Robot State Estimation

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State Estimation

- One of the main task in robotics is to estimate the **current state** of the robot given all **past states, actions** and **measurements**
- **State, Actions & Measurements** are dependent upon the task the robot performs.
- Simple example: A robot estimating the **state** of a door (open/close) using some sensor (camera / lidar !)

State Estimation

- The state is defined as a discrete random variable X_t that changes with time. Let us assume that the robot does not know the initial state of the door:

$$P(X_0 = \text{'}\textit{open}\text{'}) = 0.5 = P(X_0 = \text{'}\textit{close}\text{'})$$

  X_t = State of the door at time "t"

- What if our sensor was perfect?
  - The robot will take the measurement and will exactly know the state of the door, and we would not have been sitting here !
- Sensors are generally noisy !
  - But we can model that noise.

Sensor Model

- Let us take the example of our robot estimating the state of door, the noise in the sensor can be characterized by the following conditional probabilities:

$$P(Z_t = \textit{sensed open} \mid X_t = \textit{is open}) = 0.6$$

$$P(Z_t = \textit{sensed close} \mid X_t = \textit{is open}) = 0.4$$

$$P(Z_t = \textit{sensed open} \mid X_t = \textit{is closed}) = 0.2$$

$$P(Z_t = \textit{sensed close} \mid X_t = \textit{is closed}) = 0.8$$

Z_t = Sensor measurement at time "t"

- What does the probabilities show ?
  - The sensor is relatively better in detecting '*closed door'.*

State Estimation at "t = 1"

- Suppose at t=1 (no action is taken) the robot measures the state of the door as "*sensed open*".

- What will be the probability that the door is open given the fact that the sensor sensed it to be open ?

$$P(X_1 = \text{'}\textit{open}\text{'} \mid Z_1 = \text{'}\textit{sensed open}\text{'}) \quad ??$$

- How do we get this conditional probability ?

Bayes Rule

- Bayes rule allows us to use causal knowledge to estimate the posterior / effect

$$P(X_1 = open \mid Z_1 = sense\ open) = \frac{P(Z_1 = sense\ open \mid X_1 = open)\, P(X_1 = open)}{P(Z_1 = sense\ open)}$$

$P(Z_1 = sense\ open \mid X_1 = open)$ = Causal knowledge / Easy to obtain [Measurement model]

$P(X_1 = open)$ = Prior knowledge = $P(X_0 = open)$ [Note: Measurement do not change the state]

$P(Z_1 = sense\ open)$ = From sensor measurement model [Law of total probability]

$P(Z_1 = sense\ open)$ =

$P(Z_1 = sense\ open \mid X_1 = open)\, P(X_1 = open)$ + $P(Z_1 = sense\ open \mid X_1 = close)\, P(X_1 = close)$

$P(X_1 = open \mid Z_1 = sense\ open) = \frac{0.6 * 0.5}{0.6 * 0.5 + 0.2 * 0.5}$ = 0.75 > $P(X_1 = open)$

The conditional probability that the door is *open* increases when the robot takes a measurement

Bayesian Estimation in Dynamic Environment

- Bayesian estimation becomes easy when the environment is static, however the real world is dynamic. Robot has to carry out actions that may or may not change the world around it.
- Let us assume in our example the robot can change the state of the door using its manipulators / hands.
- That means robot is capable of taking **actions.**
- How do we incorporate these actions into the Bayesion estimation framework ?

Modelling Actions

- The Action model can be defined as: **$P(X_t \mid U_t, X_{t-1})$** where U_t is the action taken by the robot at time "t".
- For our example let us assume that the robot performs only one action i.e. "*push*" then the **<u>state transition probabilities</u>** for this action can be given as:

$$P(X_t = \textit{open} \mid U_t = \textit{push}, X_{t-1} = \textit{open}) = 1$$

$$P(X_t = \textit{close} \mid U_t = \textit{push}, X_{t-1} = \textit{open}) = 0$$

$$P(X_t = \textit{open} \mid U_t = \textit{push}, X_{t-1} = \textit{close}) = 0.8$$

$$P(X_t = \textit{close} \mid U_t = \textit{push}, X_{t-1} = \textit{close}) = 0.2$$

Robot monitoring a Door

- Robot monitors the state of the door, if the door is sensed to be closed it pushes the door and tries to keep it open.
- At any time "t" you are given:
  - All prior measurements: $\{Z_1, Z_2, \ldots.. Z_t\}$
  - All prior control actions: $\{U_1, U_2, \ldots.U_t\}$
  - Sensor Model: $P(Z_t \mid X_t)$
  - Action Model: $P(X_t \mid U_t, X_{t-1})$
  - Prior probability of system state: $P(X_0)$
- We need to estimate state X_t, i.e. posterior of the state given all the inputs as given above:
  - $Bel(X_t) = P(X_t \mid U_{\{1:t\}}, Z_{\{1:t\}})$
- How to obtain $Bel(X_t)$?

Recursive Bayes Filter

- Bel (X_t) = P(X_t | $U_{\{1:t\}}$, $Z_{\{1:t\}}$)
- Apply Bayes Rule:

$$\text{Bel}(X_t) = \frac{P(Z_t \mid X_t , U_{\{1:t\}}, Z_{\{1:t-1\}})\, P(X_t \mid U_{\{1:t\}} , Z_{\{1:t-1\}})}{P(Z_t \mid Z_{\{1:t-1\}} , U_{\{1:t\}})}$$

Normalizing Constant
(Independent of X_t)

- Apply Markov Assumption:

$$\text{Bel}(X_t) = \eta P(Z_t \mid X_t)\, P(X_t \mid U_{\{1:t\}} ,\ Z_{\{1:t-1\}})$$

- Using Law of Total Probability:

$$\text{Bel}(X_t) = \eta P(Z_t \mid X_t) \int P(X_t \mid U_{\{1:t\}} ,\ Z_{\{1:t-1\}} , X_{t-1})\, \underline{\underline{P(X_{t-1} \mid U_{\{1:t\}} ,\ Z_{\{1:t-1\}})}}\, \mathrm{d}X_{t-1}$$

Bel (X_{t-1})

$$\text{Bel}(X_t) = \eta P(Z_t \mid X_t) \int \underline{P(X_t \mid U_t , X_{t-1})}\, Bel\,(X_{t-1})\, \mathrm{d}X_{t-1}$$

Markov Assumption

Recursive Bayes Filter Algorithm

- Bayes Filter can be written as a two step process:

Prediction Step

$$\overline{\mathrm{Bel}}\,(\mathrm{X_t}) = \int \mathrm{P}(\mathrm{X_t} \mid \mathrm{U_t}\,, \mathrm{X_{t-1}}\,)\ \mathrm{Bel}\,(\mathrm{X_{t-1}})\ \mathrm{dX_{t-1}}$$

Correction Step

Bel (Xt) = $\eta \mathrm{P}(\mathrm{Z_t} \mid \mathrm{X_t}\,)\overline{\mathrm{Bel}}\,(\mathrm{X_t})$

Recursive Bayes Filter Algorithm

- Input: Bel (X_{t-1}) , U_t , Z_t
- Output: Bel (X_t)
  - $for\ all\ \ X_t\ do$
  - $\quad if\ \ (action\ U_t\ is\ not\ NULL)$
  - $\qquad \overline{\mathrm{Bel}}\,(\mathrm{X_t}) = \sum_{\mathrm{X_t} \in \chi} \mathrm{P}(\mathrm{X_t} \mid \mathrm{U_t}\,, \mathrm{X_{t-1}}\,)\ \mathrm{Bel}\ (\mathrm{X_{t-1}})$
  - $\quad else$
  - $\qquad \overline{\mathrm{Bel}}\,(\mathrm{X_t})$ = $\mathrm{Bel}\ (\mathrm{X_{t-1}}\ )$
  - $\quad end\ if$
  - $\quad if\ \ (measurement\ Z_t\ is\ not\ NULL)$
  - $\qquad$ Bel (Xt) = $\eta \mathrm{P}(\mathrm{Z_t} \mid \mathrm{X_t}\ )\overline{\mathrm{Bel}}\,(\mathrm{X_t})$
  - $\quad else$
  - $\qquad \mathrm{Bel}\ (\mathrm{X_t}) = \ \overline{\mathrm{Bel}}\,(\mathrm{X_t})$
  - $\quad end$
  - *end for*

Bayes Filter for our Robot and Door Problem

- Sensor Model:

$$P(Z_t = \textit{sensed open} \mid X_t = \textit{is open}) = 0.6$$
$$P(Z_t = \textit{sensed close} \mid X_t = \textit{is open}) = 0.4$$
$$P(Z_t = \textit{sensed open} \mid X_t = \textit{is closed}) = 0.2$$
$$P(Z_t = \textit{sensed close} \mid X_t = \textit{is closed}) = 0.8$$

- Action Model:

$$P(X_t = \textit{open} \mid U_t = \textit{push}, X_{t-1} = \textit{open}) = 1$$
$$P(X_t = \textit{close} \mid U_t = \textit{push}, X_{t-1} = \textit{open}) = 0$$
$$P(X_t = \textit{open} \mid U_t = \textit{push}, X_{t-1} = \textit{close}) = 0.8$$
$$P(X_t = \textit{close} \mid U_t = \textit{push}, X_{t-1} = \textit{close}) = 0.2$$

"t = 1"

- <u>U_1 = *No Action* , Z_1 = *sensed open*</u>

$$Bel\ (X_1 = open\ ) = \eta\ P(Z_1 = sense\ open \mid X_1 = open)\ \overline{Bel}(X_1 = open)$$
$$= \eta\ 0.6\ * 0.5 = \eta 0.3$$
$$Bel\ (X_1 = close) = \eta\ P(Z_1 = sense\ open \mid X_1 = close)\ \overline{Bel}(X_1 = close)$$
$$= \eta\ 0.2\ * 0.5 = \eta 0.1$$

$$Bel\ (X_1 = open\ ) + Bel\ (X_1 = close\ ) = 1$$
$$\Rightarrow \eta 0.3 + \eta 0.1 = 1\ \Rightarrow \eta = 2.5$$
$$\left.\begin{array}{l} \Rightarrow Bel\ (X_1 = open\ ) = 0.75 \\ \Rightarrow Bel(X_1 = close) = 0.25 \end{array}\right\}$$

Same as applying Bayes Rule !!

Recall Bayes Rule

- Bayes rule allows us to use causal knowledge to estimate the posterior / effect

$$P(X_1 = open \mid Z_1 = sense\ open) = \frac{P(Z_1 = sense\ open \mid X_1 = open)\, P(X_1 = open)}{P(Z_1 = sense\ open)}$$

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$P(Z_1 = sense\ open \mid X_1 = open)\, P(X_1 = open)$ + $P(Z_1 = sense\ open \mid X_1 = close)\, P(X_1 = close)$

$P(X_1 = open \mid Z_1 = sense\ open) = \frac{0.6 * 0.5}{0.6 * 0.5 + 0.2 * 0.5}$ = 0.75 > $P(X_1 = open)$

The conditional probability that the door is *open* increases when the robot takes a measurement

"t = 2"

- U_2 = *push* , Z_2 = *sense open* , Bel (X_1 = *open*) = 0.75
- Prediction Step:

$$\overline{\mathrm{Bel}}\,(\mathrm{X}_2) = \sum_{\mathrm{X}_1 \in \chi} \mathrm{P}(\mathrm{X}_2 \mid \mathrm{U}_2 , \mathrm{X}_1)\ \mathrm{Bel}\ (\mathrm{X}_1)$$

$$\overline{\mathrm{Bel}}(\mathrm{X}_2 = \mathrm{open}\ ) = \mathrm{P}(\mathrm{X}_2 = \mathrm{open} \mid \mathrm{U}_2 = \mathrm{push} , \mathrm{X}_1 = \mathrm{open}\ )\ \mathrm{Bel}\ (\mathrm{X}_1 = \mathrm{open}) + \mathrm{P}(\mathrm{X}_2 = \mathrm{open} \mid \mathrm{U}_2 = \mathrm{push} , \mathrm{X}_1 = \mathrm{close})\ \mathrm{Bel}\ (\mathrm{X}_1 = \mathrm{close})$$

$$= 0 * 0.75 + 0.8 * 0.25 = 0.95$$

$$\overline{\mathrm{Bel}}(\mathrm{X}_2 = \mathrm{close}\ ) = 0 * 0.75 + 0.2 * 0.25 = 0.05$$

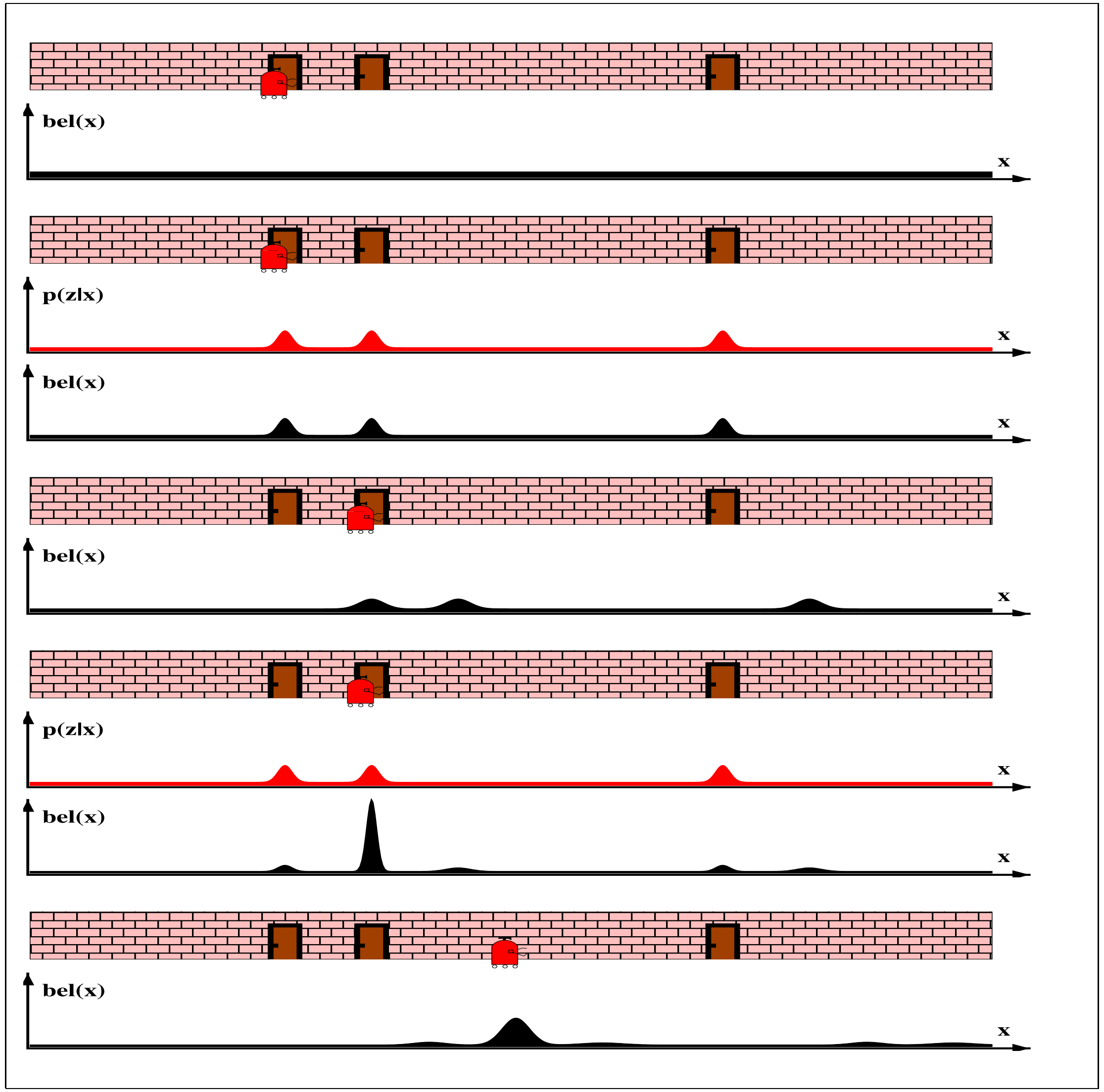
- Correction Step:

$$\mathrm{Bel}\ (\mathrm{X}_2) = \eta \mathrm{P}(\mathrm{Z}_2 | \mathrm{X}_2) \overline{\mathrm{Bel}}\,(\mathrm{X}_2)$$

$$Bel\ (X_2 = open\ ) = \eta P(Z_2 = sensed\ open \mid X_2 = open) \overline{\mathrm{Bel}}\,(\mathrm{X}_2 = open)$$

$$\left.\begin{aligned} &= \eta * 0.6 * 0.95 \\ Bel\ (X_2 = close) &= \eta * 0.2 * 0.05 \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} &Bel\ (X_2 = open\ ) = 0.983 \\ &Bel\ (X_2 = open\ ) = 0.017 \end{aligned}}$$

Localization using Bayes Filter



Summary

- Bayes rule allows us to use causal knowledge to estimate the posterior / effect.
- Bayes filter allows us to estimate robot state in a dynamic environment.
- Sensor Model: Models the sensor measurement.
- Action Model: Models the robot actions i.e. how the actions change the state of the robot. Defined as state transition probabilities.
- Markov Assumption: The current robot state only depends on the previous state.