Probabilistic Mobile Robotics - EE698G

Assignment- 1

- Due Date: **January 22, 2017, 11:59pm**
- Late submission penalty is 20% for every late day.
- Please submit MATLAB codes to moodle. You can submit other solutions by
  - (i) **(Recommended)** Typing it in latex and uploading the PDF to moodle along with codes.
  - (ii) Dropping handwritten solutions in the mailbox.
  - (iii) Creating a PDF by scanning handwritten documents and uploading it to moodle – should be clear.
- **MATLAB**
  - (i) Submit a single .zip file for this assigment.
  - (ii) Include a README textfile in the .zip file. It should mention which scripts to run to generate the desired results for each question.

Q.1 Show that a set of nonzero vectors $\{p_1, p_2, ...p_n\}$ that are mutually orthogonal, i.e. $< p_i, p_j >= 0$ if $i \neq j$, is linearly independent (i.e. orthogonality implies linear independence). [5 Marks]

Q.2 There are 3 buildings A, B, and C. Suppose you have an instrument that measures the relative height of the building from the point where you are standing. Since the instrument is erroneous, you take multiple measurements. Suppose the height of each of these buildings measured from ground is 24.64 mt, 38.80 mt and 48.30 mt. The height of building B measured from the top of A is 14.22 mt.The height of C measured from A is 23.55 mt, and the height of C measured from B is 9.5 mt. Find the least squares estimate of the height of buildings A, B, C. [You can use MATLAB to find the pseudoinverse of your data matrix]. [5 Marks]

Q.3 Let $Q = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$. Find the maximum value of $x^\top Q x$ such that $x^\top x = 1$, $x \in \Re^2$. [5 Marks]

Q.4 Let $\mu \in$ and $\sigma > 0$ be real constants and let X be a random variable having p.d.f. (also called a gaussian/normal distribution)

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{(x-\mu)^2}{2\sigma^2}), \ -\infty < x < \infty$$

(a) Show that $f_X(x)$ is a valid p.d.f.

(b) Show that the probability distribution function of X is symmetric about μ.

(c) Find the mean and variance of X.

[10 Marks]

Q.5 Let X and Y be i.i.d random variables having uniform distribution U[0,1]. Find the probability density functions of following random variables.

(a) $Z_1 := log(\frac{1}{X})$

(b) $Z_2 := \exp(X)$

(c) $Z_3 := X + Y$

[15 Marks]

Q.6 Let A, B and C be three events such that $P(B \cap C) > 0$. Prove or disprove each of the following:

(a) $P(A \cap B|C) = P(A|B \cap C)P(B|C)$

(b) $P(A \cap B|C) = P(A|C)P(B|C)$, if A and B are independent.

[10 Marks]

Q.7 [Proof of PCA] Prove that the principal components i.e. the direction of maximum variance is given by the eigenvector of the sample covariance matrix of the data. Assume you have n data points $\{\vec{x}_1, \vec{x}_2, ..., \vec{x}_n\}, \vec{x}_i \in \mathbb{R}^d$. What will be the size of sample covariance matrix? How many principal components are present in the data? [10 Marks]

Q.8 **[MATLAB]** In this problem, you will implement RANSAC algorithm to fit a line on a noisy data, using two different approaches. The purpose of RANSAC algorithm is to identify and exclude the outliers before fitting the model. Assuming the percentage of outliers to be 10%, calculate the number of iterations such that the probability of choosing an inlier set is 0.99.

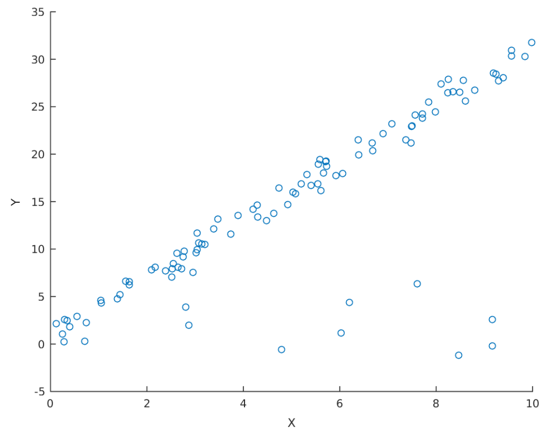

Figure 1: Data points

File `data_points_line.mat` is attached with the assignment and points are shown in the Figure 1.

(a) Assume that x-axis and y-axis represent independent and dependent variable respectively. The objective is to express Y as a linear function of X, i.e. Y $\sim$ X. Use Ordinary Least Squares (OLS) to find the coefficients of line. As is shown in the Figure 2, OLS on the whole data is deviant from the apparent relationship.

i. Implement Ordinary Least Squares on the whole data to obtain the *OLS_whole* fit. Visualize the *OLS_whole* line, and comment on the result.

ii. Implement RANSAC algorithm to exclude outliers points from the data, and obtain *OLS_RANSAC* fit line using only inliers. Visualize the line obtained from RANSAC.

iii. Compare the quality of two different solutions.

Note : The code should be self-contained and should run off the-shelf. It should generate a plot similar to Figure 2. If there is any error in code, it would be heavily penalized. Include the .mat file with the submission.

Sample : Figure 2 [10 Marks]

(b) For this part, assume that both X and Y are independent variables. Find the best fit line using PCA.

i. Implement PCA on the whole data to obtain the *PCA_whole* fit. Visualize the obtained line, and comment on the result.

ii. Implement RANSAC algorithm to exclude outliers points from the data, and obtain *PCA_RANSAC* fit line using only inliers. Visualize the line obtained from RANSAC.

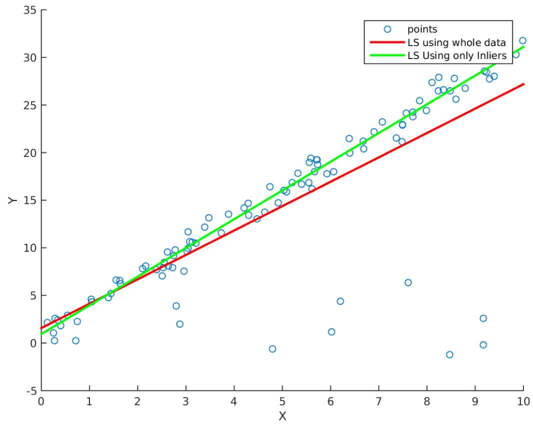

Figure 2: Sample Output with OLS

iii. Compare the quality of two different solutions.

Note : The code should be self-contained and should run off the-shelf (hence, include the .mat file with the submission). It should generate a plot similar to Figure 3. If the codes fails to run, it would be heavily penalized.

Sample : Figure 3 [10 Marks]

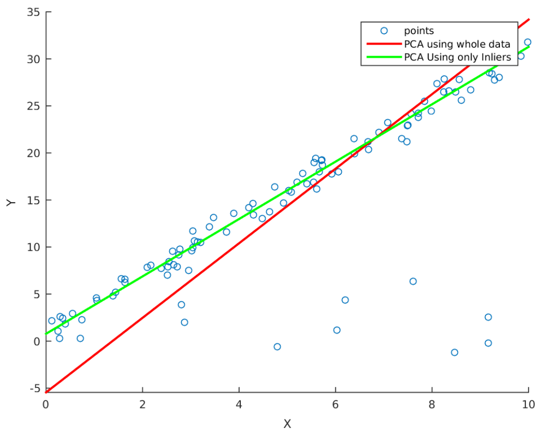

Figure 3: Sample Output with PCA

Q.9 **[MATLAB]** RANSAC is frequently used to fit a plane in point cloud data, for example, to identify ground plane in a scene. In this problem, you are given a 3D scan returned by the robot, with some noise. Implement RANSAC algorithm to find the largest plane in the given 3D point cloud data (`data_points_plane.mat`). Assume the outliers constitute 10% of the given data.
Visualize the best-fit planes obtained using (a) Whole Data and (b) Inliers (using RANSAC)

The file data_points_plane.mat is attached and is visualized in Figure 4.

Note : The code should be self-contained and should run off the-shelf (hence, include the .mat file with the submission). The code must generate a 3D-plot of points and both planes. If the code fails to run, it would be heavily penalized. [15 Marks]

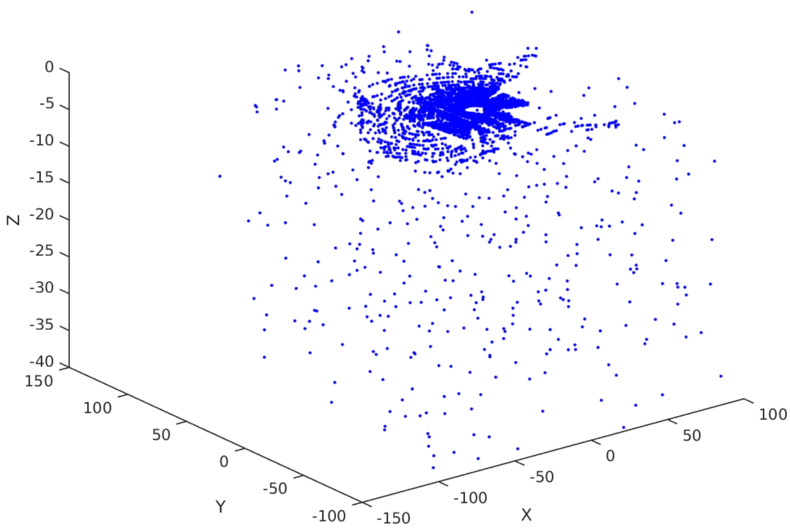

Figure 4: Point cloud