EE698G
Probabilistic Mobile Robotics

  

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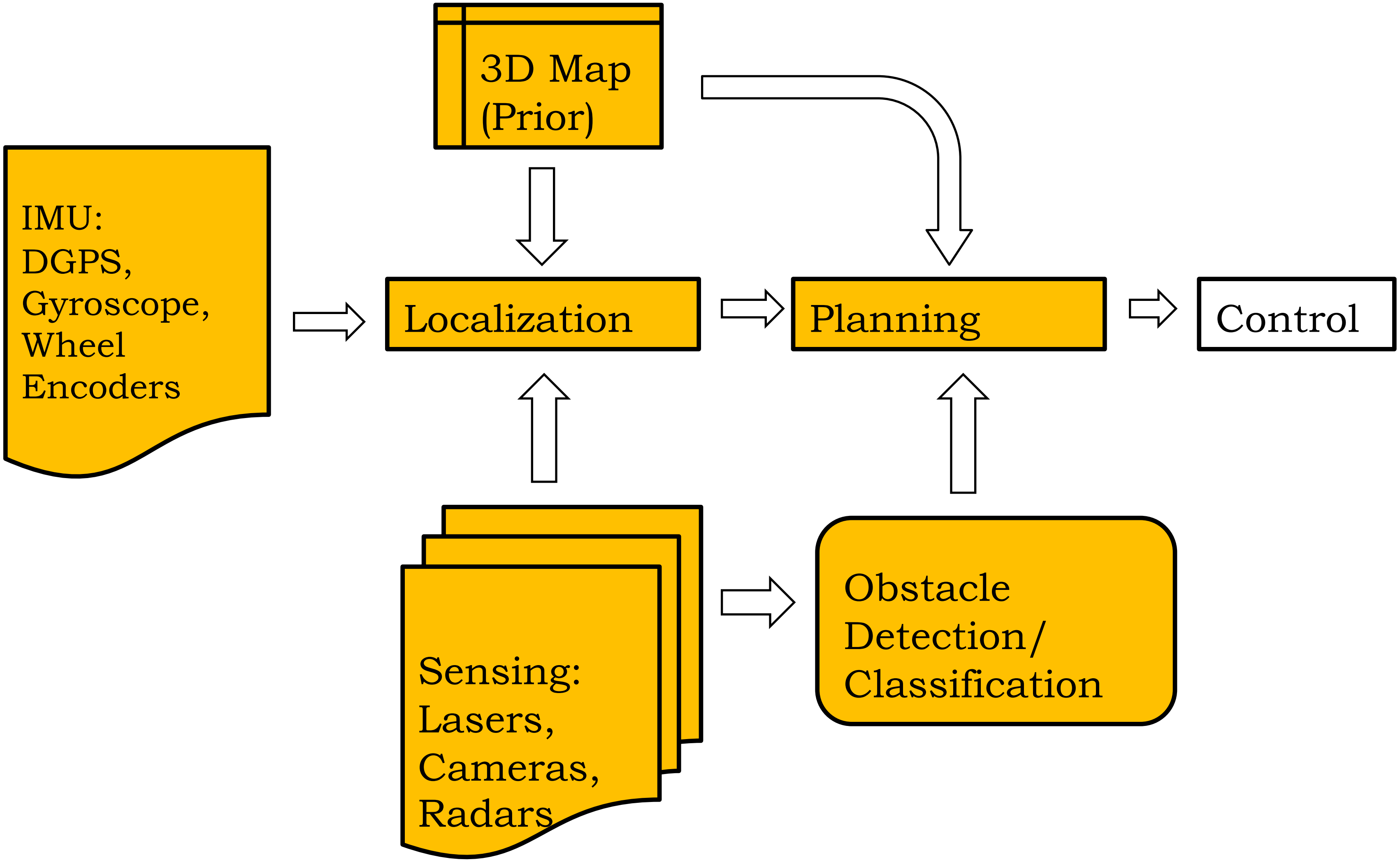
Autonomous Navigation System



Perception sensors

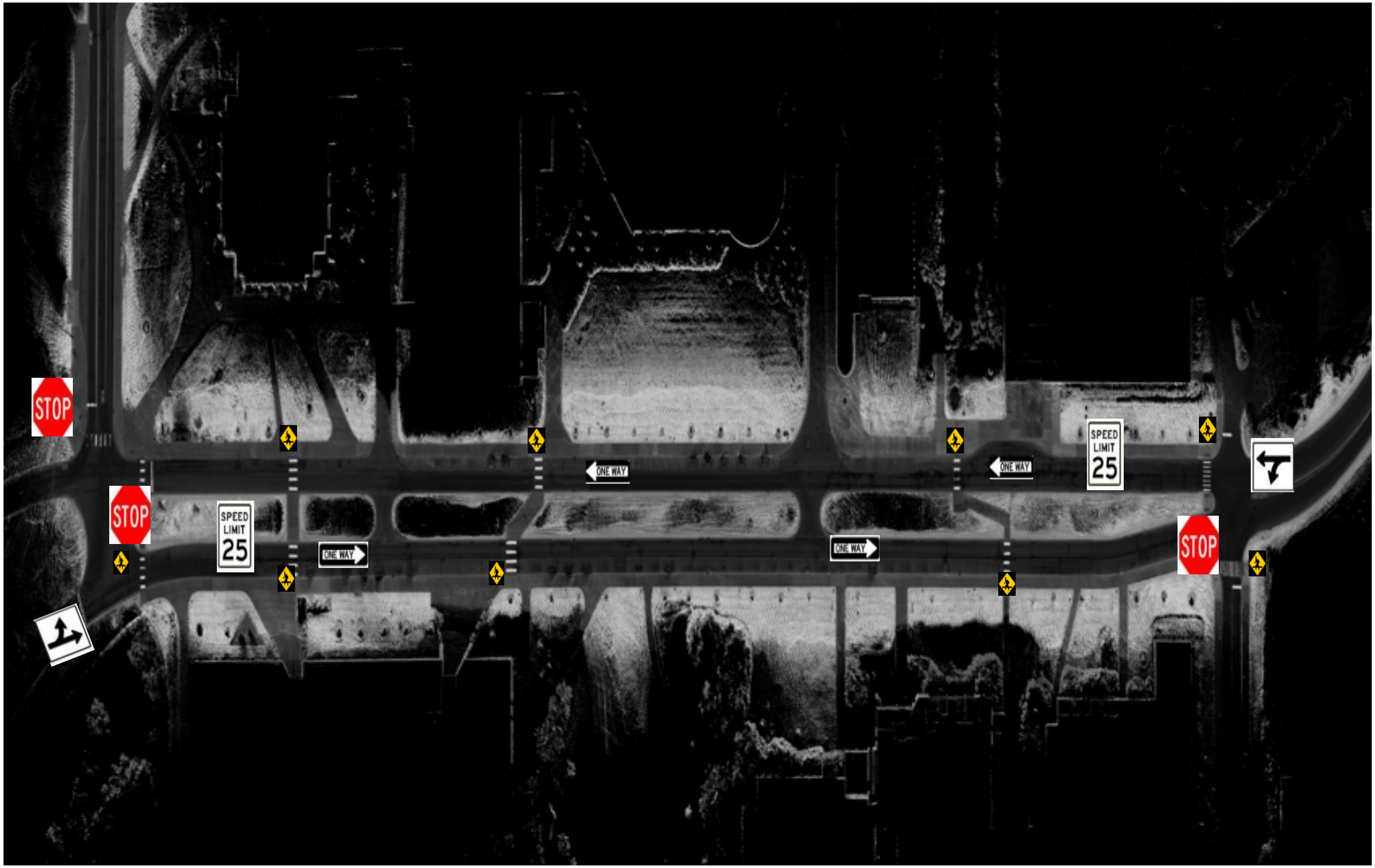
Velodyne LIDAR

Delphi RADAR

Point Grey
Camera

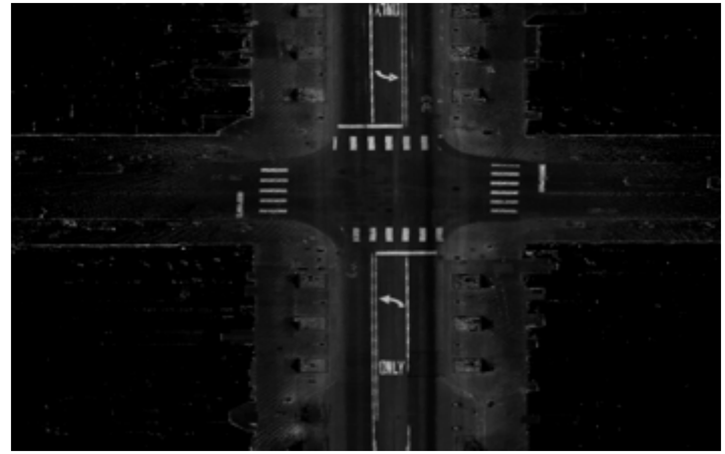
Ladybug3
Camera

Prior map

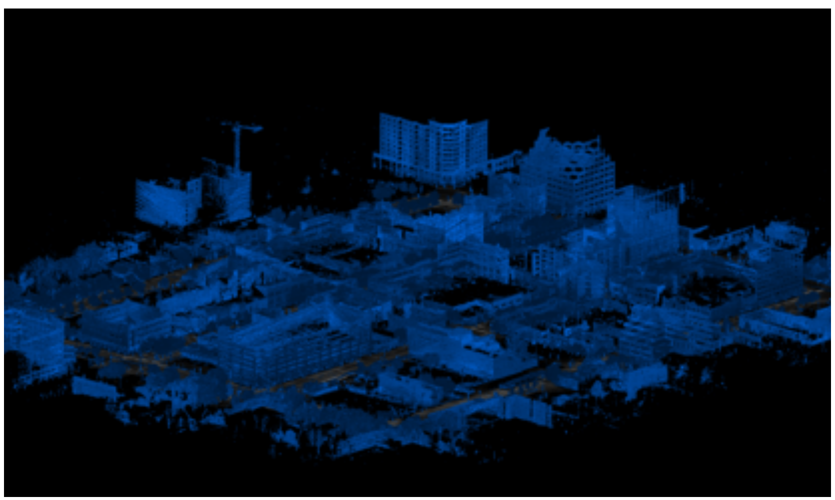

STOP
STOP
STOP
SPEED LIMIT 25
SPEED LIMIT 25
ONE WAY
ONE WAY
ONE WAY
ONE WAY

Prior map data

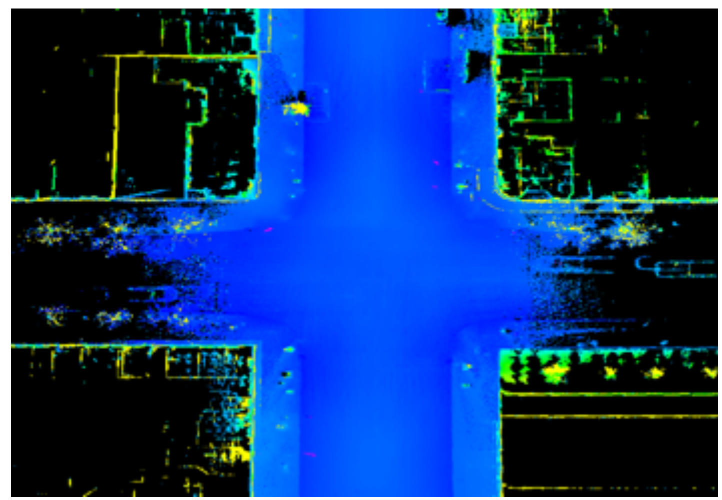
Satellite Imagery



Lidar Reflectivity



3D point cloud



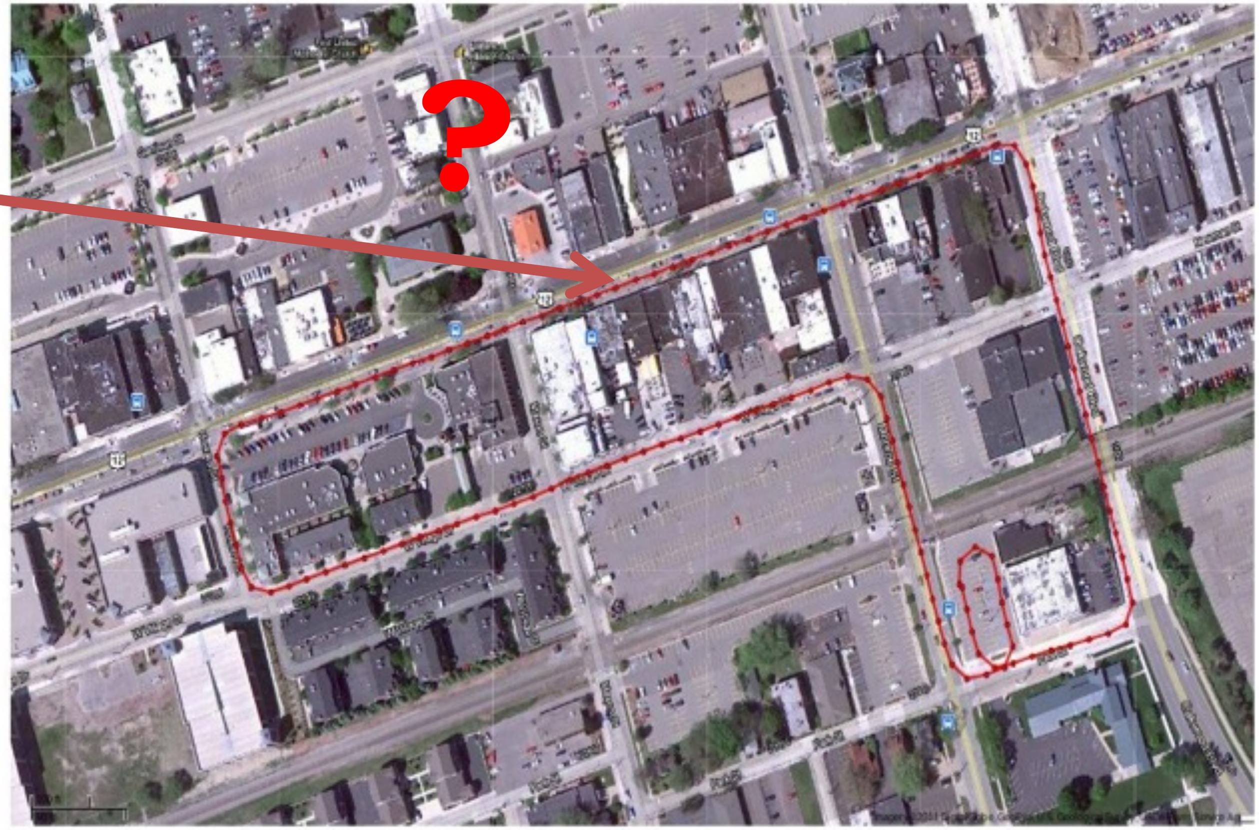
Z-height data

Localization: Where am I in the map ?

Current Observation

- GPS
- Lidar Data
    - Reflectivity
    - 3D Point cloud
- Camera Imagery

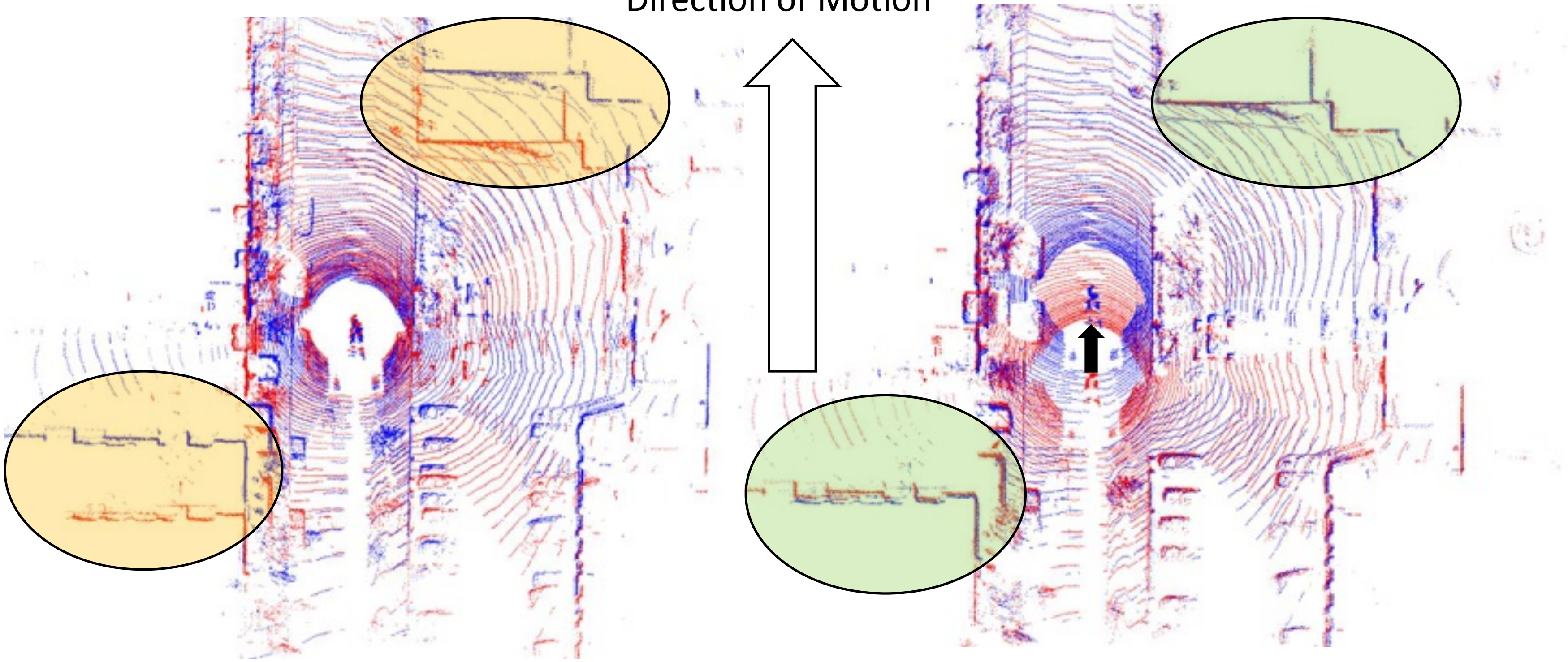
MAP



Intensity localization

3D scan matching

Direction of Motion

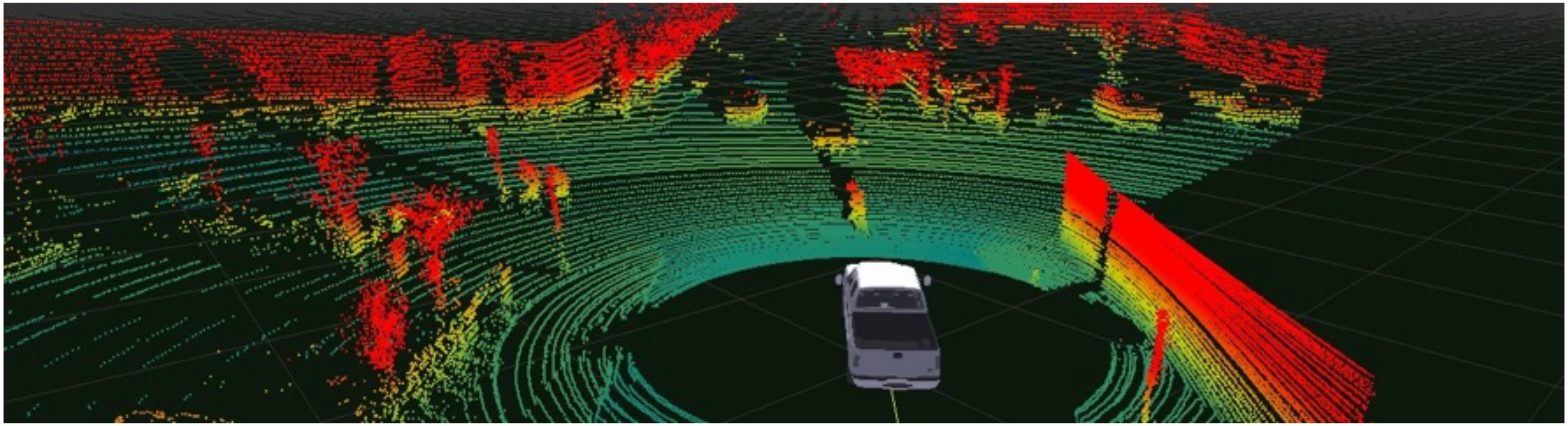


Before Scan Matching

After Scan Matching

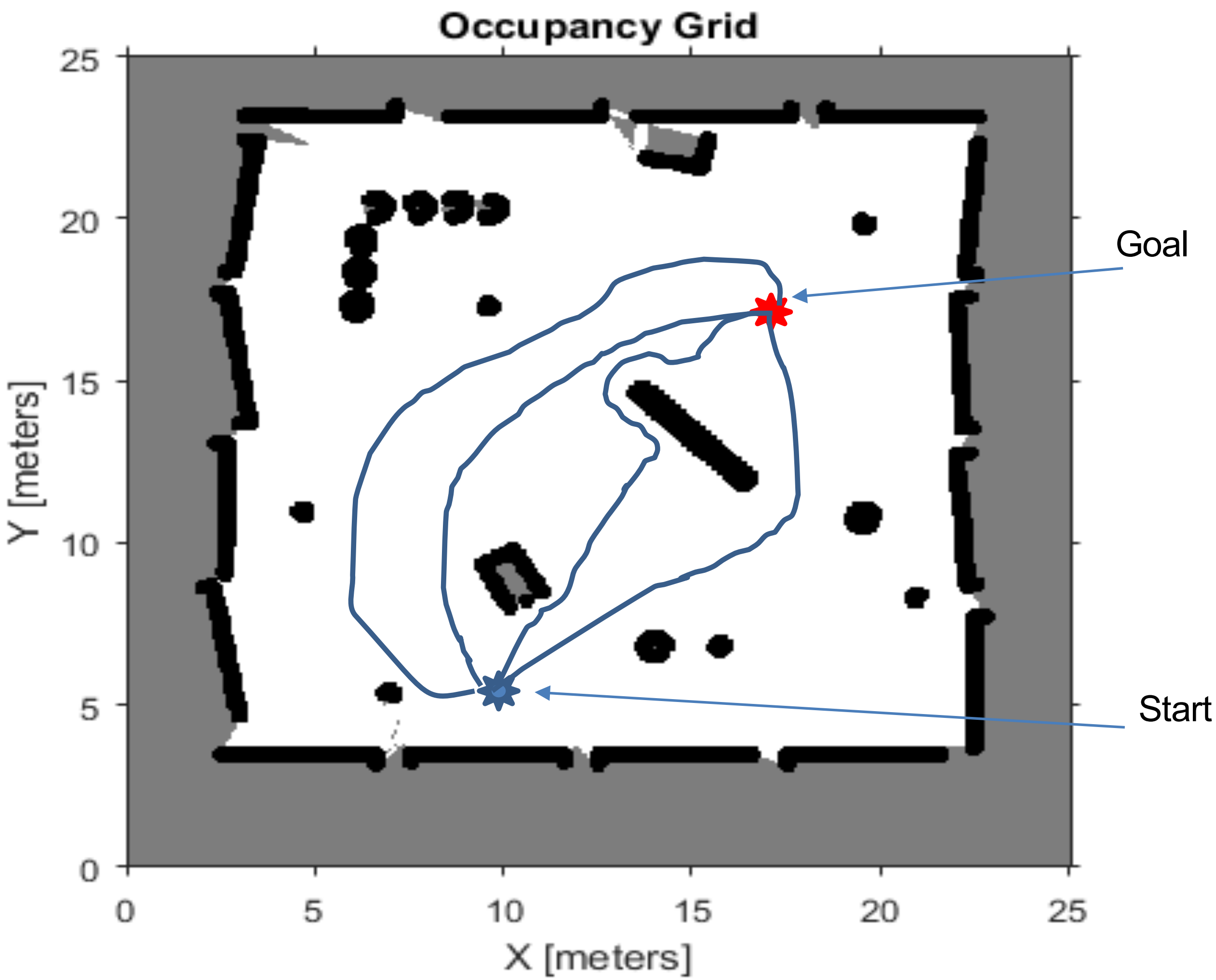
Sensor data fusion: Lidar and Camera

Lidar point cloud

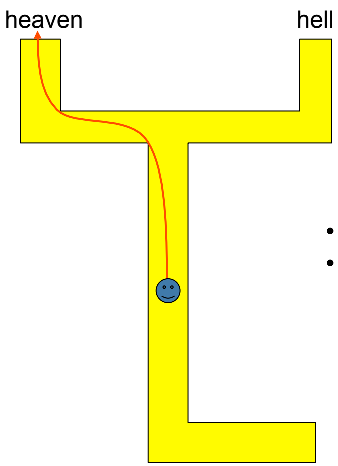


3D points projected on to image

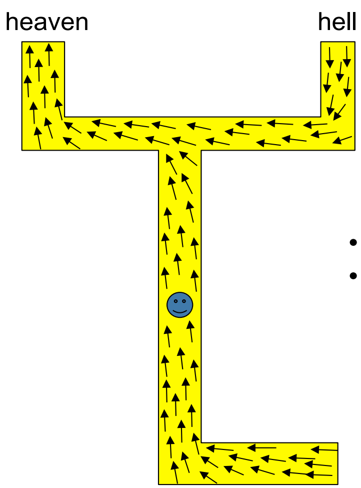
Path Planning



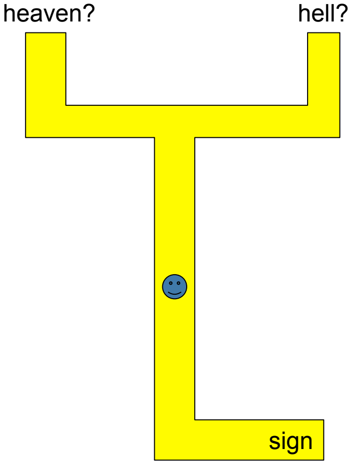
Path Planning



- World deterministic
- State observable



- World stochastic
- State observable



Lectures

- Review of Linear Algebra and Probability
  - Vector Spaces, Least Squares, RANSAC
  - Gaussian distribution, Maximum Likelihood Estimates
- Coordinate Transformations
  - Robot motion
  - Sensor calibration
  - Scan matching
- State Estimation
  - Bayes, Kalman, EKF, UKF, Information, Particle filter
- Localization
- Mapping
  - Occupancy grid map
  - Mapping as a least squares problem
  - Pose graph
- SLAM
  - Visual SLAM
- Path Planning
  - A*, MDP

Grading Policy

- Home work (including programming Assignments) – 40%
    - Bi-weekly
    - Approximately 8 HWs
- Final project – 20%
    - We will have a pool of research papers, you can also submit any paper that you want to be added in the pool.
- Mid sem – 15%
- End sem – 20%
- Class participation / surprise quiz – 5%

Resources

- Textbook:
  - Probabilistic Mobile Robotics
  - Introduction to Autonomous Robots
- Office Hours: Thursday 3pm – 5pm (ACES 401A)
- Teaching Assistants:
  - Ankit Pensia (ankitp@iitk.ac.in)
  - Mohan Krishna (nmohank@iitk.ac.in)
- Moodle: moodle.cse.iitk.ac.in

Linear Algebra (Review)

- Vector Space: It is a set **V** defined over a **scalar field F** together with two operations
  - **Vector Addition (V x V → V)**
  - **Scalar Multiplication (K x V → V)**

  such that the following properties are satisfied

Commutativity: $u + v = v + u$ for all $u, v \in V$;

Associativity: $(u + v) + w = u + (v + w)$ and $(ab)v = a(bv)$ for all $u, v, w \in V$ and $a, b \in \mathbb{F}$;

Additive identity: There exists an element $0 \in V$ such that $0 + v = v$ for all $v \in V$;

Additive inverse: For every $v \in V$, there exists an element $w \in V$ such that $v + w = 0$;

Multiplicative identity: $1v = v$ for all $v \in V$;

Distributivity: $a(u + v) = au + av$ and $(a + b)u = au + bu$ for all $u, v \in V$ and $a, b \in \mathbb{F}$.

Normed Vector Space

- A vector space equipped with a norm is called normed vector space

a <u>norm</u> is a function $\|\cdot\| : V \to \mathbb{R}$

S.t.

(a) $\|\vec{x}\| \geq 0 \quad \forall\, \vec{x} \in V$

(b) $\|\vec{x}\| = 0 \iff \vec{x} = 0$

(c) $\|a\vec{x}\| = |a|\,\|\vec{x}\| \quad \forall\, x \in V \;\&\; a \in \mathbb{R}$

(d) $\|\vec{x} + \vec{y}\| \leq \|x\| + \|y\| \quad \forall\, \vec{x}, \vec{y} \in V$

(Triangle inequality)

<u>Example:</u> l_p norm on $V = \mathbb{R}^m$ over $\mathbb{R}$

$$\|\vec{x}\|_p = \begin{cases} \left(\sum_{i=1}^{m} |x_i|^p \right)^{1/p} & 1 \leq p < \infty \\ \max_{i=1,2,\ldots m} |x_i| & p = \infty \end{cases}$$

Inner Product Space

* Inner Product : An inner product is a function $\langle \cdot , \cdot \rangle : V \times V \to K$

s.t. $\forall \vec{x}, \vec{y}, \vec{z} \in V$, $a \in K$.

IP1 $\langle x, y \rangle = \overline{\langle y, x \rangle}$

IP2 $\langle ax, y \rangle = a \langle x, y \rangle$

IP3 $\langle x+y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$

IP4 $\langle x, x \rangle \geq 0$ with equality iff $x = 0$

A vector space together with an inner product is called an <u>inner product space</u>

Example ① $V = \mathbb{R}^n$; $K = \mathbb{R}$ usual dot product

$\langle x, y \rangle = \sum_{i=1}^{n} x_i y_i$

② $V = \mathbb{C}^n$ $K = \mathbb{C}$

$\langle x, y \rangle = \sum_{i=1}^{n} x_i \overline{y_i}$

③ $V = \{ f : \mathbb{R} \to \mathbb{C} \}$ $K = \mathbb{C}$

$\langle f, g \rangle = \int_{-\infty}^{\infty} f(t) \overline{g(t)} \, dt$.

Linear Combination of Vectors

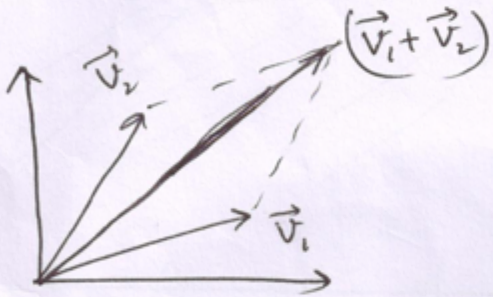
★ Linear Combination of vectors :

Let V be a vector space with scalars $K = \mathbb{R}$ or $\mathbb{C}$

We say $\vec{v} \in V$ is a linear combination of $\vec{v}_1, \vec{v}_2, \ldots \vec{v}_n \in V$ if $\exists\, a_1, \ldots a_n \in K$

s.t. $\vec{v} = \sum_{i=1}^{n} a_i \vec{v}_i$

Note that 'n' is finite.



You cannot come out of the plane. L.C. of vectors in the plane will keep you in the plane.

Linear Independence

* Linear Independence:

If $U \subseteq V$, we say U is linearly independent if, for any n and any $v_1, v_2 \ldots v_n \in U$

$$\sum a_i \vec{v}_i = \underline{0} \Rightarrow a_i = 0 \quad \forall\, i = 1, \ldots n$$

If on the other hand $\exists\, v_1, \ldots v_n \in U$ and $a_1, a_2 \ldots a_n \in K$ not all 0 s.t.

$$\sum a_i \vec{v}_i = \underline{0}$$

we say that U is linearly dependent

Example: $V = \mathbb{R}^3$, $K = \mathbb{R}$, $U = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$

If $\vec{u}_1, \vec{u}_2, \vec{u}_3$ are linearly independent

$\Rightarrow$ they are non-coplanar.

Span of Vectors

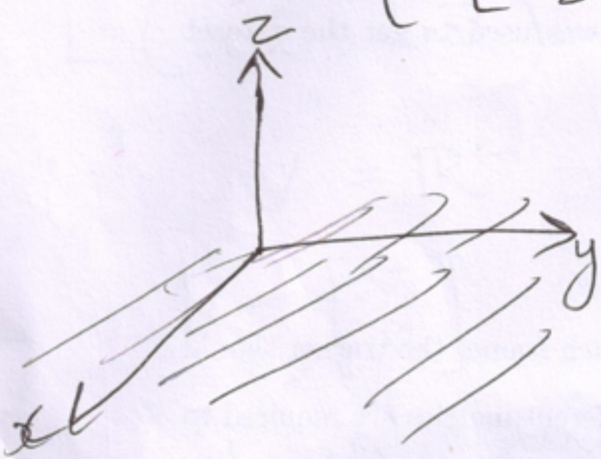
Span : Let $U \subseteq V$, the span of U is a set of all LC's of vectors in U

i.e. $\text{span}(U) = \sum_{i=1}^{m} a_i \vec{v}_i$. $m \in N$

$v_1, v_2 \ldots v_m \in U$

$a_1, a_2 - a_m \in K$.

Ex: $U = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$



$\text{span}(U) =$ <u>x-y plane</u>

Basis of Vector Space

Basis : A set of linearly independent vectors that spans the vector space are called the basis of V

Ex : $V \in \mathbb{R}^3 \Rightarrow \text{Basis} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

standard basis

Linear Transformation

Linear Transformation

Let V, W be vector spaces with the same set of scalars $K = \mathbb{R}$

A function $L : V \to W$ is a linear transformation

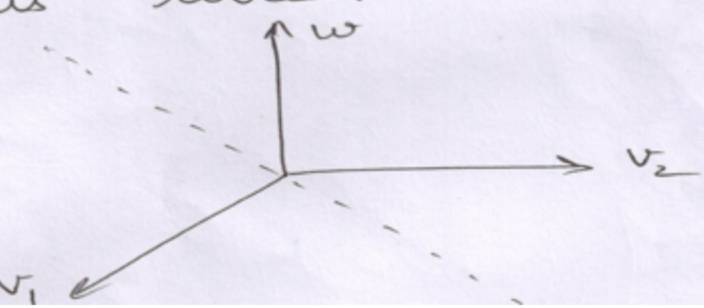
if $\forall\ \vec{u}, \vec{v} \in V$ and $\forall\ a, b \in K$

$$L(a\vec{u} + b\vec{v}) = a\,L(\vec{u}) + b\,L(\vec{v})$$

Ex: (a) $V = \mathbb{R}^2$, $W = \mathbb{R}$, $K = \mathbb{R}$

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad L(v) = w = 2v_1 - 3v_2$$

Check if its linear?



Linear Transformation

(b) More generally suppose $V = \mathbb{R}^m$ $W = \mathbb{R}$

$$L : \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix} \to c_1 v_1 + c_2 v_2 \ldots c_m v_m = \sum_{i=1}^{m} c_i v_i$$

where $c_1, c_2 \ldots c_m$ are fixed. Then

$$L(au + bv) = L\left(\begin{bmatrix} au_1 + bv_1 \\ \vdots \\ au_m + bv_m \end{bmatrix}\right)$$

$$= \sum_{i=1}^{m} c_i (au_i + bv_i)$$

$$= a \sum c_i u_i + b \sum c_i v_i$$

$$= a L(u) + b L(v)$$

Linear Transformation

(a) $V = \mathbb{R}^m$, $W = \mathbb{R}^n$

$$L : \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix} \longrightarrow \begin{bmatrix} c_{11}v_1 + \cdots c_{1m}v_m \\ \vdots \\ c_{n1}v_1 + \cdots c_{nm}v_m \end{bmatrix}$$

Is this linear ?

The last example can be represented as a matrix

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} c_{11} & \cdots & c_{1m} \\ \vdots & & \\ c_{n1} & \cdots & c_{nm} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}$$

$\Rightarrow$ Every matrix transformation is linear

When L is linear and maps one euclidean space to another then L can be represented as a matrix. The converse is also true

Subspaces Associated with a Linear Transformation

Subspaces associated with a linear transformation

* Range space : $R(L) = \{ w \in W \mid w = L(v), v \in V \}$

* Nullspace : $N(L) = \{ v \in V \mid L(v) = \underline{0} \}$

Let $L = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$

$$R(L) = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 2v_1 + 6v_2 \\ v_1 + 3v_2 \end{bmatrix} = (v_1 + 3v_2) \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow R(L) = \text{span}\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix} \right)$$

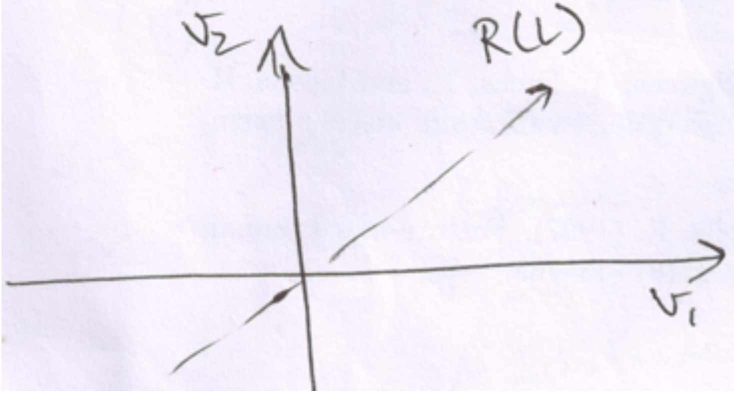
Subspace

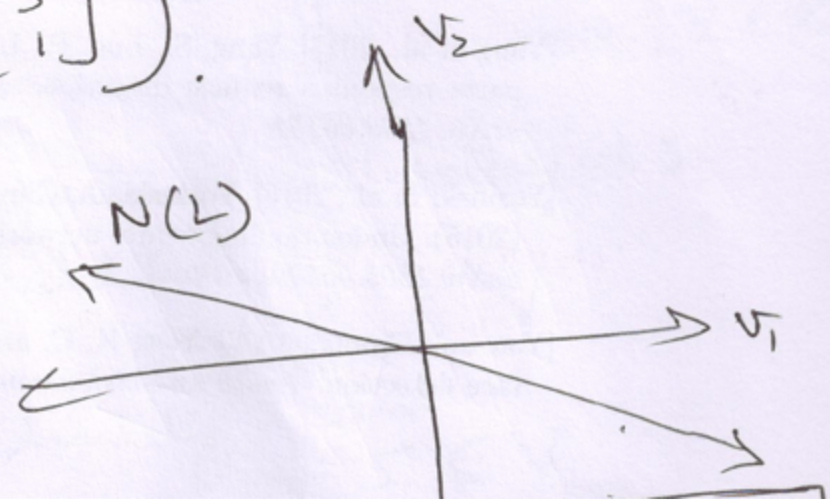
$$N(L) \Rightarrow \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\Rightarrow (v_1 + 3v_2) \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow v_1 + 3v_2 = 0$$

$$\Rightarrow N(L) = \text{span}\left(\begin{bmatrix} -3 \\ 1 \end{bmatrix} \right).$$



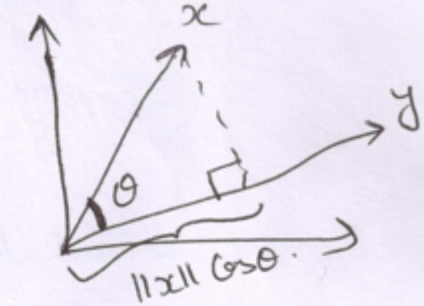


Orthogonality

Orthogonality :

For $x, y \in \mathbb{R}^2$ with $\langle x, y \rangle = x_1 y_1 + x_2 y_2$

$$\langle x, y \rangle = \|x\|\,\|y\| \cos\theta$$



θ is the angle between two vectors.

Same notion can be extended to any Inner Product space

$$\theta = \cos^{-1}\left(\frac{\langle x, y \rangle}{\|x\| \cdot \|y\|}\right)$$

This helps us define the notion of angle between non-geometric quantities like (functions)

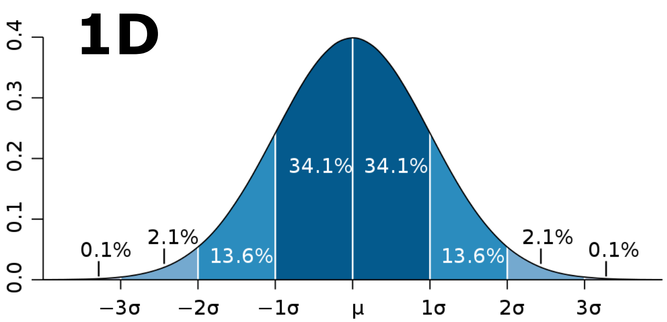
If $\langle x, y \rangle = 0 \Rightarrow \theta = \pi/2$ we say that vector x & y are orthogonal.

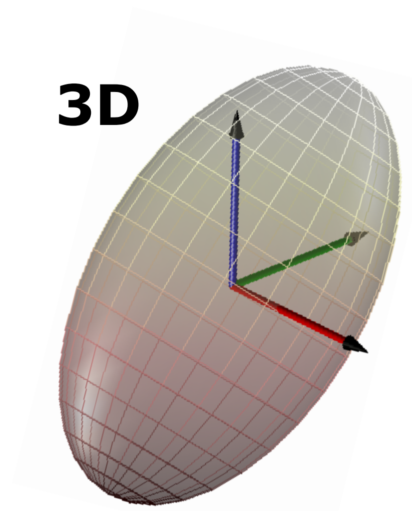
Probability Review

- Random Variables: Numerical measurements or observations that have uncertain variability each time they are repeated are called random variables
- Probability Mass Function (PMF): Joint and Marginal
- Independent Random Variables
- Independent and Identically Distributed (IID)
- Expectation (Mean) of Random Variable
- Expectation of a function of random variable
- Linearity of Expectation
- Variance, Covariance, Correlation
- Gaussian / Normal Distribution

Gaussian Distribution

$$p(x) = \det(2\pi\Sigma)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$





Slide Credit: Cyrill Stachniss