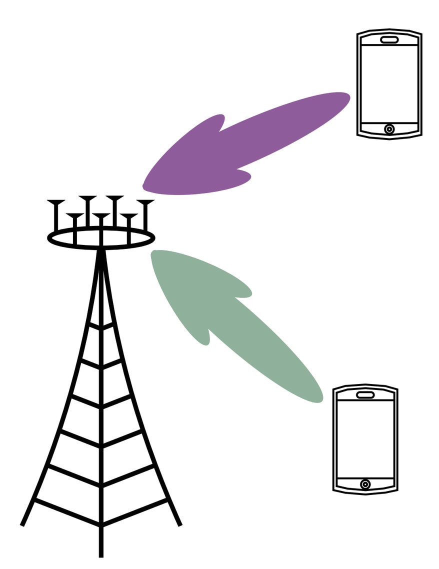
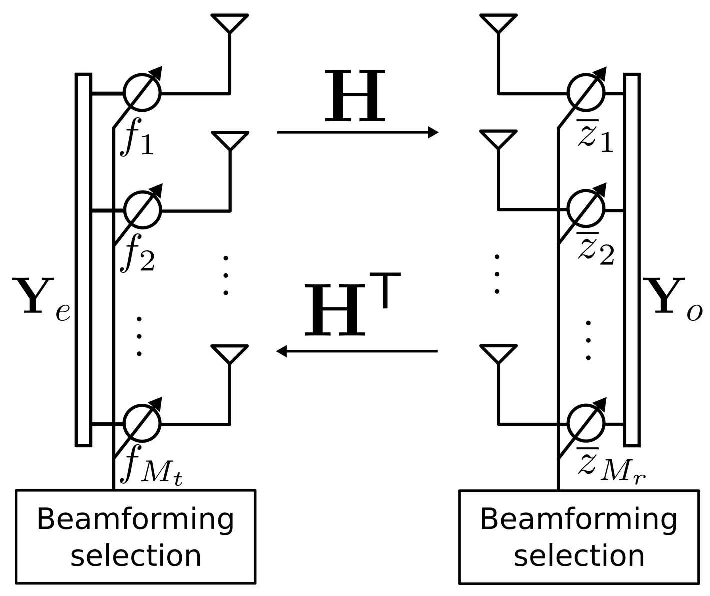
Noisy Beam Alignment Techniques for Reciprocal MIMO Channels

Dennis Ogbe, David J. Love*, Vasanthan Raghavan*[†]
*Purdue University, School of Electrical and Computer Engineering, West Lafayette, IN 47907, USA
[†]Qualcomm, Inc., Bridgewater, NJ 08807, USA

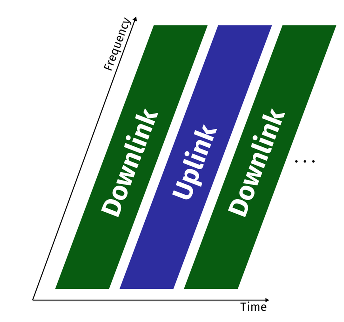
1. Background

- 5G technologies (mmWave & massive MIMO) rely on **beamforming gains** to realize data rate requirements
- However: **Optimal beamforming weights depend on the channel matrix**





- Traditional CSI acquisition (Sounding sequences $\rightarrow$ CSI feedback $\rightarrow$ SVD) is impractical with many antennas
- Solution: **Beam-based sounding**
  - Users always transmit on beams
  - Acquire beamformers using a TDD **beam alignment phase**
  - Exploit reciprocity of the wireless channel
- **Need for practical approaches to TDD-based beam alignment**
  - Additive noise
  - mmWave channel models
  - Low overhead



2. Ping-pong beam alignment

- Divide each channel use k into two time slots
- Communication nodes sound beams in their half of the slots

Ping: Node 1 sounds beam $\mathbf{f}[k]$ as

$$\mathbf{y}_o[k] = \sqrt{\rho_o}\,\mathbf{H}\mathbf{f}[k] + \mathbf{n}_o[k]$$

Pong: Node 2 sounds beam $\mathbf{z}[k]$ as

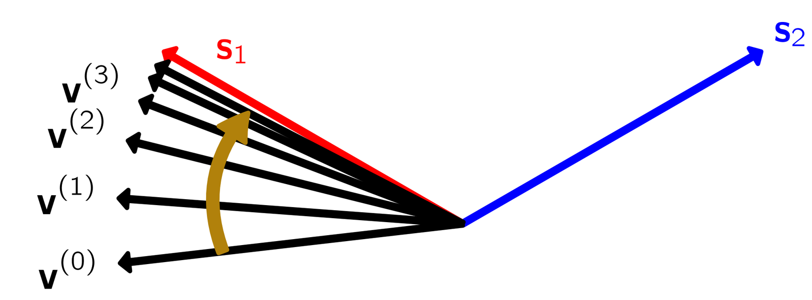
$$\mathbf{y}_e[k] = \sqrt{\rho_e}\,\mathbf{H}^{\mathsf{T}}\overline{\mathbf{z}}[k] + \mathbf{n}_e[k]$$

Notation: $\mathbf{H}$ — $M_r \times M_t$ channel matrix, ρ_e, ρ_o — beam alignment SNR, $\mathbf{n}_e[k], \mathbf{n}_o[k]$ — complex additive white Gaussian noise

3. Power Method

- We propose new beam alignment algorithms based on the **power method**
- Good performance for the noiseless case
- **Convergence can slow down dramatically under additive noise**

Example: One-way Power Method



Given: Diagonalizable $\mathbf{A} \in \mathbb{C}^{n\times n}$ and unit 2-norm $\mathbf{v}^{(0)}$
for $k = 1, 2, \ldots$ **do**
$\quad \mathbf{v}^{(k)} = \mathbf{A}\mathbf{v}^{(k-1)} / \|\mathbf{A}\mathbf{v}^{(k-1)}\|_2$
end for

4. Proposed Algorithms

Sequential Least-squares (SLS) Power Method

- Main idea: Construct a least-squares (LS) estimate of the channel matrix **using the sounding beams**
- Compute **greedy estimates** of the singular vectors
- Batch LS estimate would require all previous received beams at each iteration
- Instead, construct channel estimates sequentially:

$$\widehat{\mathbf{H}}_{o,k} = f\left(\widehat{\mathbf{H}}_{o,k-1},\ \mathbf{y}_o[k],\ \mathbf{f}[k]\right)$$

- Compute beamforming weights:

$$\mathbf{f}[k] = \frac{\widehat{\mathbf{H}}^{*}_{e,k}\mathbf{z}[k-1]}{\|\widehat{\mathbf{H}}^{*}_{e,k}\mathbf{z}[k-1]\|_2},\qquad \mathbf{z}[k] = \frac{\widehat{\mathbf{H}}_{o,k}\mathbf{f}[k]}{\|\widehat{\mathbf{H}}_{o,k}\mathbf{f}[k]\|_2}$$

Summed Power Method

- Main idea: Derive beamforming weights as a function of the **running sum of received observations**
- **Average over potentially noisy estimates** during beam alignment
- Compute beamforming weights:

$$\mathbf{f}[k+1] = \alpha_k\left[\overline{\mathbf{y}}_e[k] + \cdots + \overline{\mathbf{y}}_e[0]\right] = \alpha_k \mathbf{s}_e[k]$$

$$\mathbf{z}[k+1] = \beta_k\left[\mathbf{y}_o[k] + \cdots + \mathbf{y}_o[0]\right] = \beta_k \mathbf{s}_o[k]$$

$$\alpha_k = 1/\|\mathbf{s}_e[k]\|_2,\quad \beta_k = 1/\|\mathbf{s}_o[k]\|_2$$

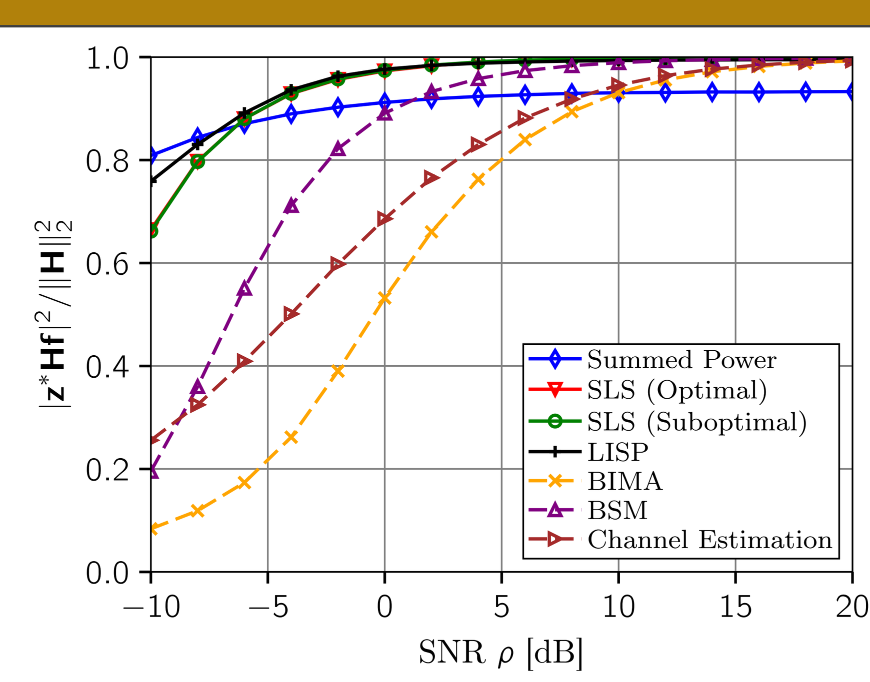
Least-squares initialized Summed Power Method (LISP method)

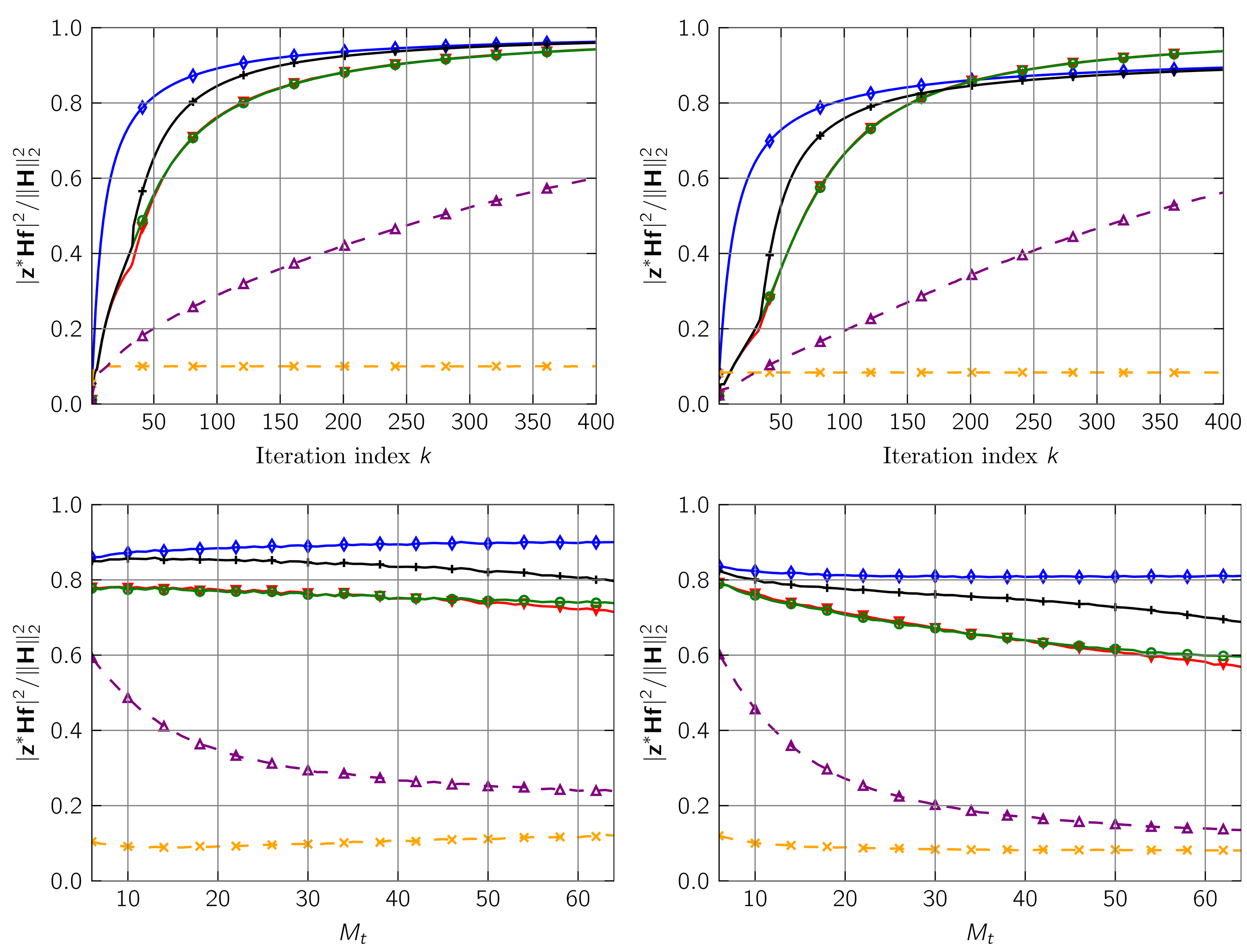
- Tradeoff between positive properties of Summed & SLS methods
- **Idea: "prime" the beamformer estimates up to period k_{switch} with the SLS method, then switch to the Summed Power Method**

	Computational Count	Feedback
Sequential Least-Squares	$k_{\max} \cdot \mathcal{O}(M^3)$	$k_{\max} \cdot \mathcal{O}(M)$
Summed Power	$k_{\max} \cdot \mathcal{O}(M)$	-
LISP	$k_{\text{switch}} \cdot \mathcal{O}(M^3) + (k_{\max} - k_{\text{switch}}) \cdot \mathcal{O}(M)$	$k_{\text{switch}} \cdot \mathcal{O}(M)$

5. Numerical Studies

- Metric of interest: Normalized effective beamforming gain $|\mathbf{z}^{*}\mathbf{H}\mathbf{f}|^2$
- Varying SNR, iteration count, and antenna dimensions
- Proposed algorithms **outperform state-of-the-art techniques** at -10 dB pre-beamforming SNR (see [**journal_paper**] for detailed discussion)





Sparse mmWave model   I.I.D Rayleigh fading model

6. Publications

This material is based upon work supported in part by the National Science Foundation under Grant No. CNS-1642982.