```
> with(LinearAlgebra):
> read `cha.txt`:
  read `coef.txt`:
  read `test.txt`:

> # deg 3 relations
  # initial i=1..8
  # by periodicity, i=9 to 26
  # are enough.

  for n from 3 to 26 do
    print(n);
    print(truncate(listkPartitions(n, 3), 4));
    print(deg3(n));
    print("--------------------------------");
  end do:
```

$$3$$

$$[[1, 1, 1]]$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{"------------------------------"}$$

$$4$$

$$[[2, 1, 1]]$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{"------------------------------"}$$

$$5$$

$$[[2, 2, 1], [3, 1, 1]]$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{"------------------------------"}$$

$$6$$

$$[[2, 2, 2], [3, 2, 1], [4, 1, 1]]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

"----------------------------"

7

[[3, 2, 2], [3, 3, 1], [4, 2, 1], [5, 1, 1]]

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"----------------------------"

8

[[3, 3, 2], [4, 2, 2], [4, 3, 1], [5, 2, 1]]

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"----------------------------"

9

[[3, 3, 3], [4, 3, 2], [4, 4, 1], [5, 2, 2]]

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"----------------------------"

10

[[4, 3, 3], [4, 4, 2], [5, 3, 2], [5, 4, 1]]

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"----------------------------"

11

[[4, 4, 3], [5, 3, 3], [5, 4, 2], [5, 5, 1]]

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"----------------------------"

12

$[[4, 4, 4], [5, 4, 3], [5, 5, 2], [6, 3, 3]]$

$$\begin{bmatrix} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & \dfrac{5}{2} \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"----------------------------"

13

$[[5, 4, 4], [5, 5, 3], [6, 4, 3], [6, 5, 2]]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"----------------------------"

14

$[[5, 5, 4], [6, 4, 4], [6, 5, 3], [6, 6, 2]]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \dfrac{3}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"----------------------------"

15

$[[5, 5, 5], [6, 5, 4], [6, 6, 3], [7, 4, 4]]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"----------------------------"

16

$[[6, 5, 5], [6, 6, 4], [7, 5, 4], [7, 6, 3]]$

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"------------------------------"

17

$[[6, 6, 5], [7, 5, 5], [7, 6, 4], [7, 7, 3]]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"------------------------------"

18

$[[6, 6, 6], [7, 6, 5], [7, 7, 4], [8, 5, 5]]$

$$\begin{bmatrix} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & \frac{5}{2} \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"------------------------------"

19

$[[7, 6, 6], [7, 7, 5], [8, 6, 5], [8, 7, 4]]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"------------------------------"

20

$[[7, 7, 6], [8, 6, 6], [8, 7, 5], [8, 8, 4]]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"------------------------------"

21

$[[7, 7, 7], [8, 7, 6], [8, 8, 5], [9, 6, 6]]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"-----------------------------"

22

$[[8, 7, 7], [8, 8, 6], [9, 7, 6], [9, 8, 5]]$

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"-----------------------------"

23

$[[8, 8, 7], [9, 7, 7], [9, 8, 6], [9, 9, 5]]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"-----------------------------"

24

$[[8, 8, 8], [9, 8, 7], [9, 9, 6], [10, 7, 7]]$

$$\begin{bmatrix} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & \frac{5}{2} \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

"-----------------------------"

25

$[[9, 8, 8], [9, 9, 7], [10, 8, 7], [10, 9, 6]]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

"-----------------------------"

$$26$$

$$[\,[9, 9, 8], [10, 8, 8], [10, 9, 7], [10, 10, 6]\,]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \dfrac{3}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{"----------------------------"}$$

(1)

```
> # Computation of the Matrix from the proof of the
  # degree 3 exceptional forbidden triplets.

  for n from 13 to 25 by 6 do
    print(n);
    print(truncate(listkPartitions(n, 3), 6));
    print(truncate(listkPartitions(n+1, 3), 4));
    print(deg3ex(n));
    print("--------------------------------");
  end do:
```

$$13$$

$$[\,[5, 4, 4], [5, 5, 3], [6, 4, 3], [6, 5, 2], [6, 6, 1], [7, 3, 3]\,]$$

$$[\,[5, 5, 4], [6, 4, 4], [6, 5, 3], [6, 6, 2]\,]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & \dfrac{3}{2} \\ 0 & 0 & 1 & 0 & -\dfrac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \dfrac{3}{2} & 0 & 0 & 0 & 0 & -\dfrac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dfrac{3}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{"----------------------------"}$$

$$19$$

$$[\,[7, 6, 6], [7, 7, 5], [8, 6, 5], [8, 7, 4], [8, 8, 3], [9, 5, 5]\,]$$

$$[\,[7, 7, 6], [8, 6, 6], [8, 7, 5], [8, 8, 4]\,]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & \dfrac{3}{2} \\ 0 & 0 & 1 & 0 & -\dfrac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \dfrac{3}{2} & 0 & 0 & 0 & 0 & -\dfrac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dfrac{3}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"------------------------------"

25

$[[9, 8, 8], [9, 9, 7], [10, 8, 7], [10, 9, 6], [10, 10, 5], [11, 7, 7]]$

$[[9, 9, 8], [10, 8, 8], [10, 9, 7], [10, 10, 6]]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & \dfrac{3}{2} \\ 0 & 0 & 1 & 0 & -\dfrac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \dfrac{3}{2} & 0 & 0 & 0 & 0 & -\dfrac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dfrac{3}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"------------------------------"

(2)

```
> # Computation of the matrix for the initial condition
  # of the (0,2)-module showing that (7,4,2) is reducible.

  n := 13;
  L := truncate(listkPartitions(n, 3), 7);
  for i from 1 to 4 do
    r[i] := map(c||i, L);
  end do:
```

```
r[5] := [0$7]:
L1 := listkPartitions(7,2);
for i from 1 to 7 do
  for x in L1 do
    if L[i] = sort([6, op(x)], `>`) then
      r[5][i] := r[5][i] + c(x);
    end if;
  end do;
end do:

L2 := listkPartitions(6,2);
for i from 1 to 7 do
  for x in L2 do
    if L[i] = sort([7, op(x)], `>`) then
      r[5][i] := r[5][i] + 6*cb(x);
    end if;
  end do;
end do:

M := Matrix([seq(r[i], i=1..5)]):
ReducedRowEchelonForm(M);
```

$$n := 13$$

$$L := [[5, 4, 4], [5, 5, 3], [6, 4, 3], [6, 5, 2], [6, 6, 1], [7, 3, 3], [7, 4, 2]]$$

$$L1 := [[4, 3], [5, 2], [6, 1]]$$

$$L2 := [[3, 3], [4, 2], [5, 1]]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 & 0 & 2 \\ 0 & 1 & 0 & 0 & -3 & 0 & -6 \\ 0 & 0 & 1 & 0 & -\dfrac{1}{2} & 0 & 2 \\ 0 & 0 & 0 & 1 & \dfrac{3}{2} & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(3)