EE698G – Assignment 2

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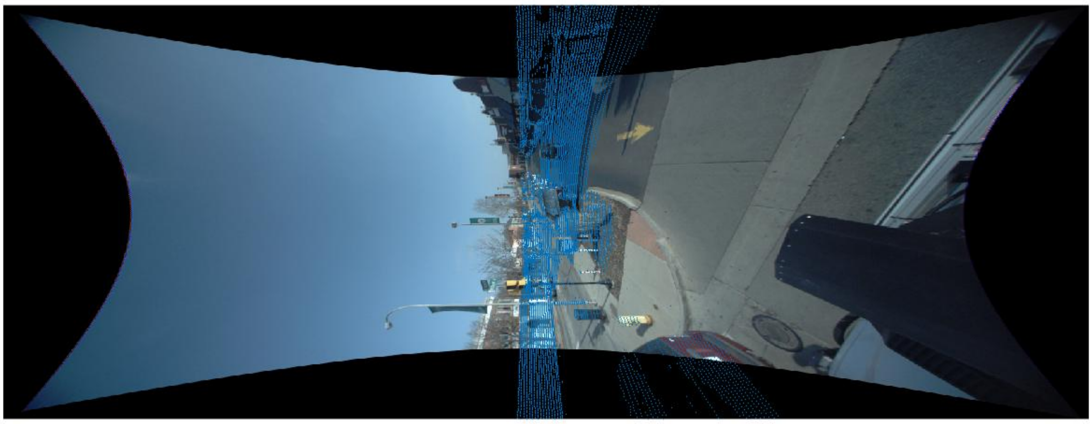
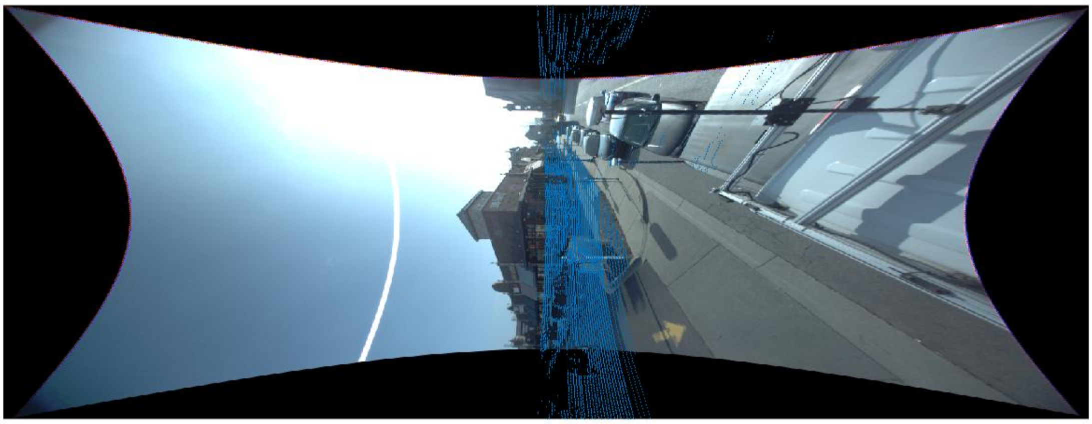
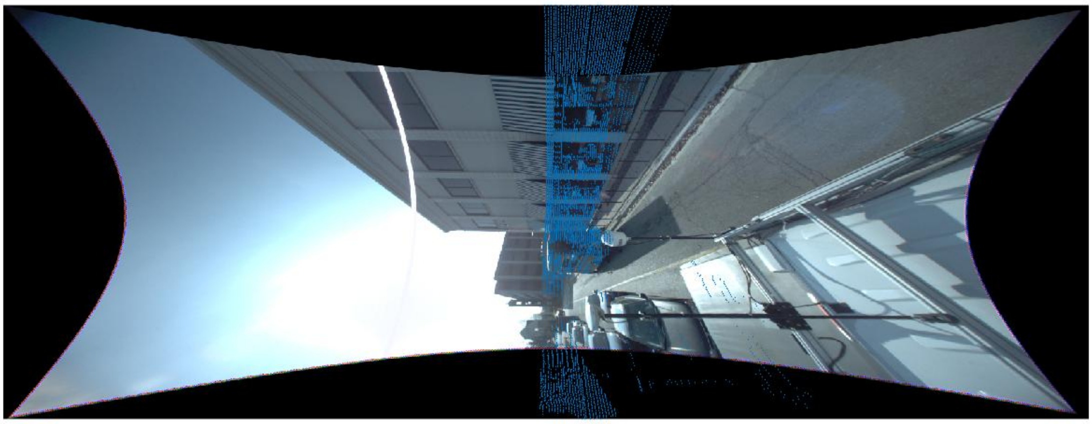
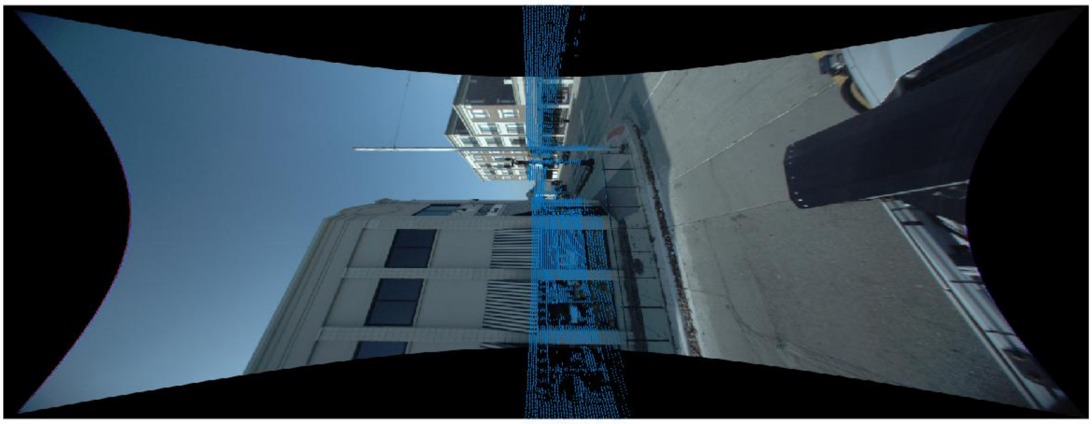
1

Sample Outputs for ground threshold = 2:



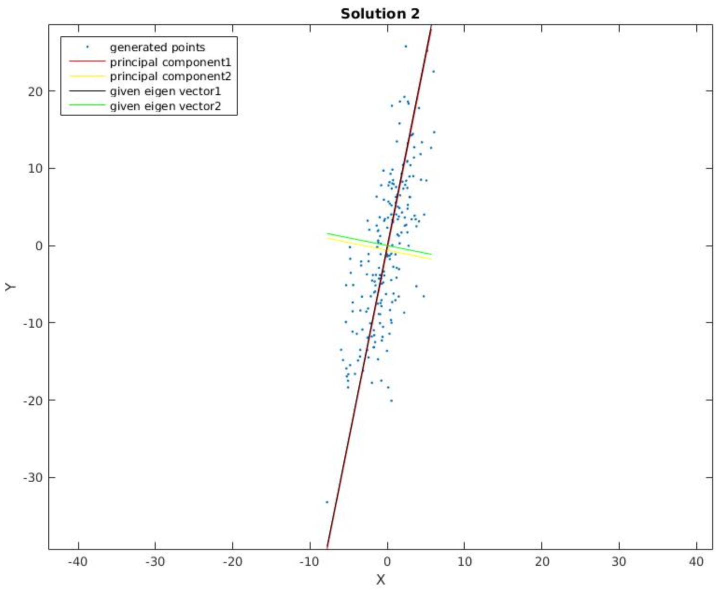






2

Yes, the direction of eigenvectors of the generated data match with that of original eigenvectors. sample output of the attached MATLAB code is given below.



3

3.1

No,

As transformation between polar and euclidean coordinates contains cosine and sine terms that is non-linear, euclidean coordinates will not be normally distributed. But it can be approximated by a normal distribution by linearising the transformation using taylor series expansion.

It can also be seen by joint probability distribution function of euclidean coordinates which is calculated in the following parts.

joint probability distribution function of euclidean

Let r, θ be the polar coordinates and the x, y be the corresponding euclidean coordinates.

$$\Rightarrow r = \sqrt{(x^2 + y^2)} \text{ and } \theta = \arctan\left(\frac{y}{x}\right)$$

$$\text{Jacobian } J = \left\{ \begin{array}{cc} \frac{\delta r}{\delta x} & \frac{\delta r}{\delta y} \\ \frac{\delta \theta}{\delta x} & \frac{\delta \theta}{\delta y} \end{array} \right\} = \left\{ \begin{array}{cc} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} \\ \frac{\frac{-y}{x}}{1+\frac{y^2}{x^2}} & \frac{\frac{1}{x}}{1+\frac{y^2}{x^2}} \end{array} \right\}$$

$$\Rightarrow |J| = \frac{1}{x^2 + y^2}$$

$\mu = [\mu_\theta, \mu_r]^T$ and $\Sigma = \begin{Bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_r^2 \end{Bmatrix}$ is:

$$f(r,\theta) = \frac{1}{\sqrt{4\pi^2\sigma_r^2\sigma_\theta}} e^{(-\frac{1}{2}[r-\mu_r, \theta-\mu_\theta]\ \sum\ [r-\mu_r, \theta-\mu_\theta]^T)}$$

$$\Rightarrow f(r,\theta) = \frac{1}{\sqrt{4\pi^2\sigma_r^2\sigma_\theta}} e^{(-\frac{1}{2}(\frac{(r-\mu)^2}{\sigma_\theta^2} + \frac{(\theta-\mu_\theta)^2}{\sigma_r^2}))}$$

$$\Rightarrow f(r(x,y),\theta(x,y)) = \frac{1}{\sqrt{4\pi^2\sigma_r^2\sigma_\theta}} e^{(-\frac{1}{2}(\frac{\sqrt{x^2+y^2}-\mu)^2}{\sigma_\theta^2} + \frac{(\arctan\frac{y}{x}-\mu_\theta)^2}{\sigma_r^2}))}$$

The joint pdf of the euclidean coordinates $f(x,y) = |J| f(r(x,y),\theta(x,y))$

$$\Rightarrow f(x,y) = \frac{1}{\sqrt{4\pi^2\sigma_r^2\sigma_\theta(x^2+y^2)}} e^{(-\frac{1}{2}(\frac{r-\mu)^2}{\sigma_\theta^2} + \frac{(\theta-\mu_\theta)^2}{\sigma_r^2}))}$$

Approx covariance after linearisation

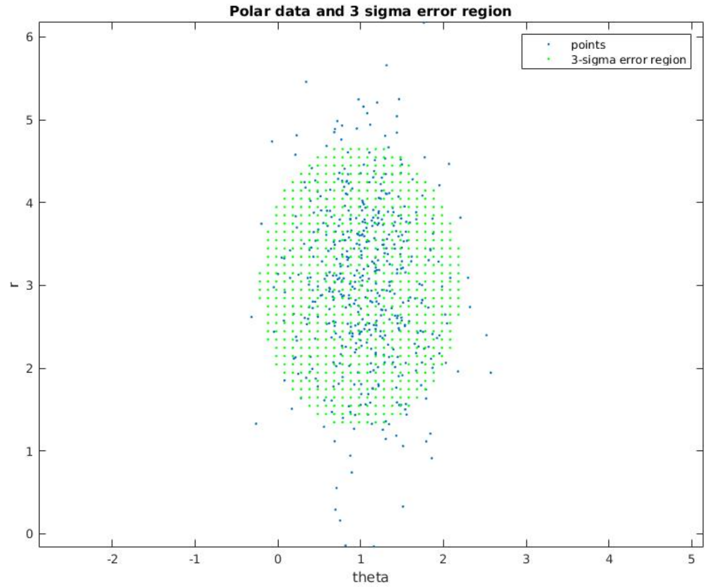
$$\Sigma_{xy} = J\Sigma_{r\theta}J^T$$

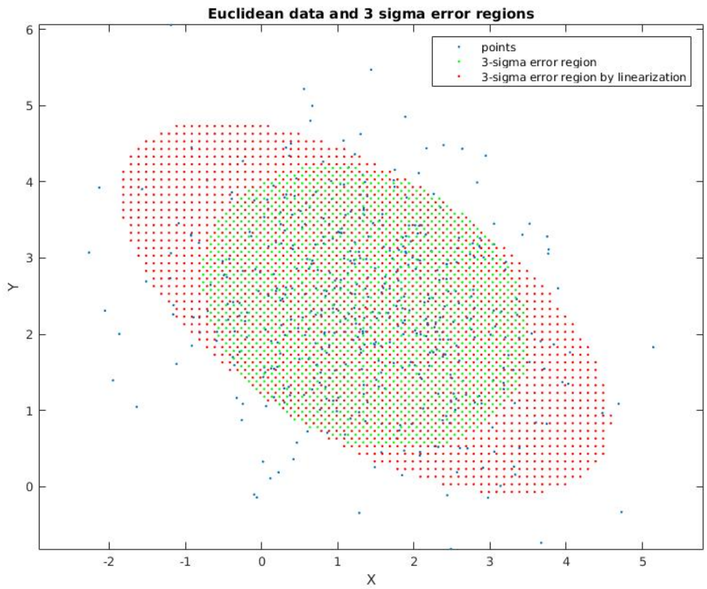
$$J = \begin{Bmatrix} -r\sin(\theta) & cos(\theta) \\ r\cos(\theta) & \sin(\theta) \end{Bmatrix}_\mu$$

$$\Rightarrow \Sigma_{xy} = \begin{Bmatrix} -\mu_r\sin(\mu_\theta) & cos(\mu_\theta) \\ \mu_r\cos(\mu_\theta) & \sin(\mu_\theta) \end{Bmatrix} \begin{Bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_r^2 \end{Bmatrix} \begin{Bmatrix} -\mu_r\sin(\mu_\theta) & \mu_r\cos(\mu_\theta) \\ cos(\mu_\theta) & \sin(\mu_\theta) \end{Bmatrix}$$

$$\Rightarrow \Sigma_{xy} = \begin{Bmatrix} \mu_r^2\sigma_\theta^2 + \sin^2(\sigma_r^2 - \mu_r^2\sigma_\theta^2) & cos(\mu_\theta)sin(\mu_\theta)(\sigma_r^2 - \mu_r^2\sigma_\theta^2) \\ cos(\mu_\theta)sin(\mu_\theta)(\sigma_r^2 - \mu_r^2\sigma_\theta^2) & \mu_r^2\sigma_\theta^2 + \cos^2(\sigma_r^2 - \mu_r^2\sigma_\theta^2) \end{Bmatrix}$$

3.2





from the above plots it is clear that $\sum_{lin} < \sum_{sam}$ i.e. $\sum_{lin}$ is less accurate but more faster than $\sum_{sam}$. In other words it is a good approximation in real time usage.

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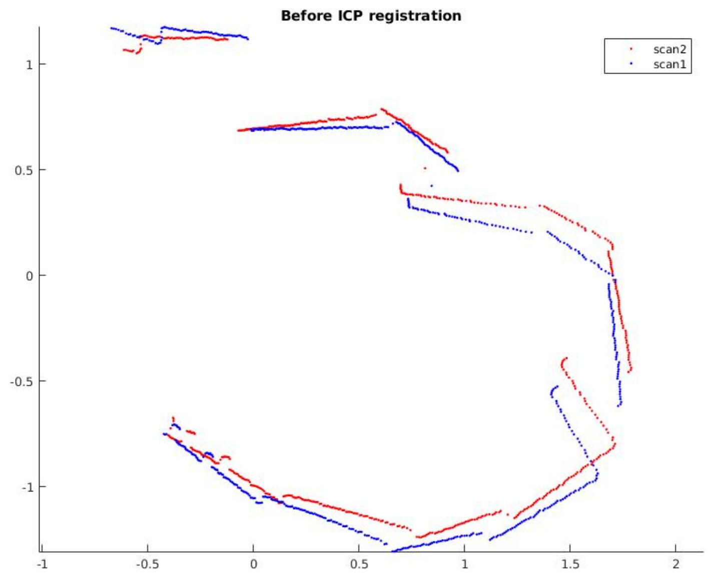
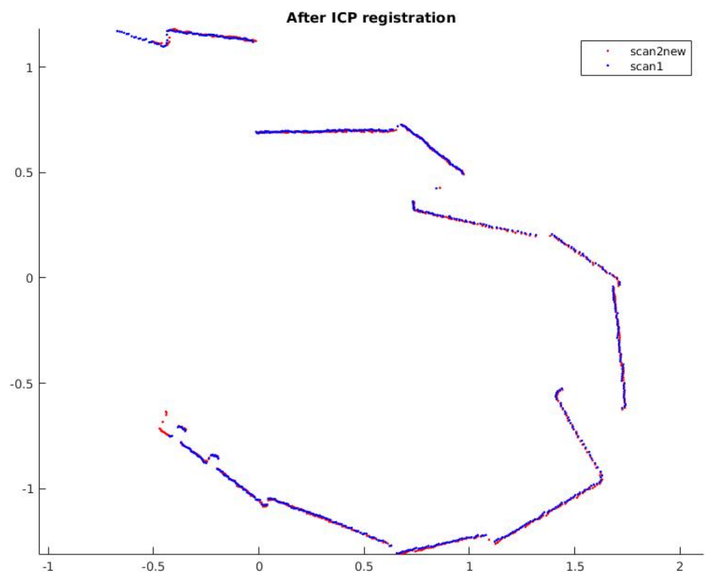
ICP is very computation extensive due to traversing both scans each time to compute correspondences hence it is not useful in real time without any further modifications.

For maximum threshold = 0.01 output of ICP is following -





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$$H_{ij} = \left\{ \begin{array}{cc} R_{ij} & t_{ij} \\ 0 & 1 \end{array} \right\}$$

similarly,

$$H_{ji} = \left\{ \begin{array}{cc} R_{ji} & t_{ji} \\ 0 & 1 \end{array} \right\}$$

$$\Rightarrow H_{ij}H_{ji} = \left\{ \begin{array}{cc} R_{ij}R_{ji} & R_{ij}t_{ji} + t_{ij} \\ 0 & 1 \end{array} \right\}$$

$t_{ij} = -R_{ji}^T t_{ji}$ and$R_{ij} = R_{ji}^T$,

$$\Rightarrow H_{ij}H_{ji} = \left\{ \begin{array}{cc} R_{ij}R_{ij}^T & R_{ji}^T t_{ji} - R_{ji}^T t_{ji} \\ 0 & 1 \end{array} \right\}$$

$RR^T = 1$ as R is an orthogonal matrix with norm 1.

$$\Rightarrow H_{ij}H_{ji} = \left\{ \begin{array}{cc} I & 0 \\ 0 & 1 \end{array} \right\}$$

$$\Rightarrow H_{ij}H_{ji} = I$$

$$\Rightarrow H_{ij} = H_{ji}^{-1}$$