# Probabilistic Mobile Robotics - EE698G

## Assignment- 3

- Due Date: **February 19, 2017, 11:59pm**
- Late submission penalty is 20% for every late day.
- Please submit MATLAB codes to moodle. You can submit other solutions by
  - (i) **(Recommended)** Typing it in latex and uploading the PDF to moodle along with codes.
  - (ii) Dropping handwritten solutions in the mailbox.
  - (iii) Creating a single PDF by scanning handwritten documents and uploading it to moodle – should be clear.
- **MATLAB**
  - (i) Submit a single .zip file for this assignment with every question in different directory.
  - (ii) The name of the top-level directory should be your roll number.
  - (iii) Include a **README textfile** in the .zip file. It should mention which scripts to run to generate the desired results for each question.

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**Q.1** You have two coins one biased (P(Heads) = 0.6) and one fair (P(Heads) = 0.5). You select a coin at random (with a probability of 0.5), you flip it twice and observe that it shows 'heads' in both the tosses. What is the probability that you picked a biased coin? [5 Marks]

Q.2 Suppose you are a witness to a nighttime hit-and-run accident involving a taxi in Athens. All taxi cars in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and green is 75% reliable. Is it possible to calculate the most likely color for the taxi? (Hint: distinguish carefully between the proposition that the taxi is blue and the proposition that the taxi appears blue.) What is your resulting estimate, given that 9 out of 10 Athenian taxis are green? [10 Marks]

Q.3 [**Conditioning of Gaussians**] Let $X$ and $Y$ denote two random variables that are jointly gaussian:

$$P(X,Y) = \frac{1}{2\pi\sqrt{|\Sigma|}} \exp\left\{-\frac{1}{2}\begin{bmatrix} X-\mu_X \\ Y-\mu_Y \end{bmatrix}^\top \Sigma^{-1} \begin{bmatrix} X-\mu_X \\ Y-\mu_Y \end{bmatrix}\right\}$$

where $\mu_X$, $\mu_Y$ are mean and $\Sigma = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{bmatrix}$ is the covariance matrix of the random variables $X$ and $Y$. Show that:

$$P(X|Y=y) = \frac{1}{\sqrt{2\pi\sigma_*^2}} \exp\left\{-\frac{(X-\mu_*)^2}{2\sigma_*^2}\right\}$$

where $\mu_* = \mu_X + \frac{\sigma_{XY}}{\sigma_Y^2}(y-\mu_Y)$ and $\sigma_*^2 = \sigma_X^2 - \frac{\sigma_{XY}^2}{\sigma_Y^2}$.

[10 Marks]

Q.4 Suppose that we live at a place where days are either sunny, cloudy or rainy. The weather transition function is a Markov chain with the transition table given in Table 1.

Suppose that we cannot observe the weather directly, but instead realy on a sensor. The problem is that our sensor is noisy. Its measurements are governed by the measurement model given in Table 2:

Suppose Day 1 is sunny(this is known for a fact), and in the subsequent four days our sensor observes *cloudy,cloudy,rainy,sunny*. What is the probability that Day 5 is indeed sunny as predicted by our sensor ?

[15 Marks]

Table 1: State Transition Matrix

| | | tomorrow will be | | |
|---|---|---|---|---|
| | | sunny | cloudy | rainy |
| | sunny | 0.8 | 0.2 | 0 |
| today it is .. | cloudy | 0.4 | 0.4 | 0.2 |
| | rainy | 0.2 | 0.6 | 0.2 |

Table 2: Measurement Model

| | | our sensors tell us | | |
|---|---|---|---|---|
| | | sunny | cloudy | rainy |
| | sunny | 0.6 | 0.4 | 0 |
| the actual weather is | cloudy | 0.3 | 0.7 | 0 |
| | rainy | 0 | 0 | 1 |

Q.5 [**Epipolar Constraint**] Epipolar constraint can help us in reducing the search space for Points of Interest. For any two cameras, a point $P$ which is projected to the pixel location $p_1 = (p_{x_1}, p_{y_1}, 1)$ in camera 1, is bound to be projected on a specific line in camera 2, given by $p_2' F p_1 = 0$, where $p_2$ is pixel location of point $P$ in camera 2 and F is the fundamental matrix. Given the rigid body transformation between two cameras and their intrinsic parameters, we can calculate the fundamental matrix $F$. As seen in the previous assignment, the autonomous vehicle had images from multiple cameras with the required details.

Attached with this assignment is `data_epipolar.mat` and it contains:

- Image1 : RGB Image captured from Camera1
- Image2 : RGB Image captured from Camera2
- K1 : Intrinsic parameters of camera 1
- K2 : Intrinsic parameters of camera 2
- pointOfInterest : pixel location of Point of Interest
- X_hc1 : transformation from head of camera frame to Camera 1
- X_hc2 : transformation from head of camera frame to Camera 2

For this assignment, we will take bottom of the parking meter as our point of Interest (see Fig. 1) in camera 1. Plot the line corresponding to epipolar constraint as shown in Fig. 2

Plot the epipolar line on Image 2 corresponding to pointOfInterest. [20 Marks]


Figure 1: Point of Interest in Camera 1

Q.6 [**MATLAB**] In this problem, you will implement Kalman Filter to localize an object where only an approximate model of object's dynamics is known.

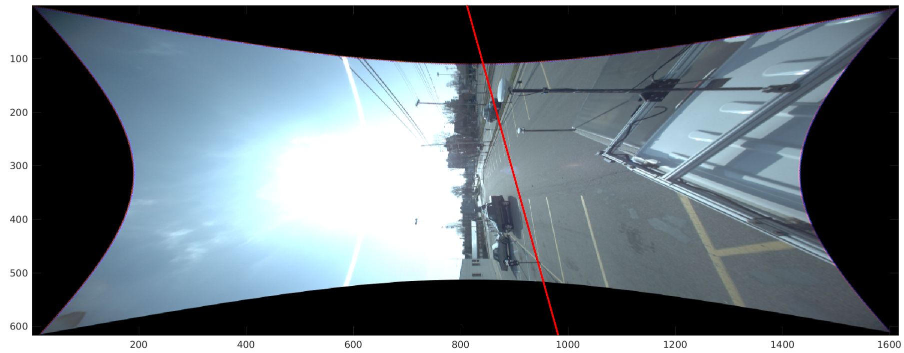

Figure 2: Epipolar constraint line in Camera 2

An object is known to be at height 0m and going upwards at speed of 300 m$s^{-1}$ at t = 0. At t=0, it starts its trajectory from this state (position and velocity) under the influence of gravity and other un-modelled external factors (here, drag caused by air). Assume we measure the height of the object directly with an error following zero-mean Gaussian distribution with a standard deviation ($\sqrt{Q_t}$) of 200m. Note : $Q_t$ is measurement model noise variance and $R_t$ is process model noise variance.

Model the given scenario as a discrete time linear system with time-step of 0.1 s (i.e $\Delta t = 0.1s$) and take g to be $9.8ms^{-2}$ (ignore other external forces acting on object). In the prediction step of position and velocity of object, take error covariance, $R_t$ to be $\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$.

**Data** :- data_kalman.mat is attached is with the assignment includes following.

- data.z : contains the height measurements at 10Hz as shown in Fig. 3.
- data.orig_state : contains the actual position(column 1) and velocity(column 2) of the object at 10Hz.

(a) Write the state equations to model the given conditions. Neglect all the external forces except gravity.

(b) Plot the following estimated trajectories of object alongside measurements and original trajectory as shown in Fig. 4.

  (i) Without using any measurements.

  (ii) Using Kalman Filter and measurements

(c) **Optional** Take $R_t$ to be diag($\|V_t\|^2 * 10^{-5}, \|V_t\|^2 * 10^{-5}$). Here, we have modelled the prediction error to be proportional to square of instantaneous velocity which closely resembles the actual drag experienced by the object. Plot normalized Kalman Gain and normalized $\|R_t\|$ on same figure. Comment on the relation between the two.

[20 Marks]

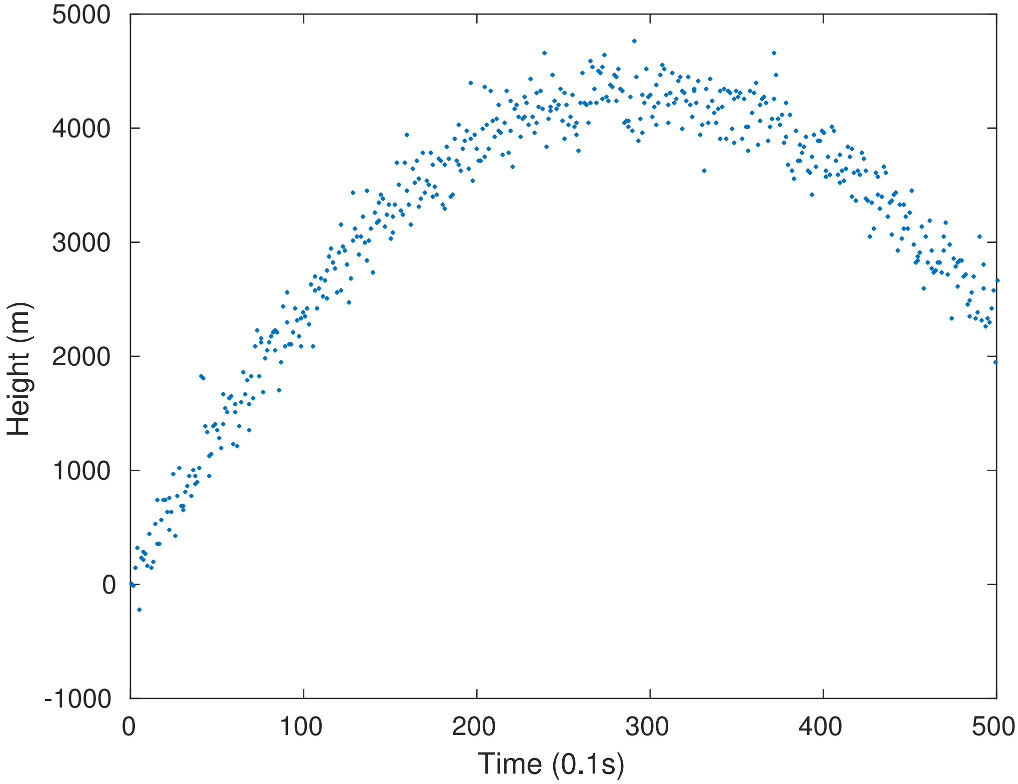


Figure 3: Height Measurements

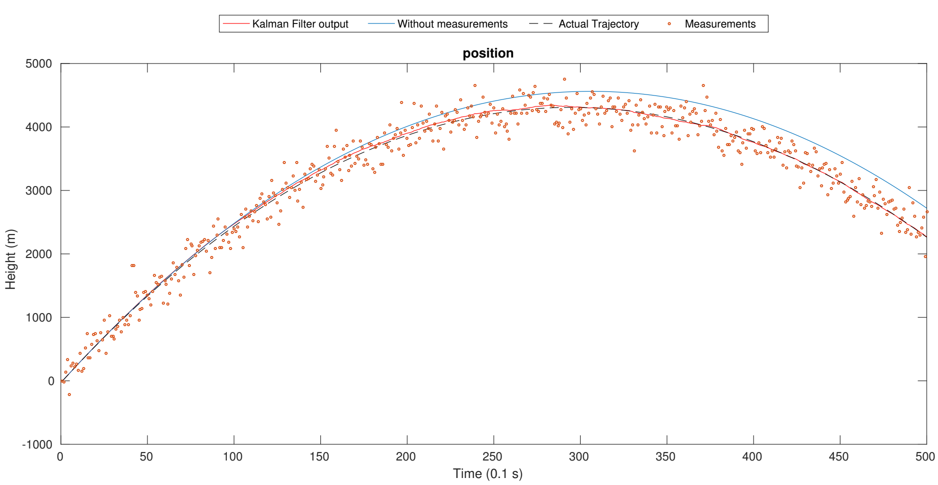


Figure 4: Trajectories