THE CARROLL-ARABIE TAXONOMY OF SCALING METHODS

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A large number of computerized scaling techniques were developed in the wake of the pioneering work of Shepard, Kruskal, and Guttman [4, 5, 6, 8, 9]. There have been various attempts to bring some order into this bewildering variety of techniques. Books such as Roskam [7], or review articles such as De Leeuw and Heiser [3] are organized with a clear taxonomy in mind, but the most well-known and comprehensive organization of scaling methods is due to Carroll and Arabie [1].

Before we discuss the taxonomy, we have to emphasize two important points. First, the proposed organization is clearly inspired by earlier work of Coombs [2] and Shepard [10]. The exquisite theoretical work of Coombs was written before the computer revolution, and the scaling methods he proposed were antiquated before they were ever seriously used. This had the

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unfortunate consequence that the theoretical work was also largely ignored. More or less the same thing is true for the work of Shepard, who actually did proposed computerized algorithms, but never got them beyond the stage of research software. Again, this implied that his seminal contributions to multidimensional scaling have been undervalued. Both Coombs and Shepard had some followers, but they did not have an army of consumers using their name and referring to their papers. The second important aspect of the Carroll-Shepard taxonomy is that it was written around 1980. In the subsequent 25 years hundreds of additional metric and nonmetric scaling methods have been developed, and some of them fall outside the boundaries of the taxonomy. It is also probably true that the messianistic zeal with which the nonmetric methods were presented around 1970 has subsided. They are now much more widely employed, in many different disciplines, but shortcomings have become apparent and the magic has largely dissipated.

The actual taxonomy is given in Table . We give a brief explanation of the concept that are not self-evident. The "number of ways" refers to the dimensionality of the data array, the "number of modes" to the number of sets of objects that must be represented. Thus a symmetric matrix of proximities has two ways but one mode. "Scale type" refers to the usual nominal, ordinal, numerical distinction. "Conditionality" defines which elements of the data array can be sensibly compared. Thus a matrix with preference rank

order in each row is row-conditional. Matrices with similarity rankings between, say, colors, by a number of different subjects gives three-way, two-mode, ordinal, matrix conditional data. "Completeness" refers to missing data, sometimes in the more theoretical sense in which we say that unfolding data are incomplete, because they only define an off-diagonal submatrix of the complete similarity matrix (crossref ?).

The taxonomy of models is somewhat dated. It is clear the authors set out to classify the existing scaling techniques, more specifically the computerized one they and their co-workers had developed (which happened to be a pretty complete coverage of the field at the time). We can clearly distinguish the nonmetric scaling methods, the influence of using Minkovski power metrics, the work on cluster analysis and additive partitioning, and the work on internal and external analysis of preferences. Some clarifications are perhaps needed. "Number of spaces" refers to either a joint or a separate representation of the two modes of a matrix (or the multiple modes of an array). Such considerations are especially important in off-diagonal methods such as unfolding or correspondence analysis. "External" analysis implies that coordinates in one of the spaces in which we are representing our data are fixed (usually found by some previous analysis, or defined by theoretical considerations). We only fit the coordinates of the points in other

spaces, for instance we have a two-dimensional space of objects and we fit individual preferences as either points or lines in that space.

In summary, we can say that the Carroll-Arabie taxonomy can be used to describe and classify a large number of scaling methods, especially scaling methods developed at Bell Telephone Laboratories and its immediate vicinity between 1960 and 1980. Since 1980 the field of scaling has moved away to some extent from the geometrical methods and the heavy emphasis on solving very complicated optimization problems. Item response theory and choice modeling have become more prominent, and they are somewhat at the boundaries of the taxonomy. New types of discrete representations have been discovered. The fact that the taxonomy is still very useful and comprehensive attests to the importance of the frameworks developed in the 1960-1980 area, and to some extent also to the unfortunate fact that there no longer is a center in psychometrics and scaling with the power and creativity of Bell Labs in that area.

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- Data.
  - Number of Modes.
  - Number of Ways.
  - Scale Type of Data.
  - Conditionality.
  - Completeness.
- Model.
  - Spatiality .
    - Spatial.
      - Distance.
      - Scalar Products.
    - Non-spatial.
      - Partitions.
      - Subsets.
      - Trees.
  - Number of Sets of Points.
  - Number of Spaces.
  - External Constraints.

TABLE 1. Carroll-Arabie Taxonomy of Scaling Methods

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