Pose-graph SLAM, Occupancy Grid Mapping

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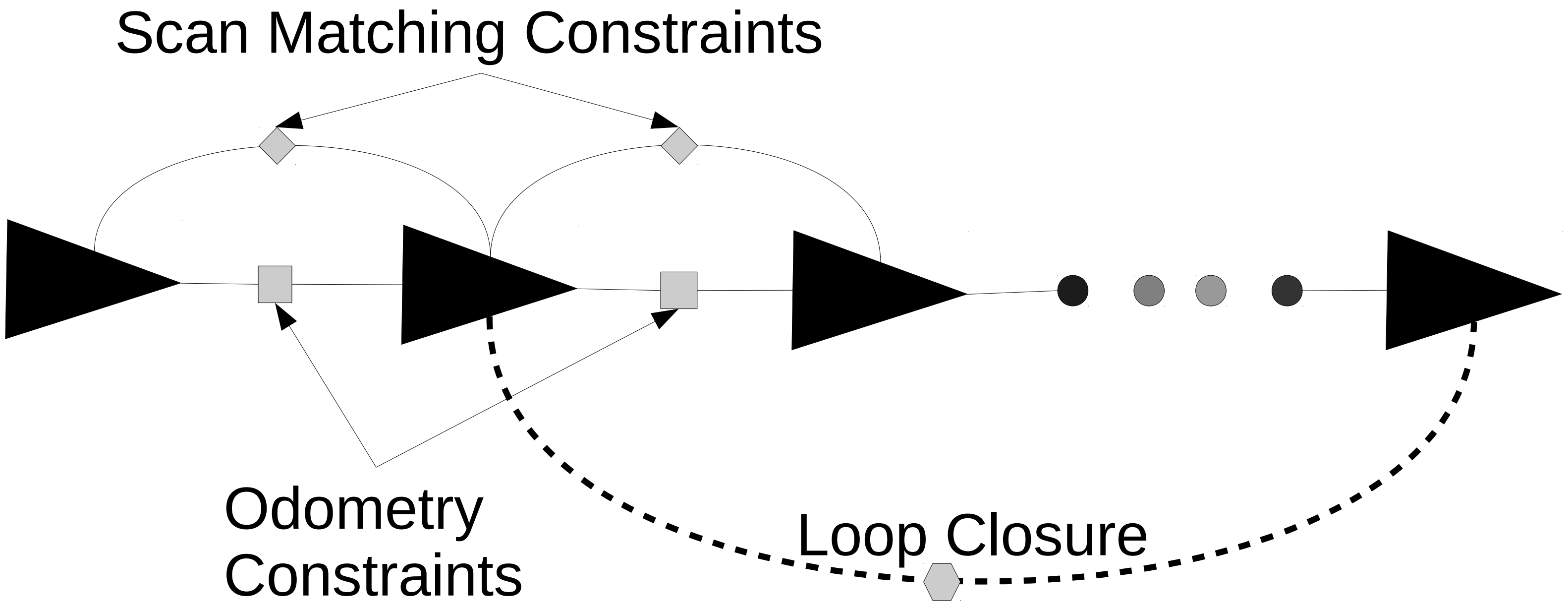
Pose Graph SLAM

- Objective is to find the complete pose of the robot



Pose graph SLAM

- Let $\mathbf{p} = (p_1, p_2, ..., p_n)^T$ represent the poses of the robot.

$$p_i = [\mathbf{t}_i, \theta_i]^T \qquad \mathbf{t}_i = [x_i, y_i, z_i]^T \qquad \theta_i = [\phi_i, \omega_i, \psi_i]$$

- Let $\mathbf{z}_{ij} = [\mathbf{t}_{ij}, \theta_{ij}]^T$ be the relative constraint between ith and jth pose (node)

$$\mathbf{z}_{ij} = \mathbf{p}_j \ominus \mathbf{p}_i + \epsilon_{ij} = \begin{bmatrix} R(\theta_i)^\top (\mathbf{t}_j - \mathbf{t}_i) \\ \theta_j - \theta_i \end{bmatrix} + \epsilon_{ij}$$

Pose graph SLAM

- The robot's task is to obtain optimal estimate of robot poses given relative pose measurements and their uncertainties. Since these measurements are modeled using Gaussian distribution, negative log likelihood of each measurement can be written as,

$$
\begin{aligned}
f(p_i, p_j) &= \epsilon_{ij}^T \Sigma_{ij}^{-1} \epsilon_{ij} \\
&= (p_j \ominus p_i - z_{ij})^T \Sigma_{ij}^{-1} (p_j \ominus p_i - z_{ij})
\end{aligned}
$$

Pose graph SLAM

- Overall negative log likelihood assuming independent measurements

$$
\begin{aligned}
F(\mathbf{p}) &= \sum_{ij} f(p_i, p_j) \\
&= \sum_{ij} \epsilon_{ij}^T \Sigma_{ij}^{-1} \epsilon_{ij} \\
&= \sum_{ij} (p_j \ominus p_i - z_{ij})^T \Sigma_{ij}^{-1} (p_j \ominus p_i - z_{ij}) \\
&= \sum_{ij} \left\| \begin{pmatrix} R(\theta_i)^T(\mathbf{t}_j - \mathbf{t}_i) - \mathbf{t}_{ij} \\ \theta_j - \theta_i - \theta_{ij} \end{pmatrix} \right\|_{\Sigma_{ij}}^2
\end{aligned}
$$

$$\mathbf{p}^* = \arg\min \ F(\mathbf{p})$$

Pose Graph SLAM

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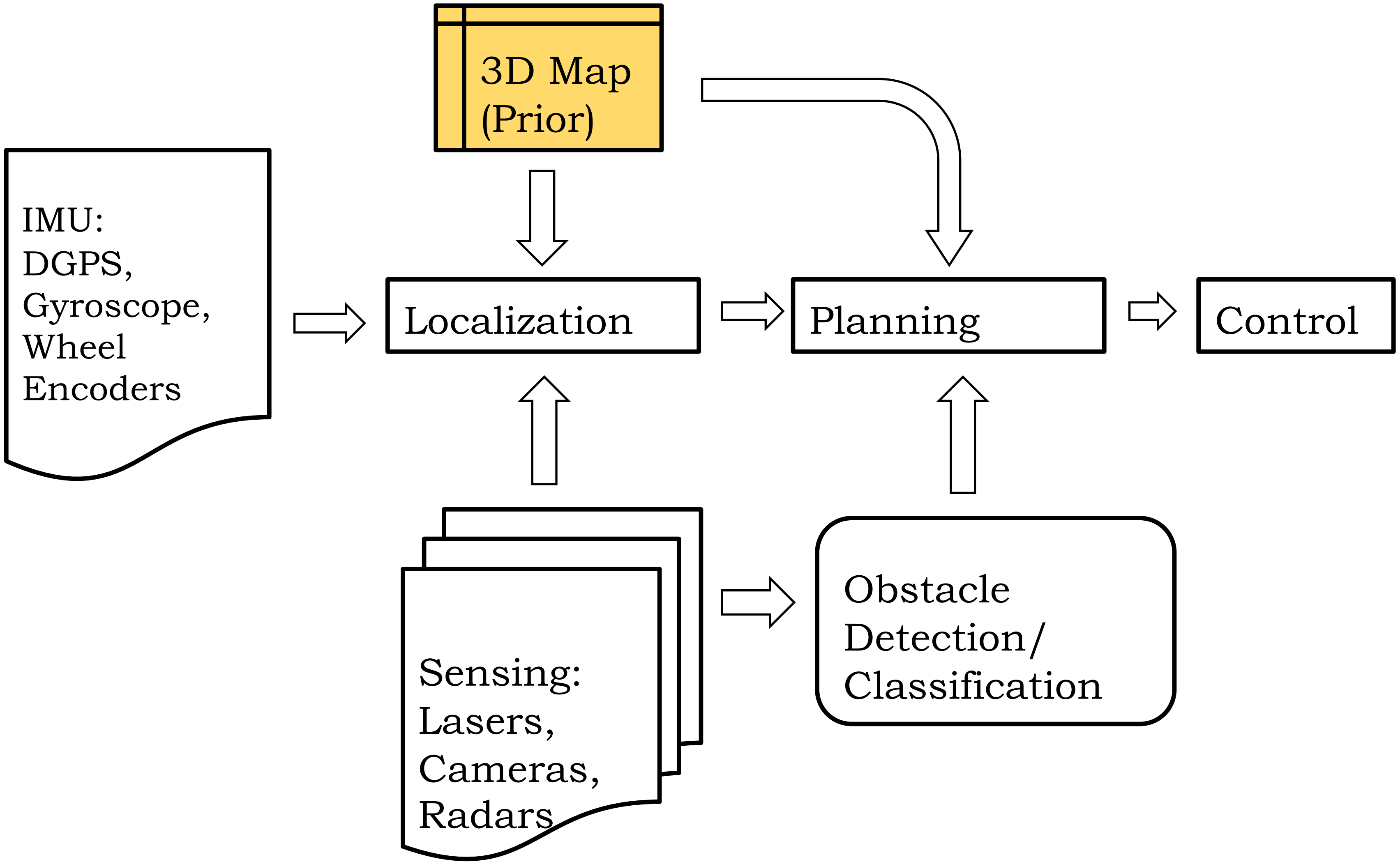
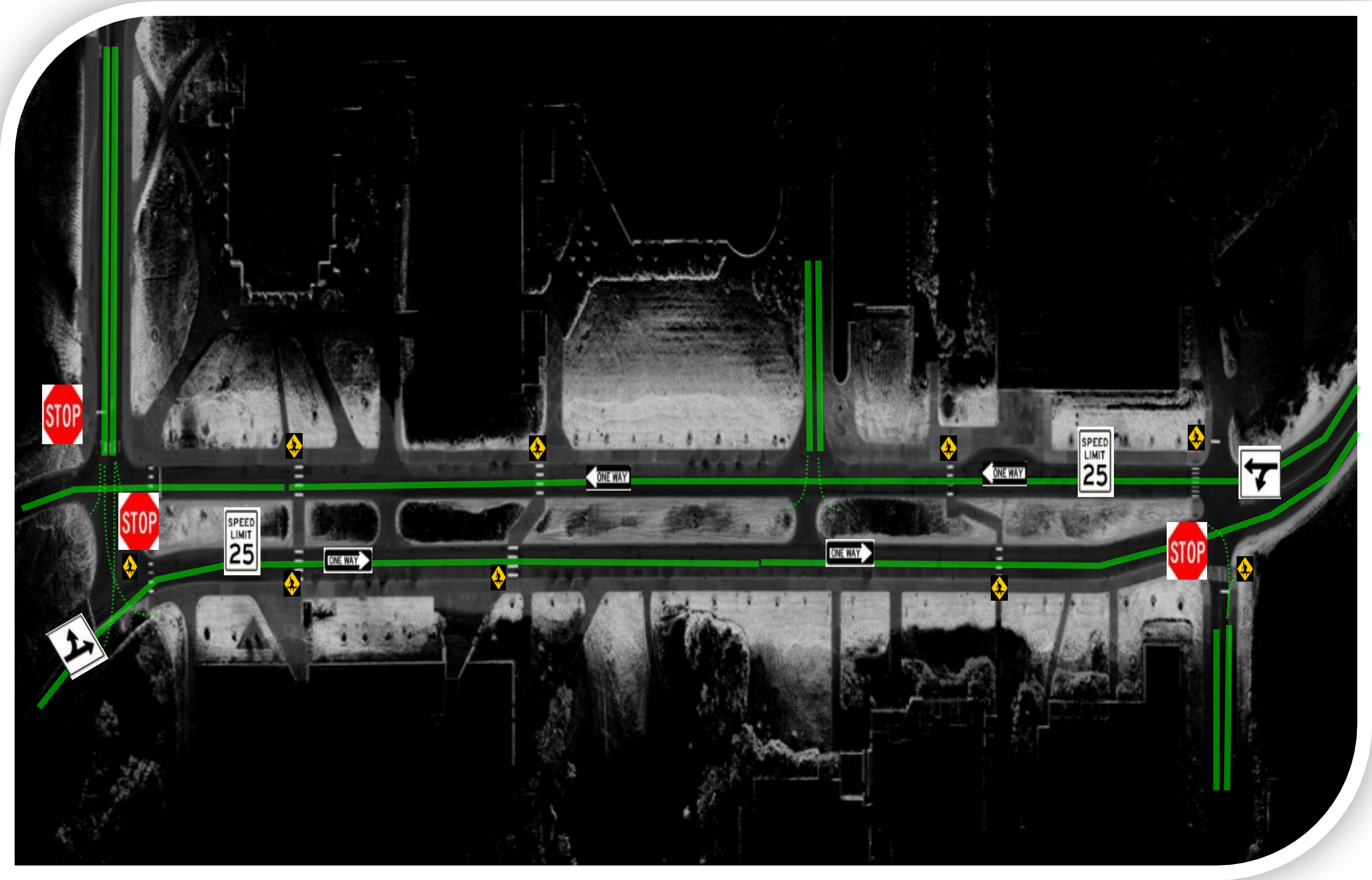
- R. Kmmerle, G. Grisetti, H. Strasdat, K. Konolige, and W. Burgard, "G2o: A general framework for graph optimization," in *Robotics and Automation (ICRA), 2011 IEEE International Conference on*, May 2011, pp. 3607–3613.
- M. Kaess, H. Johannsson, R. Roberts, V. Ila, J. J. Leonard, and F. Dellaert, "isam2: Incremental smoothing and mapping using the bayes tree," *The International Journal of Robotics Research*, p. 0278364911430419, 2011.
- L. Carlone, R. Aragues, J. A. Castellanos, and B. Bona, "A fast and accurate approximation for planar pose graph optimization," *The International Journal of Robotics Research*, vol. 33, no. 7, pp. 965–987, 2014.

Autonomous Navigation System

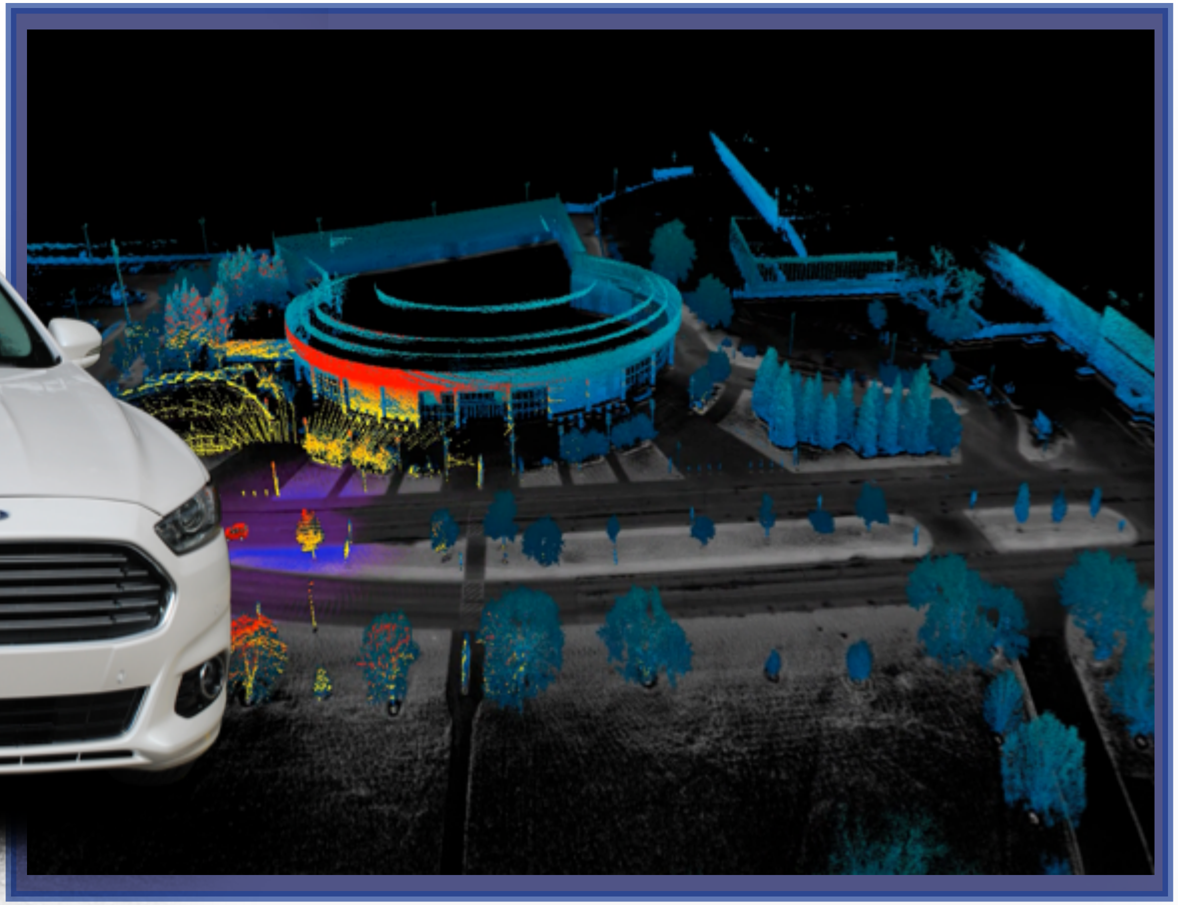


Prior map

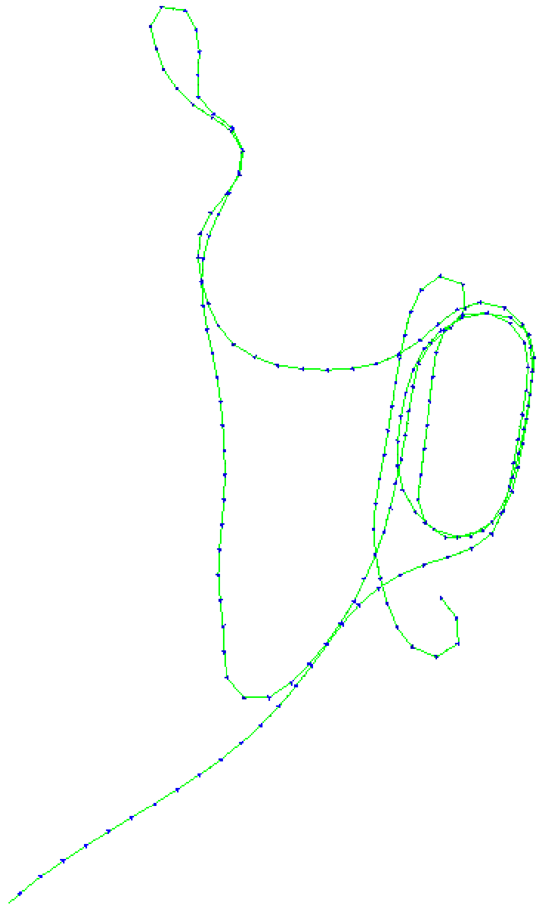


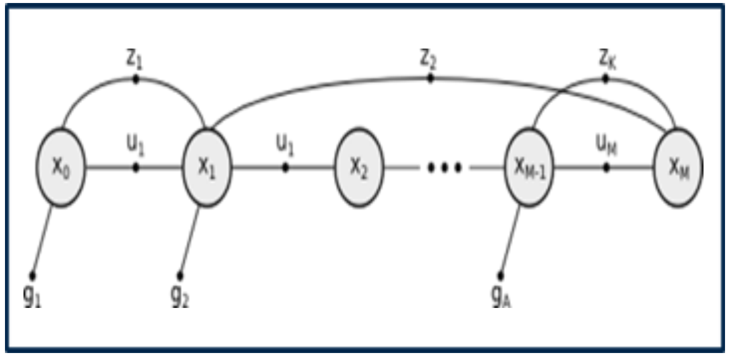
STOP
STOP
SPEED LIMIT 25
ONE WAY
ONE WAY
ONE WAY
ONE WAY
SPEED LIMIT 25
STOP

Velodyne Laser Scanner



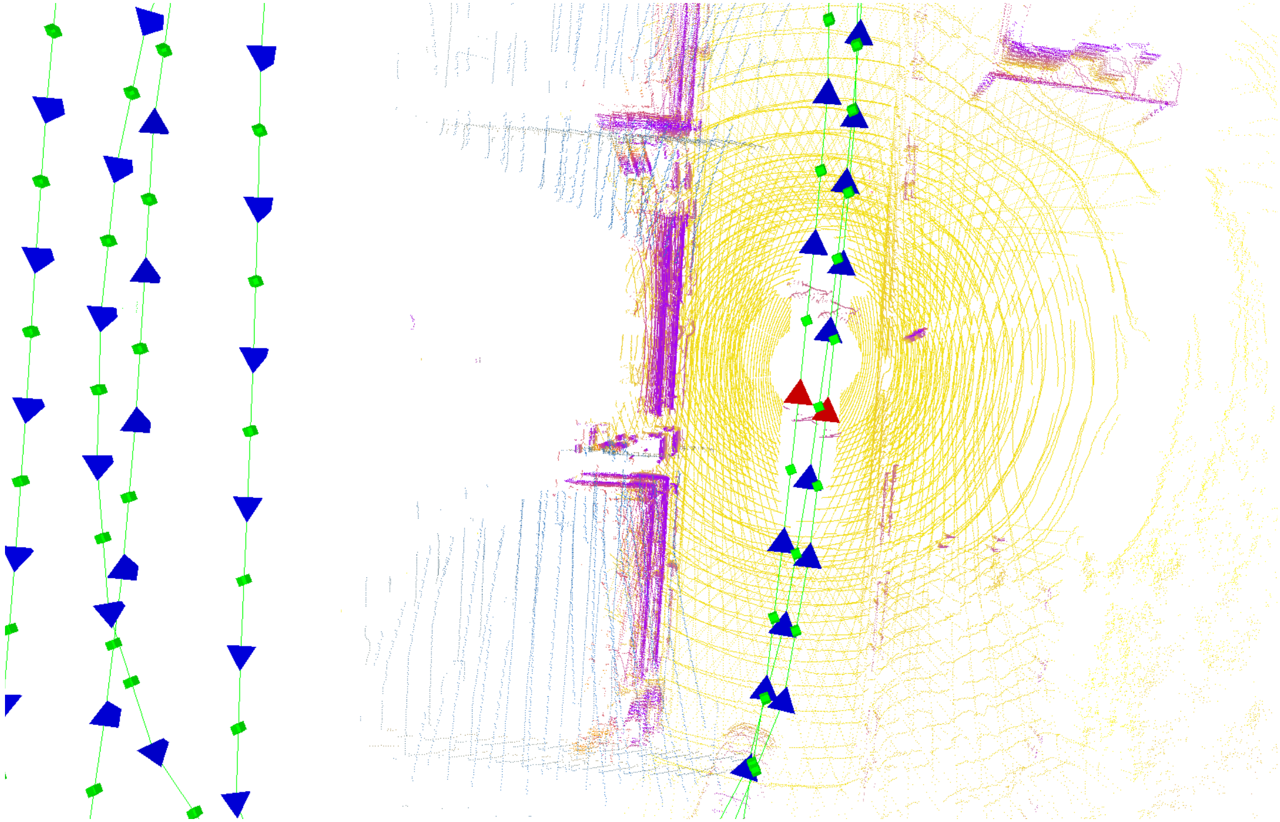
Pose-graph SLAM



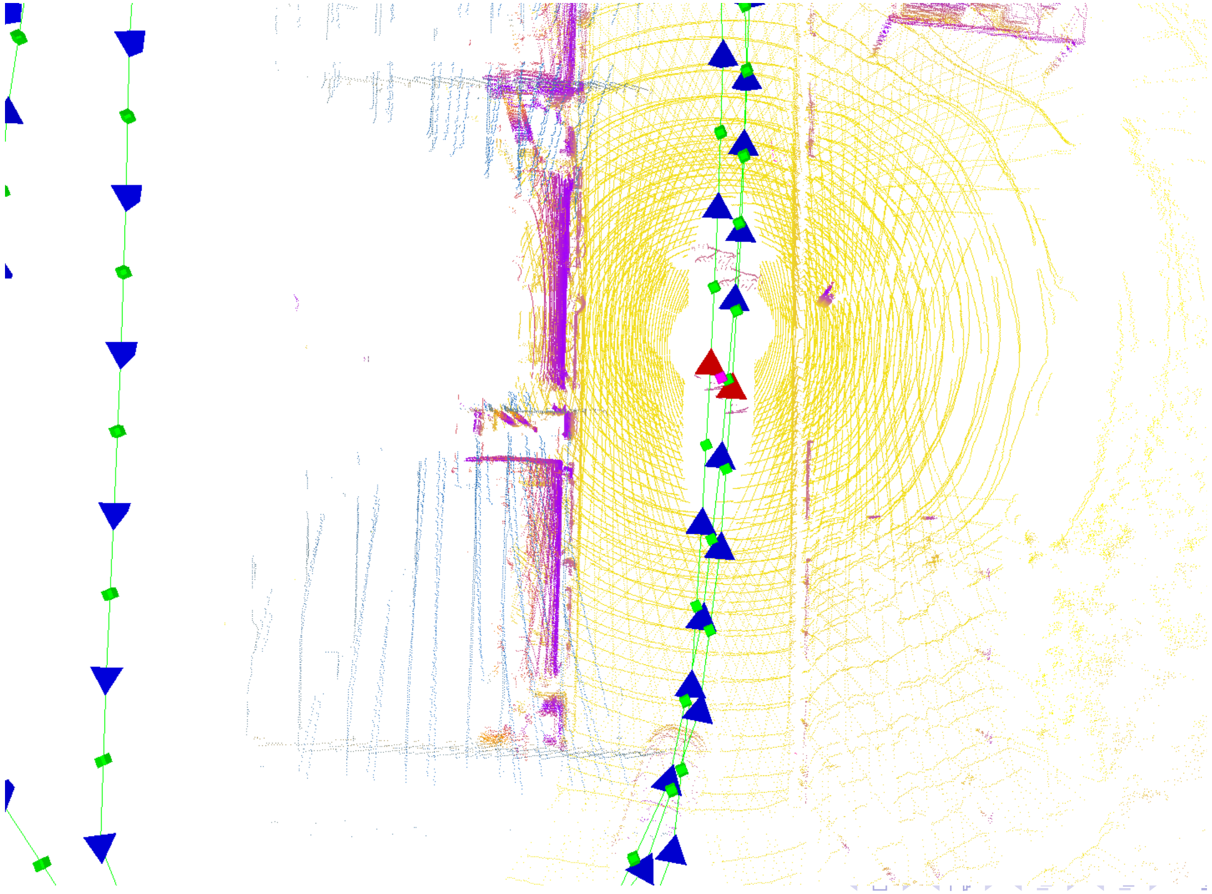


Pose-graph SLAM
X: Unknown poses
Z: Scan-matching constraints
U: Odometry constraints
G: GPS prior constraints

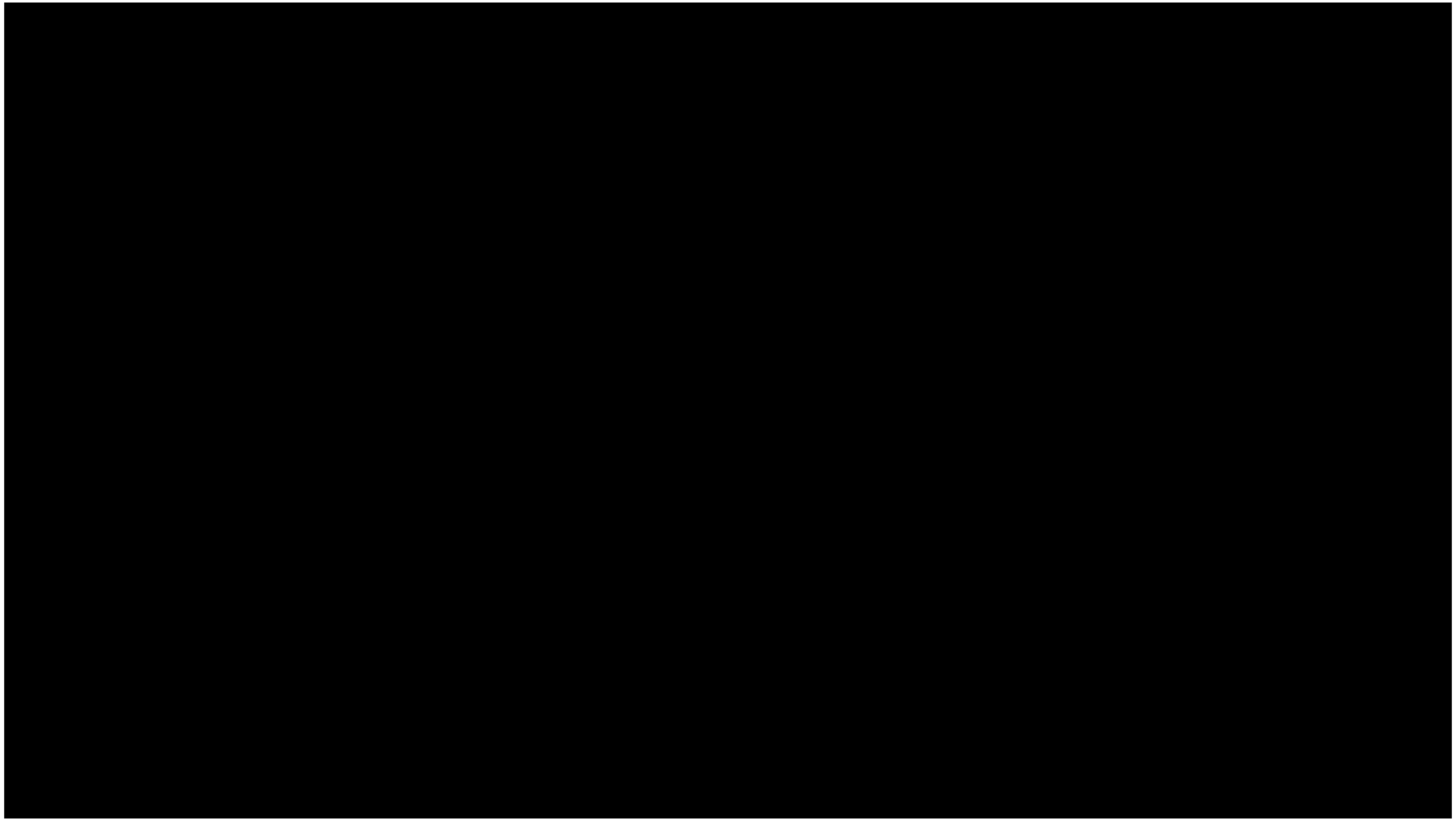
Pose graph SLAM



Pose graph SLAM



Prior map



Occupancy Grid Map

- In occupancy grid map we divide the world into grid cells and estimate the occupancy of each cell (a binary probability).
- Mapping problem posed as an optimization of the posterior as shown below, assuming pose and measurements are known:

$$m^* = \arg\max_m P(m|x_{1:t}, z_{1:t})$$

- Problem with this:
  - High dimensionality of map. Consider a map of 100x100 (=10000) grid cells with each covering 25cm x 25cm area, the total area covered is only 25m x 25m. For this map what is the total number of possible maps that you can get ??

$$2^{10000}$$

Calculating the posterior
is therefore intractable

How to make the occupancy grid mapping tractable ?

- Simplify the problem by assuming independence of grid cells. Thus the problem of estimating the full posterior breaks down to estimation of occupancy of each grid cell

$$P(m|x_{1:t}, z_{1:t}) = \prod_{i=1}^{i=n} P(m_i|z_{1:t}, x_{1:t})$$

- The estimation of the occupancy probability for each grid cell can be implemented as a **Binary Bayes Filter.** So we only need to find:

$$P(m_i|z_{1:t}, x_{1:t})$$

Occupancy of a grid cell

- Probability that i^th cell is occupied can be written as

$$P(m_i|z_{1:t}, x_{1:t}) = \frac{P(z_t|m_i, z_{1:t-1}, x_{1:t})P(m_i|z_{1:t-1}, x_{1:t})}{P(z_t|z_{1:t-1}, x_{1:t})}$$ Bayes

$$= \frac{P(z_t|m_i, x_t)P(m_i|z_{1:t-1}, x_{1:t})}{P(z_t|x_t)}$$ Markov

$$P(z_t|m_i, x_t) = \frac{P(m_i|z_t, x_t)P(z_t|x_t)}{P(m_i|x_t)}$$ Bayes

$$P(m_i|x_t) = P(m_i)$$ State is assumed to be static

Occupancy of grid cell

- Probability that i^th cell is occupied is given by

$$P(m_i|z_{1:t}, x_{1:t}) = \frac{P(m_i|z_t, x_t)P(m_i|z_{1:t-1}, x_{1:t-1})}{P(m_i)}$$

- Similarly probability that i^th cell is free is given by

$$P(\tilde{m}_i|z_{1:t}, x_{1:t}) = \frac{P(\tilde{m}_i|z_t, x_t)P(\tilde{m}_i|z_{1:t-1}, x_{1:t-1})}{P(\tilde{m}_i)}$$

$$P(\tilde{m}_i|z_{1:t}, x_{1:t}) = 1 - P(m_i|z_{1:t}, x_{1:t})$$

Binary Random Variable

Odds: Ratio of occupied vs free

- Ratio of probabilities (Odds ratio)

$$\frac{P(m_i|z_{1:t},x_{1:t})}{P(\tilde{m}_i|z_{1:t},x_{1:t})} = \frac{P(m_i|z_t,x_t)}{1-P(m_i|z_t,x_t)} \cdot \frac{P(m_i|z_{1:t-1},x_{1:t-1})}{1-P(m_i|z_{1:t-1},x_{1:t-1})} \cdot \frac{1-P(m_i)}{P(m_i)}$$

Measurement Term | Recursive Term | Prior

- Log-odds ratio: Take the log of above equation

$$\boxed{l(m_i|z_{1:t},x_{1:t}) = l(m_i|z_t,x_t) + l(m_i|z_{1:t-1},x_{1:t-1}) - l(m_i)}$$

$$l(x) = \log\frac{P(x)}{1-P(x)}$$

Complete Algorithm

Algorithm occupancy_grid_mapping($\{l_{t-1,i}\}, x_t, z_t$**):**

for all cells $\mathbf{m}_i$ do

if $\mathbf{m}_i$ in perceptual field of z_t then

$l_{t,i} = l_{t-1,i} + \textbf{inverse\_sensor\_model}(\mathbf{m}_i, x_t, z_t) - l_0$

else

$l_{t,i} = l_{t-1,i}$

endif

endfor

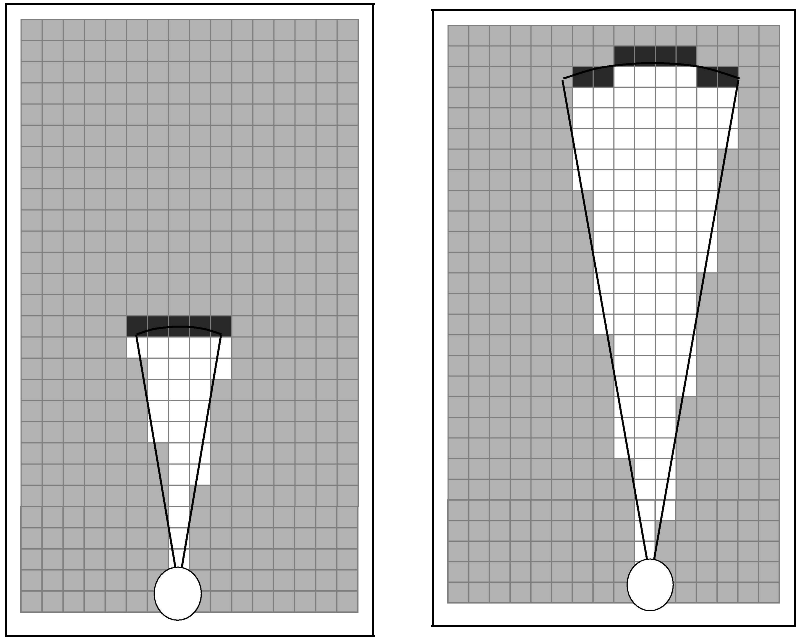
return $\{l_{t,i}\}$

Inverse Sensor Model

$$l(m_i|z_t, x_t) = \log \frac{P(m_i|z_t, x_t)}{1 - P(m_i|z_t, x_t)}$$

$$P(m_i|z_t, x_t) = \frac{P(z_t|m_i, x_t)P(m_i|x_t)}{P(z_t|x_t)}$$

$$P(m_i|z_t, x_t) = \eta P(z_t|m_i, x_t)P(m_i)$$



- Let x_i, y_i be the coordinates of ith cell then expected measurement

$$\hat{r} = \sqrt{(x_i - x_t)^2 + (y_i - y_t)^2} \qquad \hat{\phi} = tan^{-1}\frac{y_i - y_t}{x_i - x_t} - \theta_t$$

Inverse Sensor Model

- The measurement term for the occupancy grid map algorithm:

$$P(z_t = [r, \phi] | m_i = occupied, x_t) = \eta \exp(-\frac{1}{2}[r - \hat{r}, \phi - \hat{\phi}] \Sigma^{-1} \begin{bmatrix} r - \hat{r} \\ \phi - \hat{\phi} \end{bmatrix})$$

Algorithm occupancy_grid_mapping($\{l_{t-1,i}\}, x_t, z_t$**):**

for all cells $\mathbf{m}_i$ do

if $\mathbf{m}_i$ in perceptual field of z_t then

$l_{t,i} = l_{t-1,i} + \textbf{inverse\_sensor\_model}(\mathbf{m}_i, x_t, z_t) - l_0$

else

$l_{t,i} = l_{t-1,i}$

endif

endfor

return $\{l_{t,i}\}$

SLAM an important part of autonomous driving