

# ETH\_Algorithm\_Lab

Code and assessment of the course Algorithm Labs. There is one problem of the week (PoW) and multiple problems during the week. The codes are implemented in C++.

Note that only problems more (or equal) than 3 sub-questions will be tested in the final exam. So I label these more important questions with bold style in the division table.

## Divide by Week

| Week | Q1  | Q2                                 | Q3                                      | Q4                                     | PoW                                      |
|------|---|------------------------------------|---|--|--|
| 1    | <a href="#">Build the Sum</a>                   | <a href="#">Dominoes</a>           | <a href="#">Even Pairs</a>              | <a href="#">Even Matrices</a>          |  |
| 2    | <a href="#">Beach Bars</a>                      | <a href="#">Buring Coins</a>       | <a href="#">The Great Game</a>          | <a href="#">Defensive Line</a>         | <a href="#">Deck of Cards</a>            |
| 3    | <a href="#">First Hit</a>                       | <a href="#">Hit</a>                | <a href="#">Antenna</a>                 | <a href="#">Hiking Maps</a>            | <a href="#">From Russia with Love</a>    |
| 4    | <a href="#">First Step with BGL</a>             | <a href="#">Important Bridges</a>  | <a href="#">Buddy Selection</a>         | <a href="#">Ant Challenge</a>          | <a href="#">Fighting Pits of Meereen</a> |
| 5    | <a href="#">Boats</a>                           | <a href="#">Moving Books</a>       | <a href="#">Severus Snape</a>           | <a href="#">Asterix the Gaul</a>       | <a href="#">Motocycles</a>               |
| 6    | <a href="#">What is the Maximum</a>             | <a href="#">Diet</a>               | <a href="#">Inball</a>                  | <a href="#">Lannister</a>              | <a href="#">Planet Express</a>           |
| 7    | <a href="#">Shopping Trip</a>                   | <a href="#">Knights</a>            | <a href="#">Coin Tossing Tournament</a> | <a href="#">London</a>                 | <a href="#">Octopussy</a>                |
| 8    | <a href="#">Germs</a>                           | <a href="#">Bistro</a>             | <a href="#">H1N1</a>                    | <a href="#">Light the Stage</a>        | <a href="#">Suez</a>                     |
| 9    | <a href="#">Algocoon</a>                        | <a href="#">Real Estate Market</a> | <a href="#">Canteen</a>                 | <a href="#">Placing Knights</a>        | <a href="#">Kingdom Defence</a>          |
| 10   | <a href="#">Asterix and the Chariot Race</a>    | <a href="#">Evolution</a>          | <a href="#">WorldCup</a>                | <a href="#">Asterix in Switzerland</a> | <a href="#">Golden Eye</a>               |
| 11   | <a href="#">The Iron Islands</a>                | <a href="#">Return of the Jedi</a> | <a href="#">Idefix</a>                  | <a href="#">Legions</a>                | <a href="#">Phantom Menace</a>           |
| 12   | <a href="#">On Her Majesty's Secret Service</a> | <a href="#">Hong Kong</a>          | <a href="#">Car Sharing</a>             | <a href="#">Bonus Level</a>            | <a href="#">San Francisco</a>            |
| 13   | <a href="#">Hand</a>                            | <a href="#">Hagrid</a>             | <a href="#">Punch</a>                   | <a href="#">Ludo Bagman</a>            | <a href="#">Clues</a>                    |
| 14   |   |                                    |   |  | <a href="#">India</a>                    |

## Divide by Topic

| Topic               | Questions  |
|---------------------|--|
| Dynamic Programming | <a href="#">Bonus Level</a> , <a href="#">Asterix and the Chariot Race</a> , <a href="#">Even Matrices</a> , <a href="#">Fighting Pits of Meereen</a> , <a href="#">From Russia with Love</a> , <a href="#">Hagrid</a> , <a href="#">San Francisco</a> , <a href="#">The Great Game</a>  |
| Linear Programming  | <a href="#">Lannister</a> , <a href="#">Legions</a> , <a href="#">Suez</a> , <a href="#">WorldCup</a>  |
| Sliding Window      | <a href="#">Beach Bars</a> , <a href="#">Deck of Cards</a> , <a href="#">Defensive Line</a> , <a href="#">Hiking Maps</a> , <a href="#">The Iron Islands</a>   |
| Greedy              | <a href="#">Asterix the Gaul</a> , <a href="#">Boats</a> , <a href="#">Moving Books</a> , <a href="#">Severus Snape</a>  |
| Graph               | <a href="#">Ant Challenge</a> , <a href="#">Buddy Selection</a> , <a href="#">Planet Express</a>   |
| Max Flow            | <a href="#">Algocoon</a> , <a href="#">Canteen</a> , <a href="#">Car Sharing</a> , <a href="#">India</a> , <a href="#">Kingdom Defence</a> , <a href="#">Knights</a> , <a href="#">London</a> , <a href="#">Ludo Bagman</a> , <a href="#">Phantom Menace</a> , <a href="#">Placing Knights</a> , <a href="#">Real Estate Market</a> , <a href="#">Asterix in Switzerland</a> |
| CGAL                | <a href="#">Motocycles</a>   |
| Triangulation       | <a href="#">Clues</a> , <a href="#">Golden Eye</a> , <a href="#">H1N1</a> , <a href="#">Hand</a> , <a href="#">Hong Kong</a> , <a href="#">Idefix</a> , <a href="#">Light the Stage</a>  |

# Useful Codes

- Sliding Window

```
1 // [] interval
2 int head = 0, tail = 1, sum = cards[head], best_val = INT_MAX;
3 pair<int, int> solution = make_pair(head, tail-1);
4 while(true){
5     int val = abs(sum - k);
6     if(val < best_val){
7         best_val = val;
8         solution = make_pair(head, tail-1);
9     }
10    if(sum==k) break;
11    if(sum < k){
12        if(tail==cards.size()) break;
13        sum += cards[tail++];
14    } else {
15        sum -= cards[head++];
16    }
17 }
```

```
1 int sliding_window(vector<int>& costs, vector<int>& water_way, int query){
2     int left = 0, right = 0, max_result = -1, sum = 0;
3     while(true){
4         int num_of_island = right - left;
5         if(sum==query){
6             max_result = max(max_result, num_of_island);
7             sum -= costs[water_way[left++]];
8         }
9         else if(sum < query ){
10             if(right == (int)water_way.size()) break;
11             sum += costs[water_way[right++]];
12         } else {
13             sum -= costs[water_way[left++]];
14         }
15     }
16     return max_result;
17 }
```

- Binary search
  - upperbound

```
1 while(l < r){
2     int mid = (l + r + 1)/2;
3     c_map[boost::edge(v_src, k, G).first] = mid;
4     if(feasible(G, v_src, v_tar, mid)){
5         l = mid;
6     } else {
7         r = mid - 1;
8     }
9 }
```

- Lower bound

- Lambda:

```

1 void preprocess(vector<Chamber>& chamber_list, int chamber_id){
2     for(auto child : chamber_list[chamber_id].child_list){
3         preprocess(chamber_list, child.first);
4         chamber_list[chamber_id].number_of_node += chamber_list[child.first].number_of_node;
5         chamber_list[chamber_id].time_cost += child.second +
chamber_list[child.first].time_cost;
6     }
7     sort(chamber_list[chamber_id].child_list.begin(),
chamber_list[chamber_id].child_list.end(),
8         [&chamber_list, chamber_id](auto& left, auto& right)-> bool {
9         long time_cost1 = chamber_list[left.first].time_cost + left.second;
10        long number_of_node1 = chamber_list[left.first].number_of_node;
11        long time_cost2 = chamber_list[right.first].time_cost + right.second;
12        long number_of_node2 = chamber_list[right.first].number_of_node;
13        return time_cost1 * number_of_node2 < time_cost2 * number_of_node1;
14    }
15    );
16 }

```

- CGAL:

- Check Intersection: `CGAL::do_intersect`
- Find Intersection:

```

1 auto o = CGAL::intersection(ray, segments[i]);
2 if (const P* op = boost::get<P>(&*o))
3     intersect_point = *op;
4 else if (const S* os = boost::get<S>(&*o)) {
5     if(CGAL::squared_distance(start, os->source()) < CGAL::squared_distance(start, os-
>target()))
6         intersect_point = os->source();
7     else intersect_point = os->target();
8 }

```

- squared distance: `CGAL::squared_distance`
- check the cross product: `Line.oriented_side(Point)`
- Floor to double and output:

```

1 double floor_to_double(const K::FT& x) {
2     double a = floor(CGAL::to_double(x));
3     while (a > x) a -= 1;
4     while (a+1 <= x) a += 1;
5     return a;
6 }
7 double ceil_to_double(const K::FT& x){
8     double a = ceil(CGAL::to_double(x));
9     while (a < x) a += 1;
10    while (a-1 >= x) a -= 1;
11    return a;
12 }
13 cout << fixed << setprecision(0) <<...<< endl;

```

- Triangulation:
  - BFS construction

```

1 void BFS_construction(Vh vertex, Triangulation& tri, graph& G){
2     set<Vh> visited;
3     vector<Vh> queue; queue.push_back(vertex); visited.insert(vertex);
4     while(!queue.empty()){
5         auto next_vertex = queue[queue.size() - 1]; queue.pop_back();
6         if(next_vertex != vertex) boost::add_edge(vertex->info(), next_vertex->info(),
7 G);
8         auto neighbor = next_vertex->incident_vertices();
9
10        do {
11            if (!tri.is_infinite(neighbor) && visited.find(neighbor) == visited.end() &&
12 CGAL::squared_distance(vertex->point(), neighbor->point()) <= r_square) {
13                visited.insert(neighbor);
14                queue.push_back(neighbor);
15            }
16        } while(++neighbor != next_vertex->incident_vertices());
17    }
18 }

```

- Dijkstra construction

```

1 void precompute(Triangulation& tri){
2     priority_queue< pair<FT, Face> > q;
3     for(auto face = tri.all_faces_begin(); face != tri.all_faces_end(); face++){
4         if(tri.is_infinite(face)){
5             q.push(make_pair(LONG_MAX, face));
6             face->info() = LONG_MAX;
7         } else {
8             Point outheart = tri.dual(face);
9             FT distance = CGAL::squared_distance(outheart, face->vertex(0)->point());
10            q.push(make_pair(distance, face));
11            face->info() = distance;
12        }
13    }
14
15    while(!q.empty()){
16        FT distance = q.top().first;
17        Face current_face = q.top().second;
18        q.pop();
19        // current face is updated by another better face
20        if(distance < current_face->info()) continue;
21        for(int i = 0; i < 3; i++){
22            Face next_face = current_face->neighbor(i);
23            FT edge_length = tri.segment(current_face, i).squared_length();
24            FT new_distance = min(current_face->info(), edge_length);
25            if(new_distance > next_face->info()){
26                q.push(make_pair(new_distance, next_face));
27                next_face->info() = new_distance;
28            }
29        }
30    }
31 }

```

# Assessment 1: From Russia with Love

## Problem Definition

- **Input:**
  - $n$ : number of coins
  - $m$ : number of players
  - $k$ : the player you are interested in
  - values  $x_0, \dots, x_{n-1}$  s.t.  $x_i$  denotes the price of  $i$ -th coin
- **Rule:**  $m$  players take turns from player 0, pick up a coin from either the front or the end of the array of coins.
- **Output:** Largest winnings that player  $k$  can collect, **regardless of how other players play**. (even all other players play against player  $k$ , the maximum value  $k$  can get)

## Mathematical Concept

- **Recursion:** The problem can be transformed recursively into subproblems.
- **Reformulation:** This problem is similar to a zero-sum game, the difference is that there are more than two players, and all other players play against player  $k$ . We define the score that player  $k$  can get when it is player  $p$ 's turn and the remaining coins are from  $l$  to  $r$ .

$$f(l, r, p) = \begin{cases} x_l, & l = r \wedge p = k \\ 0, & l = r \wedge p \neq k \\ \max[x_l + f(l+1, r, (p+1) \bmod m), x_r + f(l, r-1, (p+1) \bmod m)], & l \neq r \wedge p = k \\ \min[f(l+1, r, (p+1) \bmod m), f(l, r-1, (p+1) \bmod m)], & \text{otherwise} \end{cases}$$

## Algorithm

- **Recursion:** Just from the mathematical formulation
  - Running time:  $O(2^n)$  (For each decision, there are two possibilities)

```

1  int score(int head, int tail, int player){
2      if(head==tail) return (player!=k?0:coins[head]);
3      int score1 = (player!=k?0:coins[head]) + score(head+1, tail, (player+1)%m);
4      int score2 = (player!=k?0:coins[tail]) + score(head, tail-1, (player+1)%m);
5      return (player==k ? max(score1, x):min(score1, score2));
6  }
```

- **Dynamic Programming (Top-Down)**
  - Since the player is fixed for a specific array, we only need to record the state as `dp[head][tail]`.
  - Running time:  $O(n^2)$  (For each `dp[head][tail]`, we only need to compute once.  $\binom{n}{2}$  in total.)

```

1  int score(int head, int tail, int player){
2      if(dp[head][tail]!=-1) return dp[head][tail];
3      if(head==tail) return (dp[head][tail] = (player!=k?0:coins[head]));
4      int score1 = (player!=k?0:coins[head]) + score(head+1, tail, (player+1)%m);
5      int score2 = (player!=k?0:coins[tail]) + score(head, tail-1, (player+1)%m);
6      return (dp[head][tail] = (player==k ? max(score1, x):min(score1, score2)));
7  }

```

## Implementation & Testing

```

1  #include <iostream>
2  #include <cstring>
3  #include <cmath>
4  using namespace std;
5  #define MAX_C 1001
6  #define MAX_P 501
7  int t, n, m, k, coins[MAX_C], dp[MAX_C][MAX_C];
8
9  int score(int head, int tail, int player){
10     if(dp[head][tail]!=-1) return dp[head][tail];
11     if(head==tail) return (dp[head][tail] = 0);
12     int score1 = (player!=k?0:coins[head]) + score(head+1, tail, (player+1)%m);
13     int score2 = (player!=k?0:coins[tail]) + score(head, tail-1, (player+1)%m);
14     return (dp[head][tail] = (player==k ? max(score1, score2):min(score1, score2)));
15 }
16
17 int main(){
18     ios_base::sync_with_stdio(false);
19     cin >> t;
20     while(t--){
21         cin >> n >> m >> k;
22         memset(dp, -1, sizeof(dp));
23         for(int i = 0; i < n; i++) cin >> coins[i];
24         cout << score(0,n-1,0) <<endl;
25     }
26     return 0;
27 }

```

- When  $m = 2$ , it is equivalent to a zero-sum game. Use min-max algorithm.

```

1  #include <iostream>
2  #include <cstring>
3  #include <cmath>
4  using namespace std;
5  #define MAX_C 1001
6  #define MAX_P 501
7  int t, n, m, k, coins[MAX_C], sum[MAX_C], dp[MAX_C][MAX_C];
8
9  int score(int head, int tail, int player){
10     if(dp[head][tail]!=-1) return dp[head][tail];
11     if(head==tail) return (dp[head][tail] = 0);
12     int score1 = (player!=k?0:coins[head]) + score(head+1, tail, (player+1)%m);
13     int score2 = (player!=k?0:coins[tail]) + score(head, tail-1, (player+1)%m);
14     return (dp[head][tail] = (player==k ? max(score1, score2):min(score1, score2)));
15 }

```

```

16
17 int score2(int head, int tail){
18     if(dp[head][tail]!=-1) return dp[head][tail];
19     if(head==tail) return (dp[head][tail] = 0);
20     int scorea = coins[head] + score2(head+1, tail);
21     int scoreb = coins[tail] + score2(head, tail-1);
22     int total = sum[tail + 1] - sum[head];
23     return (dp[head][tail] = total - max(scorea, scoreb));
24 }
25
26 int main(){
27     ios_base::sync_with_stdio(false);
28     cin >> t;
29     while(t--){
30         cin >> n >> m >> k;
31         memset(sum, 0, sizeof(sum));
32         memset(dp, -1, sizeof(dp));
33         for(int i = 0; i < n; i++){
34             cin >> coins[i];
35             sum[i+1] = sum[i] + coins[i];
36         }
37         cout << ((m==2)?score2(0, n-1):score(0,n-1,0)) <<endl;
38     }
39     return 0;
40 }

```

```

1  #include <iostream>
2  #include <cstring>
3  #include <cmath>
4  using namespace std;
5  #define MAX_C 1001
6  #define MAX_P 501
7  int t, n, m, k, coins[MAX_C], dp[MAX_C][MAX_C];
8
9  int score(int head, int tail, int player){
10     if(dp[head][tail]!=-1) return dp[head][tail];
11     if(head==tail) return (dp[head][tail] = (player!=k?0:coins[head]));
12     int score1 = (player!=k?0:coins[head]) + score(head+1, tail, (player+1)%m);
13     int score2 = (player!=k?0:coins[tail]) + score(head, tail-1, (player+1)%m);
14     return (dp[head][tail] = (player==k ? max(score1, score2):min(score1, score2)));
15 }
16
17 int main(){
18     ios_base::sync_with_stdio(false);
19     cin >> t;
20     while(t--){
21         cin >> n >> m >> k;
22         memset(dp, -1, sizeof(dp));
23         for(int i = 0; i < n; i++) cin >> coins[i];
24         cout << score(0,n-1,0) <<endl;
25     }
26     return 0;
27 }

```

## Overall

- Understand the question: I firstly understand this problem in a different way. (I thought it asks me to find the maximum score that player k can get, which means all the other players are helping him!)
- Find a bug when  $m = 2$ .

## Assessment 2: Lannister

---

### Problem Definition

- **Input:**
  - $n$  noble houses
  - $m$  common houses
  - Two canals (one for sewage one for fresh water) cross at right angles
  - sewage pipe: horizontal, fresh water pipe: vertical
  - Constraints
    - Cersei: noble houses at the LHS of the sewage cannal, common houses at the RHS of the sewage cannal.
    - Tywin: sum of sewage pipe length should be less than a threshold  $s$ .
    - Jaime: minimize the length of the longest fresh water pipe.
- **Output:**
  - check the feasibility of Cersei's and Tywin's constraints
  - minimum length of the longest fresh water pipe

### Mathematical Concept

- Assume the sewage canal is  $ax + by + c = 0$ , then the fresh canal can be represented as  $bx - ay + d = 0$ , which is perpendicular to the sewage canal.  $(x_i, y_i)$  is the position of house  $i$ .
- Cersei's constraint:

$$\begin{cases} ax_i + by_i + c \leq 0 & (i \in \text{noble}) \\ ax_i + by_i + c \geq 0 & (i \in \text{common}) \end{cases}, \quad a > 0 \text{ (enforce the LHS RHS order)}$$

- Tywin's constraint:

$$\begin{aligned} dist_{sewages} &= \sum_{i \in \text{houses}} |x_i - (-\frac{b}{a}y_i - \frac{c}{a})| = \sum_{i \in \text{houses}} |x_i + \frac{b}{a}y_i + \frac{c}{a}| \\ &\leq s \end{aligned} \tag{2}$$

- Jaime's constraint: if we set the upperbound of lengths of fresh water pipes to be  $l$

$$\begin{aligned} dist_{fresh}(i) &= |y_i - (\frac{b}{a}x_i + \frac{d}{a})| \leq l \\ &\text{minimize } l \end{aligned} \tag{3}$$

### Algorithm

---

- Linear programming: we can transform the constraints into a linear programming problem format.
  - object function:  $l$
  - Variables:  $a, b, c, d, l$
- Transformed constraints
  - Cersei's constraint



$$\begin{cases} x_i a + y_i b + c \leq 0 & (i \in \text{noble}) \\ (-x_i) a + (-y_i) b + (-1) c \leq 0 & (i \in \text{common}) \end{cases}$$

$$a > 0 \text{ (enforce the LHS RHS order)}$$

- Tywin's constraint:

$$\begin{aligned} dist_{sewages} &= \sum_{i \in \text{houses}} |x_i + \frac{b}{a} y_i + \frac{c}{a}| \\ &= \sum_{i \in \text{noble houses}} -(x_i + \frac{b}{a} y_i + \frac{c}{a}) + \sum_{i \in \text{common houses}} (x_i + \frac{b}{a} y_i + \frac{c}{a}) \\ &= (\sum_{i \in \text{common}} x_i - \sum_{i \in \text{noble}} x_i) + \frac{b}{a} (\sum_{i \in \text{common}} y_i - \sum_{i \in \text{noble}} y_i) + \frac{c}{a} (m - n) \leq s \\ a(\sum_{i \in \text{common}} x_i - \sum_{i \in \text{noble}} x_i - s) + b(\sum_{i \in \text{common}} y_i - \sum_{i \in \text{noble}} y_i) + c(m - n) &\leq 0 \end{aligned} \quad (4)$$

- Jaime's constraint:

$$\begin{aligned} dist_{fresh}(i) &= |y_i - (\frac{b}{a} x_i + \frac{d}{a})| \leq l \\ \max[(\frac{b}{a} x_i + \frac{d}{a}) - y_i, y_i - (\frac{b}{a} x_i + \frac{d}{a})] &\leq l \\ (\frac{b}{a} x_i + \frac{d}{a}) - y_i \leq l \wedge y_i - (\frac{b}{a} x_i + \frac{d}{a}) &\leq l \\ \text{minimize } l \end{aligned} \quad (5)$$

- just add two constraint to fulfill this constraint.

## Implementation & Testing

- Set  $a = 1 > 0$

```
1 | lp.set_l(a, true, 1); lp.set_u(a, true, 1);
```

- Tywin's constraint:

$$\begin{aligned} dist_{sewages} &= (\sum_{i \in \text{common}} x_i - \sum_{i \in \text{noble}} x_i) + b(\sum_{i \in \text{common}} y_i - \sum_{i \in \text{noble}} y_i) + c(m - n) \leq s \\ b(\sum_{i \in \text{common}} y_i - \sum_{i \in \text{noble}} y_i) + c(m - n) &\leq s - (\sum_{i \in \text{common}} x_i - \sum_{i \in \text{noble}} x_i) \end{aligned} \quad (6)$$

- Jaime's constraint:

$$\begin{aligned} (bx_i + d) - y_i &\leq l \wedge y_i - (bx_i + d) \leq l \\ (bx_i + d) - l &\leq y_i \wedge -(bx_i + d) - l \leq -y_i \end{aligned} \quad (7)$$

- Code
  - implement first two constraints then add Jaime's constraint
  - Use of counter to count row in matrix  $A$
  - use `long` instead of `int`

```
1 | ///3
2 | #include <iostream>
3 | #include <CGAL/QP_models.h>
4 | #include <CGAL/QP_functions.h>
```

```

5  #include <CGAL/Gmpz.h>
6  #include <cmath>
7  #include <algorithm>
8  #include <vector>
9  using namespace std;
10 typedef long IT;
11 typedef CGAL::Gmpz ET;
12 typedef CGAL::Quadratic_program<IT> Program;
13 typedef CGAL::Quadratic_program_solution<ET> Solution;
14
15 int t, n, m;
16 long s;
17
18
19 void solve(){
20     cin >> n >> m >> s;
21     vector< pair<long, long> > nobles(n);
22     vector< pair<long, long> > commons(m);
23
24     int number_of_constraints = 0;
25     long noble_sum_x = 0, noble_sum_y = 0;
26     Program lp (CGAL::SMALLER, false, 0, false, 0);
27     const int a = 0, b = 1, c = 2, d = 3, l = 4;
28     for(int i = 0; i < n; i++){
29         cin >> nobles[i].first >> nobles[i].second;
30         noble_sum_x += nobles[i].first;
31         noble_sum_y += nobles[i].second;
32         lp.set_a(a, number_of_constraints, nobles[i].first);
33         lp.set_a(b, number_of_constraints, nobles[i].second);
34         lp.set_a(c, number_of_constraints, 1);
35         number_of_constraints++;
36     }
37     long common_sum_x = 0, common_sum_y = 0;
38     for(int i = 0; i < m; i++){
39         cin >> commons[i].first >> commons[i].second;
40         common_sum_x += commons[i].first;
41         common_sum_y += commons[i].second;
42         lp.set_a(a, number_of_constraints, -commons[i].first);
43         lp.set_a(b, number_of_constraints, -commons[i].second);
44         lp.set_a(c, number_of_constraints, -1);
45         number_of_constraints++;
46     }
47     // set a = 1 to simplify the second constraint
48     lp.set_l(a, true, 1); lp.set_u(a, true, 1);
49     Solution sol = CGAL::solve_linear_program(lp, ET());
50     if(sol.is_infeasible()) {cout << "Yuck!\n"; return;}
51
52     if(s != -1){
53         lp.set_a(b, number_of_constraints, common_sum_y - noble_sum_y);
54         lp.set_a(c, number_of_constraints, m - n);
55         lp.set_b(number_of_constraints, s - common_sum_x + noble_sum_x);
56         number_of_constraints++;
57         sol = CGAL::solve_linear_program(lp, ET());
58         if(sol.is_infeasible()) {cout << "Bankrupt!\n"; return ;}
59     }
60

```

```

61     for(int i = 0; i < n; i++){
62         lp.set_a(b, number_of_constraints, -nobles[i].first);
63         lp.set_a(d, number_of_constraints, -1);
64         lp.set_a(l, number_of_constraints, -1);
65         lp.set_b(number_of_constraints, -nobles[i].second);
66         number_of_constraints++;
67
68         lp.set_a(b, number_of_constraints, nobles[i].first);
69         lp.set_a(d, number_of_constraints, 1);
70         lp.set_a(l, number_of_constraints, -1);
71         lp.set_b(number_of_constraints, nobles[i].second);
72         number_of_constraints++;
73     }
74
75     for(int i = 0; i < m; i++){
76         lp.set_a(b, number_of_constraints, -commons[i].first);
77         lp.set_a(d, number_of_constraints, -1);
78         lp.set_a(l, number_of_constraints, -1);
79         lp.set_b(number_of_constraints, -commons[i].second);
80         number_of_constraints++;
81
82         lp.set_a(b, number_of_constraints, commons[i].first);
83         lp.set_a(d, number_of_constraints, 1);
84         lp.set_a(l, number_of_constraints, -1);
85         lp.set_b(number_of_constraints, commons[i].second);
86         number_of_constraints++;
87     }
88     lp.set_l(l, true, 0);
89     lp.set_c(l, 1);
90
91     sol = CGAL::solve_linear_program(lp, ET());
92     cout << fixed << setprecision(0) << ceil(CGAL::to_double(sol.objective_value())) << endl;
93 }
94
95 int main(){
96     ios_base::sync_with_stdio(false);
97     cin.tie(0);
98     cin >> t;
99     while(t--){
100         solve();
101     }
102     return 0;
103 }

```

## Overall

- Understand the question: too long...
- Set up the constraints in linear programming format (need trick for all the constraints)
  - Cersei: linearly separable **but should be on the same side**
  - Tywin : **need to deal with the absolute sign properly.** (by dividing into two cases)
  - Jamie:
    - **need to deal with the absolute sign properly** (by adding two constraints)
    - **need to find special value for a** (otherwise becomes quadratic problem)
- Debug: use `long` instead of `int`

# Assessment 3: Kingdom Defense

## Problem Definition

- **Inputs:**
  - $l$ : number of locations ( $1 \leq l \leq 500$ )
    - Each one corresponds to a pair of  $g$  (number of solder stationed),  $d$  (number of solder needed to defend the city)
  - $p$ : number of paths ( $1 \leq p \leq l^2$ )
    - min flow  $c$  and max flow  $C$
- **Outputs:**
  - For every test case output a single line containing the word `yes`, if the soldiers can be moved such that during the move enough military presence is displayed along every path and after moving every location is well defended, and the word `no` otherwise.

## Mathematical Concept & Algorithm

- This problem can be formulated as a max flow problem (Circulation Problem)
  - Connect source to every supply (all the locations) with capacity  $g$
  - Connect every demand (all the locations) to target with capacity  $d$
  - Connect the locations according to the path information
- Minimum edge constraints
  - Assume all the minimum constraints are fulfilled:  $c$  soldiers are moved from  $u$  to  $v$
  - Simulate the procedure:
    - Generate a flow from sink to source
    - increase the demand of the supply node by  $c$
    - increase the supply of the demand node by  $c$
- Check whether the max flow equals to the demand

## Implementation & Testing

```
1  #include <iostream>
2  #include <vector>
3  #include <algorithm>
4  #include <boost/graph/adjacency_list.hpp>
5  #include <boost/graph/push_relabel_max_flow.hpp>
6  using namespace std;
7  // Graph Type with nested interior edge properties for flow algorithms
8  typedef boost::adjacency_list_traits<boost::vecS, boost::vecS, boost::directedS> traits;
9  typedef boost::adjacency_list<boost::vecS, boost::vecS, boost::directedS, boost::no_property,
10     boost::property<boost::edge_capacity_t, long,
11     boost::property<boost::edge_residual_capacity_t, long,
12     boost::property<boost::edge_reverse_t, traits::edge_descriptor > > > graph;
13
14  typedef traits::vertex_descriptor vertex_desc;
15  typedef traits::edge_descriptor edge_desc;
16  using namespace std;
17
18  // Custom edge adder class, highly recommended
19  class edge_adder {
20     graph &G;
21     public:
```

```

22     explicit edge_adder(graph &G) : G(G) {}
23     void add_edge(int from, int to, long capacity) {
24         auto c_map = boost::get(boost::edge_capacity, G);
25         auto r_map = boost::get(boost::edge_reverse, G);
26         const auto e = boost::add_edge(from, to, G).first;
27         const auto rev_e = boost::add_edge(to, from, G).first;
28         c_map[e] = capacity;
29         c_map[rev_e] = 0; // reverse edge has no capacity!
30         r_map[e] = rev_e;
31         r_map[rev_e] = e;
32     }
33 };
34
35 int t, l, p;
36 void solve(){
37     cin >> l >> p;
38     graph G(l);
39     const vertex_desc v_src = boost::add_vertex(G);
40     const vertex_desc v_tar = boost::add_vertex(G);
41     edge_adder adder(G);
42     vector< pair<int, int> > node_info(l);
43     for(int i = 0; i < l; i++) cin >> node_info[i].first >> node_info[i].second;
44     for(int i = 0; i < p; i++){
45         int from, to, c, C; cin >> from >> to >> c >> C;
46         adder.add_edge(from, to, C-c);
47         node_info[from].second += c;
48         node_info[to].first += c;
49     }
50     long demands_sum = 0;
51     for(int i = 0; i < l; i++){
52         adder.add_edge(v_src, i, node_info[i].first);
53         adder.add_edge(i, v_tar, node_info[i].second);
54         demands_sum += node_info[i].second;
55     }
56     long flow = boost::push_relabel_max_flow(G, v_src, v_tar);
57     cout << (flow==demands_sum? "yes\n":"no\n");
58 }
59
60
61 int main(){
62     ios_base::sync_with_stdio(false);
63     cin.tie(0);
64     cin >> t;
65     while(t--){
66         solve();
67     }
68     return 0;
69 }

```