



Robot Vision

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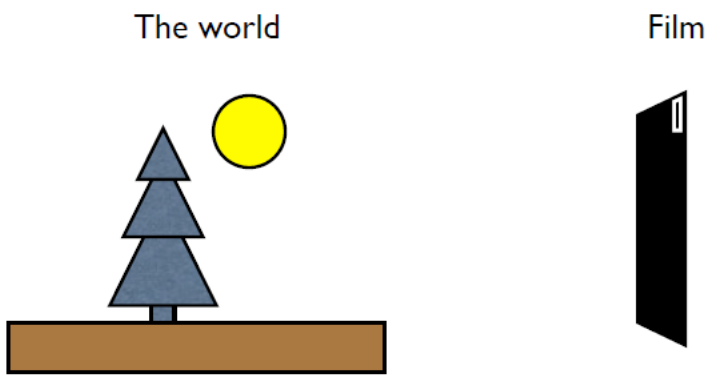
Outline

- Cameras
  - Calibration
- Lidars
  - Scan Matching
- Extrinsic Calibration of Lidar and Camera
  - Target based
  - Targetless
- Stereo Camera
- Epipolar Geometry

World Simplest Camera?



- Just hold up a piece of film
- Do we get an image on the film?
  - For each piece of the film, where do the photons come from?

Slide Credit: Edwin Olson, University of Michigan

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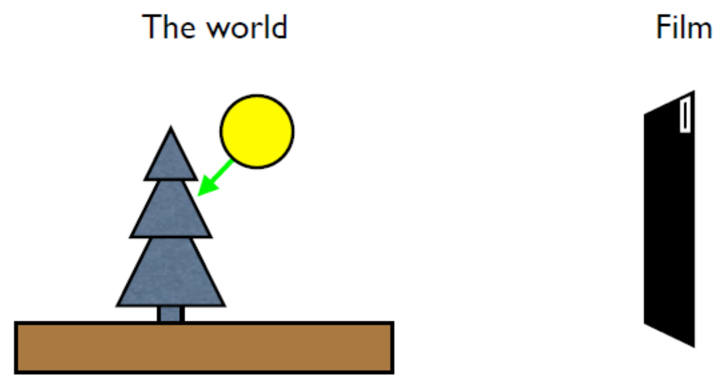

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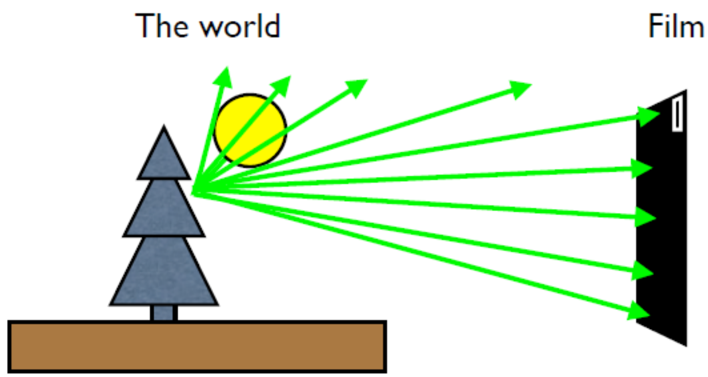

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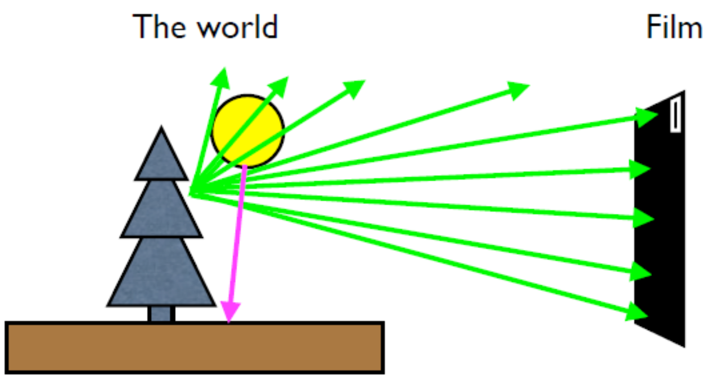

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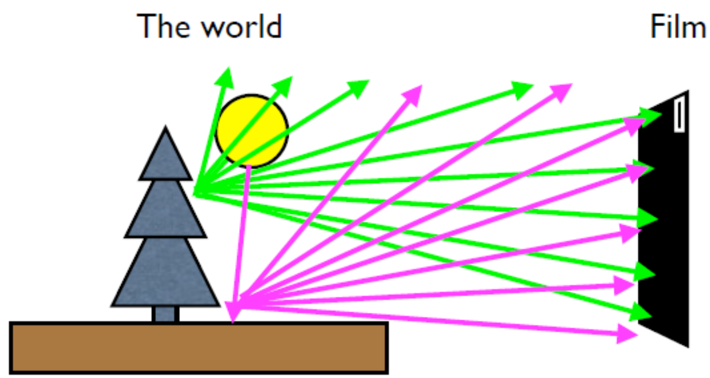

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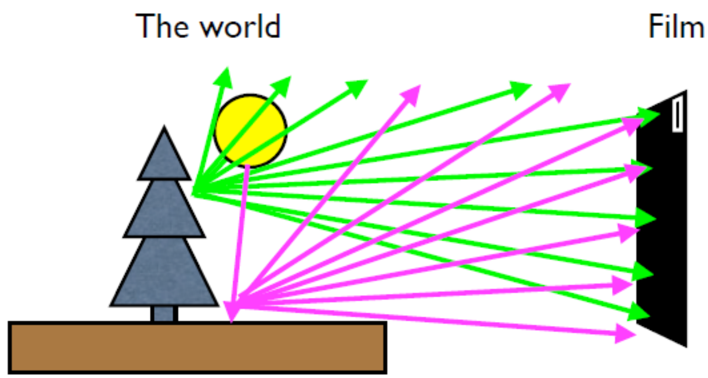

- Just hold up a piece of film

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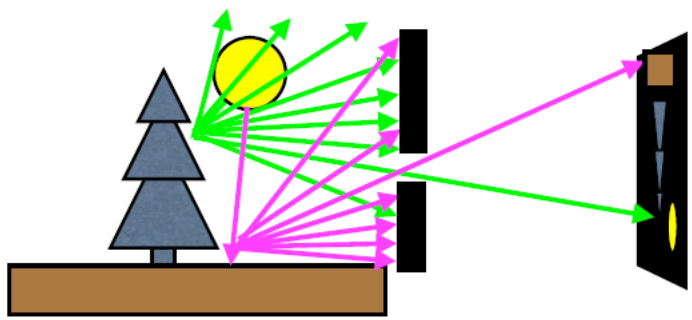
Slide Credit: Edwin Olson, University of Michigan

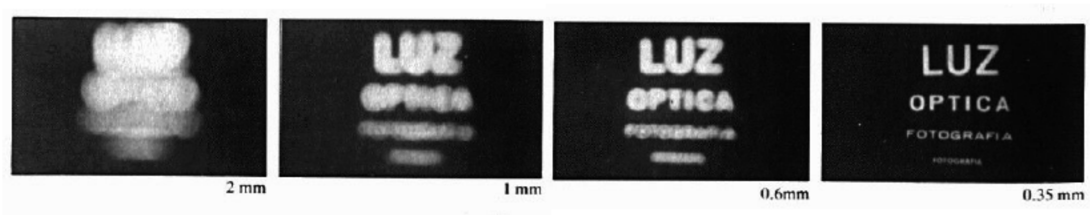
Let's add an aperture



- An aperture blocks all but a small subset of the rays
  - Causes the image to appear in focus!

Slide Credit: Edwin Olson, University of Michigan

Aperture Size

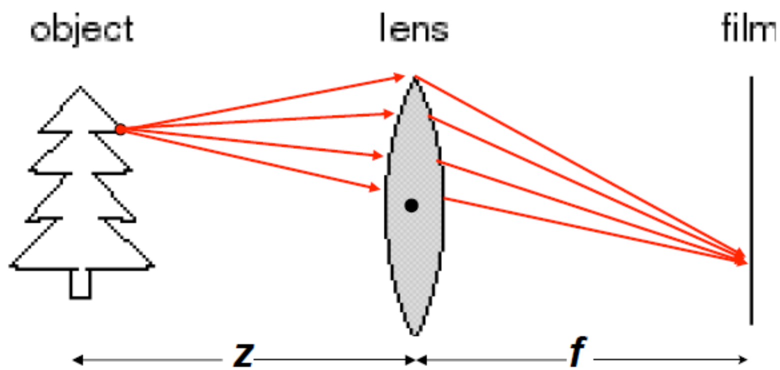


- Why not make the aperture super small?
  - A "pin-hole" lens.
  - Not enough light to "register" on our film
- What happens when the aperture is bigger?
  - More rays can fit through--- blurrier image

- Is there any way of getting a sharp image, but allow more light through?
  - Yes! A *lens*.

Slide Credit: Edwin Olson, University of Michigan

Lenses

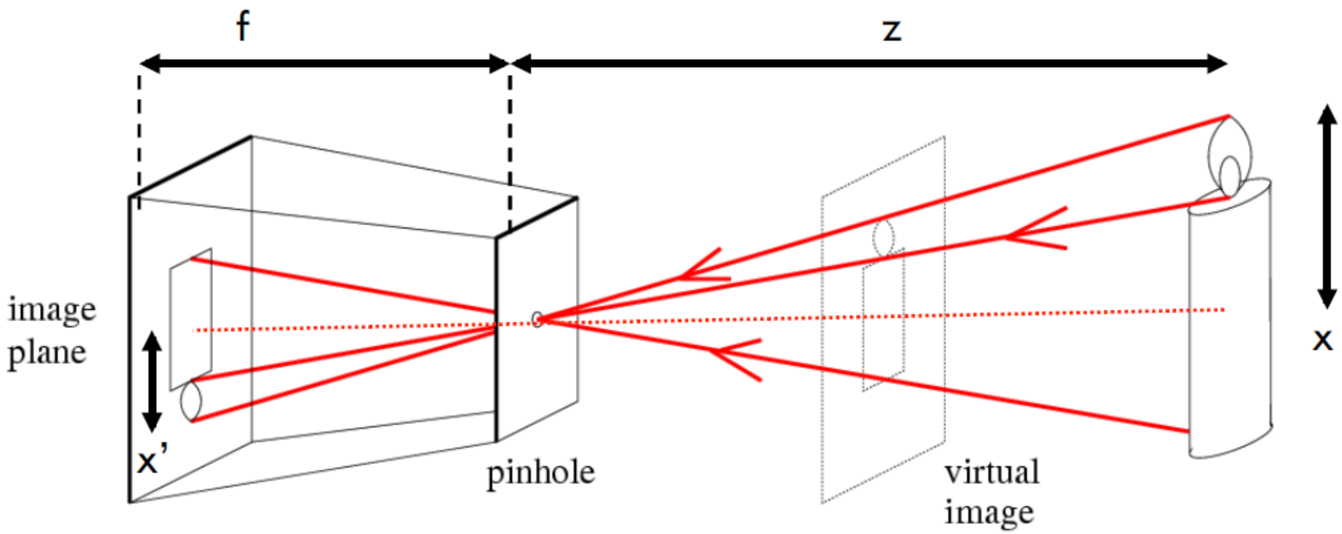


- A lens collects rays with a particular divergence and refocuses them to a point.
  - But points at the "wrong" distance won't be refocused exactly.
- *Depth of field:* how much of the scene is in focus

- We're going to ignore this today, however--- we're going to assume a "pin-hole" model.

Slide Credit: Edwin Olson, University of Michigan

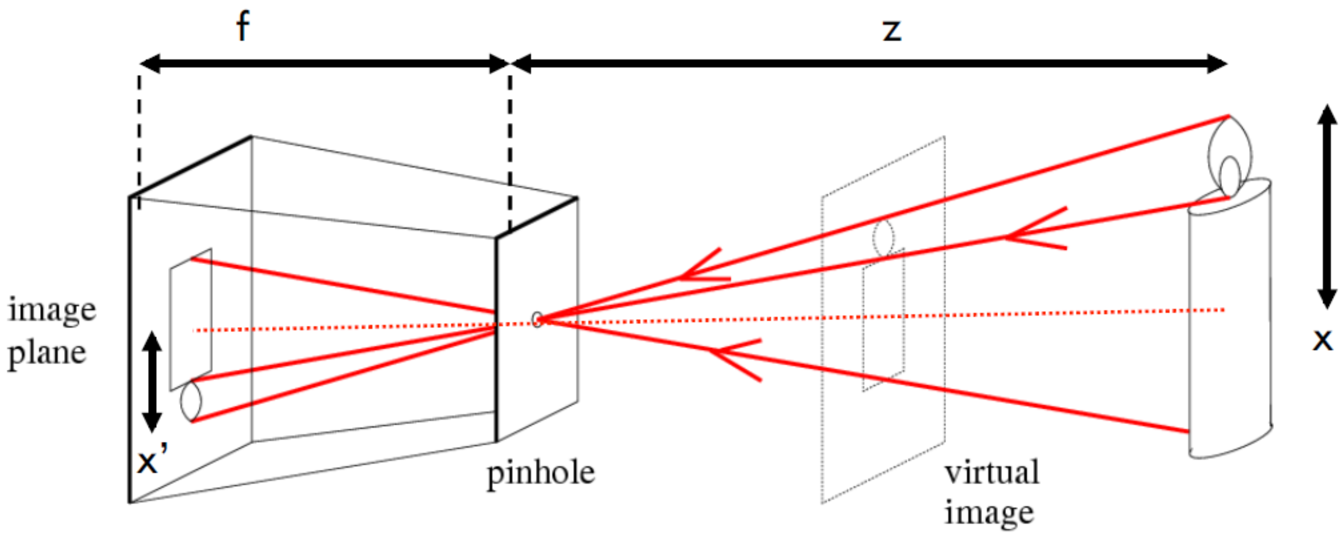
Perspective Projection



- The pinhole creates two similar triangles
  - Allows us to determine x' in terms of x

Slide Credit: Edwin Olson, University of Michigan

Perspective Projection

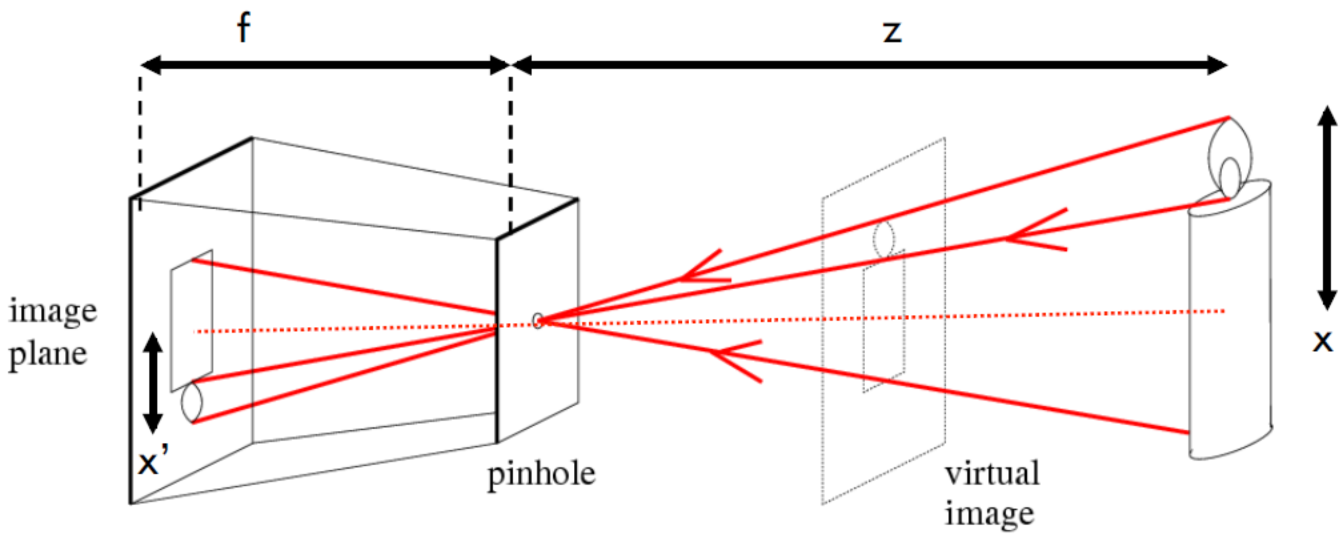


- The pinhole creates two similar triangles
  - Allows us to determine x' in terms of x

$$x' = -xf/z$$

(why is it negative? we'll assume from here on out that the camera "unflips" the image.)

Perspective Projection



- What are the pixel coordinates where the flame appears?
  - x' = fx/z + c
  - Measure f in "pixels" and add an offset (so that the "middle" pixel is in the middle of the image)

The Perspective Matrix

- Suppose we write a point in the world (like the position of the candle flame) as a vector:

$$p = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- Can we write a matrix so that p' = Mp?

$$p' = \begin{bmatrix} fx/z + c_x \\ fy/z + c_y \end{bmatrix}$$

Slide Credit: Edwin Olson, University of Michigan

Homogenous Coordinates

- We'll introduce a new convention, *homogenous coordinates*.
- We write points just the way we did before, but add an extra row:
  - The extra row is a *scale factor* for the whole vector.

$$p = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{becomes} \quad p = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide Credit: Edwin Olson, University of Michigan

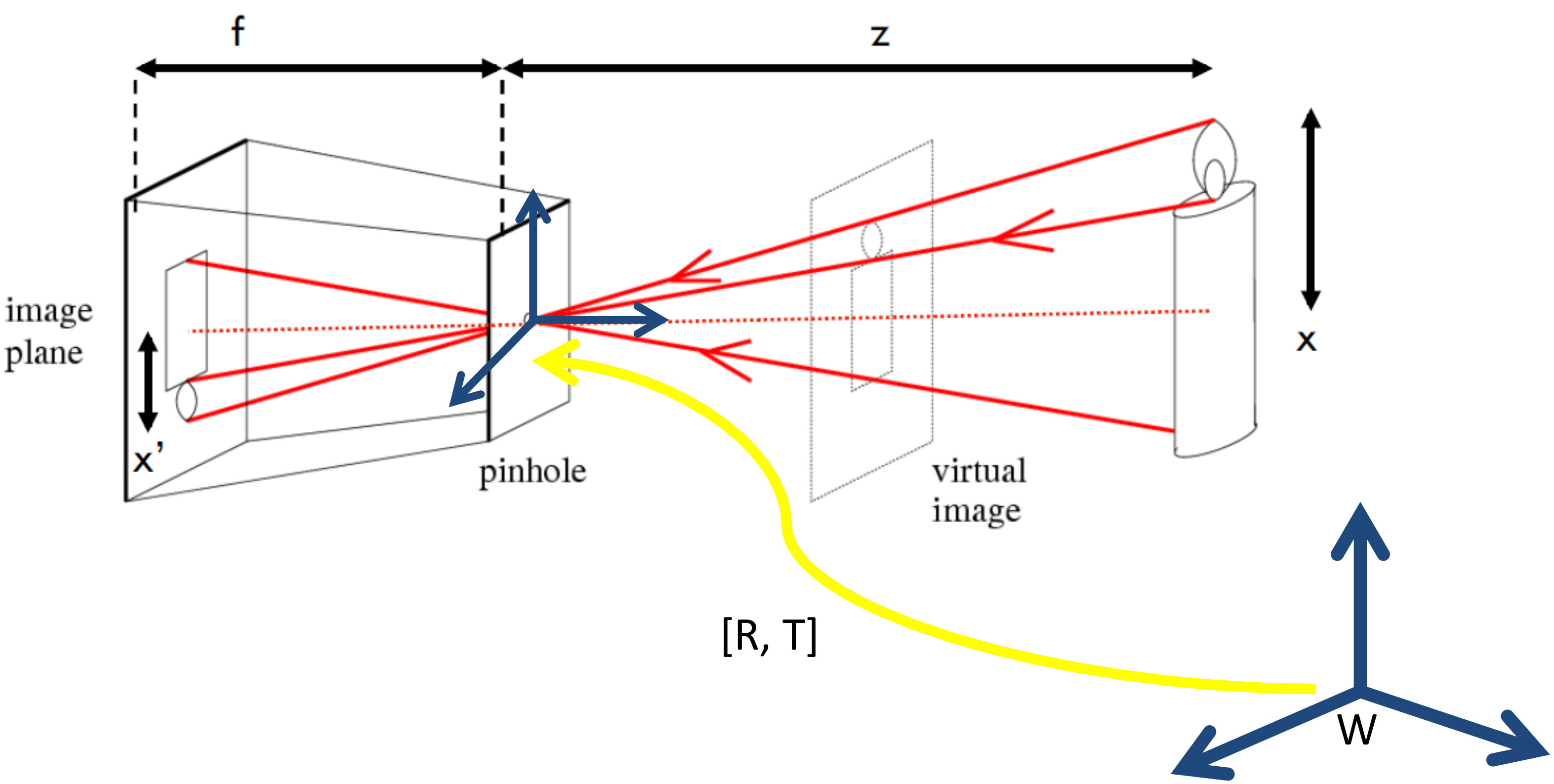
Do homogeneous coordinates help?

- Eureka!

$$p' = \underbrace{\begin{bmatrix} fx/z + c_x \\ fy/z + c_y \\ 1 \end{bmatrix}}_{\text{2d homogeneous coordinates}} = \underbrace{\begin{bmatrix} fx + c_x z \\ fy + c_y z \\ z \end{bmatrix}}_{\text{2d homogeneous coordinates}} = \underbrace{\begin{bmatrix} f & 0 & c_x & 0 \\ 0 & f & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{perspective transform}} \underbrace{\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}_{\text{3d homogeneous coordinates}}$$

Slide Credit: Edwin Olson, University of Michigan

World Coordinate Frame



Rigid-Body Transformations

- The product of two rigid-body transformations is ***always*** another rigid-body transformation!
- So no matter how the object has been translated or rotated, we can describe its position with a single 4x4 matrix, which has the structure:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} R_{00} & R_{01} & R_{02} & T_x \\ R_{10} & R_{11} & R_{12} & T_y \\ R_{20} & R_{21} & R_{22} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide Credit: Edwin Olson, University of Michigan

Putting it all together

$$\begin{bmatrix} x' \\ y' \\ s \end{bmatrix} = \begin{bmatrix} f & 0 & c_x & 0 \\ 0 & f & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{00} & R_{01} & R_{02} & T_x \\ R_{10} & R_{11} & R_{12} & T_y \\ R_{20} & R_{21} & R_{22} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

2d homogeneous pixel (camera) coordinates

Focal length and focal center of camera

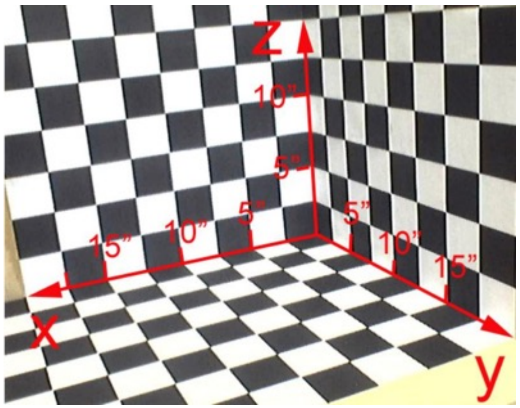
Rigidly move every object in the world to simulate the camera's true position

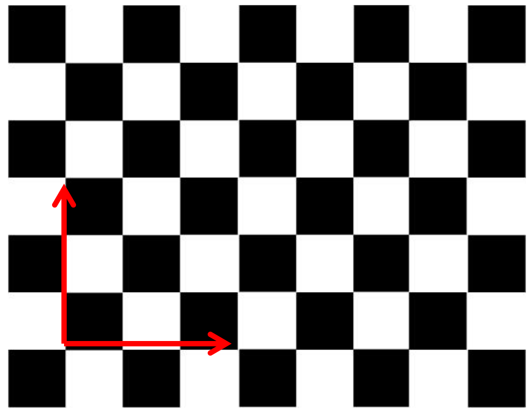
3d homogenous (world) coordinates

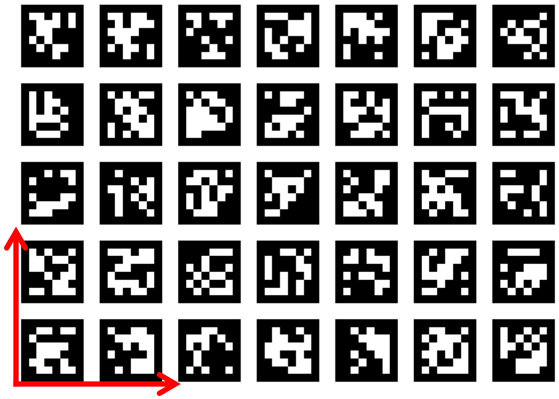
“Intrinsics” “Extrinsics”

Slide Credit: Edwin Olson, University of Michigan

Target based calibration



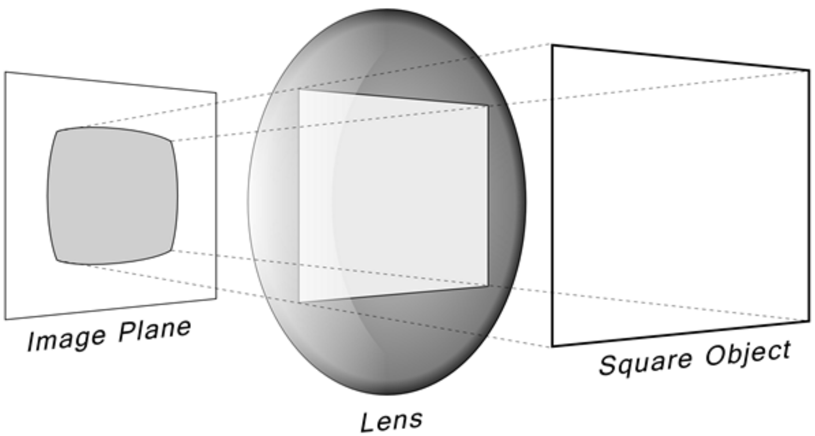


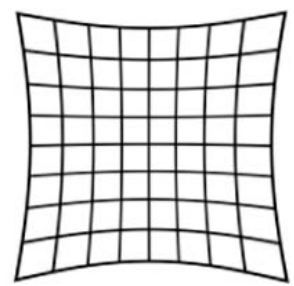


$$M = \arg\min_{M} \sum \|P_{ci} - M P_i\|^2$$

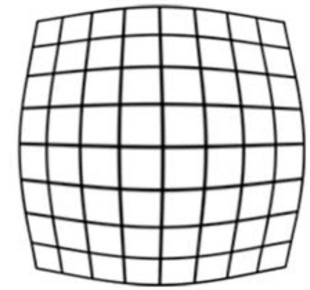
$$P_{ci} \Leftrightarrow P_i$$

Lens distortion



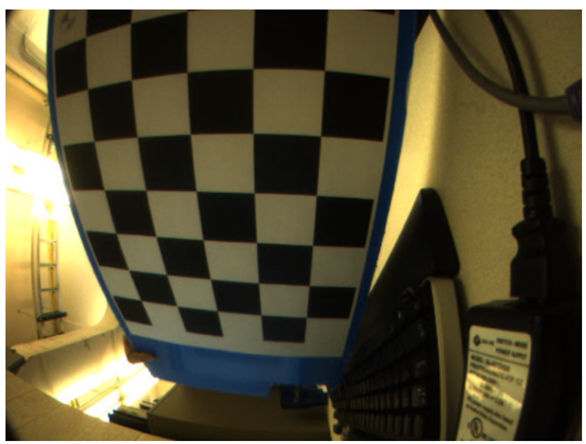


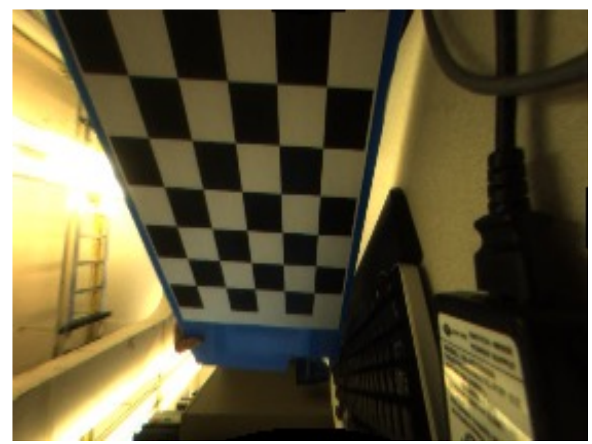
Pin-cushion



Barrel

Distorted





Undistorted

$$x_{\mathrm{d}} = x_{\mathrm{u}}(1+K_1 r^2 + K_2 r^4 + \cdots)$$
$$y_{\mathrm{d}} = y_{\mathrm{u}}(1+K_1 r^2 + K_2 r^4 + \cdots)$$

References & Resources

- **A versatile camera calibration technique for high accuracy 3D machine vision metrology using off-the-shelf TV cameras and lenses -** R. Y. Tsai, IEEE J. Robotics Automat. , pages 323-344, Vol. RA-3, No. 4 1987.

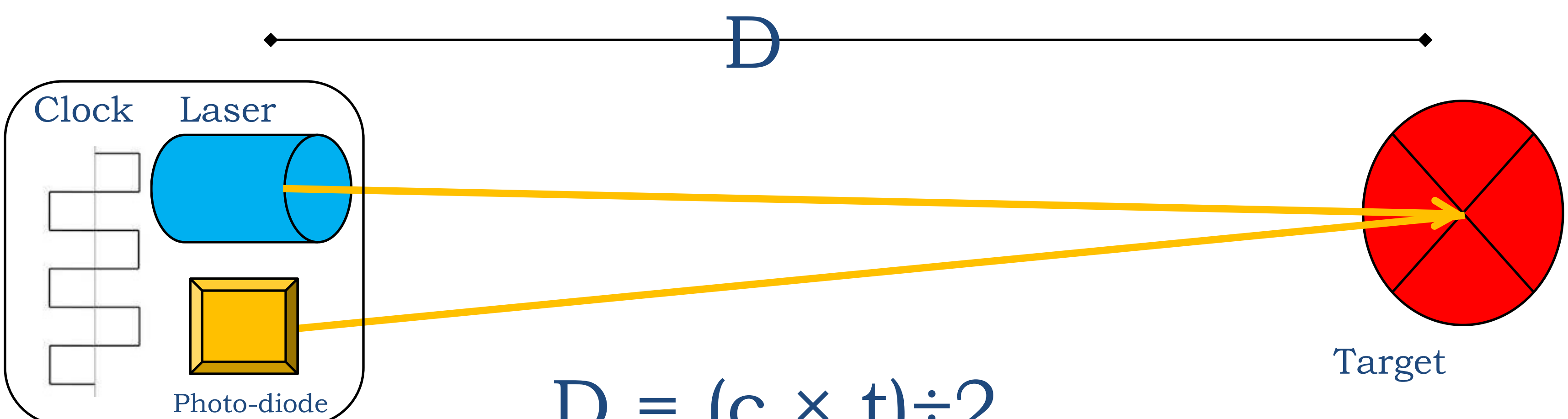
- **A Four-step Camera Calibration Procedure with Implicit Image Correction -** Heikkil and Silven, CVPR97

- **On Plane-Based Camera Calibration: A General Algorithm, Singularities, Applications -** Sturm and Maybank, CVPR99.

- **Flexible Camera Calibration by Viewing a Plane from Unknown Orientations -** Zhang, ICCV99 *[MATLAB & OpenCV implementation available online]*

- **AprilCal: Assisted and repeatable camera calibration**- Richardson et al, IROS 2013. *[Open Source, software available online]* ***Show Video***

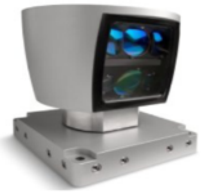
Lidar: Light Detection and Ranging

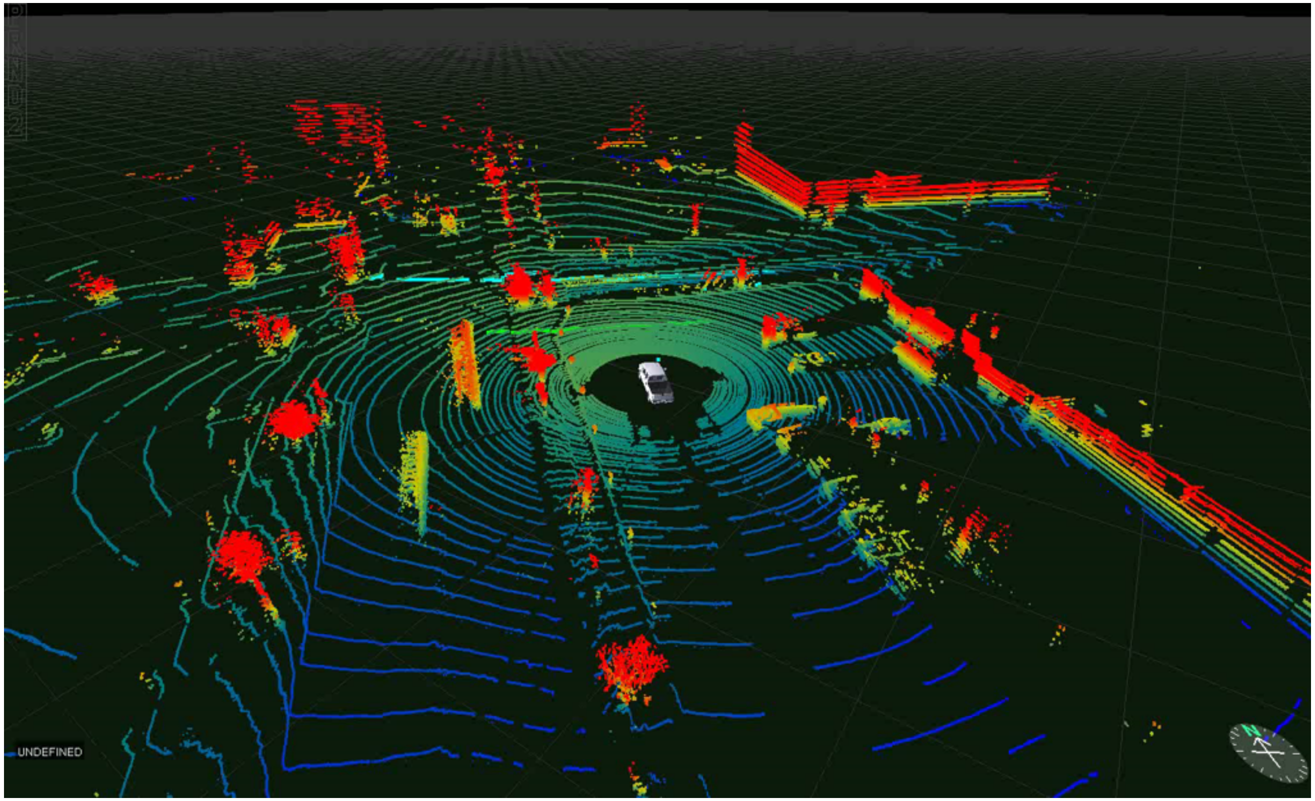


$$D = (c \times t) \div 2$$

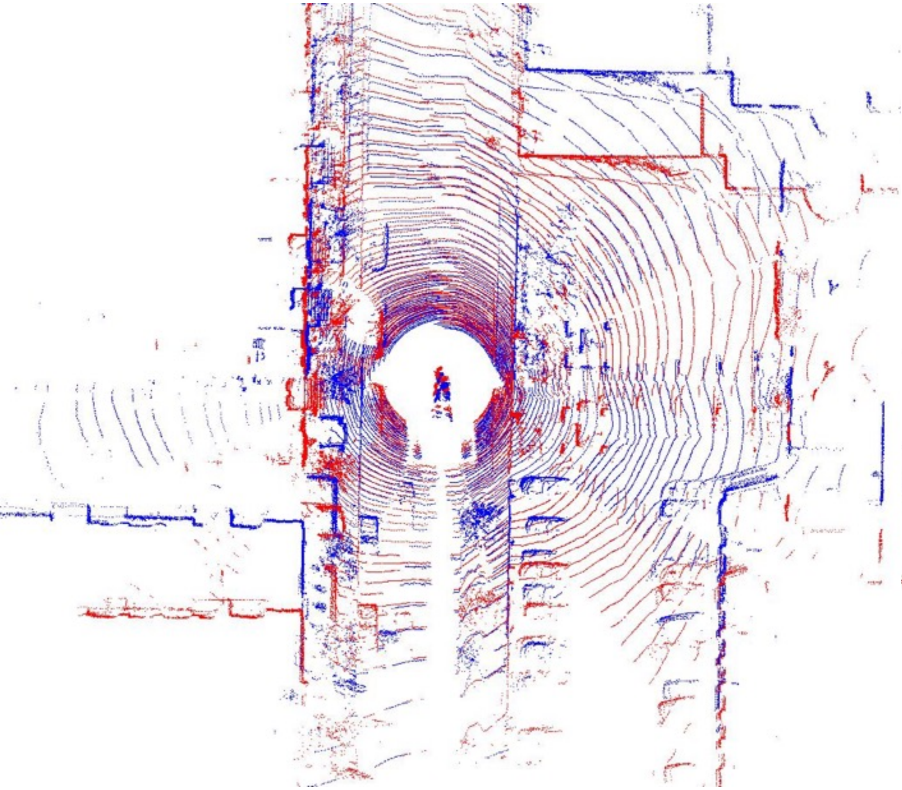
c = speed of light; t = time of flight

Velodyne Laser Scanner

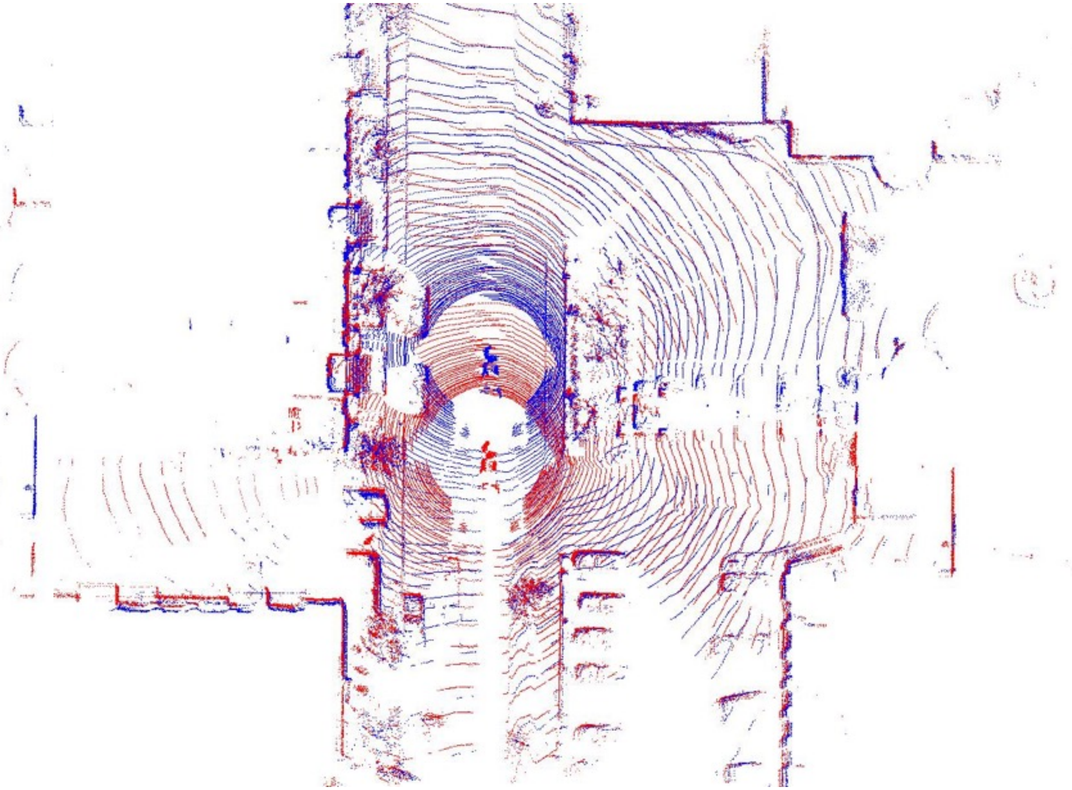




Scan Alignment



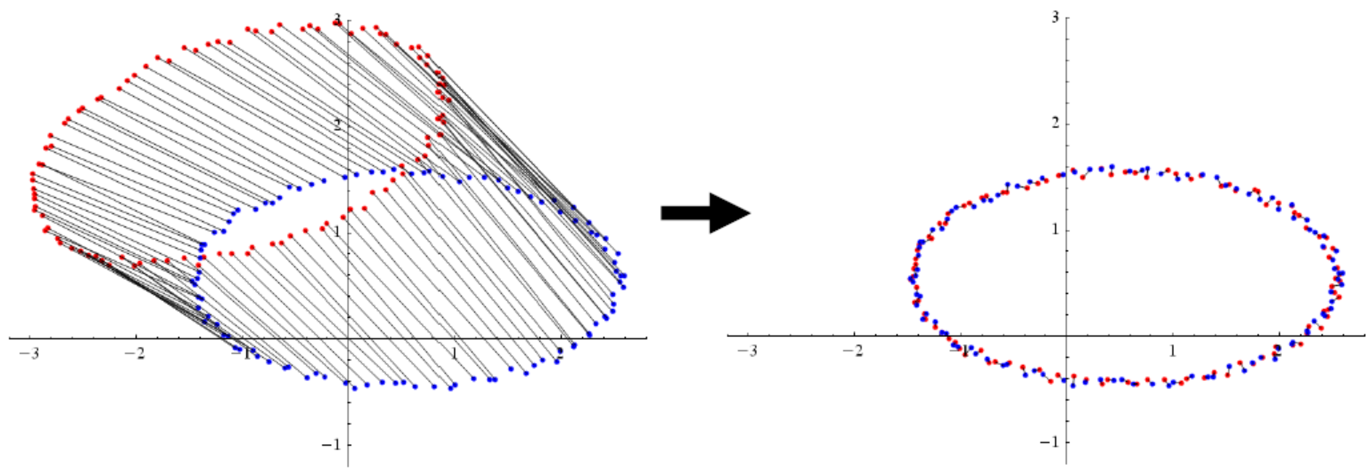
Not-Aligned



Aligned

Scan Alignment: SVD

- Optimal alignment between corresponding points
  - Assuming that for each source point, we know where the corresponding target point is



Slide Credit: http://www.cse.wustl.edu/~taoju/cse554/lectures/lect05_Alignment

Scan Alignment: SVD

- SVD algorithm:
  - Let P be a matrix whose *i*-th column is vector $p_i - c_S$
  - Let Q be a matrix whose *i*-th column is vector $q_i - c_T$
  - Consider the cross-covariance matrix:

$$M = PQ^\top$$

  - Find SVD of M:

$$M = U\Sigma V^\top$$

  - Find Rotation R:

$$R = UV^\top$$

  - Find Translation:

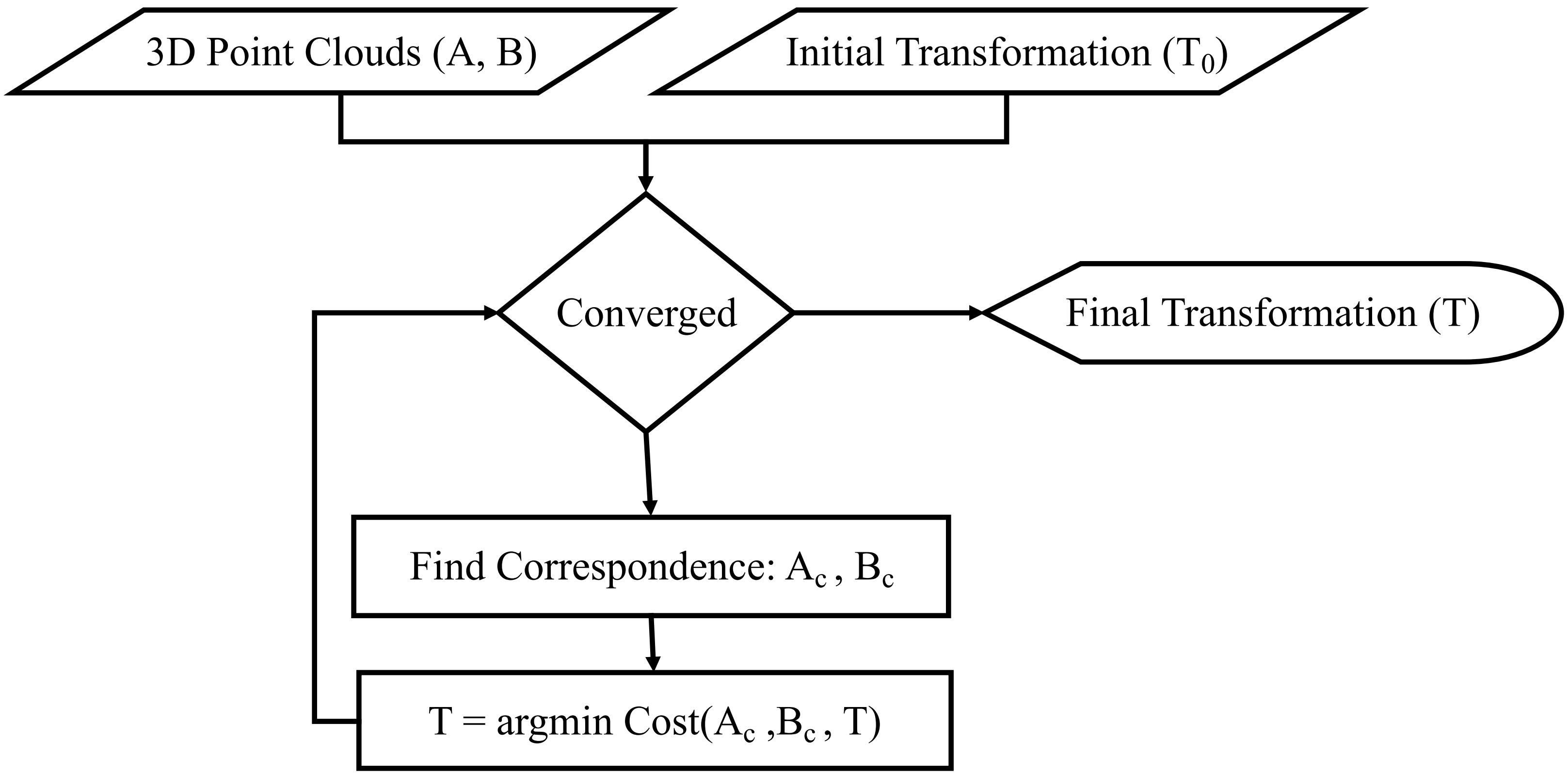
$$\vec{t} = c_T - R * c_S$$

Limitations of SVD

- Requires correct point correspondences
  - Generally correspondences are not known !!

Slide Credit: http://www.cse.wustl.edu/~taoju/cse554/lectures/lect05_Alignment

Iterative Closest Point (ICP)



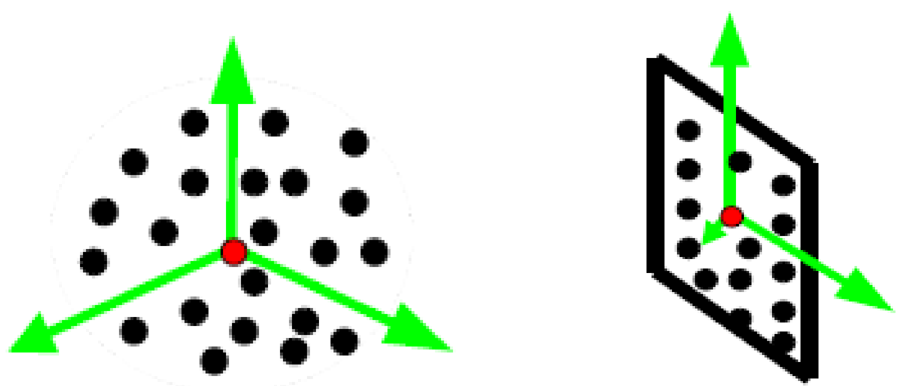
Besl, P. J. and McKay, N. D. (1992). A Method for Registration of 3-D Shapes. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 14(2):239-255.

Generalized ICP

- Generalized-ICP is based on attaching a probabilistic model to the minimization step of the standard ICP algorithm.

$$\mathrm{A} = \{a_i\}_{\,i=1,2\ldots N} \quad \mathrm{B} = \{b_i\}_{\,i=1,2\ldots N} \quad (\mathrm{st}\ a_i == b_i)$$

$$a_i \sim N(\mu_{ai}\,,\,\Sigma_{Ai}) \;\;;\;\; b_i \sim N(\mu_{bi}\,,\,\Sigma_{Bi})$$



Reference: "Generalized ICP" by A. Segal, D. Haehnel, S. Thrun. RSS 2009

Generalized ICP

- Since a_i and b_i are assumed to be drawn from independent Gaussians, the reprojection error **[**$d_i = b_i - \mathrm{T}a_i$**]** for each point correspondence should have the following distribution:

$$\boldsymbol{d_i \sim N(\, \mu_{bi} - T\mu_{ai} \,,\, \Sigma_{Bi} + T\, \Sigma_{Ai}\, T' \,)}$$

$$\boldsymbol{\mu_{bi} = T\mu_{ai}}$$

$$\boldsymbol{d_i \sim N(\, 0 \,,\, \Sigma_{Bi} + T\, \Sigma_{Ai}\, T' \,)}$$

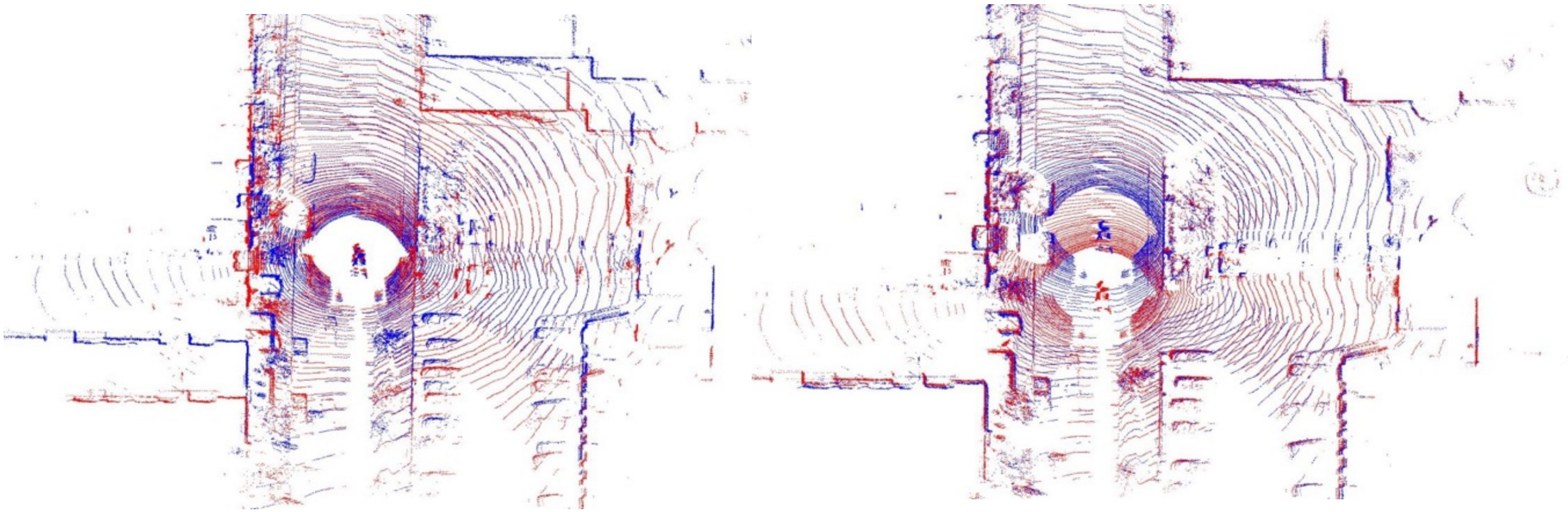
- *"So we want to maximize the likelihood that the error* $\boldsymbol{d_i}$ *is drawn from the probability distribution mentioned above"*

Generalized ICP

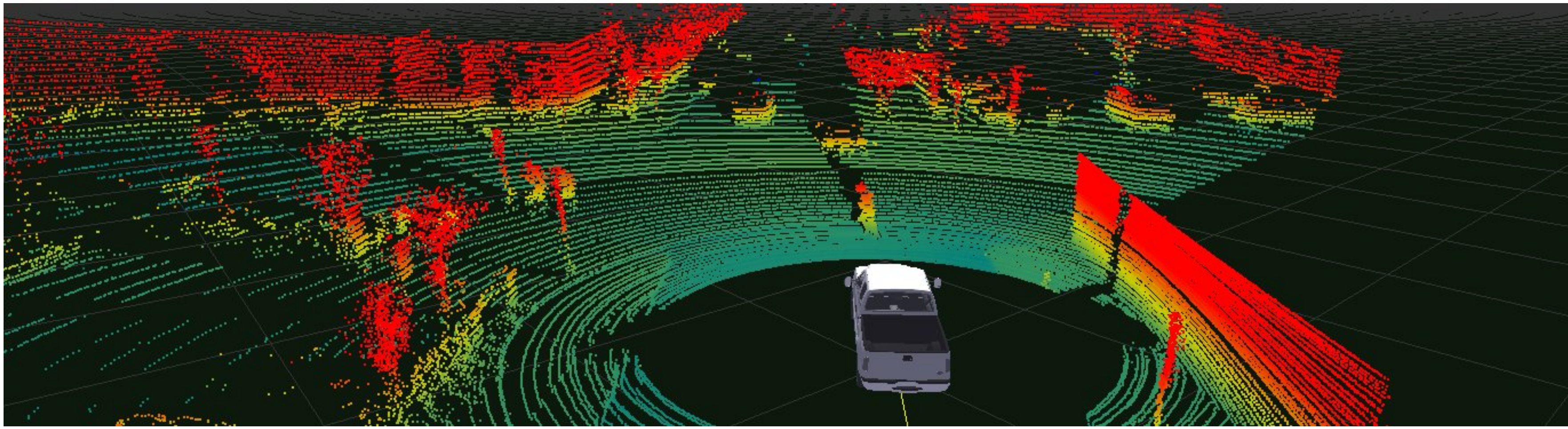
- The MLE is given by :

$$T = \operatorname{argmax} \prod p(\, d_i\,) \; = \operatorname{argmax} \sum \log (\, p(\, d_i\,)\,)$$

$$T = \operatorname{argmin} \sum d_i\,' \,(\Sigma_{Bi} + T\,\Sigma_{Ai}\,T')^{-1}\, d_i$$



Extrinsic Calibration of Perception Sensors



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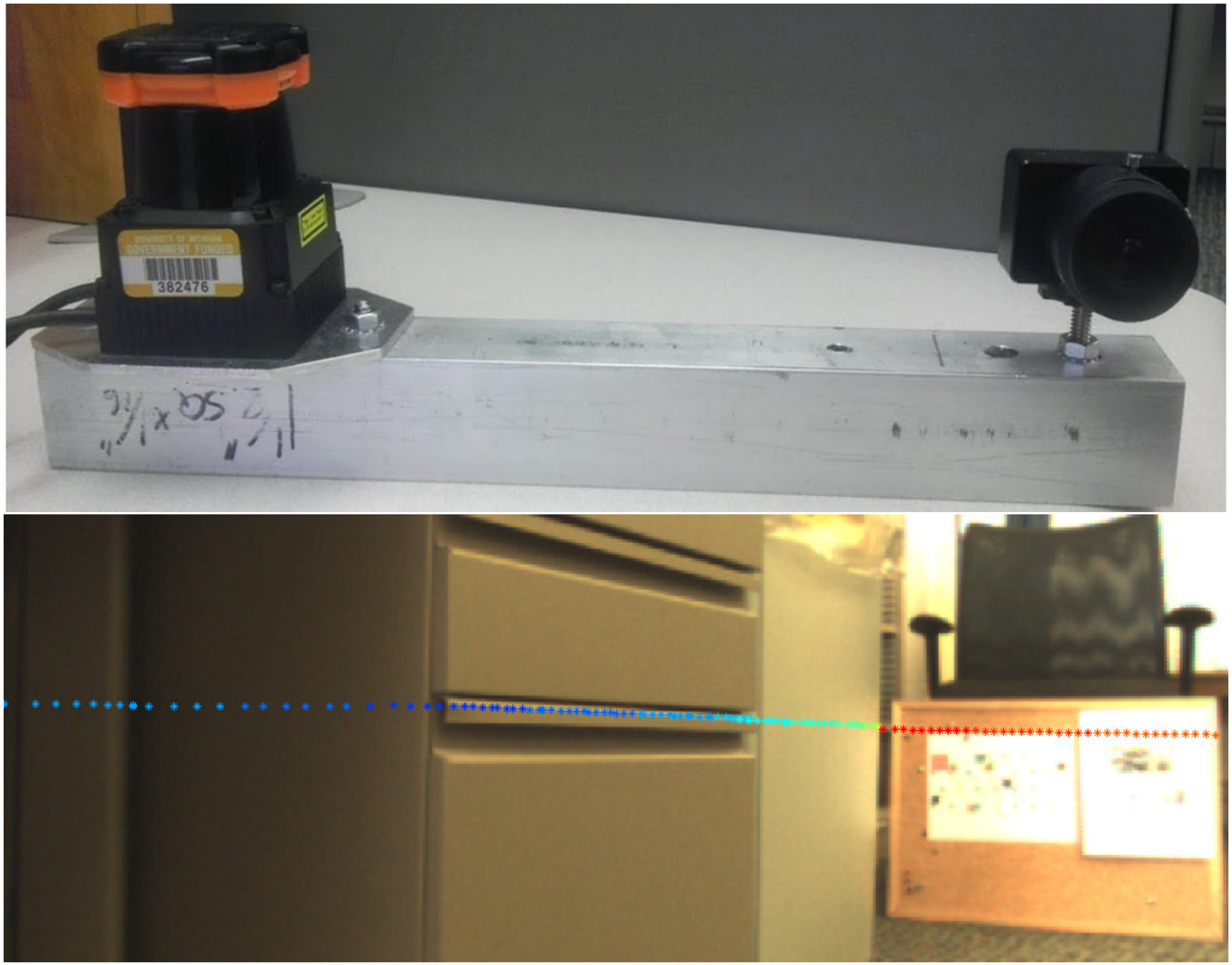
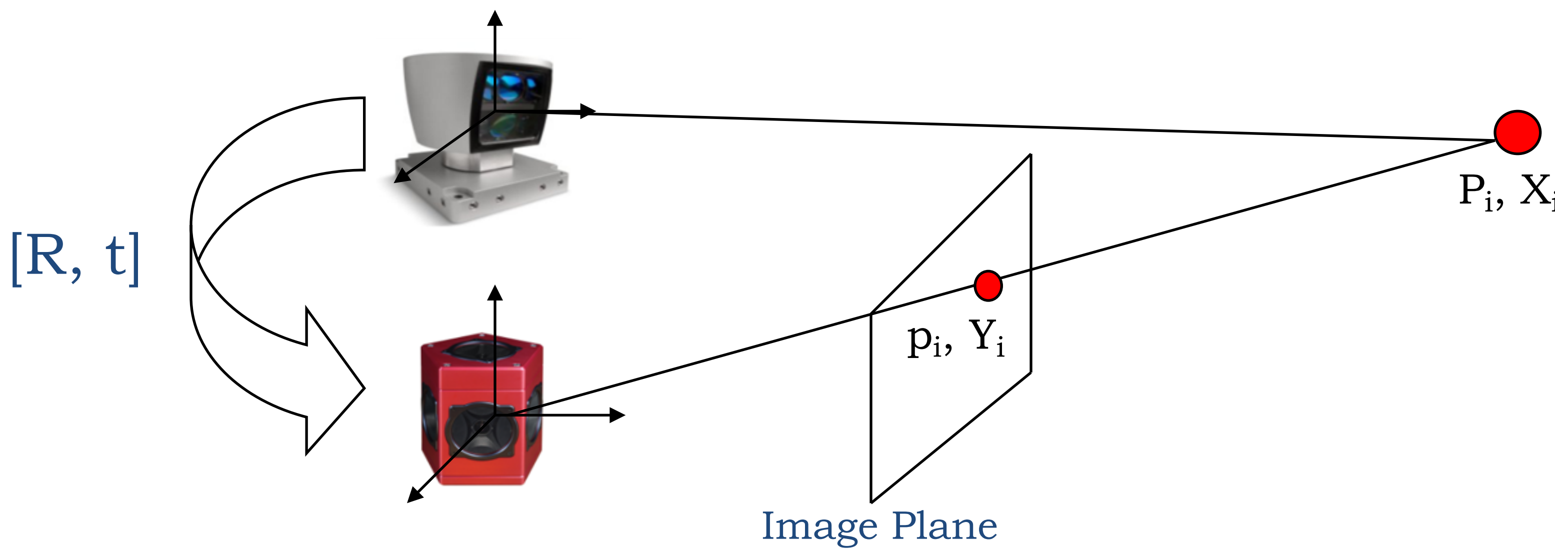


Mathematical Formulation

- $\{P_i \,;\, i = 1, 2, \ldots n\}$ = Set of 3D points
- $\{p_i \,;\, i = 1, 2, \ldots n\}$ = Projection of 3D points on image

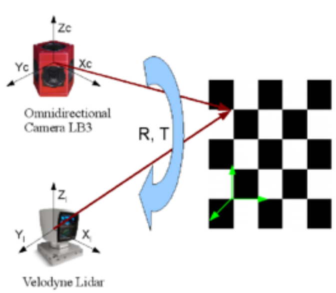
$$p_i = K[R \mid t]\, P_i$$

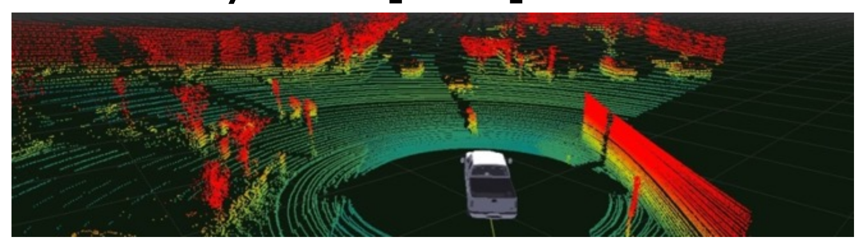


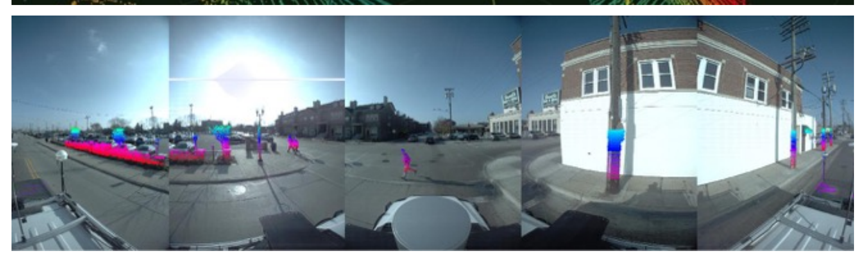
Minimize Reprojection Error !!

References

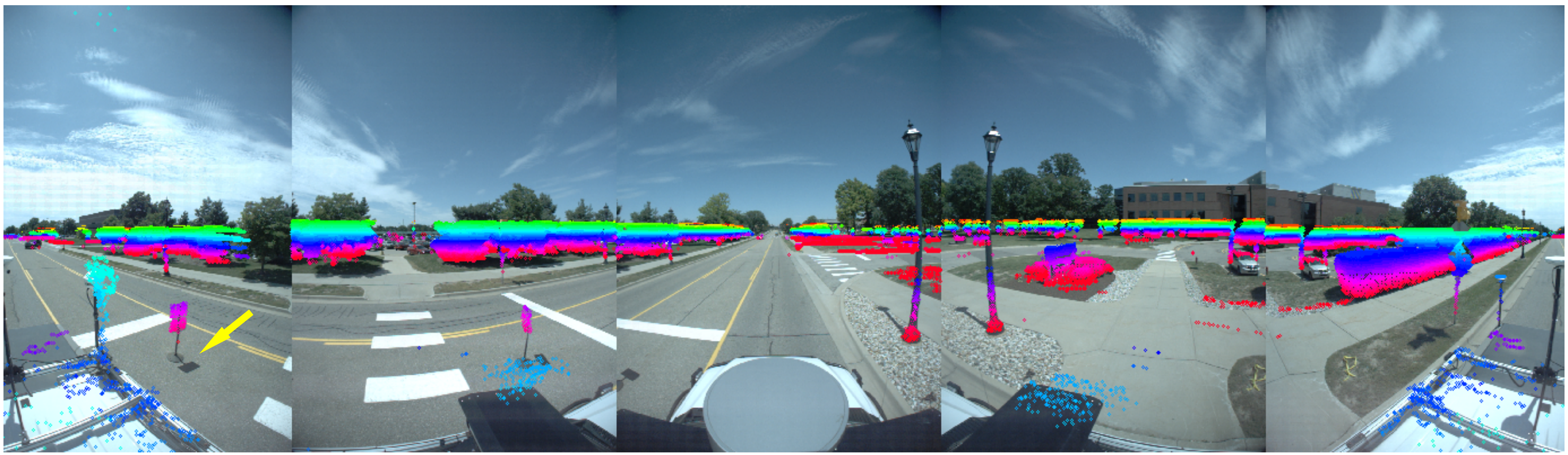
Target based	Targetless
Requires special targets for calibration	Utilizes the correlation between the sensor data for calibration.
▸Q. Zhang & R. Pless [2004]	▸Bougharbal et. al. [2000]
▸R. Unnikrishnan & M. Hebert [2005]	▸Williams et. al. [2004]
▸C. Mei & P. Rives [2006]	▸D. Scaramuzza et. al. [2007]
▸P. Nunez et. al. [2009]	▸G. Pandey et. al. [2012]
▸G. Pandey et. al. [2010]	
▸F. M. Mirzaei et al. [2012]	



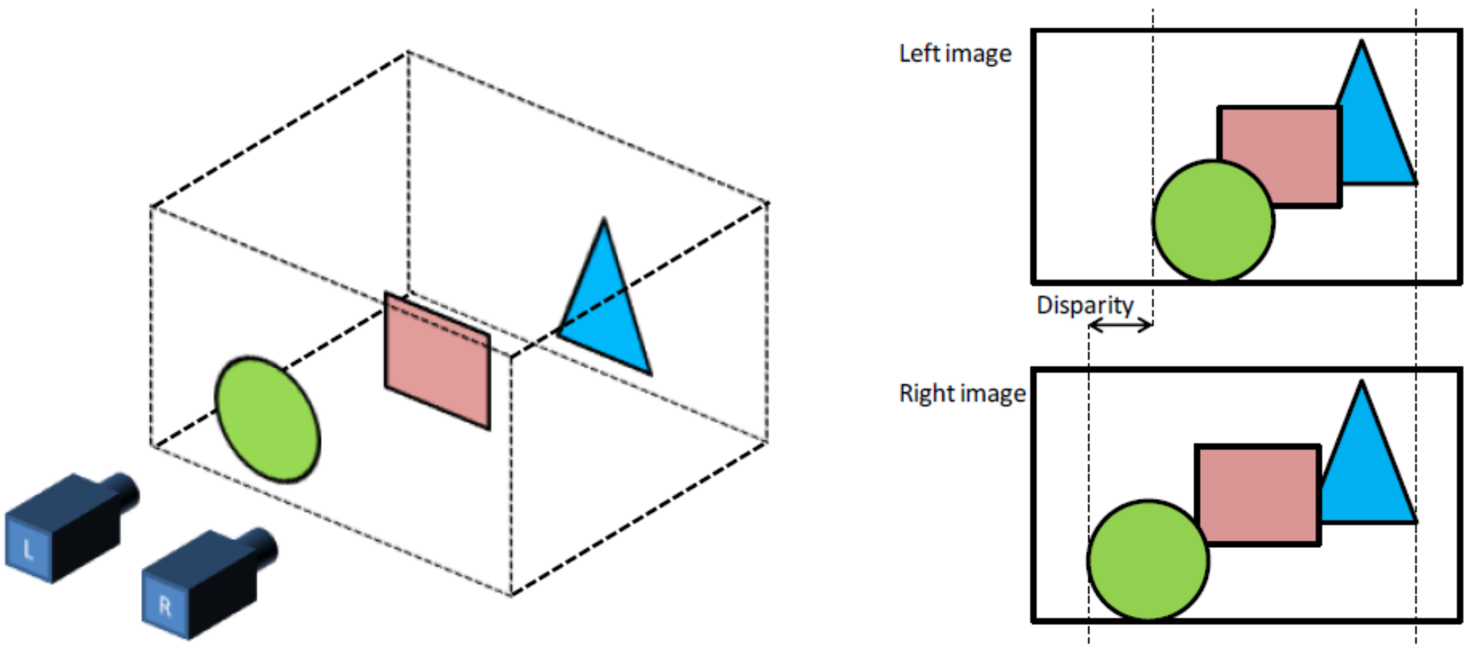




Sensor data fusion: Lidar & Camera



Stereo Cameras



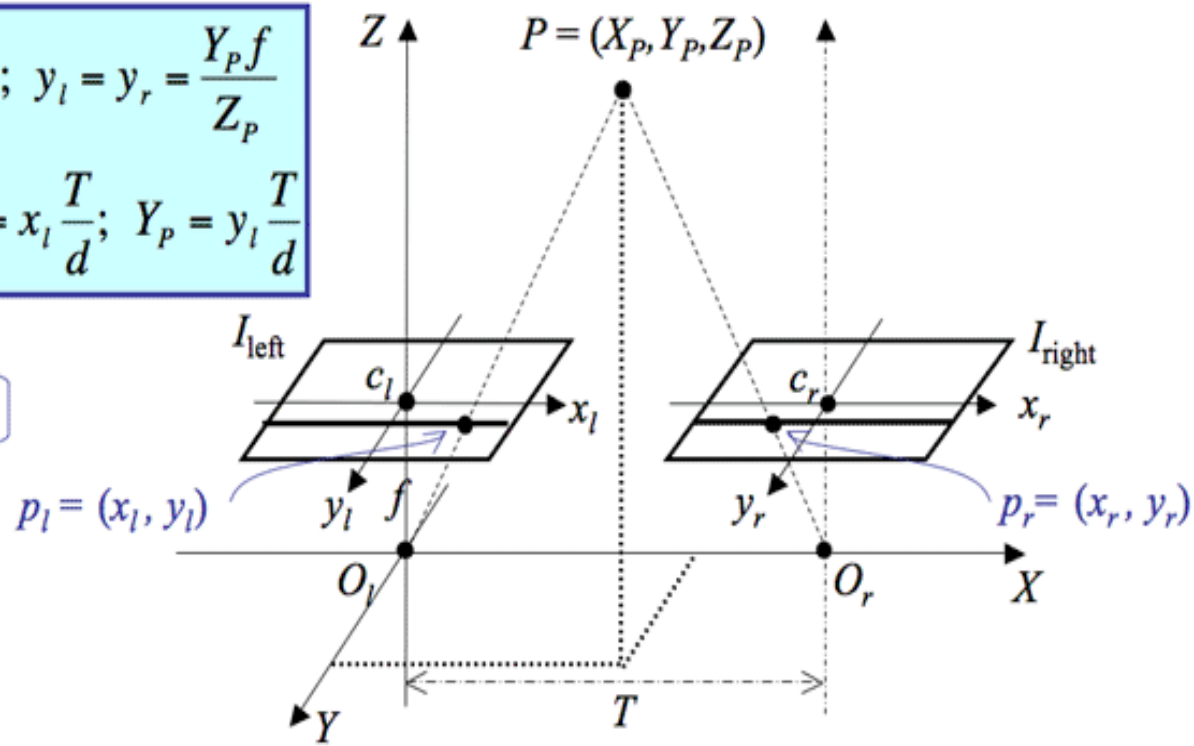
Source: http://www.cs.tut.fi/kurssit/SGN-1656/assignments/DepthEstimation/stereo_instructions.pdf

Disparity and depth from stereo camera

$$x_l = \frac{X_P f}{Z_P};\; x_r = \frac{(X_P - T) f}{Z_P};\; y_l = y_r = \frac{Y_P f}{Z_P}$$

$$Z_P = f\,\frac{T}{x_l - x_r} \equiv f\,\frac{T}{d};\; X_P = x_l \frac{T}{d};\; Y_P = y_l \frac{T}{d}$$

disparity $d = x_l - x_r$

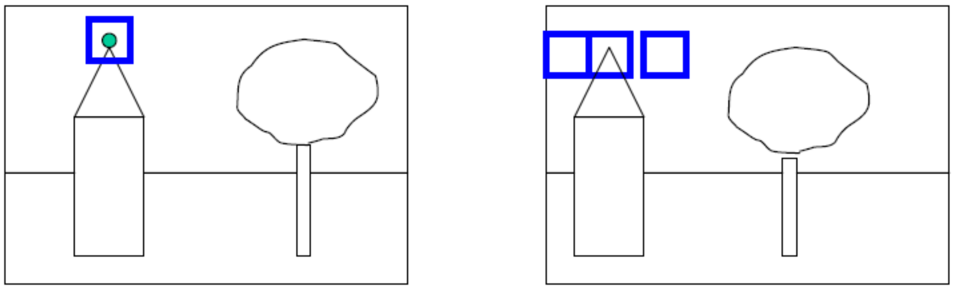


Need to find corresponding point (xr,yr) for each (xl,yl) ⇒ Correspondence problem

Figure Source: https://www.cs.auckland.ac.nz/courses/compsci773s1c/lectures/773-GG/topCS773.htm#stereovision

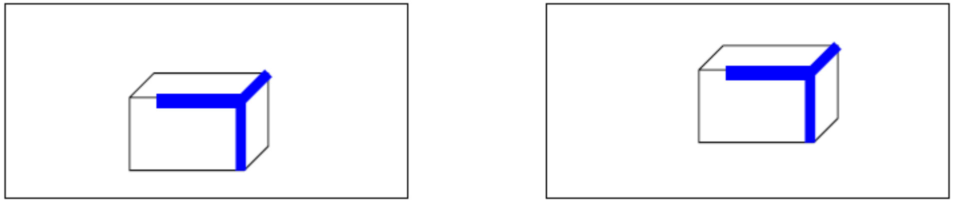
Solving Correspondence problem

1. Cross correlation or SSD using small windows.



dense

2. Symbolic feature matching, usually using segments/corners.



sparse

3. Use the newer interest operators, e.g., SIFT.

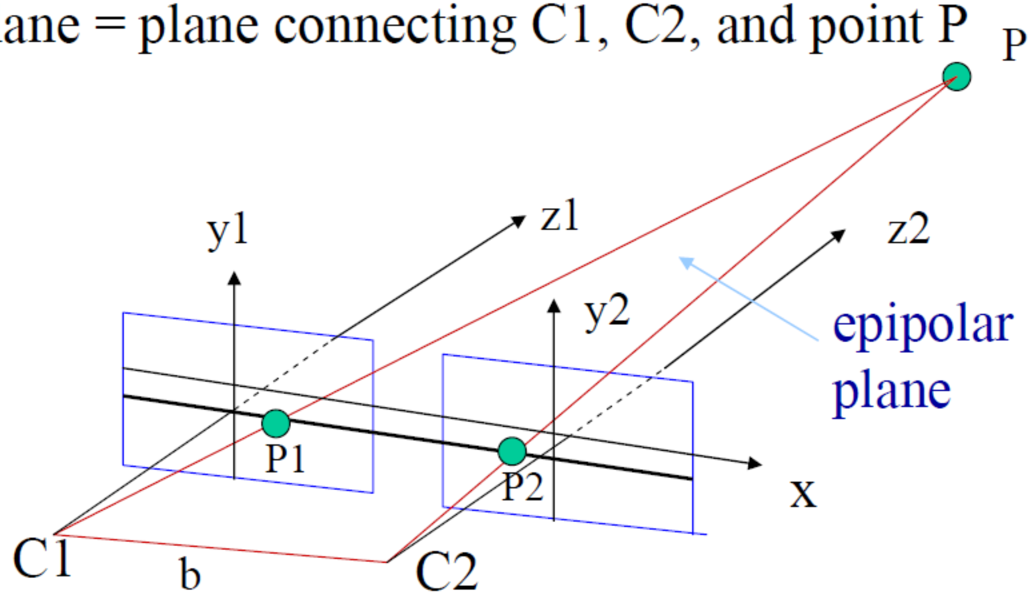
sparse

Slide Credit: https://courses.cs.washington.edu/courses/cse455/09wi/Lects/lect16.pdf

Given a point in the left image, do you need to search the entire right image for the corresponding point?

Epipolar Costraint

Epipolar plane = plane connecting C1, C2, and point P



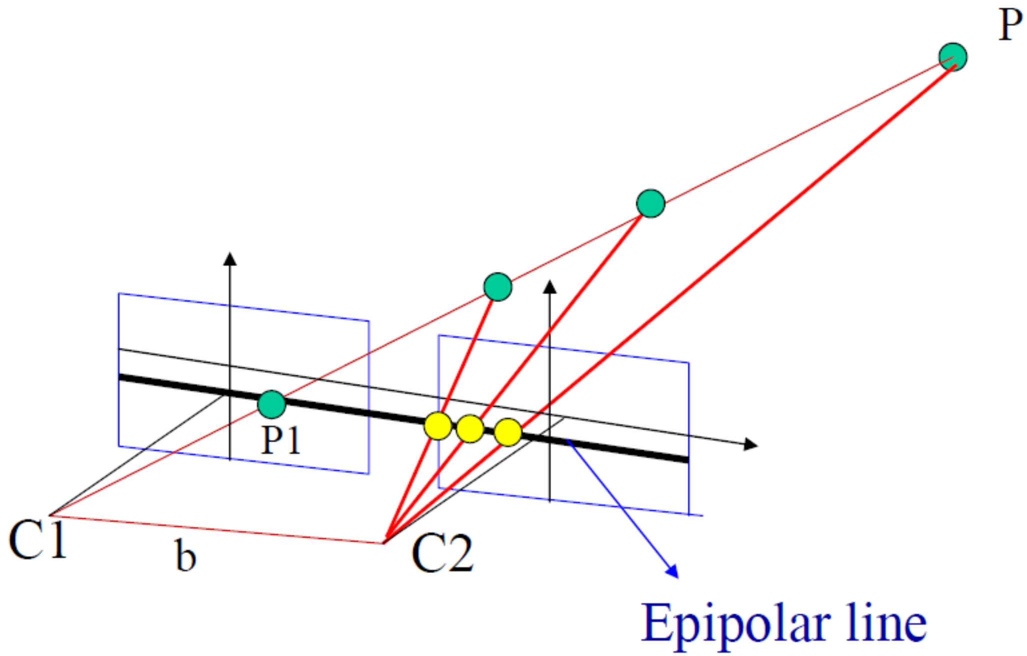
.

Epipolar plane cuts through image planes forming an epipolar line in each plane
Match for P1 (or P2) in the other image must lie on epipolar line

Slide Credit: https://courses.cs.washington.edu/courses/cse455/09wi/Lects/lect16.pdf

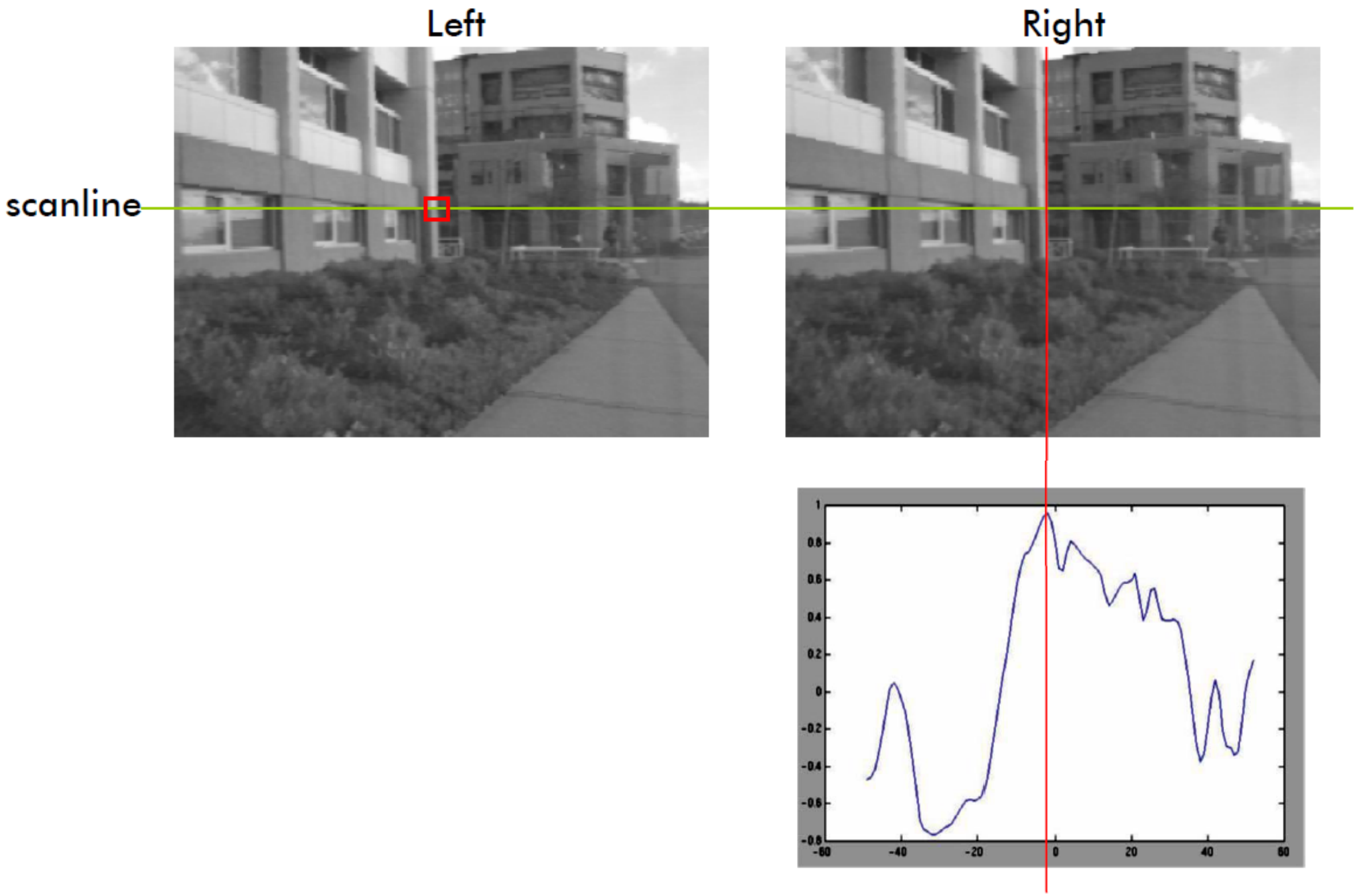
Epipolar Constraint

Match for P1 in the other image must lie on epipolar line
So need search only along this line

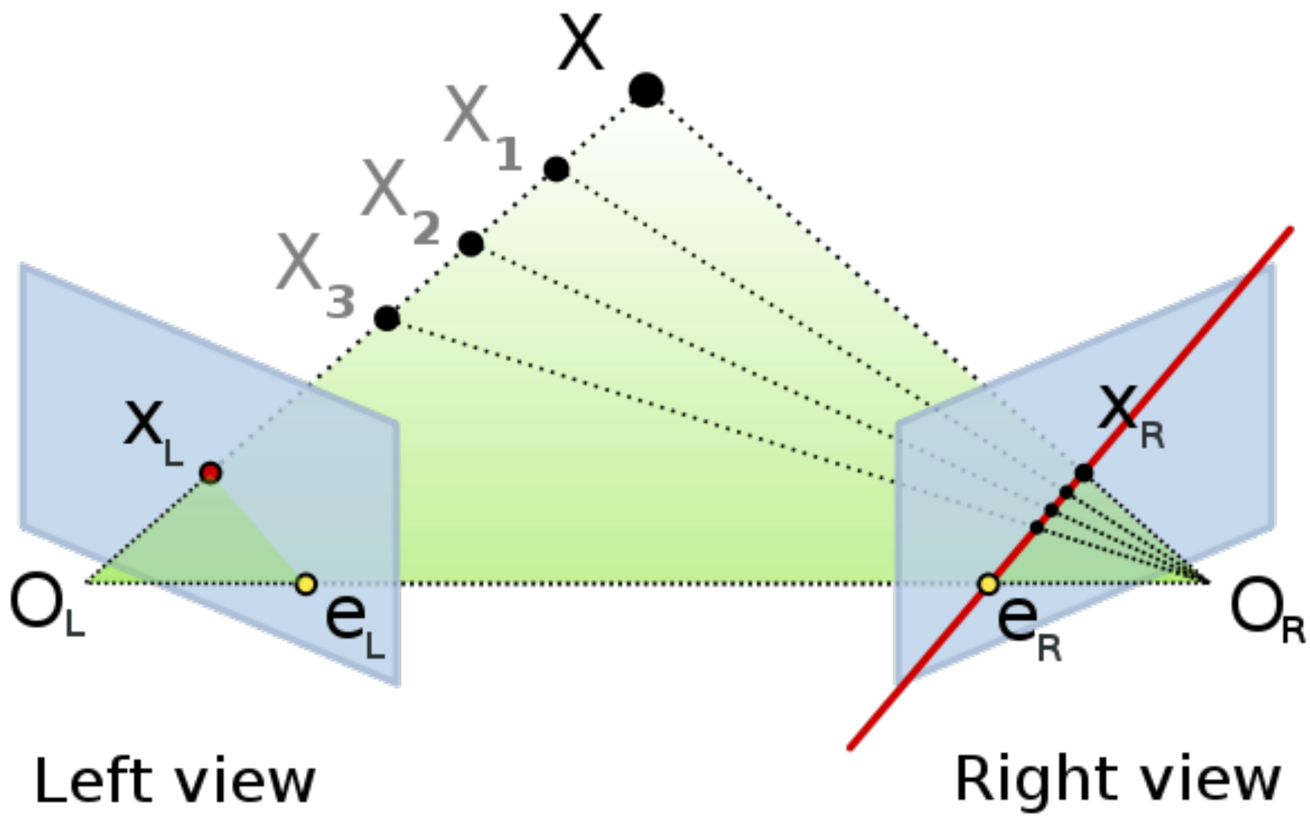


Slide Credit: https://courses.cs.washington.edu/courses/cse455/09wi/Lects/lect16.pdf

Correspondence (Correlation / SSD)

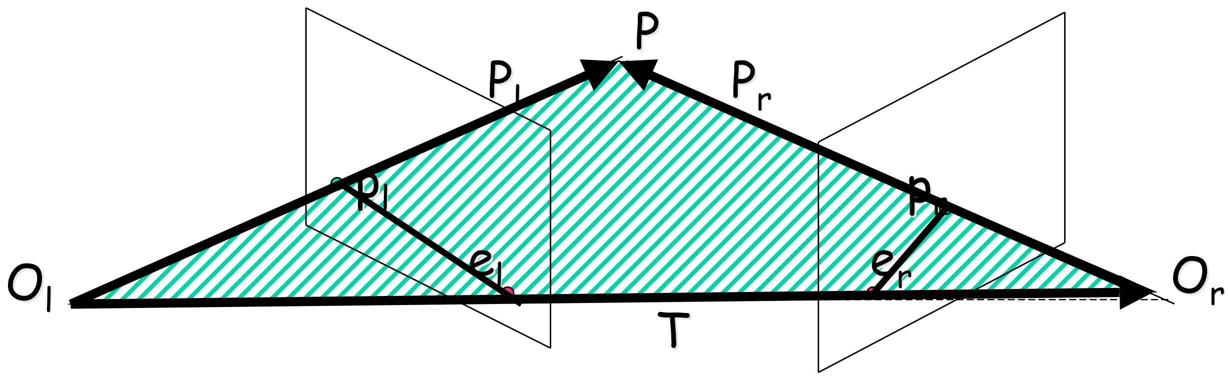


Epipolar Constraint for Non-Parallel Cameras



How to find the epipolar line ?

- Essential Matrix & Fundamental Matrix



$$P_r = RP_l + T$$
$$P_r^\top T \times P_r = 0$$
$$P_r^\top (T \times (RP_l + T)) = 0$$
$$P_r^\top T \times RP_l = 0$$
$$P_r^\top SRP_l = 0$$

Vector Cross Product as Matrix Multiplication

$$T \times P_l = \begin{vmatrix} i & j & k \\ T_x & T_y & T_z \\ P_{l_x} & P_{l_y} & P_{l_z} \end{vmatrix}$$

$$T \times P_l = (T_y P_{l_z} - T_z P_{l_y})i + (T_z P_{l_x} - T_x P_{l_z})j + (T_x P_{l_y} - T_y P_{l_x})k$$

$$T \times P_l = S P_l = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix} \begin{bmatrix} P_{l_x} \\ P_{l_y} \\ P_{l_z} \end{bmatrix} = \begin{bmatrix} T_y P_{l_z} - T_z P_{l_y} \\ T_z P_{l_x} - T_x P_{l_z} \\ T_x P_{l_y} - T_y P_{l_x} \end{bmatrix}$$

S has rank 2 ; it depends only on T

Slide Credit: Robert Collins, Penn State

Properties of Essential Matrix

- Matrix product of one rotation matrix and one skew symmetric matrix E = SR
- The skew-symmetric matrix must have two singular values which are equal and another which is zero.
- The multiplication of the rotation matrix does not change the singular values which means that also the essential matrix has two singular values which are equal and one which is zero.
- Essential Matrix has:
  - Rank 2
  - Depends only on **Extrinsic** parameters [R, T]

Fundamental Matrix

- Suppose K_l and K_r are the intrinsic parameter matrices of left and right camera
- The projection of the points onto the camera is then given by:
  - $p_l = K_l P_l \Rightarrow P_l = K_l^{-1} p_l$
  - $p_r = K_r P_r \Rightarrow P_r = K_r^{-1} p_r$
- From epipolar constraint:
  - $p_r^T [K_r^{-T} R S K_l^{-1}] p_l = 0$
- Fundamental Matrix {F} = $[K_r^{-T} R S K_l^{-1}]$
  - Rank 2
  - Depends on **Intrinsic and Extrinsic** parameters

Homogeneous equation of line

- A line $\{ax + by + c = 0\}$ can be represented by a homogeneous vector $\mathbf{L} = [a\ b\ c]^T$
- Any vector $k\mathbf{L}$ represents the same line.
- A point $\mathbf{x} = [x, y, 1]^T$ lies on the line if and only if $\mathbf{x}^T \mathbf{L} = 0$

Finding Epipolar Line

- The equation below defines a mapping between points and epipolar lines:

$$\mathbf{\bar{p}_r^T F \bar{p}_l = 0}$$

Case 1: right epipolar line u_r

the right epipolar line is represented by $u_r = F\,\bar{p}_l$

$\bar{p}_r$ lies on u_r, that is, $\bar{p}_r^T u_r = 0$ or $\mathbf{\bar{p}_r^T F \bar{p}_l = 0}$

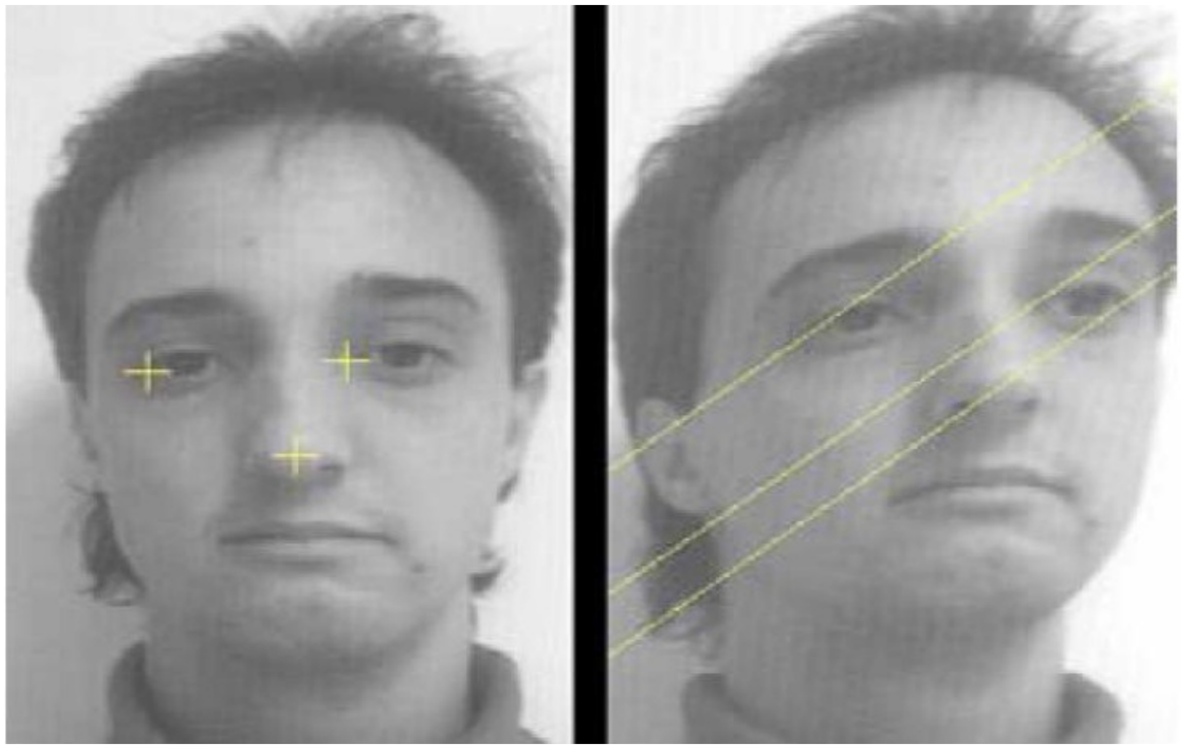
Case 2: left epipolar line u_l

$\mathbf{\bar{p}_r^T F \bar{p}_l = 0}$ is equivalent to $\mathbf{\bar{p}_l^T F^T \bar{p}_r = 0}$

the left epipolar line is represented by $u_l = F^T\,\bar{p}_r$

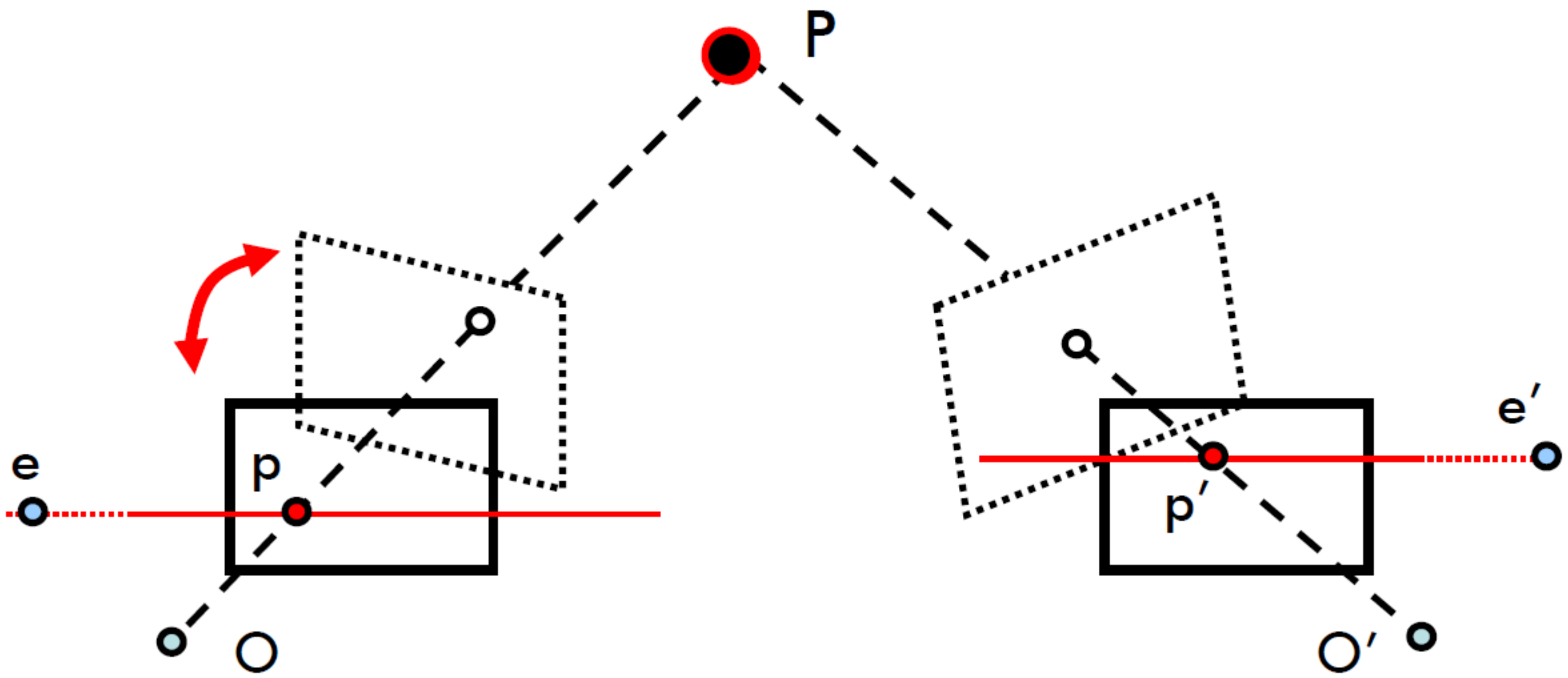
$\bar{p}_l$ lies on u_l, that is, $\bar{p}_l^T u_l = 0$ or $\mathbf{\bar{p}_l^T F^T \bar{p}_r = 0}$

Epipolar Line: Example



- For every feature point you have to find the corresponding epipolar line !

Stereo Rectification



- After rectification you only need to search along the horizontal scan lines (computationally more efficient)

Estimating Fundamental / Essential Matrix

- Assume that you have m correspondences
- Each correspondence satisfies:

$$\bar{p}_{r_i}^{T} F \bar{p}_{l_i} = 0 \quad i = 1, \ldots, m$$

- F is a 3x3 matrix (9 entries)
- Set up a **HOMOGENEOUS** linear system with 9 unknowns

Slide Credit: Robert Collins, Penn State

Estimating Fundamental Matrix

$$\bar{p}_{l_i} = (x_i \;\; y_i \;\; 1)^T \quad \bar{p}_{r_i} = (x_i' \;\; y_i' \;\; 1)^T$$

$$\bar{p}_{r_i}^T F \bar{p}_{l_i} = 0 \quad i = 1, \ldots, m$$

$$\begin{bmatrix} x_i' & y_i' & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = 0$$

Slide Credit: Robert Collins, Penn State

Estimating Fundamental Matrix

$$\begin{bmatrix} x'_i & y'_i & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = 0$$

Given m point correspondences…

$$\begin{bmatrix} x_1x'_1 & x_1y'_1 & x_1 & y_1x'_1 & y_1y'_1 & y_1 & x'_1 & y'_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_mx'_m & x_my'_m & x_m & y_mx'_m & y_my'_m & y_m & x'_m & y'_m & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{21} \\ f_{31} \\ f_{12} \\ f_{22} \\ f_{32} \\ f_{13} \\ f_{23} \\ f_{33} \end{bmatrix} = 0$$

Think: how many points do we need?

Estimating F

Assume that we need a non-trivial solution of:

$$A\mathbf{x} = 0$$

with m equations and n unknowns, m >= n – 1 and rank(A) = n-1

Since the norm of x (**||x||**) can be arbitrary we generally find the solution with norm equal to 1 in order to avoid the trivial solution.

Hence minimum 8 points are needed to solve the above equation. Therefore it is also called **8-point algorithm.**

Estimating F

Required Optimization:

$$\min_{\mathbf{x}} ||A\mathbf{x}||^2 \text{ s.t. } ||x||^2 = 1$$

$$||A\mathbf{x}||^2 = (A\mathbf{x})^T(A\mathbf{x}) = \mathbf{x}^T A^T A\mathbf{x}$$

$$||\mathbf{x}||^2 = \mathbf{x}^T\mathbf{x} = 1$$

Estimating F

Solution:

- Construct the m x 9 matrix A
- Find the SVD of A: $A = UDV^T$
- The entries of F are the components of the column of V corresponding to the least s.v.

Estimating F

Define the Lagrangian:

$$\mathcal{L}(\mathbf{x}) = \mathbf{x}^T A^T A \mathbf{x} - \lambda(\mathbf{x}^T \mathbf{x} - 1)$$

Take the derivative of Lagrangian w.r.t. x and lambda:

$$A^T A \mathbf{x} - \lambda \mathbf{x} = 0$$
$$\mathbf{x}^T \mathbf{x} - 1 = 0$$

Clearly the solution is given by the eigenvector corresponding to the smallest eigenvalue of $A^T A$.

Nister’s 5-point algorithm

“An Efficient Solution to the Five-Point Relative Pose Problem”, David Nist´er Sarnoff Corporation CN5300, Princeton, NJ 08530 dnister@sarnoff.com

Recovering [R, T] from Essential Matrix

The Singular Value Decomposition of E is:

$$\mathsf{E} = \mathsf{U} \, \mathrm{diag}(1, 1, 0) \, \mathsf{V}^\mathsf{T}$$

then the following two solutions are possible for R:

$$\begin{aligned} \mathsf{R} &= \mathsf{U}\mathsf{W}\mathsf{V}^\mathsf{T} \\ \mathsf{R} &= \mathsf{U}\mathsf{W}^\mathsf{T}\mathsf{V}^\mathsf{T} \end{aligned}$$

and for $\mathbf{t}$:

$$\mathbf{t} = \pm \mathbf{u}_3$$

Among the 4 solutions, 1 is feasible

Slide Credit: Adrien Bartoli, CNRS – LASMEA Clermont-Ferrand, France

4 Solutions

The sign of $\mathbf{t}$ is undetermined

Combining with the two possible rotations, this gives:

$$\mathsf{P}' \sim (\mathsf{U}\mathsf{W}\mathsf{V}^\mathsf{T} \;\; \mathbf{u}_3) \qquad \text{or} \qquad \mathsf{P}' \sim (\mathsf{U}\mathsf{W}\mathsf{V}^\mathsf{T} \;\; -\mathbf{u}_3)$$

$$\mathsf{P}' \sim (\mathsf{U}\mathsf{W}^\mathsf{T}\mathsf{V}^\mathsf{T} \;\; \mathbf{u}_3) \qquad \text{or} \qquad \mathsf{P}' \sim (\mathsf{U}\mathsf{W}^\mathsf{T}\mathsf{V}^\mathsf{T} \;\; -\mathbf{u}_3)$$

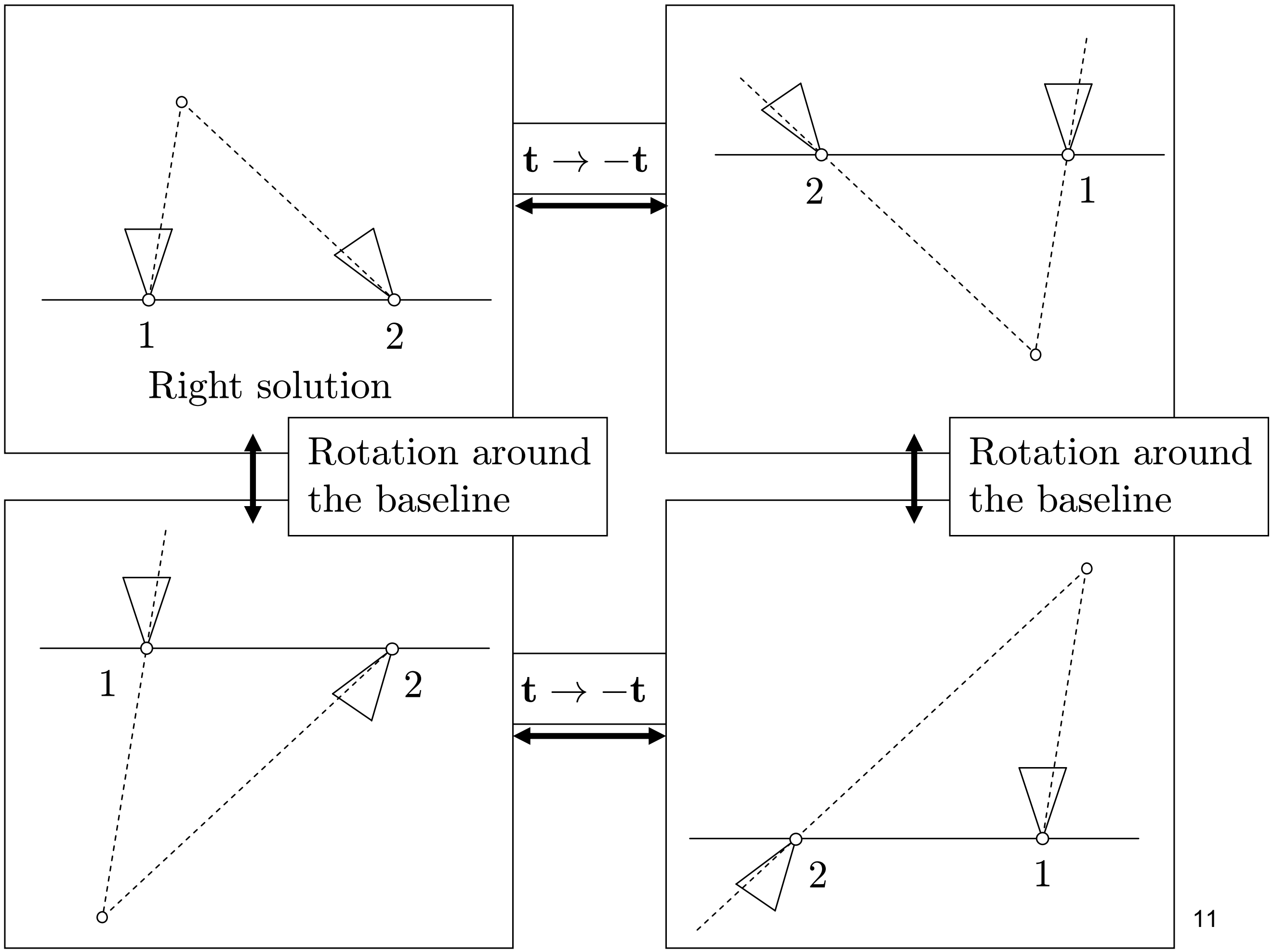
The $\mathbf{u}_3 \rightarrow -\mathbf{u}_3$ swaps the position of the cameras

The $\mathsf{U}\mathsf{W}\mathsf{V}^\mathsf{T} \rightarrow \mathsf{U}\mathsf{W}^\mathsf{T}\mathsf{V}^\mathsf{T}$ makes a rotation of π around the baseline

Only one solution is feasible

Slide Credit: Adrien Bartoli, CNRS – LASMEA Clermont-Ferrand, France

4 Solutions



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