EE698G – Assignment 3

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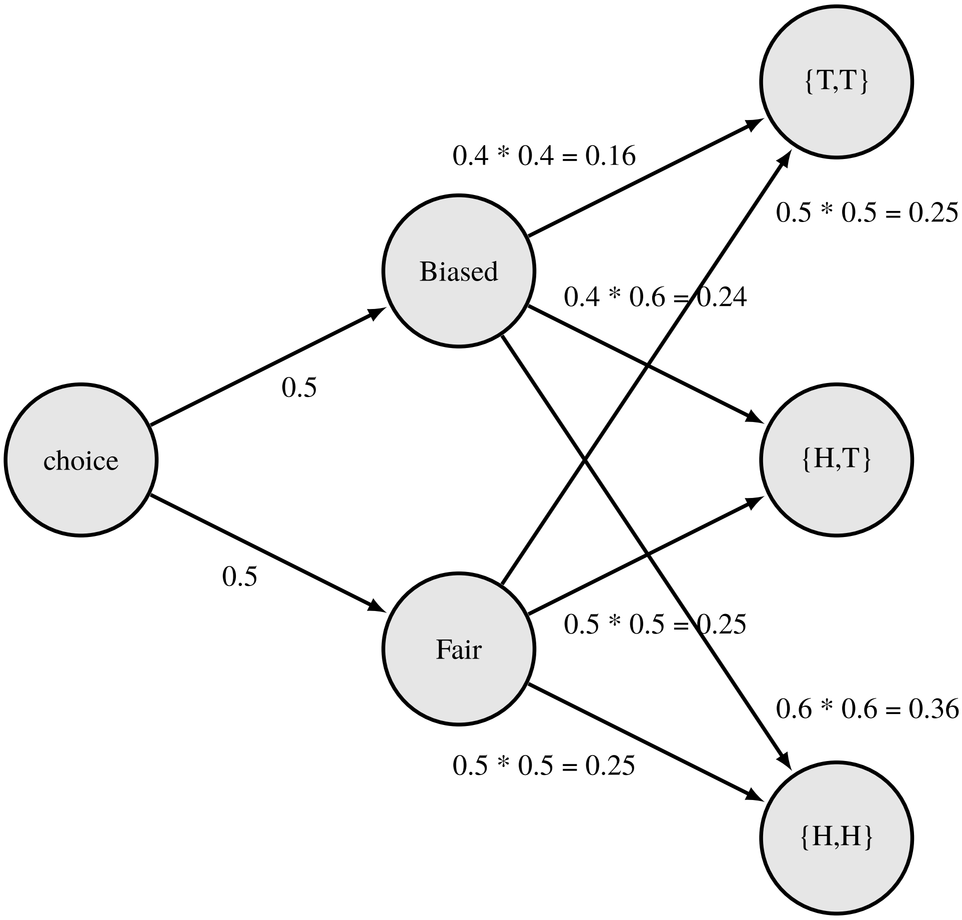
Roll No. 14208 19/02/2017

1



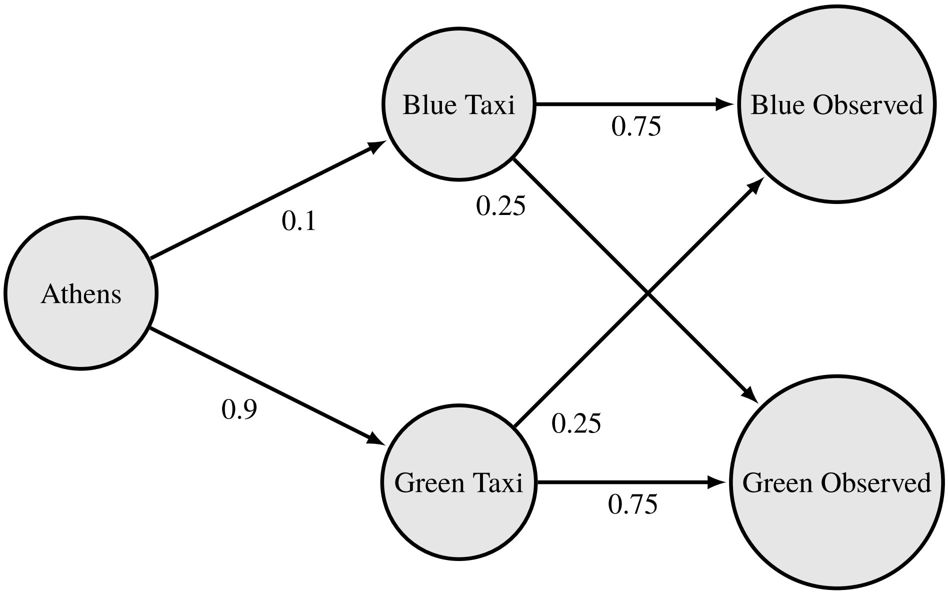
$$P(Biased|HH) = \frac{P(HH|Biased) * P(Biased)}{P(HH)}$$

$$\Rightarrow P(Biased|HH) = \frac{0.36 * 0.5}{0.36 * 0.5 + 0.25 * 0.5}$$

$$\Rightarrow P(Biased|HH) = \frac{36}{61} = \mathbf{0.59}$$

2

Yes, it is possible to calculate the most likely color for the taxi.



$$P(BlueTaxi|BlueObserved) = \frac{0.1 * 0.75}{0.1 * 0.75 + 0.9 * 0.25}$$
$$\Rightarrow P(BlueTaxi|BlueObserved) = \mathbf{0.25}$$

$$P(GreenTaxi|BlueObserved) = \frac{0.9 * 0.25}{0.1 * 0.75 + 0.9 * 0.25}$$
$$\Rightarrow P(GreenTaxi|BlueObserved) = \mathbf{0.75}$$

Probability of the taxi to be green is more likely.

3

$$P(X|Y = y) = \frac{P(X, y)}{P(y)}$$
$$= \frac{1}{\sqrt{2\pi\frac{|\Sigma|}{\sigma_Y^2}}} exp\left\{ -\frac{1}{2}\begin{bmatrix} X - \mu_X & y - \mu_Y \end{bmatrix} \frac{1}{\sigma_Y^2\sigma_*^2} \begin{bmatrix} \sigma_Y^2 & -\sigma_{XY} \\ -\sigma_{XY} & \sigma_X^2 \end{bmatrix} \begin{bmatrix} X - \mu_X \\ y - \mu_Y \end{bmatrix} + \frac{1}{2}\frac{(y - \mu_Y)^2}{\sigma_Y^2} \right\}$$

Substitute $\sigma_*^2 = \sigma_X^2 - \frac{\sigma_{XY}^2}{\sigma_Y^2}$

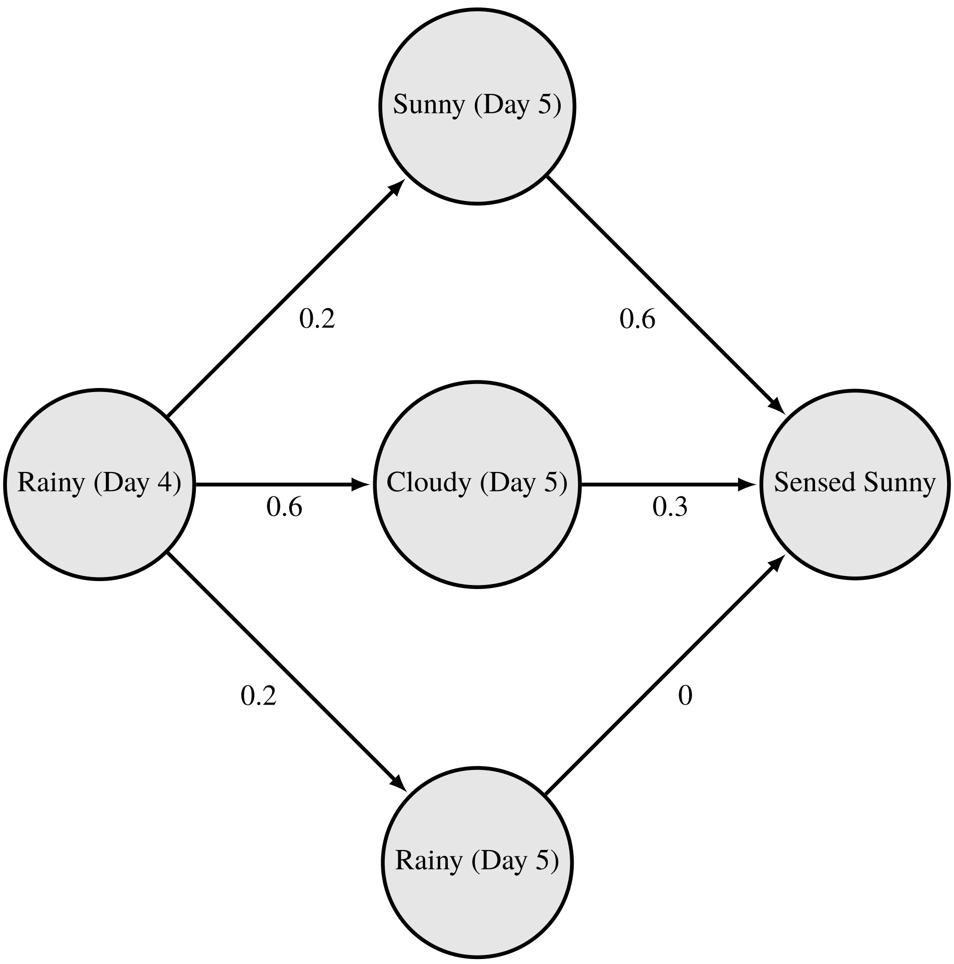
$$
\begin{aligned}
\Rightarrow P(X|Y=y) &= \frac{1}{\sqrt{2\pi\sigma_*^2}} exp\Bigg\{\frac{-1}{2\sigma_*^2}\Bigg((X-\mu_X)^2 + \frac{\sigma_X^2}{\sigma_Y^2}(y-\mu_Y)^2 - \frac{2\sigma_{XY}}{\sigma_Y^2}(X-\mu_X)(y-\mu_Y) - \frac{\sigma_*^2}{\sigma_Y^2}(y-\mu_Y)^2\Bigg)\Bigg\} \\
&= \frac{1}{\sqrt{2\pi\sigma_*^2}} exp\Bigg\{\frac{-1}{2\sigma_*^2}\Bigg((X-\mu_X)^2 - \frac{2\sigma_{XY}}{\sigma_Y^2}(X-\mu_X)(y-\mu_Y) + \frac{\sigma_{XY}^2}{\sigma_Y^4}(y-\mu_Y)^2\Bigg)\Bigg\} \\
&= \frac{1}{\sqrt{2\pi\sigma_*^2}} exp\Bigg\{\frac{-1}{2\sigma_*^2}\Bigg((X-\mu_X)^2 - 2(X-\mu_X)\Bigg(\frac{\sigma_{XY}}{\sigma_Y^2}(y-\mu_Y)\Bigg) + \Bigg(\frac{\sigma_{XY}}{\sigma_Y^2}(y-\mu_Y)\Bigg)^2\Bigg)\Bigg\} \\
&= \frac{1}{\sqrt{2\pi\sigma_*^2}} exp\Bigg\{\frac{-1}{2\sigma_*^2}\Bigg((X-\mu_X) - \frac{\sigma_{XY}}{\sigma_Y^2}(y-\mu_Y)\Bigg)^2\Bigg\} \\
&= \frac{1}{\sqrt{2\pi\sigma_*^2}} exp\Bigg\{\frac{-1}{2\sigma_*^2}\Bigg(X - \Bigg(\mu_X + \frac{\sigma_{XY}}{\sigma_Y^2}(y-\mu_Y)\Bigg)\Bigg)^2\Bigg\}
\end{aligned}
$$

substitute $\mu_* = \mu_X + \frac{\sigma_{XY}}{\sigma_Y^2}(y-\mu_Y)$

$$
\Rightarrow P(X|Y=y) = \frac{1}{\sqrt{2\pi\sigma_*^2}} exp\Bigg\{\frac{-1}{2\sigma_*^2}(X-\mu_*)^2\Bigg\}
$$

4

As the sensor senses rainy day with probability 1 and it never says rainy when it is not rainy. So the impact of days before the rainy day will be none on the day after rainy day.



$$P(Sunny_5|SensedSunny_5) = \frac{0.6 * 0.2}{0.6 * 0.2 + 0.6 * 0.3 + 0 * 0.2}$$

$$\Rightarrow P(Sunny_5|SensedSunny_5) = \frac{2}{5} = \mathbf{0.4}$$

5

Epipolar line on the Image 2 is shown in the figure.

6

6.1 Part a

$H(t) = vt + \frac{1}{2}gt^2$
$\Rightarrow H(t+dt) = v*t + v*dt + \frac{1}{2}*g*t^2 + \frac{1}{2}*g*dt^2 + g*t*dt$
$\Rightarrow H(t+dt) = H(t) + v*dt + \frac{1}{2}*g*dt^2 + g*t*dt$
Now,
$H(t) = vt + \frac{1}{2}gt^2$
$\Rightarrow \frac{\partial H(t)}{\partial t} = v + gt$
$\Rightarrow \frac{\partial H(t+dt)}{\partial t} = \frac{\partial H(t)}{\partial t} + g*dt$
Let state: $X(t) = \begin{bmatrix} H(t) \\ \frac{\partial H(t)}{\partial t} \end{bmatrix}$
Control: $u = g$
Measurement: Z

$\Rightarrow$ Process Model,

$X(t+dt) = A * X(t) + B * u + error_1$

Measurement Model,

$Z(t) = C * X(t) + error_2$

Where,

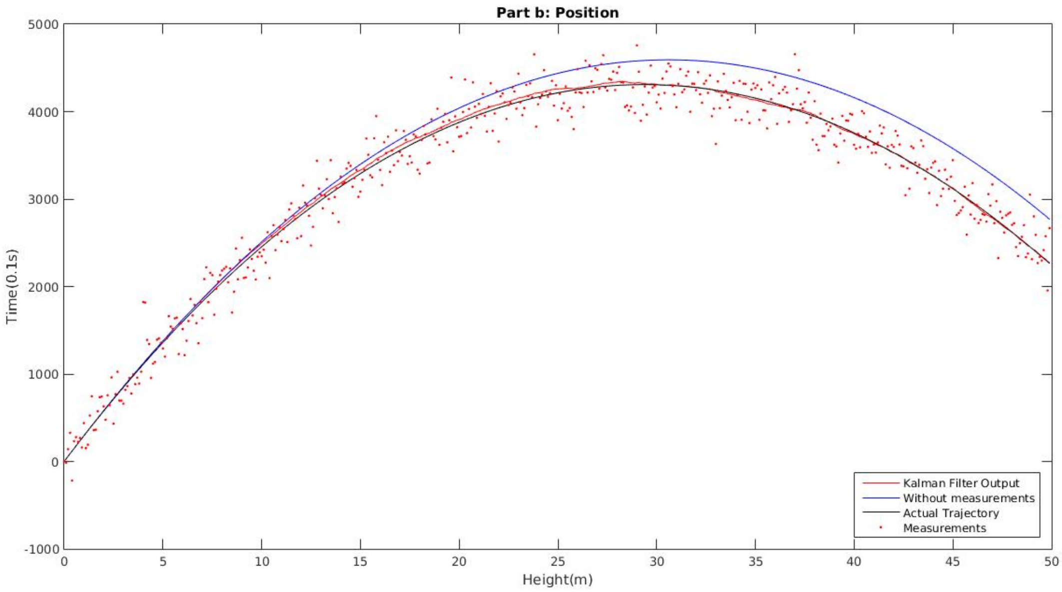
$$A = \begin{bmatrix} 1 & dt \\ 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} \frac{1}{2} * dt^2 \\ dt \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

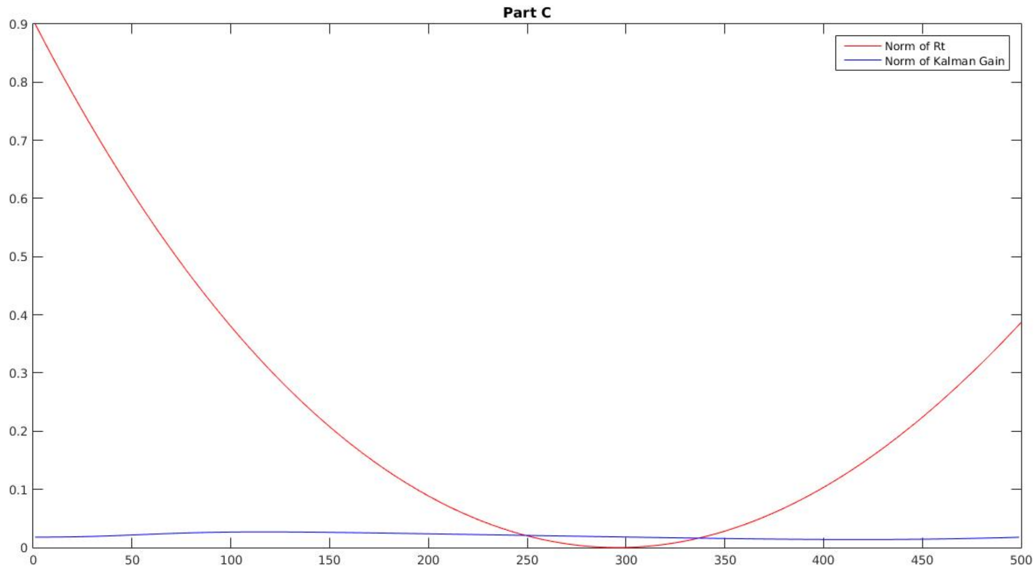
$u = g = -9.8$

variance of $error_1$ is Rt and variance of $error_2$ is Qt .

6.2 Part b



6.3 Part c


Part C
Norm of Rt
Norm of Kalman Gain
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150
200
250
300
350
400
450
500