 * b *
c * (9/12544 * alpha^4 + 3/12544 * alpha^3 + 1/50176 * alpha^2) +
2 * b * d * ((225 * alpha^4)/200704 + (45 * alpha^3)/50176 +
(9 * alpha^2)/50176) + 2 * b * e * (alpha^4/784 + (3 *
alpha^3)/1568 + (9 * alpha^2)/12544) + 2 * b * f * ((225 *
alpha^4)/200704 + (75 * alpha^3)/25088 + (25 * alpha^2)/12544) +
2 * b * g * ((9 * alpha^4)/12544 + (45 * alpha^3)/12544 +
(225 * alpha^2)/50176) + 2 * b * h * (alpha^4/4096 +
(3 * alpha^3)/1024 + (9 * alpha^2)/1024) + 2 * b * i * (1/64 *
alpha^2) + c^2 * ((81 * alpha^4)/38416 + (27 * alpha^3)/19208 +
(27 * alpha^2)/76832 + (3 * alpha)/76832 + 1/614656) + 2 *
c * d * ((2025 * alpha^4)/614656 + (2295 * alpha^3)/614656 +
(3681 * alpha^2)/2458624 + (153 * alpha)/614656 + 9/614656) +
2 * c * e * ((9 * alpha^4)/2401 + (33 * alpha^3)/4802 + (157 *
alpha^2)/38416 + (33 * alpha)/38416 + 9/153664) + 2 *
c * f * ((2025 * alpha^4)/614656 + (6075 * alpha^3)/614656 +
(21825 * alpha^2)/2458624 + (675 * alpha)/307328 + 25/153664) +
2 * c * g * ((81 * alpha^4)/38416 + (27 * alpha^3)/2401 +
(1287 * alpha^2)/76832 + (45 * alpha)/9604 + 225/614656) +
2 * c * h * ((9 * alpha^4)/12544 + (111 * alpha^3)/12544 +
(1441 * alpha^2)/50176 + (111 * alpha)/12544 + 9/12544) +
2 * c * i * (1/784 + 9/196 * alpha^2 + 3/196 * alpha) + d^2 *
((50625 * alpha^4)/9834496 + (10125 * alpha^3)/1229312 +
(6075 * alpha^2)/1229312 + (405 * alpha)/307328 + 81/614656) +
2 * d * e * ((225 * alpha^4)/38416 + (1035 * alpha^3)/76832 +

```

```

(6921 * alpha^2)/614656 + (621 * alpha)/153664 + 81/153664) +
2 * d * f * ((50625 * alpha^4)/9834496 + (43875 * alpha^3)/2458624 +
(51525 * alpha^2)/2458624 + (2925 * alpha)/307328 + 225/153664) +
2 * d * g * ((2025 * alpha^4)/614656 + (11745 * alpha^3)/614656 +
(84321 * alpha^2)/2458624 + (11745 * alpha)/614656 +
2025/614656) + 2 * d * h * ((225 * alpha^4)/200704 +
(45 * alpha^3)/3136 + (1287 * alpha^2)/25088 + (27 * alpha)/784 +
81/12544) + 2 * d * i * (9/784 + 225/3136 * alpha^2 + 45/784 *
alpha) + e^2 * ((16 * alpha^4)/2401 + (48 * alpha^3)/2401 +
(54 * alpha^2)/2401 + (27 * alpha)/2401 + 81/38416) + 2 *
e * f * ((225 * alpha^4)/38416 + (1875 * alpha^3)/76832 +
(22825 * alpha^2)/614656 + (1875 * alpha)/76832 + 225/38416) +
2 * e * g * ((9 * alpha^4)/2401 + (117 * alpha^3)/4802 +
(2061 * alpha^2)/38416 + (1755 * alpha)/38416 + 2025/153664) +
2 * e * h * (alpha^4/784 + (27 * alpha^3)/1568 + (873 * alpha^2)/12544 +
(243 * alpha)/3136 + 81/3136) + 2 * e * i * (9/196 +
4/49 * alpha^2 + 6/49 * alpha) + f^2 * ((50625 * alpha^4)/9834496 +
(16875 * alpha^3)/614656 + (16875 * alpha^2)/307328 + (1875 *
alpha)/38416 + 625/38416) + 2 * f * g * ((2025 * alpha^4)/614656 +
(15525 * alpha^3)/614656 + (173025 * alpha^2)/2458624 + (25875 *
alpha)/307328 + 5625/153664) + 2 * f * h * ((225 * alpha^4)/200704 +
(825 * alpha^3)/50176 + (3925 * alpha^2)/50176 + (825 * alpha)/6272 +
225/3136) + 2 * f * i * (25/196 + 225/3136 * alpha^2 + 75/392 *
alpha) + g^2 * ((81 * alpha^4)/38416 + (405 * alpha^3)/19208 +
(6075 * alpha^2)/76832 + (10125 * alpha)/76832 + 50625/614656) +
2 * g * h * (2025/12544 + 9/12544 * alpha^4 + 153/12544 *
alpha^3 + 3681/50176 * alpha^2 + 2295/12544 * alpha) +
2 * g * i * (225/784 + 9/196 * alpha^2 + 45/196 * alpha) +
h^2 * (81/256 + 1/4096 * alpha^4 + 3/512 * alpha^3 + 27/512 *
alpha^2 + 27/128 * alpha) + 2 * h * i * (9/16 + 1/64 *
alpha^2 + 3/16 * alpha) + i^2 * 1)

```

```
prob <- c(Q9, Q8, Q7, Q6, Q5, Q4, Q3, Q2, Q1)
```

prob and prob1 are equivalent

```
prob
```

```
## [1] 0.04948 0.09193 0.13758 0.16100 0.19942 0.14998 0.10952 0.06350 0.03759
```

```
prob1
```

```
## [1] 0.04948 0.09193 0.13758 0.16100 0.19942 0.14998 0.10952 0.06350 0.03759
```

This perturbs prob (a function of Q1,...,Q9) to get prob_new

```

d1 = (Q9 + Q1) * 0.05
d2 = (Q8 + Q2) * 0.05
d3 = (Q7 + Q3) * 0.05
d4 = (Q6 + Q4) * 0.05

```

```

P_Q9 = Q9 + d1
P_Q1 = Q1 - d1
P_Q8 = Q8 + d2
P_Q2 = Q2 - d2
P_Q7 = Q7 + d3

```

```

P_Q3 = Q3 - d3
P_Q6 = Q6 + d4
P_Q4 = Q4 - d4
P_Q5 = Q5

prob_new <- c(P_Q9, P_Q8, P_Q7, P_Q6, P_Q5, P_Q4, P_Q3, P_Q2,
              P_Q1)

```

Minor perturbations:

```

prob_new

## [1] 0.05384 0.09970 0.14993 0.17655 0.19942 0.13443 0.09716 0.05573 0.03323

prob

## [1] 0.04948 0.09193 0.13758 0.16100 0.19942 0.14998 0.10952 0.06350 0.03759

```

This simulates data from `prob_new`

```

index <- sample(c(1, 2, 3, 4, 5, 6, 7, 8, 9), 50, replace = T,
               prob = prob_new)

NN <- rep(0, 9)
st <- table(index)
NNn <- as.numeric(names(st))
NN[NNn] <- as.numeric(st)
n9 <- NN[1]
n8 <- NN[2]
n7 <- NN[3]
n6 <- NN[4]
n5 <- NN[5]
n4 <- NN[6]
n3 <- NN[7]
n2 <- NN[8]
n1 <- NN[9]
N = n9 + n8 + n7 + n6 + n5 + n4 + n3 + n2 + n1

```

This runs some iterative algorithm to estimate the genotype frequencies

```

geno.p <- c()
for (j in 1:500) {

  R9 <- (a^2 * 1 + 2 * a * b * (9/16 + 1/64 * 0^2 + 3/16 *
    0) + 2 * a * c * (225/784 + 9/196 * 0^2 + 45/196 * 0) +
    2 * a * d * (25/196 + 225/3136 * 0^2 + 75/392 * 0) +
    2 * a * e * (9/196 + 4/49 * 0^2 + 6/49 * 0) + 2 * a *
    f * (9/784 + 225/3136 * 0^2 + 45/784 * 0) + 2 * a * g *
    (1/784 + 9/196 * 0^2 + 3/196 * 0) + 2 * a * h * (1/64 *
    0^2) + 0 + b^2 * (81/256 + 1/4096 * 0^4 + 3/512 * 0^3 +
    27/512 * 0^2 + 27/128 * 0) + 2 * b * c * (2025/12544 +
    9/12544 * 0^4 + 153/12544 * 0^3 + 3681/50176 * 0^2 +
    2295/12544 * 0) + 2 * b * d * ((225 * 0^4)/200704 + (825 *
    0^3)/50176 + (3925 * 0^2)/50176 + (825 * 0)/6272 + 225/3136) +
    2 * b * e * (0^4/784 + (27 * 0^3)/1568 + (873 * 0^2)/12544 +
    (243 * 0)/3136 + 81/3136) + 2 * b * f * ((225 * 0^4)/200704 +
    (45 * 0^3)/3136 + (1287 * 0^2)/25088 + (27 * 0)/784 +

```

$$\begin{aligned}
& 81/12544) + 2 * b * g * ((9 * 0^4)/12544 + (111 * 0^3)/12544 + \\
& (1441 * 0^2)/50176 + (111 * 0)/12544 + 9/12544) + 2 * \\
& b * h * (0^4/4096 + (3 * 0^3)/1024 + (9 * 0^2)/1024) + \\
& 0 + c^2 * ((81 * 0^4)/38416 + (405 * 0^3)/19208 + (6075 * \\
& 0^2)/76832 + (10125 * 0)/76832 + 50625/614656) + 2 * \\
& c * d * ((2025 * 0^4)/614656 + (15525 * 0^3)/614656 + \\
& (173025 * 0^2)/2458624 + (25875 * 0)/307328 + 5625/153664) + \\
& 2 * c * e * ((9 * 0^4)/2401 + (117 * 0^3)/4802 + (2061 * \\
& 0^2)/38416 + (1755 * 0)/38416 + 2025/153664) + 2 * \\
& c * f * ((2025 * 0^4)/614656 + (11745 * 0^3)/614656 + \\
& (84321 * 0^2)/2458624 + (11745 * 0)/614656 + 2025/614656) + \\
& 2 * c * g * ((81 * 0^4)/38416 + (27 * 0^3)/2401 + (1287 * \\
& 0^2)/76832 + (45 * 0)/9604 + 225/614656) + 2 * c * \\
& h * ((9 * 0^4)/12544 + (45 * 0^3)/12544 + (225 * 0^2)/50176) + \\
& 0 + d^2 * ((50625 * 0^4)/9834496 + (16875 * 0^3)/614656 + \\
& (16875 * 0^2)/307328 + (1875 * 0)/38416 + 625/38416) + \\
& 2 * d * e * ((225 * 0^4)/38416 + (1875 * 0^3)/76832 + \\
& (22825 * 0^2)/614656 + (1875 * 0)/76832 + 225/38416) + \\
& 2 * d * f * ((50625 * 0^4)/9834496 + (43875 * 0^3)/2458624 + \\
& (51525 * 0^2)/2458624 + (2925 * 0)/307328 + 225/153664) + \\
& 2 * d * g * ((2025 * 0^4)/614656 + (6075 * 0^3)/614656 + \\
& (21825 * 0^2)/2458624 + (675 * 0)/307328 + 25/153664) + \\
& 2 * d * h * ((225 * 0^4)/200704 + (75 * 0^3)/25088 + \\
& (25 * 0^2)/12544) + 0 + e^2 * ((16 * 0^4)/2401 + \\
& (48 * 0^3)/2401 + (54 * 0^2)/2401 + (27 * 0)/2401 + 81/38416) + \\
& 2 * e * f * ((225 * 0^4)/38416 + (1035 * 0^3)/76832 + \\
& (6921 * 0^2)/614656 + (621 * 0)/153664 + 81/153664) + \\
& 2 * e * g * ((9 * 0^4)/2401 + (33 * 0^3)/4802 + (157 * \\
& 0^2)/38416 + (33 * 0)/38416 + 9/153664) + 2 * e * \\
& h * (0^4/784 + (3 * 0^3)/1568 + (9 * 0^2)/12544) + 0 + \\
& f^2 * ((50625 * 0^4)/9834496 + (10125 * 0^3)/1229312 + \\
& (6075 * 0^2)/1229312 + (405 * 0)/307328 + 81/614656) + \\
& 2 * f * g * ((2025 * 0^4)/614656 + (2295 * 0^3)/614656 + \\
& (3681 * 0^2)/2458624 + (153 * 0)/614656 + 9/614656) + \\
& 2 * f * h * ((225 * 0^4)/200704 + (45 * 0^3)/50176 + \\
& (9 * 0^2)/50176) + 0 + g^2 * ((81 * 0^4)/38416 + \\
& (27 * 0^3)/19208 + (27 * 0^2)/76832 + (3 * 0)/76832 + \\
& 1/614656) + 2 * g * h * (9/12544 * 0^4 + 3/12544 * 0^3 + \\
& 1/50176 * 0^2) + 0 + h^2 * (1/4096 * 0^4) + 0 + i^2 * \\
& 0)
\end{aligned}$$

```

R8 <- (a^2 * 0 + 2 * a * b * (3/8 - 1/16 * 0^2 - 5/16 * 0) +
2 * a * c * (45/98 - 9/49 * 0^2 - 27/98 * 0) + 2 * a *
d * (75/196 - 225/784 * 0^2 - 75/784 * 0) + 2 * a * e *
(12/49 - 16/49 * 0^2 + 4/49 * 0) + 2 * a * f * (45/392 -
225/784 * 0^2 + 135/784 * 0) + 2 * a * g * (3/98 - 9/49 *
0^2 + 15/98 * 0) + 2 * a * h * (1/16 * 0 - 1/16 * 0^2) +
0 + b^2 * (27/64 - 1/512 * 0^4 - 17/512 * 0^3 - 45/256 *
0^2 - 27/128 * 0) + 2 * b * c * (2295/6272 - 9/1568 *
0^4 - 423/6272 * 0^3 - 2763/12544 * 0^2 - 909/12544 *
0) + 2 * b * d * ((-(225 * 0^4)/25088) - (1125 * 0^3)/12544 -
(5375 * 0^2)/25088 + (625 * 0)/12544 + 825/3136) + 2 *
b * e * ((-0^4/98) - (73 * 0^3)/784 - (549 * 0^2)/3136 +

```

$$\begin{aligned}
& (387 * 0)/3136 + 243/1568) + 2 * b * f * ((-(225 * 0^4)/25088) - \\
& (1935 * 0^3)/25088 - (747 * 0^2)/6272 + (855 * 0)/6272 + \\
& 27/392) + 2 * b * g * ((-(9 * 0^4)/1568) - (297 * 0^3)/6272 - \\
& (775 * 0^2)/12544 + (1219 * 0)/12544 + 111/6272) + 2 * \\
& b * h * ((-0^4/512) - 0^3/64 - (9 * 0^2)/512 + (9 * 0)/256) + \\
& 0 + c^2 * ((-(81 * 0^4)/4802) - (1053 * 0^3)/9604 - (3645 * \\
& 0^2)/19208 + (2025 * 0)/38416 + 10125/38416) + 2 * c * \\
& d * ((-(2025 * 0^4)/76832) - (38475 * 0^3)/307328 - (79875 * \\
& 0^2)/614656 + (69525 * 0)/614656 + 25875/153664) + 2 * \\
& c * e * ((-(72 * 0^4)/2401) - (279 * 0^3)/2401 - (657 * \\
& 0^2)/9604 + (2367 * 0)/19208 + 1755/19208) + 2 * c * \\
& f * ((-(2025 * 0^4)/76832) - (27135 * 0^3)/307328 - (13851 * \\
& 0^2)/614656 + (60831 * 0)/614656 + 11745/307328) + 2 * \\
& c * g * ((-(81 * 0^4)/4802) - (243 * 0^3)/4802 + (9 * \\
& 0^2)/19208 + (1107 * 0)/19208 + 45/4802) + 2 * c * h * \\
& ((-(9 * 0^4)/1568) - (99 * 0^3)/6272 + (45 * 0^2)/12544 + \\
& (225 * 0)/12544) + 0 + d^2 * ((-(50625 * 0^4)/1229312) - \\
& (151875 * 0^3)/1229312 - (16875 * 0^2)/307328 + (9375 * \\
& 0)/76832 + 1875/19208) + 2 * d * e * ((-(225 * 0^4)/4802) - \\
& (3825 * 0^3)/38416 - (325 * 0^2)/153664 + (15325 * 0)/153664 + \\
& 1875/38416) + 2 * d * f * ((-(50625 * 0^4)/1229312) - \\
& (10125 * 0^3)/153664 + (28575 * 0^2)/1229312 + (39825 * \\
& 0)/614656 + 2925/153664) + 2 * d * g * ((-(2025 * 0^4)/76832) - \\
& (10125 * 0^3)/307328 + (14625 * 0^2)/614656 + (19125 * \\
& 0)/614656 + 675/153664) + 2 * d * h * ((-(225 * 0^4)/25088) - \\
& (225 * 0^3)/25088 + (125 * 0^2)/12544 + (25 * 0)/3136) + \\
& 0 + e^2 * ((-(128 * 0^4)/2401) - (160 * 0^3)/2401 + (72 * \\
& 0^2)/2401 + (162 * 0)/2401 + 54/2401) + 2 * e * f * ((-(225 * \\
& 0^4)/4802) - (1305 * 0^3)/38416 + (5499 * 0^2)/153664 + \\
& (5679 * 0)/153664 + 621/76832) + 2 * e * g * ((-(72 * \\
& 0^4)/2401) - (27 * 0^3)/2401 + (239 * 0^2)/9604 + (281 * \\
& 0)/19208 + 33/19208) + 2 * e * h * ((-0^4/98) - 0^3/784 + \\
& (27 * 0^2)/3136 + (9 * 0)/3136) + 0 + f^2 * ((-(50625 * \\
& 0^4)/1229312) - (10125 * 0^3)/1229312 + (18225 * 0^2)/614656 + \\
& (5265 * 0)/307328 + 405/153664) + 2 * f * g * ((-(2025 * \\
& 0^4)/76832) + (1215 * 0^3)/307328 + (10089 * 0^2)/614656 + \\
& (3375 * 0)/614656 + 153/307328) + 2 * f * h * ((-(225 * \\
& 0^4)/25088) + (45 * 0^3)/12544 + (117 * 0^2)/25088 + \\
& (9 * 0)/12544) + 0 + g^2 * ((-(81 * 0^4)/4802) + (81 * \\
& 0^3)/9604 + (135 * 0^2)/19208 + (51 * 0)/38416 + 3/38416) + \\
& 2 * g * h * ((-9/1568 * 0^4 + 27/6272 * 0^3 + 17/12544 * \\
& 0^2 + 1/12544 * 0) + 0 + h^2 * (-1/512 * 0^4 + 1/512 * \\
& 0^3) + 0 + i^2 * 0)
\end{aligned}$$

R7 <- (a^2 * 0 + 2 * a * b * (1/16 + 3/32 * 0^2 + 1/16 *
0) + 2 * a * c * (87/392 + 27/98 * 0^2 - 6/49 * 0) +
2 * a * d * (285/784 + 675/1568 * 0^2 - 255/784 * 0) +
2 * a * e * (41/98 + 24/49 * 0^2 - 20/49 * 0) + 2 * a *
f * (285/784 + 675/1568 * 0^2 - 255/784 * 0) + 2 * a *
g * (87/392 + 27/98 * 0^2 - 6/49 * 0) + 2 * a * h * (1/16 +
3/32 * 0^2 + 1/16 * 0) + 0 + b^2 * (27/128 + 7/1024 *
0^4 + 39/512 * 0^3 + 93/512 * 0^2 - 9/64 * 0) + 2 * b *
c * (3951/12544 + 9/448 * 0^4 + 933/6272 * 0^3 + 2109/12544 *

$$\begin{aligned}
& 0^2 - 3033/12544 * 0) + 2 * b * d * ((225 * 0^4)/7168 + \\
& (1215 * 0^3)/6272 + (1425 * 0^2)/12544 - (785 * 0)/3136 + \\
& 4465/12544) + 2 * b * e * (0^4/28 + (39 * 0^3)/196 + \\
& (93 * 0^2)/1568 - (585 * 0)/3136 + 1035/3136) + 2 * b * \\
& f * ((225 * 0^4)/7168 + (4125 * 0^3)/25088 + (687 * 0^2)/25088 - \\
& (513 * 0)/6272 + 1557/6272) + 2 * b * g * ((9 * 0^4)/448 + \\
& (639 * 0^3)/6272 + (261 * 0^2)/12544 + (271 * 0)/12544 + \\
& 1711/12544) + 2 * b * h * ((7 * 0^4)/1024 + (9 * 0^3)/256 + \\
& (3 * 0^2)/128 + (9 * 0)/128 + 9/256) + 0 + c^2 * ((81 * \\
& 0^4)/1372 + (297 * 0^3)/1372 + (81 * 0^2)/2744 - (1215 * \\
& 0)/5488 + 7425/21952) + 2 * c * d * ((2025 * 0^4)/21952 + \\
& (9855 * 0^3)/43904 - (1035 * 0^2)/12544 - (11625 * 0)/87808 + \\
& 27075/87808) + 2 * c * e * ((36 * 0^4)/343 + (66 * 0^3)/343 - \\
& (93 * 0^2)/686 - (81 * 0)/2744 + 333/1372) + 2 * c * \\
& f * ((2025 * 0^4)/21952 + (6075 * 0^3)/43904 - (1539 * \\
& 0^2)/12544 + (4293 * 0)/87808 + 14013/87808) + 2 * c * \\
& g * ((81 * 0^4)/1372 + (27 * 0^3)/343 - (171 * 0^2)/2744 + \\
& (111 * 0)/1372 + 1713/21952) + 2 * c * h * ((9 * 0^4)/448 + \\
& (177 * 0^3)/6272 + (3 * 0^2)/1792 + (765 * 0)/12544 + \\
& 225/12544) + 0 + d^2 * ((50625 * 0^4)/351232 + (30375 * \\
& 0^3)/175616 - (30375 * 0^2)/175616 - (375 * 0)/21952 + \\
& 375/1568) + 2 * d * e * ((225 * 0^4)/1372 + (135 * 0^3)/1372 - \\
& (1965 * 0^2)/10976 + (205 * 0)/3136 + 3595/21952) + 2 * \\
& d * f * ((50625 * 0^4)/351232 + (3375 * 0^3)/87808 - \\
& (5625 * 0^2)/43904 + (4395 * 0)/43904 + 1185/12544) + \\
& 2 * d * g * ((2025 * 0^4)/21952 + (405 * 0^3)/43904 - \\
& (675 * 0^2)/12544 + (7695 * 0)/87808 + 3555/87808) + \\
& 2 * d * h * ((225 * 0^4)/7168 + (135 * 0^3)/25088 + (15 * \\
& 0^2)/3584 + (275 * 0)/6272 + 25/3136) + 0 + e^2 * \\
& ((64 * 0^4)/343 - (48 * 0^2)/343 + (36 * 0)/343 + 135/1372) + \\
& 2 * e * f * ((225 * 0^4)/1372 - (75 * 0^3)/1372 - (789 * \\
& 0^2)/10976 + (45 * 0)/448 + 1089/21952) + 2 * e * \\
& g * ((36 * 0^4)/343 - (18 * 0^3)/343 - (9 * 0^2)/686 + \\
& (185 * 0)/2744 + 25/1372) + 2 * e * h * (0^4/28 - (3 * \\
& 0^3)/196 + (3 * 0^2)/224 + (81 * 0)/3136 + 9/3136) + \\
& 0 + f^2 * ((50625 * 0^4)/351232 - (16875 * 0^3)/175616 - \\
& (2025 * 0^2)/175616 + (405 * 0)/5488 + 135/6272) + 2 * \\
& f * g * ((2025 * 0^4)/21952 - (3375 * 0^3)/43904 + (261 * \\
& 0^2)/12544 + (3453 * 0)/87808 + 573/87808) + 2 * f * \\
& h * ((225 * 0^4)/7168 - (75 * 0^3)/3136 + (33 * 0^2)/1792 + \\
& (9 * 0)/784 + 9/12544) + 0 + g^2 * ((81 * 0^4)/1372 - \\
& (81 * 0^3)/1372 + (81 * 0^2)/2744 + (87 * 0)/5488 + 33/21952) + \\
& 2 * g * h * (1/12544 + 9/448 * 0^4 - 117/6272 * 0^3 + \\
& 27/1792 * 0^2 + 37/12544 * 0) + 0 + h^2 * (7/1024 * \\
& 0^4 - 3/512 * 0^3 + 3/512 * 0^2) + 0 + i^2 * 0)
\end{aligned}$$

```

R6 <- (a^2 * 0 + 2 * a * b * (1/16 * 0 - 1/16 * 0^2) + 2 *
a * c * (3/98 - 9/49 * 0^2 + 15/98 * 0) + 2 * a * d *
(45/392 - 225/784 * 0^2 + 135/784 * 0) + 2 * a * e *
(12/49 - 16/49 * 0^2 + 4/49 * 0) + 2 * a * f * (75/196 -
225/784 * 0^2 - 75/784 * 0) + 2 * a * g * (45/98 - 9/49 *
0^2 - 27/98 * 0) + 2 * a * h * (3/8 - 1/16 * 0^2 - 5/16 *
0) + 0 + b^2 * (3/64 - 7/512 * 0^4 - 45/512 * 0^3 - 3/128 *

```



```

0^2 + 5/64 * 0) + 2 * b * c * (405/3136 - 9/224 * 0^4 -
999/6272 * 0^3 + 807/12544 * 0^2 + 75/12544 * 0) + 2 *
b * d * ((-(225 * 0^4)/3584) - (2505 * 0^3)/12544 + (3235 *
0^2)/25088 - (1145 * 0)/12544 + 705/3136) + 2 * b * e *
((-0^4/14) - (159 * 0^3)/784 + (417 * 0^2)/3136 - (529 *
0)/3136 + 243/784) + 2 * b * f * ((-(225 * 0^4)/3584) -
(4275 * 0^3)/25088 + (921 * 0^2)/12544 - (39 * 0)/196 +
1125/3136) + 2 * b * g * ((-(9 * 0^4)/224) - (705 * 0^3)/6272 -
(229 * 0^2)/12544 - (2165 * 0)/12544 + 1077/3136) + 2 *
b * h * ((-(7 * 0^4)/512) - (3 * 0^3)/64 - (39 * 0^2)/512 -
(25 * 0)/256 + 15/64) + 0 + c^2 * ((-(81 * 0^4)/686) -
(243 * 0^3)/1372 + (81 * 0^2)/392 - (729 * 0)/5488 +
1215/5488) + 2 * c * d * ((-(2025 * 0^4)/10976) - (5805 *
0^3)/43904 + (21465 * 0^2)/87808 - (2715 * 0)/12544 +
12675/43904) + 2 * c * e * ((-(72 * 0^4)/343) - (27 *
0^3)/343 + (267 * 0^2)/1372 - (615 * 0)/2744 + 873/2744) +
2 * c * f * ((-(2025 * 0^4)/10976) - (2025 * 0^3)/43904 +
(7857 * 0^2)/87808 - (2025 * 0)/12544 + 3321/10976) +
2 * c * g * ((-(81 * 0^4)/686) - (27 * 0^3)/686 - (9 *
0^2)/392 - (165 * 0)/2744 + 165/686) + 2 * c * h *
((-(9 * 0^4)/224) - (243 * 0^3)/6272 - (993 * 0^2)/12544 +
(39 * 0)/1792 + 855/6272) + 0 + d^2 * ((-(50625 *
0^4)/175616) + (3375 * 0^3)/175616 + (16875 * 0^2)/87808 -
(10125 * 0)/43904 + 3375/10976) + 2 * d * e * ((-(225 *
0^4)/686) + (15 * 0^3)/112 + (1655 * 0^2)/21952 - (3665 *
0)/21952 + 3135/10976) + 2 * d * f * ((-(50625 * 0^4)/175616) +
(3375 * 0^3)/21952 - (5625 * 0^2)/175616 - (5655 * 0)/87808 +
2535/10976) + 2 * d * g * ((-(2025 * 0^4)/10976) + (3645 *
0^3)/43904 - (7515 * 0^2)/87808 + (405 * 0)/12544 + 6795/43904) +
2 * d * h * ((-(225 * 0^4)/3584) - (285 * 0^3)/25088 -
(55 * 0^2)/784 + (65 * 0)/896 + 225/3136) + 0 + e^2 *
((-(128 * 0^4)/343) + (96 * 0^3)/343 - (24 * 0^2)/343 -
(22 * 0)/343 + 78/343) + 2 * e * f * ((-(225 * 0^4)/686) +
(225 * 0^3)/784 - (3273 * 0^2)/21952 + (717 * 0)/21952 +
54/343) + 2 * e * g * ((-(72 * 0^4)/343) + (57 * 0^3)/343 -
(181 * 0^2)/1372 + (239 * 0)/2744 + 243/2744) + 2 * e *
h * ((-0^4/14) + (9 * 0^3)/784 - (159 * 0^2)/3136 + (5 *
0)/64 + 51/1568) + 0 + f^2 * ((-(50625 * 0^4)/175616) +
(50625 * 0^3)/175616 - (2025 * 0^2)/10976 + (2025 * 0)/21952 +
2025/21952) + 2 * f * g * ((-(2025 * 0^4)/10976) + (7425 *
0^3)/43904 - (11043 * 0^2)/87808 + (177 * 0)/1792 + 465/10976) +
2 * f * h * ((-(225 * 0^4)/3584) + (225 * 0^3)/12544 -
(633 * 0^2)/25088 + (15 * 0)/256 + 9/784) + 0 + g^2 *
((-(81 * 0^4)/686) + (135 * 0^3)/1372 - (27 * 0^2)/392 +
(405 * 0)/5488 + 81/5488) + 2 * g * h * (15/6272 -
9/224 * 0^4 + 51/6272 * 0^3 - 13/12544 * 0^2 + 55/1792 *
0) + 0 + h^2 * (-7/512 * 0^4 - 3/512 * 0^3 + 3/256 *
0^2 + 1/128 * 0) + 0 + i^2 * 0)

```

```

R5 <- (a^2 * 0 + 2 * a * b * (1/64 * 0^2) + 2 * a * c * (1/784 +
9/196 * 0^2 + 3/196 * 0) + 2 * a * d * (9/784 + 225/3136 *
0^2 + 45/784 * 0) + 2 * a * e * (9/196 + 4/49 * 0^2 +
6/49 * 0) + 2 * a * f * (25/196 + 225/3136 * 0^2 + 75/392 *

```

$$\begin{aligned}
& 0) + 2 * a * g * (225/784 + 9/196 * 0^2 + 45/196 * 0) + \\
& 2 * a * h * (9/16 + 1/64 * 0^2 + 3/16 * 0) + 2 * a * \\
& i * 1 + b^2 * (1/256 + 35/2048 * 0^4 + 25/512 * 0^3 - \\
& 27/512 * 0^2 + 7/128 * 0) + 2 * b * c * (327/12544 + \\
& 45/896 * 0^4 + 225/3136 * 0^3 - 2533/25088 * 0^2 + 1149/12544 * \\
& 0) + 2 * b * d * ((1125 * 0^4)/14336 + (2025 * 0^3)/25088 - \\
& (2623 * 0^2)/25088 + (561 * 0)/6272 + 453/6272) + 2 * \\
& b * e * ((5 * 0^4)/56 + (65 * 0^3)/784 - (369 * 0^2)/6272 + \\
& (149 * 0)/3136 + 451/3136) + 2 * b * f * ((1125 * 0^4)/14336 + \\
& (2025 * 0^3)/25088 + (527 * 0^2)/25088 - (45 * 0)/3136 + \\
& 2985/12544) + 2 * b * g * ((45 * 0^4)/896 + (225 * 0^3)/3136 + \\
& (2507 * 0^2)/25088 - (699 * 0)/12544 + 4359/12544) + \\
& 2 * b * h * ((35 * 0^4)/2048 + (25 * 0^3)/512 + (63 * \\
& 0^2)/512 - 0/64 + 59/128) + 2 * b * i * (9/16 + 1/64 * \\
& 0^2 + 3/16 * 0) + c^2 * ((405 * 0^4)/2744 - (513 * 0^2)/5488 + \\
& (27 * 0)/343 + 3429/43904) + 2 * c * d * ((10125 * 0^4)/43904 - \\
& (2025 * 0^3)/21952 - (2691 * 0^2)/175616 + (675 * 0)/87808 + \\
& 13045/87808) + 2 * c * e * ((90 * 0^4)/343 - (45 * 0^3)/343 + \\
& (197 * 0^2)/2744 - (117 * 0)/1372 + 2481/10976) + 2 * \\
& c * f * ((10125 * 0^4)/43904 - (2025 * 0^3)/21952 + (22509 * \\
& 0^2)/175616 - (13941 * 0)/87808 + 26037/87808) + 2 * \\
& c * g * ((405 * 0^4)/2744 + (747 * 0^2)/5488 - (57 * \\
& 0)/343 + 15077/43904) + 2 * c * h * ((45 * 0^4)/896 + \\
& (225 * 0^3)/3136 + (2507 * 0^2)/25088 - (699 * 0)/12544 + \\
& 4359/12544) + 2 * c * i * (225/784 + 9/196 * 0^2 + 45/196 * \\
& 0) + d^2 * ((253125 * 0^4)/702464 - (50625 * 0^3)/175616 + \\
& (24975 * 0^2)/175616 - (675 * 0)/6272 + 19575/87808) + \\
& 2 * d * e * ((1125 * 0^4)/2744 - (2025 * 0^3)/5488 + \\
& (10649 * 0^2)/43904 - (4317 * 0)/21952 + 879/3136) + \\
& 2 * d * f * ((253125 * 0^4)/702464 - (50625 * 0^3)/175616 + \\
& (40725 * 0^2)/175616 - (345 * 0)/1568 + 13529/43904) + \\
& 2 * d * g * ((10125 * 0^4)/43904 - (2025 * 0^3)/21952 + \\
& (22509 * 0^2)/175616 - (13941 * 0)/87808 + 26037/87808) + \\
& 2 * d * h * ((1125 * 0^4)/14336 + (2025 * 0^3)/25088 + \\
& (527 * 0^2)/25088 - (45 * 0)/3136 + 2985/12544) + \\
& 2 * d * i * (25/196 + 225/3136 * 0^2 + 75/392 * 0) + \\
& e^2 * ((160 * 0^4)/343 - (160 * 0^3)/343 + (108 * 0^2)/343 - \\
& (82 * 0)/343 + 821/2744) + 2 * e * f * ((1125 * 0^4)/2744 - \\
& (2025 * 0^3)/5488 + (10649 * 0^2)/43904 - (4317 * 0)/21952 + \\
& 879/3136) + 2 * e * g * ((90 * 0^4)/343 - (45 * 0^3)/343 + \\
& (197 * 0^2)/2744 - (117 * 0)/1372 + 2481/10976) + 2 * \\
& e * h * ((5 * 0^4)/56 + (65 * 0^3)/784 - (369 * 0^2)/6272 + \\
& (149 * 0)/3136 + 451/3136) + 2 * e * i * (9/196 + 4/49 * \\
& 0^2 + 6/49 * 0) + f^2 * ((253125 * 0^4)/702464 - (50625 * \\
& 0^3)/175616 + (24975 * 0^2)/175616 - (675 * 0)/6272 + \\
& 19575/87808) + 2 * f * g * ((10125 * 0^4)/43904 - (2025 * \\
& 0^3)/21952 - (2691 * 0^2)/175616 + (675 * 0)/87808 + \\
& 13045/87808) + 2 * f * h * ((1125 * 0^4)/14336 + (2025 * \\
& 0^3)/25088 - (2623 * 0^2)/25088 + (561 * 0)/6272 + 453/6272) + \\
& 2 * f * i * (9/784 + 225/3136 * 0^2 + 45/784 * 0) + g^2 * \\
& ((405 * 0^4)/2744 - (513 * 0^2)/5488 + (27 * 0)/343 + \\
& 3429/43904) + 2 * g * h * (327/12544 + 45/896 * 0^4 + \\
& 225/3136 * 0^3 - 2533/25088 * 0^2 + 1149/12544 * 0) +
\end{aligned}$$

$$2 * g * i * (1/784 + 9/196 * 0^2 + 3/196 * 0) + h^2 * (1/256 + 35/2048 * 0^4 + 25/512 * 0^3 - 27/512 * 0^2 + 7/128 * 0) + 2 * h * i * (1/64 * 0^2) + i^2 * 0$$

```
R4 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a *
d * 0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 +
2 * a * h * 0 + 2 * a * i * 0 + b^2 * (-7/512 * 0^4 -
3/512 * 0^3 + 3/256 * 0^2 + 1/128 * 0) + 2 * b * c *
(15/6272 - 9/224 * 0^4 + 51/6272 * 0^3 - 13/12544 * 0^2 +
55/1792 * 0) + 2 * b * d * ((-(225 * 0^4)/3584) +
(225 * 0^3)/12544 - (633 * 0^2)/25088 + (15 * 0)/256 +
9/784) + 2 * b * e * ((-0^4/14) + (9 * 0^3)/784 - (159 *
0^2)/3136 + (5 * 0)/64 + 51/1568) + 2 * b * f * ((-(225 *
0^4)/3584) - (285 * 0^3)/25088 - (55 * 0^2)/784 + (65 *
0)/896 + 225/3136) + 2 * b * g * ((-(9 * 0^4)/224) -
(243 * 0^3)/6272 - (993 * 0^2)/12544 + (39 * 0)/1792 +
855/6272) + 2 * b * h * ((-(7 * 0^4)/512) - (3 * 0^3)/64 -
(39 * 0^2)/512 - (25 * 0)/256 + 15/64) + 2 * b * i *
(3/8 - 1/16 * 0^2 - 5/16 * 0) + c^2 * ((-(81 * 0^4)/686) +
(135 * 0^3)/1372 - (27 * 0^2)/392 + (405 * 0)/5488 +
81/5488) + 2 * c * d * ((-(2025 * 0^4)/10976) + (7425 *
0^3)/43904 - (11043 * 0^2)/87808 + (177 * 0)/1792 + 465/10976) +
2 * c * e * ((-(72 * 0^4)/343) + (57 * 0^3)/343 - (181 *
0^2)/1372 + (239 * 0)/2744 + 243/2744) + 2 * c *
f * ((-(2025 * 0^4)/10976) + (3645 * 0^3)/43904 - (7515 *
0^2)/87808 + (405 * 0)/12544 + 6795/43904) + 2 * c *
g * ((-(81 * 0^4)/686) - (27 * 0^3)/686 - (9 * 0^2)/392 -
(165 * 0)/2744 + 165/686) + 2 * c * h * ((-(9 * 0^4)/224) -
(705 * 0^3)/6272 - (229 * 0^2)/12544 - (2165 * 0)/12544 +
1077/3136) + 2 * c * i * (45/98 - 9/49 * 0^2 - 27/98 *
0) + d^2 * ((-(50625 * 0^4)/175616) + (50625 * 0^3)/175616 -
(2025 * 0^2)/10976 + (2025 * 0)/21952 + 2025/21952) +
2 * d * e * ((-(225 * 0^4)/686) + (225 * 0^3)/784 - (3273 *
0^2)/21952 + (717 * 0)/21952 + 54/343) + 2 * d *
f * ((-(50625 * 0^4)/175616) + (3375 * 0^3)/21952 - (5625 *
0^2)/175616 - (5655 * 0)/87808 + 2535/10976) + 2 * d *
g * ((-(2025 * 0^4)/10976) - (2025 * 0^3)/43904 + (7857 *
0^2)/87808 - (2025 * 0)/12544 + 3321/10976) + 2 * d *
h * ((-(225 * 0^4)/3584) - (4275 * 0^3)/25088 + (921 *
0^2)/12544 - (39 * 0)/196 + 1125/3136) + 2 * d * i *
(75/196 - 225/784 * 0^2 - 75/784 * 0) + e^2 * ((-(128 *
0^4)/343) + (96 * 0^3)/343 - (24 * 0^2)/343 - (22 * 0)/343 +
78/343) + 2 * e * f * ((-(225 * 0^4)/686) + (15 * 0^3)/112 +
(1655 * 0^2)/21952 - (3665 * 0)/21952 + 3135/10976) +
2 * e * g * ((-(72 * 0^4)/343) - (27 * 0^3)/343 + (267 *
0^2)/1372 - (615 * 0)/2744 + 873/2744) + 2 * e *
h * ((-0^4/14) - (159 * 0^3)/784 + (417 * 0^2)/3136 -
(529 * 0)/3136 + 243/784) + 2 * e * i * (12/49 - 16/49 *
0^2 + 4/49 * 0) + f^2 * ((-(50625 * 0^4)/175616) + (3375 *
0^3)/175616 + (16875 * 0^2)/87808 - (10125 * 0)/43904 +
3375/10976) + 2 * f * g * ((-(2025 * 0^4)/10976) - (5805 *
0^3)/43904 + (21465 * 0^2)/87808 - (2715 * 0)/12544 +
12675/43904) + 2 * f * h * ((-(225 * 0^4)/3584) - (2505 *
0^3)/12544 + (15 * 0)/12544 + 15/12544) + 2 * f * i *
(15/12544 - (15 * 0)/12544 + (15 * 0)/12544 + 15/12544) +
2 * g * h * ((-(225 * 0^4)/3584) - (2505 * 0^3)/12544 +
(15 * 0)/12544 + 15/12544) + 2 * g * i * (1/784 + 9/196 *
0^2 + 3/196 * 0) + h^2 * (1/256 + 35/2048 * 0^4 + 25/512 *
0^3 - 27/512 * 0^2 + 7/128 * 0) + 2 * h * i * (1/64 * 0^2) +
i^2 * 0
```

$$\begin{aligned} & 0^3)/12544 + (3235 * 0^2)/25088 - (1145 * 0)/12544 + \\ & 705/3136) + 2 * f * i * (45/392 - 225/784 * 0^2 + 135/784 * \\ & 0) + g^2 * ((- (81 * 0^4)/686) - (243 * 0^3)/1372 + (81 * \\ & 0^2)/392 - (729 * 0)/5488 + 1215/5488) + 2 * g * h * \\ & (405/3136 - 9/224 * 0^4 - 999/6272 * 0^3 + 807/12544 * \\ & 0^2 + 75/12544 * 0) + 2 * g * i * (3/98 - 9/49 * \\ & 0^2 + 15/98 * 0) + h^2 * (3/64 - 7/512 * 0^4 - 45/512 * \\ & 0^3 - 3/128 * 0^2 + 5/64 * 0) + 2 * h * i * (1/16 * 0 - \\ & 1/16 * 0^2) + i^2 * 0) \end{aligned}$$

```
R3 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a *
d * 0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 +
2 * a * h * 0 + 2 * a * i * 0 + b^2 * (7/1024 * 0^4 -
3/512 * 0^3 + 3/512 * 0^2) + 2 * b * c * (1/12544 + 9/448 *
0^4 - 117/6272 * 0^3 + 27/1792 * 0^2 + 37/12544 * 0) +
2 * b * d * ((225 * 0^4)/7168 - (75 * 0^3)/3136 + (33 *
0^2)/1792 + (9 * 0)/784 + 9/12544) + 2 * b * e *
(0^4/28 - (3 * 0^3)/196 + (3 * 0^2)/224 + (81 * 0)/3136 +
9/3136) + 2 * b * f * ((225 * 0^4)/7168 + (135 *
0^3)/25088 + (15 * 0^2)/3584 + (275 * 0)/6272 + 25/3136) +
2 * b * g * ((9 * 0^4)/448 + (177 * 0^3)/6272 + (3 *
0^2)/1792 + (765 * 0)/12544 + 225/12544) + 2 * b *
h * ((7 * 0^4)/1024 + (9 * 0^3)/256 + (3 * 0^2)/128 +
(9 * 0)/128 + 9/256) + 2 * b * i * (1/16 + 3/32 * 0^2 +
1/16 * 0) + c^2 * ((81 * 0^4)/1372 - (81 * 0^3)/1372 +
(81 * 0^2)/2744 + (87 * 0)/5488 + 33/21952) + 2 * c *
d * ((2025 * 0^4)/21952 - (3375 * 0^3)/43904 + (261 *
0^2)/12544 + (3453 * 0)/87808 + 573/87808) + 2 * c *
e * ((36 * 0^4)/343 - (18 * 0^3)/343 - (9 * 0^2)/686 +
(185 * 0)/2744 + 25/1372) + 2 * c * f * ((2025 * 0^4)/21952 +
(405 * 0^3)/43904 - (675 * 0^2)/12544 + (7695 * 0)/87808 +
3555/87808) + 2 * c * g * ((81 * 0^4)/1372 + (27 * 0^3)/343 -
(171 * 0^2)/2744 + (111 * 0)/1372 + 1713/21952) + 2 *
c * h * ((9 * 0^4)/448 + (639 * 0^3)/6272 + (261 * 0^2)/12544 +
(271 * 0)/12544 + 1711/12544) + 2 * c * i * (87/392 +
27/98 * 0^2 - 6/49 * 0) + d^2 * ((50625 * 0^4)/351232 -
(16875 * 0^3)/175616 - (2025 * 0^2)/175616 + (405 * 0)/5488 +
135/6272) + 2 * d * e * ((225 * 0^4)/1372 - (75 * 0^3)/1372 -
(789 * 0^2)/10976 + (45 * 0)/448 + 1089/21952) + 2 *
d * f * ((50625 * 0^4)/351232 + (3375 * 0^3)/87808 -
(5625 * 0^2)/43904 + (4395 * 0)/43904 + 1185/12544) +
2 * d * g * ((2025 * 0^4)/21952 + (6075 * 0^3)/43904 -
(1539 * 0^2)/12544 + (4293 * 0)/87808 + 14013/87808) +
2 * d * h * ((225 * 0^4)/7168 + (4125 * 0^3)/25088 +
(687 * 0^2)/25088 - (513 * 0)/6272 + 1557/6272) +
2 * d * i * (285/784 + 675/1568 * 0^2 - 255/784 * 0) +
e^2 * ((64 * 0^4)/343 - (48 * 0^2)/343 + (36 * 0)/343 +
135/1372) + 2 * e * f * ((225 * 0^4)/1372 + (135 *
0^3)/1372 - (1965 * 0^2)/10976 + (205 * 0)/3136 + 3595/21952) +
2 * e * g * ((36 * 0^4)/343 + (66 * 0^3)/343 - (93 *
0^2)/686 - (81 * 0)/2744 + 333/1372) + 2 * e * h *
(0^4/28 + (39 * 0^3)/196 + (93 * 0^2)/1568 - (585 * 0)/3136 +
1035/3136) + 2 * e * i * (41/98 + 24/49 * 0^2 - 20/49 *
```

```

0) + f^2 * ((50625 * 0^4)/351232 + (30375 * 0^3)/175616 -
(30375 * 0^2)/175616 - (375 * 0)/21952 + 375/1568) +
2 * f * g * ((2025 * 0^4)/21952 + (9855 * 0^3)/43904 -
(1035 * 0^2)/12544 - (11625 * 0)/87808 + 27075/87808) +
2 * f * h * ((225 * 0^4)/7168 + (1215 * 0^3)/6272 + (1425 *
0^2)/12544 - (785 * 0)/3136 + 4465/12544) + 2 * f *
i * (285/784 + 675/1568 * 0^2 - 255/784 * 0) + g^2 *
((81 * 0^4)/1372 + (297 * 0^3)/1372 + (81 * 0^2)/2744 -
(1215 * 0)/5488 + 7425/21952) + 2 * g * h * (3951/12544 +
9/448 * 0^4 + 933/6272 * 0^3 + 2109/12544 * 0^2 - 3033/12544 *
0) + 2 * g * i * (87/392 + 27/98 * 0^2 - 6/49 * 0) +
h^2 * (27/128 + 7/1024 * 0^4 + 39/512 * 0^3 + 93/512 *
0^2 - 9/64 * 0) + 2 * h * i * (1/16 + 3/32 * 0^2 +
1/16 * 0) + i^2 * 0)

```

```

R2 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a *
d * 0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 +
2 * a * h * 0 + 2 * a * i * 0 + b^2 * (-1/512 * 0^4 +
1/512 * 0^3) + 2 * b * c * (-9/1568 * 0^4 + 27/6272 *
0^3 + 17/12544 * 0^2 + 1/12544 * 0) + 2 * b * d * ((-225 *
0^4)/25088) + (45 * 0^3)/12544 + (117 * 0^2)/25088 +
(9 * 0)/12544) + 2 * b * e * ((-0^4/98) - 0^3/784 + (27 *
0^2)/3136 + (9 * 0)/3136) + 2 * b * f * ((-225 * 0^4)/25088) -
(225 * 0^3)/25088 + (125 * 0^2)/12544 + (25 * 0)/3136) +
2 * b * g * ((-9 * 0^4)/1568) - (99 * 0^3)/6272 + (45 *
0^2)/12544 + (225 * 0)/12544) + 2 * b * h * ((-0^4/512) -
0^3/64 - (9 * 0^2)/512 + (9 * 0)/256) + 2 * b * i * (1/16 *
0 - 1/16 * 0^2) + c^2 * ((-81 * 0^4)/4802) + (81 * 0^3)/9604 +
(135 * 0^2)/19208 + (51 * 0)/38416 + 3/38416) + 2 * c *
d * ((-2025 * 0^4)/76832) + (1215 * 0^3)/307328 + (10089 *
0^2)/614656 + (3375 * 0)/614656 + 153/307328) + 2 * c *
e * ((-72 * 0^4)/2401) - (27 * 0^3)/2401 + (239 * 0^2)/9604 +
(281 * 0)/19208 + 33/19208) + 2 * c * f * ((-2025 *
0^4)/76832) - (10125 * 0^3)/307328 + (14625 * 0^2)/614656 +
(19125 * 0)/614656 + 675/153664) + 2 * c * g * ((-81 *
0^4)/4802) - (243 * 0^3)/4802 + (9 * 0^2)/19208 + (1107 *
0)/19208 + 45/4802) + 2 * c * h * ((-9 * 0^4)/1568) -
(297 * 0^3)/6272 - (775 * 0^2)/12544 + (1219 * 0)/12544 +
111/6272) + 2 * c * i * (3/98 - 9/49 * 0^2 + 15/98 *
0) + d^2 * ((-50625 * 0^4)/1229312) - (10125 * 0^3)/1229312 +
(18225 * 0^2)/614656 + (5265 * 0)/307328 + 405/153664) +
2 * d * e * ((-225 * 0^4)/4802) - (1305 * 0^3)/38416 +
(5499 * 0^2)/153664 + (5679 * 0)/153664 + 621/76832) +
2 * d * f * ((-50625 * 0^4)/1229312) - (10125 * 0^3)/153664 +
(28575 * 0^2)/1229312 + (39825 * 0)/614656 + 2925/153664) +
2 * d * g * ((-2025 * 0^4)/76832) - (27135 * 0^3)/307328 -
(13851 * 0^2)/614656 + (60831 * 0)/614656 + 11745/307328) +
2 * d * h * ((-225 * 0^4)/25088) - (1935 * 0^3)/25088 -
(747 * 0^2)/6272 + (855 * 0)/6272 + 27/392) + 2 *
d * i * (45/392 - 225/784 * 0^2 + 135/784 * 0) + e^2 *
((-(128 * 0^4)/2401) - (160 * 0^3)/2401 + (72 * 0^2)/2401 +
(162 * 0)/2401 + 54/2401) + 2 * e * f * ((-225 *
0^4)/4802) - (3825 * 0^3)/38416 - (325 * 0^2)/153664 +

```

$$\begin{aligned}
& (15325 * 0)/153664 + 1875/38416) + 2 * e * g * ((- (72 * \\
& 0^4)/2401) - (279 * 0^3)/2401 - (657 * 0^2)/9604 + (2367 * \\
& 0)/19208 + 1755/19208) + 2 * e * h * ((- (0^4/98) - (73 * \\
& 0^3)/784 - (549 * 0^2)/3136 + (387 * 0)/3136 + 243/1568) + \\
& 2 * e * i * (12/49 - 16/49 * 0^2 + 4/49 * 0) + f^2 * \\
& ((- (50625 * 0^4)/1229312) - (151875 * 0^3)/1229312 - \\
& (16875 * 0^2)/307328 + (9375 * 0)/76832 + 1875/19208) + \\
& 2 * f * g * ((- (2025 * 0^4)/76832) - (38475 * 0^3)/307328 - \\
& (79875 * 0^2)/614656 + (69525 * 0)/614656 + 25875/153664) + \\
& 2 * f * h * ((- (225 * 0^4)/25088) - (1125 * 0^3)/12544 - \\
& (5375 * 0^2)/25088 + (625 * 0)/12544 + 825/3136) + \\
& 2 * f * i * (75/196 - 225/784 * 0^2 - 75/784 * 0) + g^2 * \\
& ((- (81 * 0^4)/4802) - (1053 * 0^3)/9604 - (3645 * 0^2)/19208 + \\
& (2025 * 0)/38416 + 10125/38416) + 2 * g * h * (2295/6272 - \\
& 9/1568 * 0^4 - 423/6272 * 0^3 - 2763/12544 * 0^2 - 909/12544 * \\
& 0) + 2 * g * i * (45/98 - 9/49 * 0^2 - 27/98 * 0) + h^2 * \\
& (27/64 - 1/512 * 0^4 - 17/512 * 0^3 - 45/256 * 0^2 - \\
& 27/128 * 0) + 2 * h * i * (3/8 - 1/16 * 0^2 - 5/16 * \\
& 0) + i^2 * 0)
\end{aligned}$$

```

R1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a *
d * 0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 +
2 * a * h * 0 + 2 * a * i * 0 + b^2 * (1/4096 * 0^4) +
2 * b * c * (9/12544 * 0^4 + 3/12544 * 0^3 + 1/50176 *
0^2) + 2 * b * d * ((225 * 0^4)/200704 + (45 * 0^3)/50176 +
(9 * 0^2)/50176) + 2 * b * e * (0^4/784 + (3 * 0^3)/1568 +
(9 * 0^2)/12544) + 2 * b * f * ((225 * 0^4)/200704 +
(75 * 0^3)/25088 + (25 * 0^2)/12544) + 2 * b * g * ((9 *
0^4)/12544 + (45 * 0^3)/12544 + (225 * 0^2)/50176) +
2 * b * h * (0^4/4096 + (3 * 0^3)/1024 + (9 * 0^2)/1024) +
2 * b * i * (1/64 * 0^2) + c^2 * ((81 * 0^4)/38416 +
(27 * 0^3)/19208 + (27 * 0^2)/76832 + (3 * 0)/76832 +
1/614656) + 2 * c * d * ((2025 * 0^4)/614656 + (2295 *
0^3)/614656 + (3681 * 0^2)/2458624 + (153 * 0)/614656 +
9/614656) + 2 * c * e * ((9 * 0^4)/2401 + (33 * 0^3)/4802 +
(157 * 0^2)/38416 + (33 * 0)/38416 + 9/153664) + 2 *
c * f * ((2025 * 0^4)/614656 + (6075 * 0^3)/614656 +
(21825 * 0^2)/2458624 + (675 * 0)/307328 + 25/153664) +
2 * c * g * ((81 * 0^4)/38416 + (27 * 0^3)/2401 + (1287 *
0^2)/76832 + (45 * 0)/9604 + 225/614656) + 2 * c *
h * ((9 * 0^4)/12544 + (111 * 0^3)/12544 + (1441 * 0^2)/50176 +
(111 * 0)/12544 + 9/12544) + 2 * c * i * (1/784 + 9/196 *
0^2 + 3/196 * 0) + d^2 * ((50625 * 0^4)/9834496 + (10125 *
0^3)/1229312 + (6075 * 0^2)/1229312 + (405 * 0)/307328 +
81/614656) + 2 * d * e * ((225 * 0^4)/38416 + (1035 *
0^3)/76832 + (6921 * 0^2)/614656 + (621 * 0)/153664 +
81/153664) + 2 * d * f * ((50625 * 0^4)/9834496 + (43875 *
0^3)/2458624 + (51525 * 0^2)/2458624 + (2925 * 0)/307328 +
225/153664) + 2 * d * g * ((2025 * 0^4)/614656 + (11745 *
0^3)/614656 + (84321 * 0^2)/2458624 + (11745 * 0)/614656 +
2025/614656) + 2 * d * h * ((225 * 0^4)/200704 + (45 *
0^3)/3136 + (1287 * 0^2)/25088 + (27 * 0)/784 + 81/12544) +
2 * d * i * (9/784 + 225/3136 * 0^2 + 45/784 * 0) + e^2 *

```


$$\begin{aligned}
& ((16 * 0^4)/2401 + (48 * 0^3)/2401 + (54 * 0^2)/2401 + \\
& (27 * 0)/2401 + 81/38416) + 2 * e * f * ((225 * 0^4)/38416 + \\
& (1875 * 0^3)/76832 + (22825 * 0^2)/614656 + (1875 * 0)/76832 + \\
& 225/38416) + 2 * e * g * ((9 * 0^4)/2401 + (117 * 0^3)/4802 + \\
& (2061 * 0^2)/38416 + (1755 * 0)/38416 + 2025/153664) + \\
& 2 * e * h * (0^4/784 + (27 * 0^3)/1568 + (873 * 0^2)/12544 + \\
& (243 * 0)/3136 + 81/3136) + 2 * e * i * (9/196 + \\
& 4/49 * 0^2 + 6/49 * 0) + f^2 * ((50625 * 0^4)/9834496 + \\
& (16875 * 0^3)/614656 + (16875 * 0^2)/307328 + (1875 * \\
& 0)/38416 + 625/38416) + 2 * f * g * ((2025 * 0^4)/614656 + \\
& (15525 * 0^3)/614656 + (173025 * 0^2)/2458624 + (25875 * \\
& 0)/307328 + 5625/153664) + 2 * f * h * ((225 * 0^4)/200704 + \\
& (825 * 0^3)/50176 + (3925 * 0^2)/50176 + (825 * 0)/6272 + \\
& 225/3136) + 2 * f * i * (25/196 + 225/3136 * 0^2 + 75/392 * \\
& 0) + g^2 * ((81 * 0^4)/38416 + (405 * 0^3)/19208 + (6075 * \\
& 0^2)/76832 + (10125 * 0)/76832 + 50625/614656) + 2 * \\
& g * h * (2025/12544 + 9/12544 * 0^4 + 153/12544 * 0^3 + \\
& 3681/50176 * 0^2 + 2295/12544 * 0) + 2 * g * i * (225/784 + \\
& 9/196 * 0^2 + 45/196 * 0) + h^2 * (81/256 + 1/4096 * \\
& 0^4 + 3/512 * 0^3 + 27/512 * 0^2 + 27/128 * 0) + 2 * \\
& h * i * (9/16 + 1/64 * 0^2 + 3/16 * 0) + i^2 * 1) \\
\\
& \text{fai9.9} = (2 * a^2 * 1 + 2 * a * b * (9/16) + 2 * a * c * \\
& (225/784) + 2 * a * d * (25/196) + 2 * a * e * (9/196) + \\
& 2 * a * f * (9/784) + 2 * a * g * (1/784))/R9 \\
& \text{fai9.8} = (2 * a^2 * 0 + 2 * a * b * (3/8) + 2 * a * c * (45/98) + \\
& 2 * a * d * (75/196) + 2 * a * e * (12/49) + 2 * a * \\
& f * (45/392) + 2 * a * g * (3/98))/R8 \\
& \text{fai9.7} = (2 * a^2 * 0 + 2 * a * b * (1/16) + 2 * a * c * \\
& (87/392) + 2 * a * d * (285/784) + 2 * a * e * (41/98) + \\
& 2 * a * f * (285/784) + 2 * a * g * (87/392) + 2 * a * \\
& h * (1/16))/R7 \\
& \text{fai9.6} = (2 * a^2 * 0 + 2 * a * b * 0 + 2 * a * c * (3/98) + \\
& 2 * a * d * (45/392) + 2 * a * e * (12/49) + 2 * a * \\
& f * (75/196) + 2 * a * g * (45/98) + 2 * a * h * (3/8))/R6 \\
& \text{fai9.5} = (2 * a^2 * 0 + 2 * a * b * 0 + 2 * a * c * (1/784) + \\
& 2 * a * d * (9/784) + 2 * a * e * (9/196) + 2 * a * f * \\
& (25/196) + 2 * a * g * (225/784) + 2 * a * h * (9/16) + \\
& 2 * a * i * 1)/R5 \\
\\
& \text{fai8.9} = (2 * a * b * (9/16) + 2 * b^2 * (81/256) + 2 * b * \\
& c * (2025/12544) + 2 * b * d * (225/3136) + 2 * b * e * \\
& (81/3136) + 2 * b * f * (81/12544) + 2 * b * g * (9/12544))/R9 \\
& \text{fai8.8} = (2 * a * b * (3/8) + 2 * b^2 * (27/64) + 2 * b * \\
& c * (2295/6272) + 2 * b * d * (825/3136) + 2 * b * e * \\
& (243/1568) + 2 * b * f * (27/392) + 2 * b * g * (111/6272))/R8 \\
& \text{fai8.7} = (2 * a * b * (1/16) + 2 * b^2 * (27/128) + 2 * b * \\
& c * (3951/12544) + 2 * b * d * (4465/12544) + 2 * b * \\
& e * (1035/3136) + 2 * b * f * (1557/6272) + 2 * b * g * \\
& (1711/12544) + 2 * b * h * (9/256))/R7 \\
& \text{fai8.6} = (2 * a * b * 0 + 2 * b^2 * (3/64) + 2 * b * c * \\
& (405/3136) + 2 * b * d * (705/3136) + 2 * b * e * (243/784) + \\
& 2 * b * f * (1125/3136) + 2 * b * g * (1077/3136) + 2 *
\end{aligned}$$

```

    b * h * (15/64))/R6
fai8.5 = (2 * a * b * 0 + 2 * b^2 * (1/256) + 2 * b * c *
(327/12544) + 2 * b * d * (453/6272) + 2 * b * e * (451/3136) +
2 * b * f * (2985/12544) + 2 * b * g * (4359/12544) +
2 * b * h * (59/128))/R5
fai8.4 = (2 * a * b * 0 + 2 * b^2 * 0 + 2 * b * c * (15/6272) +
2 * b * d * (9/784) + 2 * b * e * (51/1568) + 2 * b *
f * (225/3136) + 2 * b * g * (855/6272) + 2 * b * h *
(15/64) + 2 * b * i * (3/8))/R4
fai8.3 = (2 * a * b * 0 + 2 * b^2 * 0 + 2 * b * c * (1/12544) +
2 * b * d * (9/12544) + 2 * b * e * (9/3136) + 2 * b *
f * (25/3136) + 2 * b * g * (225/12544) + 2 * b * h *
(9/256) + 2 * b * i * (1/16))/R3

fai7.9 = (2 * a * c * (225/784) + 2 * b * c * (2025/12544) +
2 * c^2 * (50625/614656) + 2 * c * d * (5625/153664) +
2 * c * e * (2025/153664) + 2 * c * f * (2025/614656) +
2 * c * g * (225/614656))/R9
fai7.8 = (2 * a * c * (45/98) + 2 * b * c * (2295/6272) +
2 * c^2 * (10125/38416) + 2 * c * d * (25875/153664) +
2 * c * e * (1755/19208) + 2 * c * f * (11745/307328) +
2 * c * g * (45/4802))/R8
fai7.7 = (2 * a * c * (87/392) + 2 * b * c * (3951/12544) +
2 * c^2 * (7425/21952) + 2 * c * d * (27075/87808) +
2 * c * e * (333/1372) + 2 * c * f * (14013/87808) +
2 * c * g * (1713/21952) + 2 * c * h * (225/12544))/R7
fai7.6 = (2 * a * c * (3/98) + 2 * b * c * (405/3136) + 2 *
c^2 * (1215/5488) + 2 * c * d * (12675/43904) + 2 * c *
e * (873/2744) + 2 * c * f * (3321/10976) + 2 * c * g *
(165/686) + 2 * c * h * (855/6272))/R6
fai7.5 = (2 * a * c * (1/784) + 2 * b * c * (327/12544) +
2 * c^2 * (3429/43904) + 2 * c * d * (13045/87808) +
2 * c * e * (2481/10976) + 2 * c * f * (26037/87808) +
2 * c * g * (15077/43904) + 2 * c * h * (4359/12544) +
2 * c * i * (225/784))/R5
fai7.4 = (2 * a * c * 0 + 2 * b * c * (15/6272) + 2 * c^2 *
(81/5488) + 2 * c * d * (465/10976) + 2 * c * e * (243/2744) +
2 * c * f * (6795/43904) + 2 * c * g * (165/686) + 2 *
c * h * (1077/3136) + 2 * c * i * (45/98))/R4
fai7.3 = (2 * a * c * 0 + 2 * b * c * (1/12544) + 2 * c^2 *
(33/21952) + 2 * c * d * (573/87808) + 2 * c * e * (25/1372) +
2 * c * f * (3555/87808) + 2 * c * g * (1713/21952) +
2 * c * h * (1711/12544) + 2 * c * i * (87/392))/R3
fai7.2 = (2 * a * c * 0 + 2 * b * c * 0 + 2 * c^2 * (3/38416) +
2 * c * d * (153/307328) + 2 * c * e * (33/19208) + 2 *
c * f * (675/153664) + 2 * c * g * (45/4802) + 2 * c *
h * (111/6272) + 2 * c * i * (3/98))/R2
fai7.1 = (2 * a * c * 0 + 2 * b * c * 0 + 2 * c^2 * (1/614656) +
2 * c * d * (9/614656) + 2 * c * e * (9/153664) + 2 *
c * f * (25/153664) + 2 * c * g * (225/614656) + 2 *
c * h * (9/12544) + 2 * c * i * (1/784))/R1

fai6.9 = (2 * a * d * (25/196) + 2 * b * d * (225/3136) +

```

$$\begin{aligned}
& 2 * c * d * (5625/153664) + 2 * d^2 * (625/38416) + 2 * \\
& d * e * (225/38416) + 2 * d * f * (225/153664) + 2 * \\
& d * g * (25/153664) + 2 * d * h * (0) + 2 * d * i * (0)) / R9 \\
\text{fai6.8} &= (2 * a * d * (75/196) + 2 * b * d * (825/3136) + \\
& 2 * c * d * (25875/153664) + 2 * d^2 * (1875/19208) + \\
& 2 * d * e * (1875/38416) + 2 * d * f * (2925/153664) + \\
& 2 * d * g * (675/153664) + 2 * d * h * (0) + 2 * d * \\
& i * (0)) / R8 \\
\text{fai6.7} &= (2 * a * d * (285/784) + 2 * b * d * (4465/12544) + \\
& 2 * c * d * (27075/87808) + 2 * d^2 * (375/1568) + 2 * \\
& d * e * (3595/21952) + 2 * d * f * (1185/12544) + 2 * \\
& d * g * (3555/87808) + 2 * d * h * (25/3136) + 2 * d * \\
& i * (0)) / R7 \\
\text{fai6.6} &= (2 * a * d * (45/392) + 2 * b * d * (705/3136) + \\
& 2 * c * d * (12675/43904) + 2 * d^2 * (3375/10976) + \\
& 2 * d * e * (3135/10976) + 2 * d * f * (2535/10976) + \\
& 2 * d * g * (6795/43904) + 2 * d * h * (225/3136) + 2 * \\
& d * i * (0)) / R6 \\
\text{fai6.5} &= (2 * a * d * (9/784) + 2 * b * d * (453/6272) + \\
& 2 * c * d * (13045/87808) + 2 * d^2 * (19575/87808) + \\
& 2 * d * e * (879/3136) + 2 * d * f * (13529/43904) + \\
& 2 * d * g * (26037/87808) + 2 * d * h * (2985/12544) + \\
& 2 * d * i * (25/196)) / R5 \\
\text{fai6.4} &= (2 * a * d * (0) + 2 * b * d * (9/784) + 2 * c * \\
& d * (465/10976) + 2 * d^2 * (2025/21952) + 2 * d * e * \\
& (54/343) + 2 * d * f * (2535/10976) + 2 * d * g * (3321/10976) + \\
& 2 * d * h * (1125/3136) + 2 * d * i * (75/196)) / R4 \\
\text{fai6.3} &= (2 * a * d * (0) + 2 * b * d * (9/12544) + 2 * c * \\
& d * (573/87808) + 2 * d^2 * (135/6272) + 2 * d * e * \\
& (1089/21952) + 2 * d * f * (1185/12544) + 2 * d * g * \\
& (14013/87808) + 2 * d * h * (1557/6272) + 2 * d * i * \\
& (285/784)) / R3 \\
\text{fai6.2} &= (2 * a * d * (0) + 2 * b * d * (0) + 2 * c * d * \\
& (153/307328) + 2 * d^2 * (405/153664) + 2 * d * e * (621/76832) + \\
& 2 * d * f * (2925/153664) + 2 * d * g * (11745/307328) + \\
& 2 * d * h * (27/392) + 2 * d * i * (45/392)) / R2 \\
\text{fai6.1} &= (2 * a * d * (0) + 2 * b * d * (0) + 2 * c * d * \\
& (9/614656) + 2 * d^2 * (81/614656) + 2 * d * e * (81/153664) + \\
& 2 * d * f * (225/153664) + 2 * d * g * (2025/614656) + \\
& 2 * d * h * (81/12544) + 2 * d * i * (9/784)) / R1 \\
\\
\text{fai5.9} &= (2 * a * e * (9/196) + 2 * b * e * (81/3136) + 2 * \\
& c * e * (2025/153664) + 2 * d * e * (225/38416) + 2 * \\
& e^2 * (81/38416) + 2 * e * f * (81/153664) + 2 * e * \\
& g * (9/153664) + 2 * e * h * (0) + 2 * e * i * (0)) / R9 \\
\text{fai5.8} &= (2 * a * e * (12/49) + 2 * b * e * (243/1568) + \\
& 2 * c * e * (1755/19208) + 2 * d * e * (1875/38416) + \\
& 2 * e^2 * (54/2401) + 2 * e * f * (621/76832) + 2 * e * \\
& g * (33/19208) + 2 * e * h * (0) + 2 * e * i * (0)) / R8 \\
\text{fai5.7} &= (2 * a * e * (41/98) + 2 * b * e * (1035/3136) + \\
& 2 * c * e * (333/1372) + 2 * d * e * (3595/21952) + 2 * \\
& e^2 * (135/1372) + 2 * e * f * (1089/21952) + 2 * e * \\
& g * (25/1372) + 2 * e * h * (9/3136) + 2 * e * i * (0)) / R7
\end{aligned}$$

```

fai5.6 = (2 * a * e * (12/49) + 2 * b * e * (243/784) + 2 *
c * e * (873/2744) + 2 * d * e * (3135/10976) + 2 * e^2 *
(78/343) + 2 * e * f * (54/343) + 2 * e * g * (243/2744) +
2 * e * h * (51/1568) + 2 * e * i * (0))/R6
fai5.5 = (2 * a * e * (9/196) + 2 * b * e * (451/3136) +
2 * c * e * (2481/10976) + 2 * d * e * (879/3136) + 2 *
e^2 * (821/2744) + 2 * e * f * (879/3136) + 2 * e * g *
(2481/10976) + 2 * e * h * (451/3136) + 2 * e * i * (9/196))/R5
fai5.4 = (2 * a * e * (0) + 2 * b * e * (51/1568) + 2 * c *
e * (243/2744) + 2 * d * e * (54/343) + 2 * e^2 * (78/343) +
2 * e * f * (3135/10976) + 2 * e * g * (873/2744) + 2 *
e * h * (243/784) + 2 * e * i * (12/49))/R4
fai5.3 = (2 * a * e * (0) + 2 * b * e * (9/3136) + 2 * c *
e * (25/1372) + 2 * d * e * (1089/21952) + 2 * e^2 *
(135/1372) + 2 * e * f * (3595/21952) + 2 * e * g * (333/1372) +
2 * e * h * (1035/3136) + 2 * e * i * (41/98))/R3
fai5.2 = (2 * a * e * (0) + 2 * b * e * (0) + 2 * c * e *
(33/19208) + 2 * d * e * (621/76832) + 2 * e^2 * (54/2401) +
2 * e * f * (1875/38416) + 2 * e * g * (1755/19208) +
2 * e * h * (243/1568) + 2 * e * i * (12/49))/R2
fai5.1 = (2 * a * e * (0) + 2 * b * e * (0) + 2 * c * e *
(9/153664) + 2 * d * e * (81/153664) + 2 * e^2 * (81/38416) +
2 * e * f * (225/38416) + 2 * e * g * (2025/153664) +
2 * e * h * (81/3136) + 2 * e * i * (9/196))/R1

fai4.9 = (2 * a * f * (9/784) + 2 * b * f * (81/12544) +
2 * c * f * (2025/614656) + 2 * d * f * (225/153664) +
2 * e * f * (81/153664) + 2 * f^2 * (81/614656) + 2 *
f * g * (9/614656) + 2 * f * h * (0) + 2 * f * i * (0))/R9
fai4.8 = (2 * a * f * (45/392) + 2 * b * f * (27/392) + 2 *
c * f * (11745/307328) + 2 * d * f * (2925/153664) +
2 * e * f * (621/76832) + 2 * f^2 * (405/153664) + 2 *
f * g * (153/307328) + 2 * f * h * (0) + 2 * f * i *
(0))/R8
fai4.7 = (2 * a * f * (285/784) + 2 * b * f * (1557/6272) +
2 * c * f * (14013/87808) + 2 * d * f * (1185/12544) +
2 * e * f * (1089/21952) + 2 * f^2 * (135/6272) + 2 *
f * g * (573/87808) + 2 * f * h * (9/12544) + 2 * f *
i * (0))/R7
fai4.6 = (2 * a * f * (75/196) + 2 * b * f * (1125/3136) +
2 * c * f * (3321/10976) + 2 * d * f * (2535/10976) +
2 * e * f * (54/343) + 2 * f^2 * (2025/21952) + 2 * f *
g * (465/10976) + 2 * f * h * (9/784) + 2 * f * i * (0))/R6
fai4.5 = (2 * a * f * (25/196) + 2 * b * f * (2985/12544) +
2 * c * f * (26037/87808) + 2 * d * f * (13529/43904) +
2 * e * f * (879/3136) + 2 * f^2 * (19575/87808) + 2 *
f * g * (13045/87808) + 2 * f * h * (453/6272) + 2 *
f * i * (9/784))/R5
fai4.4 = (2 * a * f * (0) + 2 * b * f * (225/3136) + 2 *
c * f * (6795/43904) + 2 * d * f * (2535/10976) + 2 *
e * f * (3135/10976) + 2 * f^2 * (3375/10976) + 2 * f *
g * (12675/43904) + 2 * f * h * (705/3136) + 2 * f *
i * (45/392))/R4

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```

fai4.3 = (2 * a * f * (0) + 2 * b * f * (25/3136) + 2 * c *
f * (3555/87808) + 2 * d * f * (1185/12544) + 2 * e *
f * (3595/21952) + 2 * f^2 * (375/1568) + 2 * f * g *
(27075/87808) + 2 * f * h * (4465/12544) + 2 * f * i *
(285/784))/R3
fai4.2 = (2 * a * f * (0) + 2 * b * f * (0) + 2 * c * f *
(675/153664) + 2 * d * f * (2925/153664) + 2 * e * f *
(1875/38416) + 2 * f^2 * (1875/19208) + 2 * f * g * (25875/153664) +
2 * f * h * (825/3136) + 2 * f * i * (75/196))/R2
fai4.1 = (2 * a * f * (0) + 2 * b * f * (0) + 2 * c * f *
(25/153664) + 2 * d * f * (225/153664) + 2 * e * f *
(225/38416) + 2 * f^2 * (625/38416) + 2 * f * g * (5625/153664) +
2 * f * h * (225/3136) + 2 * f * i * (25/196))/R1

fai3.9 = (2 * a * g * (1/784) + 2 * b * g * (9/12544) + 2 *
c * g * (225/614656) + 2 * d * g * (25/153664) + 2 *
e * g * (9/153664) + 2 * f * g * (9/614656) + 2 * g^2 *
(1/614656) + 2 * g * h * (0) + 2 * g * i * (0))/R9
fai3.8 = (2 * a * g * (3/98) + 2 * b * g * (111/6272) + 2 *
c * g * (45/4802) + 2 * d * g * (675/153664) + 2 * e *
g * (33/19208) + 2 * f * g * (153/307328) + 2 * g^2 *
(3/38416) + 2 * g * h * (0) + 2 * g * i * (0))/R8
fai3.7 = (2 * a * g * (87/392) + 2 * b * g * (1711/12544) +
2 * c * g * (1713/21952) + 2 * d * g * (3555/87808) +
2 * e * g * (25/1372) + 2 * f * g * (573/87808) + 2 *
g^2 * (33/21952) + 2 * g * h * (1/12544) + 2 * g * i *
(0))/R7
fai3.6 = (2 * a * g * (45/98) + 2 * b * g * (1077/3136) +
2 * c * g * (165/686) + 2 * d * g * (6795/43904) + 2 *
e * g * (243/2744) + 2 * f * g * (465/10976) + 2 * g^2 *
(81/5488) + 2 * g * h * (15/6272) + 2 * g * i * (0))/R6
fai3.5 = (2 * a * g * (225/784) + 2 * b * g * (4359/12544) +
2 * c * g * (15077/43904) + 2 * d * g * (26037/87808) +
2 * e * g * (2481/10976) + 2 * f * g * (13045/87808) +
2 * g^2 * (3429/43904) + 2 * g * h * (327/12544) + 2 *
g * i * (1/784))/R5
fai3.4 = (2 * a * g * (0) + 2 * b * g * (855/6272) + 2 *
c * g * (165/686) + 2 * d * g * (3321/10976) + 2 * e *
g * (873/2744) + 2 * f * g * (12675/43904) + 2 * g^2 *
(1215/5488) + 2 * g * h * (405/3136) + 2 * g * i * (3/98))/R4
fai3.3 = (2 * a * g * (0) + 2 * b * g * (225/12544) + 2 *
c * g * (1713/21952) + 2 * d * g * (14013/87808) + 2 *
e * g * (333/1372) + 2 * f * g * (27075/87808) + 2 *
g^2 * (7425/21952) + 2 * g * h * (3951/12544) + 2 * g *
i * (87/392))/R3
fai3.2 = (2 * a * g * (0) + 2 * b * g * (0) + 2 * c * g *
(45/4802) + 2 * d * g * (11745/307328) + 2 * e * g *
(1755/19208) + 2 * f * g * (25875/153664) + 2 * g^2 *
(10125/38416) + 2 * g * h * (2295/6272) + 2 * g * i *
(45/98))/R2
fai3.1 = (2 * a * g * (0) + 2 * b * g * (0) + 2 * c * g *
(225/614656) + 2 * d * g * (2025/614656) + 2 * e * g *
(2025/153664) + 2 * f * g * (5625/153664) + 2 * g^2 *

```

```

(50625/614656) + 2 * g * h * (2025/12544) + 2 * g * i *
(225/784))/R1

fai2.9 = (2 * a * h * (0) + 2 * b * h * (0) + 2 * c * h *
(0) + 2 * d * h * (0) + 2 * e * h * (0) + 2 * f * h *
(0) + 2 * g * h * (0) + 2 * h^2 * (0) + 2 * h * i * (0))/R9
fai2.8 = (2 * a * h * (0) + 2 * b * h * (0) + 2 * c * h *
(0) + 2 * d * h * (0) + 2 * e * h * (0) + 2 * f * h *
(0) + 2 * g * h * (0) + 2 * h^2 * (0) + 2 * h * i * (0))/R8
fai2.7 = (2 * a * h * (1/16) + 2 * b * h * (9/256) + 2 *
c * h * (225/12544) + 2 * d * h * (25/3136) + 2 * e *
h * (9/3136) + 2 * f * h * (9/12544) + 2 * g * h * (1/12544) +
2 * h^2 * (0) + 2 * h * i * (0))/R7
fai2.6 = (2 * a * h * (3/8) + 2 * b * h * (15/64) + 2 * c *
h * (855/6272) + 2 * d * h * (225/3136) + 2 * e * h *
(51/1568) + 2 * f * h * (9/784) + 2 * g * h * (15/6272) +
2 * h^2 * (0) + 2 * h * i * (0))/R6
fai2.5 = (2 * a * h * (9/16) + 2 * b * h * (59/128) + 2 *
c * h * (4359/12544) + 2 * d * h * (2985/12544) + 2 *
e * h * (451/3136) + 2 * f * h * (453/6272) + 2 * g *
h * (327/12544) + 2 * h^2 * (1/256) + 2 * h * i * (0))/R5
fai2.4 = (2 * a * h * (0) + 2 * b * h * (15/64) + 2 * c *
h * (1077/3136) + 2 * d * h * (1125/3136) + 2 * e * h *
(243/784) + 2 * f * h * (705/3136) + 2 * g * h * (405/3136) +
2 * h^2 * (3/64) + 2 * h * i * (0))/R4
fai2.3 = (2 * a * h * (0) + 2 * b * h * (9/256) + 2 * c *
h * (1711/12544) + 2 * d * h * (1557/6272) + 2 * e *
h * (1035/3136) + 2 * f * h * (4465/12544) + 2 * g *
h * (3951/12544) + 2 * h^2 * (27/128) + 2 * h * i * (1/16))/R3
fai2.2 = (2 * a * h * (0) + 2 * b * h * (0) + 2 * c * h *
(111/6272) + 2 * d * h * (27/392) + 2 * e * h * (243/1568) +
2 * f * h * (825/3136) + 2 * g * h * (2295/6272) + 2 *
h^2 * (27/64) + 2 * h * i * (3/8))/R2
fai2.1 = (2 * a * h * (0) + 2 * b * h * (0) + 2 * c * h *
(9/12544) + 2 * d * h * (81/12544) + 2 * e * h * (81/3136) +
2 * f * h * (225/3136) + 2 * g * h * (2025/12544) + 2 *
h^2 * (81/256) + 2 * h * i * (9/16))/R1

fai1.9 = (2 * a * i * (0) + 2 * b * i * (0) + 2 * c * i *
(0) + 2 * d * i * (0) + 2 * e * i * (0) + 2 * f * i *
(0) + 2 * g * i * (0) + 2 * h * i * (0) + 2 * i^2 * (0))/R9
fai1.8 = (2 * a * i * (0) + 2 * b * i * (0) + 2 * c * i *
(0) + 2 * d * i * (0) + 2 * e * i * (0) + 2 * f * i *
(0) + 2 * g * i * (0) + 2 * h * i * (0) + 2 * i^2 * (0))/R8
fai1.7 = (2 * a * i * (0) + 2 * b * i * (0) + 2 * c * i *
(0) + 2 * d * i * (0) + 2 * e * i * (0) + 2 * f * i *
(0) + 2 * g * i * (0) + 2 * h * i * (0) + 2 * i^2 * (0))/R7
fai1.6 = (2 * a * i * (0) + 2 * b * i * (0) + 2 * c * i *
(0) + 2 * d * i * (0) + 2 * e * i * (0) + 2 * f * i *
(0) + 2 * g * i * (0) + 2 * h * i * (0) + 2 * i^2 * (0))/R6
fai1.5 = (2 * a * i * (1) + 2 * b * i * (9/16) + 2 * c *
i * (225/784) + 2 * d * i * (25/196) + 2 * e * i * (9/196) +
2 * f * i * (9/784) + 2 * g * i * (1/784) + 2 * h * i *

```



```

      (0) + 2 * i^2 * (0))/R5
fai1.4 = (2 * a * i * (0) + 2 * b * i * (3/8) + 2 * c * i *
      (45/98) + 2 * d * i * (75/196) + 2 * e * i * (12/49) +
      2 * f * i * (45/392) + 2 * g * i * (3/98) + 2 * h * i *
      (0) + 2 * i^2 * (0))/R4
fai1.3 = (2 * a * i * (0) + 2 * b * i * (1/16) + 2 * c *
      i * (87/392) + 2 * d * i * (285/784) + 2 * e * i * (41/98) +
      2 * f * i * (285/784) + 2 * g * i * (87/392) + 2 * h *
      i * (1/16) + 2 * i^2 * (0))/R3
fai1.2 = (2 * a * i * (0) + 2 * b * i * (0) + 2 * c * i *
      (3/98) + 2 * d * i * (45/392) + 2 * e * i * (12/49) +
      2 * f * i * (75/196) + 2 * g * i * (45/98) + 2 * h *
      i * (3/8) + 2 * i^2 * (0))/R2
fai1.1 = (2 * a * i * (0) + 2 * b * i * (0) + 2 * c * i *
      (1/784) + 2 * d * i * (9/784) + 2 * e * i * (9/196) +
      2 * f * i * (25/196) + 2 * g * i * (225/784) + 2 * h *
      i * (9/16) + 2 * i^2 * (1))/R1

P.9 = (fai9.9 * n9 + fai9.8 * n8 + fai9.7 * n7 + fai9.6 *
      n6 + fai9.5 * n5)/(2 * N)
P.8 = (fai8.9 * n9 + fai8.8 * n8 + fai8.7 * n7 + fai8.6 *
      n6 + fai8.5 * n5 + fai8.4 * n4 + fai8.3 * n3)/(2 * N)
P.7 = (fai7.9 * n9 + fai7.8 * n8 + fai7.7 * n7 + fai7.6 *
      n6 + fai7.5 * n5 + fai7.4 * n4 + fai7.3 * n3 + fai7.2 *
      n2 + fai7.1 * n1)/(2 * N)
P.6 = (fai6.9 * n9 + fai6.8 * n8 + fai6.7 * n7 + fai6.6 *
      n6 + fai6.5 * n5 + fai6.4 * n4 + fai6.3 * n3 + fai6.2 *
      n2 + fai6.1 * n1)/(2 * N)
P.5 = (fai5.9 * n9 + fai5.8 * n8 + fai5.7 * n7 + fai5.6 *
      n6 + fai5.5 * n5 + fai5.4 * n4 + fai5.3 * n3 + fai5.2 *
      n2 + fai5.1 * n1)/(2 * N)
P.4 = (fai4.9 * n9 + fai4.8 * n8 + fai4.7 * n7 + fai4.6 *
      n6 + fai4.5 * n5 + fai4.4 * n4 + fai4.3 * n3 + fai4.2 *
      n2 + fai4.1 * n1)/(2 * N)
P.3 = (fai3.9 * n9 + fai3.8 * n8 + fai3.7 * n7 + fai3.6 *
      n6 + fai3.5 * n5 + fai3.4 * n4 + fai3.3 * n3 + fai3.2 *
      n2 + fai3.1 * n1)/(2 * N)
P.2 = (fai2.9 * n9 + fai2.8 * n8 + fai2.7 * n7 + fai2.6 *
      n6 + fai2.5 * n5 + fai2.4 * n4 + fai2.3 * n3 + fai2.2 *
      n2 + fai2.1 * n1)/(2 * N)
P.1 = (fai1.9 * n9 + fai1.8 * n8 + fai1.7 * n7 + fai1.6 *
      n6 + fai1.5 * n5 + fai1.4 * n4 + fai1.3 * n3 + fai1.2 *
      n2 + fai1.1 * n1)/(2 * N)

geno.p <- rbind(geno.p, c(a, b, c, d, e, f, g, h, i))
a <- P.9
b <- P.8
c <- P.7
d <- P.6
e <- P.5
f <- P.4
g <- P.3
h <- P.2

```

```

    i <- P.1
  }
a <- geno.p[500, 1]
b <- geno.p[500, 2]
c <- geno.p[500, 3]
d <- geno.p[500, 4]
e <- geno.p[500, 5]
f <- geno.p[500, 6]
g <- geno.p[500, 7]
h <- geno.p[500, 8]
i <- geno.p[500, 9]

geno.L <- rbind(geno.L, c(a, b, c, d, e, f, g, h, i))

```

This obtains the final genotype frequency estimates, which they call `exp`

```

R.9 <- (a^2 * 1 + 2 * a * b * (9/16 + 1/64 * 0^2 + 3/16 * 0) +
  2 * a * c * (225/784 + 9/196 * 0^2 + 45/196 * 0) + 2 * a *
  d * (25/196 + 225/3136 * 0^2 + 75/392 * 0) + 2 * a * e *
  (9/196 + 4/49 * 0^2 + 6/49 * 0) + 2 * a * f * (9/784 + 225/3136 *
  0^2 + 45/784 * 0) + 2 * a * g * (1/784 + 9/196 * 0^2 + 3/196 *
  0) + 2 * a * h * (1/64 * 0^2) + 0 + b^2 * (81/256 + 1/4096 *
  0^4 + 3/512 * 0^3 + 27/512 * 0^2 + 27/128 * 0) + 2 * b *
  c * (2025/12544 + 9/12544 * 0^4 + 153/12544 * 0^3 + 3681/50176 *
  0^2 + 2295/12544 * 0) + 2 * b * d * ((225 * 0^4)/200704 +
  (825 * 0^3)/50176 + (3925 * 0^2)/50176 + (825 * 0)/6272 +
  225/3136) + 2 * b * e * (0^4/784 + (27 * 0^3)/1568 + (873 *
  0^2)/12544 + (243 * 0)/3136 + 81/3136) + 2 * b * f * ((225 *
  0^4)/200704 + (45 * 0^3)/3136 + (1287 * 0^2)/25088 + (27 *
  0)/784 + 81/12544) + 2 * b * g * ((9 * 0^4)/12544 + (111 *
  0^3)/12544 + (1441 * 0^2)/50176 + (111 * 0)/12544 + 9/12544) +
  2 * b * h * (0^4/4096 + (3 * 0^3)/1024 + (9 * 0^2)/1024) +
  0 + c^2 * ((81 * 0^4)/38416 + (405 * 0^3)/19208 + (6075 *
  0^2)/76832 + (10125 * 0)/76832 + 50625/614656) + 2 * c *
  d * ((2025 * 0^4)/614656 + (15525 * 0^3)/614656 + (173025 *
  0^2)/2458624 + (25875 * 0)/307328 + 5625/153664) + 2 * c *
  e * ((9 * 0^4)/2401 + (117 * 0^3)/4802 + (2061 * 0^2)/38416 +
  (1755 * 0)/38416 + 2025/153664) + 2 * c * f * ((2025 * 0^4)/614656 +
  (11745 * 0^3)/614656 + (84321 * 0^2)/2458624 + (11745 * 0)/614656 +
  2025/614656) + 2 * c * g * ((81 * 0^4)/38416 + (27 * 0^3)/2401 +
  (1287 * 0^2)/76832 + (45 * 0)/9604 + 225/614656) + 2 * c *
  h * ((9 * 0^4)/12544 + (45 * 0^3)/12544 + (225 * 0^2)/50176) +
  0 + d^2 * ((50625 * 0^4)/9834496 + (16875 * 0^3)/614656 +
  (16875 * 0^2)/307328 + (1875 * 0)/38416 + 625/38416) + 2 *
  d * e * ((225 * 0^4)/38416 + (1875 * 0^3)/76832 + (22825 *
  0^2)/614656 + (1875 * 0)/76832 + 225/38416) + 2 * d * f *
  ((50625 * 0^4)/9834496 + (43875 * 0^3)/2458624 + (51525 *
  0^2)/2458624 + (2925 * 0)/307328 + 225/153664) + 2 *
  d * g * ((2025 * 0^4)/614656 + (6075 * 0^3)/614656 + (21825 *
  0^2)/2458624 + (675 * 0)/307328 + 25/153664) + 2 * d * h *
  ((225 * 0^4)/200704 + (75 * 0^3)/25088 + (25 * 0^2)/12544) +
  0 + e^2 * ((16 * 0^4)/2401 + (48 * 0^3)/2401 + (54 * 0^2)/2401 +
  (27 * 0)/2401 + 81/38416) + 2 * e * f * ((225 * 0^4)/38416 +
  (1035 * 0^3)/76832 + (6921 * 0^2)/614656 + (621 * 0)/153664 +

```

```

81/153664) + 2 * e * g * ((9 * 0^4)/2401 + (33 * 0^3)/4802 +
(157 * 0^2)/38416 + (33 * 0)/38416 + 9/153664) + 2 * e *
h * (0^4/784 + (3 * 0^3)/1568 + (9 * 0^2)/12544) + 0 + f^2 *
((50625 * 0^4)/9834496 + (10125 * 0^3)/1229312 + (6075 *
0^2)/1229312 + (405 * 0)/307328 + 81/614656) + 2 * f *
g * ((2025 * 0^4)/614656 + (2295 * 0^3)/614656 + (3681 *
0^2)/2458624 + (153 * 0)/614656 + 9/614656) + 2 * f * h *
((225 * 0^4)/200704 + (45 * 0^3)/50176 + (9 * 0^2)/50176) +
0 + g^2 * ((81 * 0^4)/38416 + (27 * 0^3)/19208 + (27 * 0^2)/76832 +
(3 * 0)/76832 + 1/614656) + 2 * g * h * (9/12544 * 0^4 +
3/12544 * 0^3 + 1/50176 * 0^2) + 0 + h^2 * (1/4096 * 0^4) +
0 + i^2 * 0)

```

```

R.8 <- (a^2 * 0 + 2 * a * b * (3/8 - 1/16 * 0^2 - 5/16 * 0) +
2 * a * c * (45/98 - 9/49 * 0^2 - 27/98 * 0) + 2 * a * d *
(75/196 - 225/784 * 0^2 - 75/784 * 0) + 2 * a * e * (12/49 -
16/49 * 0^2 + 4/49 * 0) + 2 * a * f * (45/392 - 225/784 *
0^2 + 135/784 * 0) + 2 * a * g * (3/98 - 9/49 * 0^2 + 15/98 *
0) + 2 * a * h * (1/16 * 0 - 1/16 * 0^2) + 0 + b^2 * (27/64 -
1/512 * 0^4 - 17/512 * 0^3 - 45/256 * 0^2 - 27/128 * 0) +
2 * b * c * (2295/6272 - 9/1568 * 0^4 - 423/6272 * 0^3 -
2763/12544 * 0^2 - 909/12544 * 0) + 2 * b * d * ((-225 *
0^4)/25088) - (1125 * 0^3)/12544 - (5375 * 0^2)/25088 + (625 *
0)/12544 + 825/3136) + 2 * b * e * ((-0^4)/98) - (73 * 0^3)/784 -
(549 * 0^2)/3136 + (387 * 0)/3136 + 243/1568) + 2 * b * f *
((-225 * 0^4)/25088) - (1935 * 0^3)/25088 - (747 * 0^2)/6272 +
(855 * 0)/6272 + 27/392) + 2 * b * g * ((-9 * 0^4)/1568) -
(297 * 0^3)/6272 - (775 * 0^2)/12544 + (1219 * 0)/12544 +
111/6272) + 2 * b * h * ((-0^4)/512) - 0^3/64 - (9 * 0^2)/512 +
(9 * 0)/256) + 0 + c^2 * ((-81 * 0^4)/4802) - (1053 * 0^3)/9604 -
(3645 * 0^2)/19208 + (2025 * 0)/38416 + 10125/38416) + 2 *
c * d * ((-2025 * 0^4)/76832) - (38475 * 0^3)/307328 - (79875 *
0^2)/614656 + (69525 * 0)/614656 + 25875/153664) + 2 * c *
e * ((-72 * 0^4)/2401) - (279 * 0^3)/2401 - (657 * 0^2)/9604 +
(2367 * 0)/19208 + 1755/19208) + 2 * c * f * ((-2025 * 0^4)/76832) -
(27135 * 0^3)/307328 - (13851 * 0^2)/614656 + (60831 * 0)/614656 +
11745/307328) + 2 * c * g * ((-81 * 0^4)/4802) - (243 *
0^3)/4802 + (9 * 0^2)/19208 + (1107 * 0)/19208 + 45/4802) +
2 * c * h * ((-9 * 0^4)/1568) - (99 * 0^3)/6272 + (45 *
0^2)/12544 + (225 * 0)/12544) + 0 + d^2 * ((-50625 *
0^4)/1229312) - (151875 * 0^3)/1229312 - (16875 * 0^2)/307328 +
(9375 * 0)/76832 + 1875/19208) + 2 * d * e * ((-225 * 0^4)/4802) -
(3825 * 0^3)/38416 - (325 * 0^2)/153664 + (15325 * 0)/153664 +
1875/38416) + 2 * d * f * ((-50625 * 0^4)/1229312) - (10125 *
0^3)/153664 + (28575 * 0^2)/1229312 + (39825 * 0)/614656 +
2925/153664) + 2 * d * g * ((-2025 * 0^4)/76832) - (10125 *
0^3)/307328 + (14625 * 0^2)/614656 + (19125 * 0)/614656 +
675/153664) + 2 * d * h * ((-225 * 0^4)/25088) - (225 *
0^3)/25088 + (125 * 0^2)/12544 + (25 * 0)/3136) + 0 + e^2 *
((-128 * 0^4)/2401) - (160 * 0^3)/2401 + (72 * 0^2)/2401 +
(162 * 0)/2401 + 54/2401) + 2 * e * f * ((-225 * 0^4)/4802) -
(1305 * 0^3)/38416 + (5499 * 0^2)/153664 + (5679 * 0)/153664 +

```

$$\begin{aligned}
& 621/76832) + 2 * e * g * ((-(72 * 0^4)/2401) - (27 * 0^3)/2401 + \\
& (239 * 0^2)/9604 + (281 * 0)/19208 + 33/19208) + 2 * e * \\
& h * ((-0^4/98) - 0^3/784 + (27 * 0^2)/3136 + (9 * 0)/3136) + \\
& 0 + f^2 * ((-(50625 * 0^4)/1229312) - (10125 * 0^3)/1229312 + \\
& (18225 * 0^2)/614656 + (5265 * 0)/307328 + 405/153664) + \\
& 2 * f * g * ((-(2025 * 0^4)/76832) + (1215 * 0^3)/307328 + \\
& (10089 * 0^2)/614656 + (3375 * 0)/614656 + 153/307328) + \\
& 2 * f * h * ((-(225 * 0^4)/25088) + (45 * 0^3)/12544 + (117 * \\
& 0^2)/25088 + (9 * 0)/12544) + 0 + g^2 * ((-(81 * 0^4)/4802) + \\
& (81 * 0^3)/9604 + (135 * 0^2)/19208 + (51 * 0)/38416 + 3/38416) + \\
& 2 * g * h * (-9/1568 * 0^4 + 27/6272 * 0^3 + 17/12544 * 0^2 + \\
& 1/12544 * 0) + 0 + h^2 * (-1/512 * 0^4 + 1/512 * 0^3) + \\
& 0 + i^2 * 0)
\end{aligned}$$

```

R.7 <- (a^2 * 0 + 2 * a * b * (1/16 + 3/32 * 0^2 + 1/16 * 0) +
2 * a * c * (87/392 + 27/98 * 0^2 - 6/49 * 0) + 2 * a * d *
(285/784 + 675/1568 * 0^2 - 255/784 * 0) + 2 * a * e * (41/98 +
24/49 * 0^2 - 20/49 * 0) + 2 * a * f * (285/784 + 675/1568 *
0^2 - 255/784 * 0) + 2 * a * g * (87/392 + 27/98 * 0^2 -
6/49 * 0) + 2 * a * h * (1/16 + 3/32 * 0^2 + 1/16 * 0) +
0 + b^2 * (27/128 + 7/1024 * 0^4 + 39/512 * 0^3 + 93/512 *
0^2 - 9/64 * 0) + 2 * b * c * (3951/12544 + 9/448 * 0^4 +
933/6272 * 0^3 + 2109/12544 * 0^2 - 3033/12544 * 0) + 2 *
b * d * ((225 * 0^4)/7168 + (1215 * 0^3)/6272 + (1425 * 0^2)/12544 -
(785 * 0)/3136 + 4465/12544) + 2 * b * e * (0^4/28 + (39 *
0^3)/196 + (93 * 0^2)/1568 - (585 * 0)/3136 + 1035/3136) +
2 * b * f * ((225 * 0^4)/7168 + (4125 * 0^3)/25088 + (687 *
0^2)/25088 - (513 * 0)/6272 + 1557/6272) + 2 * b * g *
((9 * 0^4)/448 + (639 * 0^3)/6272 + (261 * 0^2)/12544 + (271 *
0)/12544 + 1711/12544) + 2 * b * h * ((7 * 0^4)/1024 +
(9 * 0^3)/256 + (3 * 0^2)/128 + (9 * 0)/128 + 9/256) + 0 +
c^2 * ((81 * 0^4)/1372 + (297 * 0^3)/1372 + (81 * 0^2)/2744 -
(1215 * 0)/5488 + 7425/21952) + 2 * c * d * ((2025 *
0^4)/21952 + (9855 * 0^3)/43904 - (1035 * 0^2)/12544 - (11625 *
0)/87808 + 27075/87808) + 2 * c * e * ((36 * 0^4)/343 + (66 *
0^3)/343 - (93 * 0^2)/686 - (81 * 0)/2744 + 333/1372) + 2 *
c * f * ((2025 * 0^4)/21952 + (6075 * 0^3)/43904 - (1539 *
0^2)/12544 + (4293 * 0)/87808 + 14013/87808) + 2 * c * g *
((81 * 0^4)/1372 + (27 * 0^3)/343 - (171 * 0^2)/2744 + (111 *
0)/1372 + 1713/21952) + 2 * c * h * ((9 * 0^4)/448 +
(177 * 0^3)/6272 + (3 * 0^2)/1792 + (765 * 0)/12544 + 225/12544) +
0 + d^2 * ((50625 * 0^4)/351232 + (30375 * 0^3)/175616 -
(30375 * 0^2)/175616 - (375 * 0)/21952 + 375/1568) + 2 *
d * e * ((225 * 0^4)/1372 + (135 * 0^3)/1372 - (1965 * 0^2)/10976 +
(205 * 0)/3136 + 3595/21952) + 2 * d * f * ((50625 * 0^4)/351232 +
(3375 * 0^3)/87808 - (5625 * 0^2)/43904 + (4395 * 0)/43904 +
1185/12544) + 2 * d * g * ((2025 * 0^4)/21952 + (405 * 0^3)/43904 -
(675 * 0^2)/12544 + (7695 * 0)/87808 + 3555/87808) + 2 *
d * h * ((225 * 0^4)/7168 + (135 * 0^3)/25088 + (15 * 0^2)/3584 +
(275 * 0)/6272 + 25/3136) + 0 + e^2 * ((64 * 0^4)/343 - (48 *
0^2)/343 + (36 * 0)/343 + 135/1372) + 2 * e * f * ((225 *
0^4)/1372 - (75 * 0^3)/1372 - (789 * 0^2)/10976 + (45 * 0)/448 +

```

```

1089/21952) + 2 * e * g * ((36 * 0^4)/343 - (18 * 0^3)/343 -
(9 * 0^2)/686 + (185 * 0)/2744 + 25/1372) + 2 * e * h * (0^4/28 -
(3 * 0^3)/196 + (3 * 0^2)/224 + (81 * 0)/3136 + 9/3136) +
0 + f^2 * ((50625 * 0^4)/351232 - (16875 * 0^3)/175616 -
(2025 * 0^2)/175616 + (405 * 0)/5488 + 135/6272) + 2 * f *
g * ((2025 * 0^4)/21952 - (3375 * 0^3)/43904 + (261 * 0^2)/12544 +
(3453 * 0)/87808 + 573/87808) + 2 * f * h * ((225 * 0^4)/7168 -
(75 * 0^3)/3136 + (33 * 0^2)/1792 + (9 * 0)/784 + 9/12544) +
0 + g^2 * ((81 * 0^4)/1372 - (81 * 0^3)/1372 + (81 * 0^2)/2744 +
(87 * 0)/5488 + 33/21952) + 2 * g * h * (1/12544 + 9/448 *
0^4 - 117/6272 * 0^3 + 27/1792 * 0^2 + 37/12544 * 0) + 0 +
h^2 * (7/1024 * 0^4 - 3/512 * 0^3 + 3/512 * 0^2) + 0 + i^2 *
0)

```

```

R.6 <- (a^2 * 0 + 2 * a * b * (1/16 * 0 - 1/16 * 0^2) + 2 * a *
c * (3/98 - 9/49 * 0^2 + 15/98 * 0) + 2 * a * d * (45/392 -
225/784 * 0^2 + 135/784 * 0) + 2 * a * e * (12/49 - 16/49 *
0^2 + 4/49 * 0) + 2 * a * f * (75/196 - 225/784 * 0^2 - 75/784 *
0) + 2 * a * g * (45/98 - 9/49 * 0^2 - 27/98 * 0) + 2 * a *
h * (3/8 - 1/16 * 0^2 - 5/16 * 0) + 0 + b^2 * (3/64 - 7/512 *
0^4 - 45/512 * 0^3 - 3/128 * 0^2 + 5/64 * 0) + 2 * b * c *
(405/3136 - 9/224 * 0^4 - 999/6272 * 0^3 + 807/12544 * 0^2 +
75/12544 * 0) + 2 * b * d * ((-(225 * 0^4)/3584) - (2505 *
0^3)/12544 + (3235 * 0^2)/25088 - (1145 * 0)/12544 + 705/3136) +
2 * b * e * ((-0^4/14) - (159 * 0^3)/784 + (417 * 0^2)/3136 -
(529 * 0)/3136 + 243/784) + 2 * b * f * ((-(225 * 0^4)/3584) -
(4275 * 0^3)/25088 + (921 * 0^2)/12544 - (39 * 0)/196 + 1125/3136) +
2 * b * g * ((-(9 * 0^4)/224) - (705 * 0^3)/6272 - (229 *
0^2)/12544 - (2165 * 0)/12544 + 1077/3136) + 2 * b *
h * ((-(7 * 0^4)/512) - (3 * 0^3)/64 - (39 * 0^2)/512 - (25 *
0)/256 + 15/64) + 0 + c^2 * ((-(81 * 0^4)/686) - (243 * 0^3)/1372 +
(81 * 0^2)/392 - (729 * 0)/5488 + 1215/5488) + 2 * c * d *
((-(2025 * 0^4)/10976) - (5805 * 0^3)/43904 + (21465 * 0^2)/87808 -
(2715 * 0)/12544 + 12675/43904) + 2 * c * e * ((-(72 *
0^4)/343) - (27 * 0^3)/343 + (267 * 0^2)/1372 - (615 * 0)/2744 +
873/2744) + 2 * c * f * ((-(2025 * 0^4)/10976) - (2025 *
0^3)/43904 + (7857 * 0^2)/87808 - (2025 * 0)/12544 + 3321/10976) +
2 * c * g * ((-(81 * 0^4)/686) - (27 * 0^3)/686 - (9 * 0^2)/392 -
(165 * 0)/2744 + 165/686) + 2 * c * h * ((-(9 * 0^4)/224) -
(243 * 0^3)/6272 - (993 * 0^2)/12544 + (39 * 0)/1792 + 855/6272) +
0 + d^2 * ((-(50625 * 0^4)/175616) + (3375 * 0^3)/175616 +
(16875 * 0^2)/87808 - (10125 * 0)/43904 + 3375/10976) + 2 *
d * e * ((-(225 * 0^4)/686) + (15 * 0^3)/112 + (1655 * 0^2)/21952 -
(3665 * 0)/21952 + 3135/10976) + 2 * d * f * ((-(50625 *
0^4)/175616) + (3375 * 0^3)/21952 - (5625 * 0^2)/175616 -
(5655 * 0)/87808 + 2535/10976) + 2 * d * g * ((-(2025 * 0^4)/10976) +
(3645 * 0^3)/43904 - (7515 * 0^2)/87808 + (405 * 0)/12544 +
6795/43904) + 2 * d * h * ((-(225 * 0^4)/3584) - (285 * 0^3)/25088 -
(55 * 0^2)/784 + (65 * 0)/896 + 225/3136) + 0 + e^2 * ((-(128 *
0^4)/343) + (96 * 0^3)/343 - (24 * 0^2)/343 - (22 * 0)/343 +
78/343) + 2 * e * f * ((-(225 * 0^4)/686) + (225 * 0^3)/784 -
(3273 * 0^2)/21952 + (717 * 0)/21952 + 54/343) + 2 * e *

```

```

g * ((- (72 * 0^4)/343) + (57 * 0^3)/343 - (181 * 0^2)/1372 +
(239 * 0)/2744 + 243/2744) + 2 * e * h * ((-0^4/14) + (9 *
0^3)/784 - (159 * 0^2)/3136 + (5 * 0)/64 + 51/1568) + 0 +
f^2 * ((- (50625 * 0^4)/175616) + (50625 * 0^3)/175616 - (2025 *
0^2)/10976 + (2025 * 0)/21952 + 2025/21952) + 2 * f *
g * ((- (2025 * 0^4)/10976) + (7425 * 0^3)/43904 - (11043 *
0^2)/87808 + (177 * 0)/1792 + 465/10976) + 2 * f * h * ((- (225 *
0^4)/3584) + (225 * 0^3)/12544 - (633 * 0^2)/25088 + (15 *
0)/256 + 9/784) + 0 + g^2 * ((- (81 * 0^4)/686) + (135 * 0^3)/1372 -
(27 * 0^2)/392 + (405 * 0)/5488 + 81/5488) + 2 * g * h *
(15/6272 - 9/224 * 0^4 + 51/6272 * 0^3 - 13/12544 * 0^2 +
55/1792 * 0) + 0 + h^2 * (-7/512 * 0^4 - 3/512 * 0^3 +
3/256 * 0^2 + 1/128 * 0) + 0 + i^2 * 0)

```

```

R.5 <- (a^2 * 0 + 2 * a * b * (1/64 * 0^2) + 2 * a * c * (1/784 +
9/196 * 0^2 + 3/196 * 0) + 2 * a * d * (9/784 + 225/3136 *
0^2 + 45/784 * 0) + 2 * a * e * (9/196 + 4/49 * 0^2 + 6/49 *
0) + 2 * a * f * (25/196 + 225/3136 * 0^2 + 75/392 * 0) +
2 * a * g * (225/784 + 9/196 * 0^2 + 45/196 * 0) + 2 * a *
h * (9/16 + 1/64 * 0^2 + 3/16 * 0) + 2 * a * i * 1 + b^2 *
(1/256 + 35/2048 * 0^4 + 25/512 * 0^3 - 27/512 * 0^2 + 7/128 *
0) + 2 * b * c * (327/12544 + 45/896 * 0^4 + 225/3136 *
0^3 - 2533/25088 * 0^2 + 1149/12544 * 0) + 2 * b * d * ((1125 *
0^4)/14336 + (2025 * 0^3)/25088 - (2623 * 0^2)/25088 + (561 *
0)/6272 + 453/6272) + 2 * b * e * ((5 * 0^4)/56 + (65 * 0^3)/784 -
(369 * 0^2)/6272 + (149 * 0)/3136 + 451/3136) + 2 * b * f *
((1125 * 0^4)/14336 + (2025 * 0^3)/25088 + (527 * 0^2)/25088 -
(45 * 0)/3136 + 2985/12544) + 2 * b * g * ((45 * 0^4)/896 +
(225 * 0^3)/3136 + (2507 * 0^2)/25088 - (699 * 0)/12544 +
4359/12544) + 2 * b * h * ((35 * 0^4)/2048 + (25 * 0^3)/512 +
(63 * 0^2)/512 - 0/64 + 59/128) + 2 * b * i * (9/16 + 1/64 *
0^2 + 3/16 * 0) + c^2 * ((405 * 0^4)/2744 - (513 * 0^2)/5488 +
(27 * 0)/343 + 3429/43904) + 2 * c * d * ((10125 * 0^4)/43904 -
(2025 * 0^3)/21952 - (2691 * 0^2)/175616 + (675 * 0)/87808 +
13045/87808) + 2 * c * e * ((90 * 0^4)/343 - (45 * 0^3)/343 +
(197 * 0^2)/2744 - (117 * 0)/1372 + 2481/10976) + 2 * c *
f * ((10125 * 0^4)/43904 - (2025 * 0^3)/21952 + (22509 *
0^2)/175616 - (13941 * 0)/87808 + 26037/87808) + 2 * c *
g * ((405 * 0^4)/2744 + (747 * 0^2)/5488 - (57 * 0)/343 +
15077/43904) + 2 * c * h * ((45 * 0^4)/896 + (225 * 0^3)/3136 +
(2507 * 0^2)/25088 - (699 * 0)/12544 + 4359/12544) + 2 *
c * i * (225/784 + 9/196 * 0^2 + 45/196 * 0) + d^2 * ((253125 *
0^4)/702464 - (50625 * 0^3)/175616 + (24975 * 0^2)/175616 -
(675 * 0)/6272 + 19575/87808) + 2 * d * e * ((1125 * 0^4)/2744 -
(2025 * 0^3)/5488 + (10649 * 0^2)/43904 - (4317 * 0)/21952 +
879/3136) + 2 * d * f * ((253125 * 0^4)/702464 - (50625 *
0^3)/175616 + (40725 * 0^2)/175616 - (345 * 0)/1568 + 13529/43904) +
2 * d * g * ((10125 * 0^4)/43904 - (2025 * 0^3)/21952 + (22509 *
0^2)/175616 - (13941 * 0)/87808 + 26037/87808) + 2 *
d * h * ((1125 * 0^4)/14336 + (2025 * 0^3)/25088 + (527 *
0^2)/25088 - (45 * 0)/3136 + 2985/12544) + 2 * d * i * (25/196 +
225/3136 * 0^2 + 75/392 * 0) + e^2 * ((160 * 0^4)/343 - (160 *

```


$$\begin{aligned}
& 0^3)/343 + (108 * 0^2)/343 - (82 * 0)/343 + 821/2744) + 2 * \\
& e * f * ((1125 * 0^4)/2744 - (2025 * 0^3)/5488 + (10649 * \\
& 0^2)/43904 - (4317 * 0)/21952 + 879/3136) + 2 * e * g * ((90 * \\
& 0^4)/343 - (45 * 0^3)/343 + (197 * 0^2)/2744 - (117 * 0)/1372 + \\
& 2481/10976) + 2 * e * h * ((5 * 0^4)/56 + (65 * 0^3)/784 - \\
& (369 * 0^2)/6272 + (149 * 0)/3136 + 451/3136) + 2 * e * i * \\
& (9/196 + 4/49 * 0^2 + 6/49 * 0) + f^2 * ((253125 * 0^4)/702464 - \\
& (50625 * 0^3)/175616 + (24975 * 0^2)/175616 - (675 * 0)/6272 + \\
& 19575/87808) + 2 * f * g * ((10125 * 0^4)/43904 - (2025 * \\
& 0^3)/21952 - (2691 * 0^2)/175616 + (675 * 0)/87808 + 13045/87808) + \\
& 2 * f * h * ((1125 * 0^4)/14336 + (2025 * 0^3)/25088 - (2623 * \\
& 0^2)/25088 + (561 * 0)/6272 + 453/6272) + 2 * f * i * \\
& (9/784 + 225/3136 * 0^2 + 45/784 * 0) + g^2 * ((405 * 0^4)/2744 - \\
& (513 * 0^2)/5488 + (27 * 0)/343 + 3429/43904) + 2 * g * h * \\
& (327/12544 + 45/896 * 0^4 + 225/3136 * 0^3 - 2533/25088 * \\
& 0^2 + 1149/12544 * 0) + 2 * g * i * (1/784 + 9/196 * \\
& 0^2 + 3/196 * 0) + h^2 * (1/256 + 35/2048 * 0^4 + 25/512 * \\
& 0^3 - 27/512 * 0^2 + 7/128 * 0) + 2 * h * i * (1/64 * 0^2) + \\
& i^2 * 0)
\end{aligned}$$

R.4 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
h * 0 + 2 * a * i * 0 + b^2 * (-7/512 * 0^4 - 3/512 * 0^3 +
3/256 * 0^2 + 1/128 * 0) + 2 * b * c * (15/6272 - 9/224 *
0^4 + 51/6272 * 0^3 - 13/12544 * 0^2 + 55/1792 * 0) + 2 *
b * d * ((-225 * 0^4)/3584) + (225 * 0^3)/12544 - (633 *
0^2)/25088 + (15 * 0)/256 + 9/784) + 2 * b * e * ((-0^4/14) +
(9 * 0^3)/784 - (159 * 0^2)/3136 + (5 * 0)/64 + 51/1568) +
2 * b * f * ((-225 * 0^4)/3584) - (285 * 0^3)/25088 - (55 *
0^2)/784 + (65 * 0)/896 + 225/3136) + 2 * b * g * ((-9 *
0^4)/224) - (243 * 0^3)/6272 - (993 * 0^2)/12544 + (39 *
0)/1792 + 855/6272) + 2 * b * h * ((-7 * 0^4)/512) - (3 *
0^3)/64 - (39 * 0^2)/512 - (25 * 0)/256 + 15/64) + 2 * b *
i * (3/8 - 1/16 * 0^2 - 5/16 * 0) + c^2 * ((-81 * 0^4)/686) +
(135 * 0^3)/1372 - (27 * 0^2)/392 + (405 * 0)/5488 + 81/5488) +
2 * c * d * ((-2025 * 0^4)/10976) + (7425 * 0^3)/43904 -
(11043 * 0^2)/87808 + (177 * 0)/1792 + 465/10976) + 2 *
c * e * ((-72 * 0^4)/343) + (57 * 0^3)/343 - (181 * 0^2)/1372 +
(239 * 0)/2744 + 243/2744) + 2 * c * f * ((-2025 * 0^4)/10976) +
(3645 * 0^3)/43904 - (7515 * 0^2)/87808 + (405 * 0)/12544 +
6795/43904) + 2 * c * g * ((-81 * 0^4)/686) - (27 * 0^3)/686 -
(9 * 0^2)/392 - (165 * 0)/2744 + 165/686) + 2 * c * h * ((-9 *
0^4)/224) - (705 * 0^3)/6272 - (229 * 0^2)/12544 - (2165 *
0)/12544 + 1077/3136) + 2 * c * i * (45/98 - 9/49 * 0^2 -
27/98 * 0) + d^2 * ((-50625 * 0^4)/175616) + (50625 * 0^3)/175616 -
(2025 * 0^2)/10976 + (2025 * 0)/21952 + 2025/21952) + 2 *
d * e * ((-225 * 0^4)/686) + (225 * 0^3)/784 - (3273 * 0^2)/21952 +
(717 * 0)/21952 + 54/343) + 2 * d * f * ((-50625 * 0^4)/175616) +
(3375 * 0^3)/21952 - (5625 * 0^2)/175616 - (5655 * 0)/87808 +
2535/10976) + 2 * d * g * ((-2025 * 0^4)/10976) - (2025 *
0^3)/43904 + (7857 * 0^2)/87808 - (2025 * 0)/12544 + 3321/10976) +
2 * d * h * ((-225 * 0^4)/3584) - (4275 * 0^3)/25088 + (921 *

$$\begin{aligned}
& 0^2)/12544 - (39 * 0)/196 + 1125/3136) + 2 * d * i * \\
& (75/196 - 225/784 * 0^2 - 75/784 * 0) + e^2 * ((-(128 * 0^4)/343) + \\
& (96 * 0^3)/343 - (24 * 0^2)/343 - (22 * 0)/343 + 78/343) + \\
& 2 * e * f * ((-(225 * 0^4)/686) + (15 * 0^3)/112 + (1655 * \\
& 0^2)/21952 - (3665 * 0)/21952 + 3135/10976) + 2 * e * \\
& g * ((-(72 * 0^4)/343) - (27 * 0^3)/343 + (267 * 0^2)/1372 - \\
& (615 * 0)/2744 + 873/2744) + 2 * e * h * ((-0^4/14) - (159 * \\
& 0^3)/784 + (417 * 0^2)/3136 - (529 * 0)/3136 + 243/784) + \\
& 2 * e * i * (12/49 - 16/49 * 0^2 + 4/49 * 0) + f^2 * ((-(50625 * \\
& 0^4)/175616) + (3375 * 0^3)/175616 + (16875 * 0^2)/87808 - \\
& (10125 * 0)/43904 + 3375/10976) + 2 * f * g * ((-(2025 * \\
& 0^4)/10976) - (5805 * 0^3)/43904 + (21465 * 0^2)/87808 - \\
& (2715 * 0)/12544 + 12675/43904) + 2 * f * h * ((-(225 * 0^4)/3584) - \\
& (2505 * 0^3)/12544 + (3235 * 0^2)/25088 - (1145 * 0)/12544 + \\
& 705/3136) + 2 * f * i * (45/392 - 225/784 * 0^2 + 135/784 * \\
& 0) + g^2 * ((-(81 * 0^4)/686) - (243 * 0^3)/1372 + (81 * \\
& 0^2)/392 - (729 * 0)/5488 + 1215/5488) + 2 * g * h * (405/3136 - \\
& 9/224 * 0^4 - 999/6272 * 0^3 + 807/12544 * 0^2 + 75/12544 * \\
& 0) + 2 * g * i * (3/98 - 9/49 * 0^2 + 15/98 * 0) + h^2 * \\
& (3/64 - 7/512 * 0^4 - 45/512 * 0^3 - 3/128 * 0^2 + 5/64 * \\
& 0) + 2 * h * i * (1/16 * 0 - 1/16 * 0^2) + i^2 * 0)
\end{aligned}$$

```

R.3 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
h * 0 + 2 * a * i * 0 + b^2 * (7/1024 * 0^4 - 3/512 * 0^3 +
3/512 * 0^2) + 2 * b * c * (1/12544 + 9/448 * 0^4 - 117/6272 *
0^3 + 27/1792 * 0^2 + 37/12544 * 0) + 2 * b * d * ((225 *
0^4)/7168 - (75 * 0^3)/3136 + (33 * 0^2)/1792 + (9 * 0)/784 +
9/12544) + 2 * b * e * (0^4/28 - (3 * 0^3)/196 + (3 * 0^2)/224 +
(81 * 0)/3136 + 9/3136) + 2 * b * f * ((225 * 0^4)/7168 +
(135 * 0^3)/25088 + (15 * 0^2)/3584 + (275 * 0)/6272 + 25/3136) +
2 * b * g * ((9 * 0^4)/448 + (177 * 0^3)/6272 + (3 * 0^2)/1792 +
(765 * 0)/12544 + 225/12544) + 2 * b * h * ((7 * 0^4)/1024 +
(9 * 0^3)/256 + (3 * 0^2)/128 + (9 * 0)/128 + 9/256) + 2 *
b * i * (1/16 + 3/32 * 0^2 + 1/16 * 0) + c^2 * ((81 * 0^4)/1372 -
(81 * 0^3)/1372 + (81 * 0^2)/2744 + (87 * 0)/5488 + 33/21952) +
2 * c * d * ((2025 * 0^4)/21952 - (3375 * 0^3)/43904 + (261 *
0^2)/12544 + (3453 * 0)/87808 + 573/87808) + 2 * c *
e * ((36 * 0^4)/343 - (18 * 0^3)/343 - (9 * 0^2)/686 + (185 *
0)/2744 + 25/1372) + 2 * c * f * ((2025 * 0^4)/21952 + (405 *
0^3)/43904 - (675 * 0^2)/12544 + (7695 * 0)/87808 + 3555/87808) +
2 * c * g * ((81 * 0^4)/1372 + (27 * 0^3)/343 - (171 * 0^2)/2744 +
(111 * 0)/1372 + 1713/21952) + 2 * c * h * ((9 * 0^4)/448 +
(639 * 0^3)/6272 + (261 * 0^2)/12544 + (271 * 0)/12544 +
1711/12544) + 2 * c * i * (87/392 + 27/98 * 0^2 - 6/49 *
0) + d^2 * ((50625 * 0^4)/351232 - (16875 * 0^3)/175616 -
(2025 * 0^2)/175616 + (405 * 0)/5488 + 135/6272) + 2 * d *
e * ((225 * 0^4)/1372 - (75 * 0^3)/1372 - (789 * 0^2)/10976 +
(45 * 0)/448 + 1089/21952) + 2 * d * f * ((50625 * 0^4)/351232 +
(3375 * 0^3)/87808 - (5625 * 0^2)/43904 + (4395 * 0)/43904 +
1185/12544) + 2 * d * g * ((2025 * 0^4)/21952 + (6075 * 0^3)/43904 -
(1539 * 0^2)/12544 + (4293 * 0)/87808 + 14013/87808) + 2 *

```

```

d * h * ((225 * 0^4)/7168 + (4125 * 0^3)/25088 + (687 * 0^2)/25088 -
(513 * 0)/6272 + 1557/6272) + 2 * d * i * (285/784 + 675/1568 *
0^2 - 255/784 * 0) + e^2 * ((64 * 0^4)/343 - (48 * 0^2)/343 +
(36 * 0)/343 + 135/1372) + 2 * e * f * ((225 * 0^4)/1372 +
(135 * 0^3)/1372 - (1965 * 0^2)/10976 + (205 * 0)/3136 +
3595/21952) + 2 * e * g * ((36 * 0^4)/343 + (66 * 0^3)/343 -
(93 * 0^2)/686 - (81 * 0)/2744 + 333/1372) + 2 * e * h *
(0^4/28 + (39 * 0^3)/196 + (93 * 0^2)/1568 - (585 * 0)/3136 +
1035/3136) + 2 * e * i * (41/98 + 24/49 * 0^2 - 20/49 *
0) + f^2 * ((50625 * 0^4)/351232 + (30375 * 0^3)/175616 -
(30375 * 0^2)/175616 - (375 * 0)/21952 + 375/1568) + 2 *
f * g * ((2025 * 0^4)/21952 + (9855 * 0^3)/43904 - (1035 *
0^2)/12544 - (11625 * 0)/87808 + 27075/87808) + 2 * f * h *
((225 * 0^4)/7168 + (1215 * 0^3)/6272 + (1425 * 0^2)/12544 -
(785 * 0)/3136 + 4465/12544) + 2 * f * i * (285/784 +
675/1568 * 0^2 - 255/784 * 0) + g^2 * ((81 * 0^4)/1372 +
(297 * 0^3)/1372 + (81 * 0^2)/2744 - (1215 * 0)/5488 + 7425/21952) +
2 * g * h * (3951/12544 + 9/448 * 0^4 + 933/6272 * 0^3 +
2109/12544 * 0^2 - 3033/12544 * 0) + 2 * g * i * (87/392 +
27/98 * 0^2 - 6/49 * 0) + h^2 * (27/128 + 7/1024 * 0^4 +
39/512 * 0^3 + 93/512 * 0^2 - 9/64 * 0) + 2 * h * i * (1/16 +
3/32 * 0^2 + 1/16 * 0) + i^2 * 0)

```

```

R.2 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
h * 0 + 2 * a * i * 0 + b^2 * (-1/512 * 0^4 + 1/512 * 0^3) +
2 * b * c * (-9/1568 * 0^4 + 27/6272 * 0^3 + 17/12544 * 0^2 +
1/12544 * 0) + 2 * b * d * ((-225 * 0^4)/25088) + (45 *
0^3)/12544 + (117 * 0^2)/25088 + (9 * 0)/12544) + 2 * b *
e * ((-0^4/98) - 0^3/784 + (27 * 0^2)/3136 + (9 * 0)/3136) +
2 * b * f * ((-225 * 0^4)/25088) - (225 * 0^3)/25088 + (125 *
0^2)/12544 + (25 * 0)/3136) + 2 * b * g * ((-9 * 0^4)/1568) -
(99 * 0^3)/6272 + (45 * 0^2)/12544 + (225 * 0)/12544) + 2 *
b * h * ((-0^4/512) - 0^3/64 - (9 * 0^2)/512 + (9 * 0)/256) +
2 * b * i * (1/16 * 0 - 1/16 * 0^2) + c^2 * ((-81 * 0^4)/4802) +
(81 * 0^3)/9604 + (135 * 0^2)/19208 + (51 * 0)/38416 + 3/38416) +
2 * c * d * ((-2025 * 0^4)/76832) + (1215 * 0^3)/307328 +
(10089 * 0^2)/614656 + (3375 * 0)/614656 + 153/307328) +
2 * c * e * ((-72 * 0^4)/2401) - (27 * 0^3)/2401 + (239 *
0^2)/9604 + (281 * 0)/19208 + 33/19208) + 2 * c * f *
((-(2025 * 0^4)/76832) - (10125 * 0^3)/307328 + (14625 *
0^2)/614656 + (19125 * 0)/614656 + 675/153664) + 2 *
c * g * ((-81 * 0^4)/4802) - (243 * 0^3)/4802 + (9 * 0^2)/19208 +
(1107 * 0)/19208 + 45/4802) + 2 * c * h * ((-9 * 0^4)/1568) -
(297 * 0^3)/6272 - (775 * 0^2)/12544 + (1219 * 0)/12544 +
111/6272) + 2 * c * i * (3/98 - 9/49 * 0^2 + 15/98 * 0) +
d^2 * ((-50625 * 0^4)/1229312) - (10125 * 0^3)/1229312 +
(18225 * 0^2)/614656 + (5265 * 0)/307328 + 405/153664) +
2 * d * e * ((-225 * 0^4)/4802) - (1305 * 0^3)/38416 + (5499 *
0^2)/153664 + (5679 * 0)/153664 + 621/76832) + 2 * d *
f * ((-50625 * 0^4)/1229312) - (10125 * 0^3)/153664 + (28575 *
0^2)/1229312 + (39825 * 0)/614656 + 2925/153664) + 2 * d *

```

```

g * ((-(2025 * 0^4)/76832) - (27135 * 0^3)/307328 - (13851 *
0^2)/614656 + (60831 * 0)/614656 + 11745/307328) + 2 * d *
h * ((-(225 * 0^4)/25088) - (1935 * 0^3)/25088 - (747 * 0^2)/6272 +
(855 * 0)/6272 + 27/392) + 2 * d * i * (45/392 - 225/784 *
0^2 + 135/784 * 0) + e^2 * ((-(128 * 0^4)/2401) - (160 *
0^3)/2401 + (72 * 0^2)/2401 + (162 * 0)/2401 + 54/2401) +
2 * e * f * ((-(225 * 0^4)/4802) - (3825 * 0^3)/38416 - (325 *
0^2)/153664 + (15325 * 0)/153664 + 1875/38416) + 2 *
e * g * ((-(72 * 0^4)/2401) - (279 * 0^3)/2401 - (657 * 0^2)/9604 +
(2367 * 0)/19208 + 1755/19208) + 2 * e * h * ((-0^4/98) -
(73 * 0^3)/784 - (549 * 0^2)/3136 + (387 * 0)/3136 + 243/1568) +
2 * e * i * (12/49 - 16/49 * 0^2 + 4/49 * 0) + f^2 * ((-(50625 *
0^4)/1229312) - (151875 * 0^3)/1229312 - (16875 * 0^2)/307328 +
(9375 * 0)/76832 + 1875/19208) + 2 * f * g * ((-(2025 * 0^4)/76832) -
(38475 * 0^3)/307328 - (79875 * 0^2)/614656 + (69525 * 0)/614656 +
25875/153664) + 2 * f * h * ((-(225 * 0^4)/25088) - (1125 *
0^3)/12544 - (5375 * 0^2)/25088 + (625 * 0)/12544 + 825/3136) +
2 * f * i * (75/196 - 225/784 * 0^2 - 75/784 * 0) + g^2 *
((-(81 * 0^4)/4802) - (1053 * 0^3)/9604 - (3645 * 0^2)/19208 +
(2025 * 0)/38416 + 10125/38416) + 2 * g * h * (2295/6272 -
9/1568 * 0^4 - 423/6272 * 0^3 - 2763/12544 * 0^2 - 909/12544 *
0) + 2 * g * i * (45/98 - 9/49 * 0^2 - 27/98 * 0) + h^2 *
(27/64 - 1/512 * 0^4 - 17/512 * 0^3 - 45/256 * 0^2 - 27/128 *
0) + 2 * h * i * (3/8 - 1/16 * 0^2 - 5/16 * 0) + i^2 *
0)

```

```

R.1 <- (a^2 * 0 + 2 * a * b * 0 + 2 * a * c * 0 + 2 * a * d *
0 + 2 * a * e * 0 + 2 * a * f * 0 + 2 * a * g * 0 + 2 * a *
h * 0 + 2 * a * i * 0 + b^2 * (1/4096 * 0^4) + 2 * b * c *
(9/12544 * 0^4 + 3/12544 * 0^3 + 1/50176 * 0^2) + 2 * b *
d * ((225 * 0^4)/200704 + (45 * 0^3)/50176 + (9 * 0^2)/50176) +
2 * b * e * (0^4/784 + (3 * 0^3)/1568 + (9 * 0^2)/12544) +
2 * b * f * ((225 * 0^4)/200704 + (75 * 0^3)/25088 + (25 *
0^2)/12544) + 2 * b * g * ((9 * 0^4)/12544 + (45 * 0^3)/12544 +
(225 * 0^2)/50176) + 2 * b * h * (0^4/4096 + (3 * 0^3)/1024 +
(9 * 0^2)/1024) + 2 * b * i * (1/64 * 0^2) + c^2 * ((81 *
0^4)/38416 + (27 * 0^3)/19208 + (27 * 0^2)/76832 + (3 * 0)/76832 +
1/614656) + 2 * c * d * ((2025 * 0^4)/614656 + (2295 * 0^3)/614656 +
(3681 * 0^2)/2458624 + (153 * 0)/614656 + 9/614656) + 2 *
c * e * ((9 * 0^4)/2401 + (33 * 0^3)/4802 + (157 * 0^2)/38416 +
(33 * 0)/38416 + 9/153664) + 2 * c * f * ((2025 * 0^4)/614656 +
(6075 * 0^3)/614656 + (21825 * 0^2)/2458624 + (675 * 0)/307328 +
25/153664) + 2 * c * g * ((81 * 0^4)/38416 + (27 * 0^3)/2401 +
(1287 * 0^2)/76832 + (45 * 0)/9604 + 225/614656) + 2 * c *
h * ((9 * 0^4)/12544 + (111 * 0^3)/12544 + (1441 * 0^2)/50176 +
(111 * 0)/12544 + 9/12544) + 2 * c * i * (1/784 + 9/196 *
0^2 + 3/196 * 0) + d^2 * ((50625 * 0^4)/9834496 + (10125 *
0^3)/1229312 + (6075 * 0^2)/1229312 + (405 * 0)/307328 +
81/614656) + 2 * d * e * ((225 * 0^4)/38416 + (1035 * 0^3)/76832 +
(6921 * 0^2)/614656 + (621 * 0)/153664 + 81/153664) + 2 *
d * f * ((50625 * 0^4)/9834496 + (43875 * 0^3)/2458624 +
(51525 * 0^2)/2458624 + (2925 * 0)/307328 + 225/153664) +

```

```

2 * d * g * ((2025 * 0^4)/614656 + (11745 * 0^3)/614656 +
(84321 * 0^2)/2458624 + (11745 * 0)/614656 + 2025/614656) +
2 * d * h * ((225 * 0^4)/200704 + (45 * 0^3)/3136 + (1287 *
0^2)/25088 + (27 * 0)/784 + 81/12544) + 2 * d * i * (9/784 +
225/3136 * 0^2 + 45/784 * 0) + e^2 * ((16 * 0^4)/2401 + (48 *
0^3)/2401 + (54 * 0^2)/2401 + (27 * 0)/2401 + 81/38416) +
2 * e * f * ((225 * 0^4)/38416 + (1875 * 0^3)/76832 + (22825 *
0^2)/614656 + (1875 * 0)/76832 + 225/38416) + 2 * e *
g * ((9 * 0^4)/2401 + (117 * 0^3)/4802 + (2061 * 0^2)/38416 +
(1755 * 0)/38416 + 2025/153664) + 2 * e * h * (0^4/784 +
(27 * 0^3)/1568 + (873 * 0^2)/12544 + (243 * 0)/3136 + 81/3136) +
2 * e * i * (9/196 + 4/49 * 0^2 + 6/49 * 0) + f^2 * ((50625 *
0^4)/9834496 + (16875 * 0^3)/614656 + (16875 * 0^2)/307328 +
(1875 * 0)/38416 + 625/38416) + 2 * f * g * ((2025 * 0^4)/614656 +
(15525 * 0^3)/614656 + (173025 * 0^2)/2458624 + (25875 *
0)/307328 + 5625/153664) + 2 * f * h * ((225 * 0^4)/200704 +
(825 * 0^3)/50176 + (3925 * 0^2)/50176 + (825 * 0)/6272 +
225/3136) + 2 * f * i * (25/196 + 225/3136 * 0^2 + 75/392 *
0) + g^2 * ((81 * 0^4)/38416 + (405 * 0^3)/19208 + (6075 *
0^2)/76832 + (10125 * 0)/76832 + 50625/614656) + 2 * g *
h * (2025/12544 + 9/12544 * 0^4 + 153/12544 * 0^3 + 3681/50176 *
0^2 + 2295/12544 * 0) + 2 * g * i * (225/784 + 9/196 * 0^2 +
45/196 * 0) + h^2 * (81/256 + 1/4096 * 0^4 + 3/512 * 0^3 +
27/512 * 0^2 + 27/128 * 0) + 2 * h * i * (9/16 + 1/64 * 0^2 +
3/16 * 0) + i^2 * 1)

```

```
exp <- c(R.9, R.8, R.7, R.6, R.5, R.4, R.3, R.2, R.1)
```

This constructs the test statistic. Notice that it uses `prob1`, a perturbation of the true genotype frequencies. This makes no sense.

```

KF <- ((prob1[1] - exp[1])^2/exp[1] + (prob1[2] - exp[2])^2/exp[2] +
(prob1[3] - exp[3])^2/exp[3] + (prob1[4] - exp[4])^2/exp[4] +
(prob1[5] - exp[5])^2/exp[5] + (prob1[6] - exp[6])^2/exp[6] +
(prob1[7] - exp[7])^2/exp[7] + (prob1[8] - exp[8])^2/exp[8] +
(prob1[9] - exp[9])^2/exp[9]) * N
kff <- c(kff, KF)

```

We can verify that `prob1` is the same as above

```
prob1
```

```
## [1] 0.04948 0.09193 0.13758 0.16100 0.19942 0.14998 0.10952 0.06350 0.03759
```

References

Wang, Jing, Xuemin Lv, Li Feng, Ang Dong, Dan Liang, and Rongling Wu. 2021. "A Tracing Model for the Evolutionary Equilibrium of Octoploids." *Frontiers in Genetics* 12. <https://doi.org/10.3389/fgene.2021.794907>.