MACM 316 - June 3

Workshop in AW 4135

Know: Lagrange exact formula
- divided difference method
- set up linear system
- Hermite only divided difference
- No exact formula
- DO need to know definition

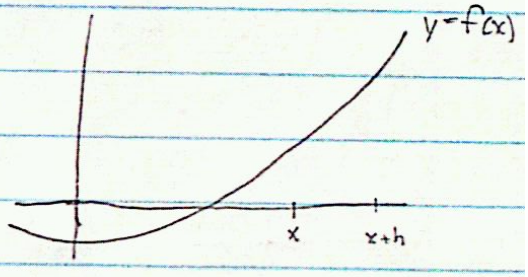
4.1 - Numerical Analysis

Finite Differences

$f'(x) \approx \frac{f(x+h) - f(x)}{h}$ forward difference

$\approx \frac{f(x+h) - f(x-h)}{2h}$ center difference

$\approx \frac{f(x) - f(x-h)}{h}$ backward difference



$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

or

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$

Accuracy (via taylor series)

$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{3!} f'''(x) + O(h^4)$

$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{3!} f'''(x) + O(h^4)$

forward difference

$\frac{f(x+h) - f(x)}{h} = f'(x) + \frac{h}{2} f''(x) + O(h^2)$

center difference

$\frac{f(x+h) - f(x-h)}{2h} = \frac{2hf'(x) + \frac{2h^3}{3!} f'''(x) + O(h^5)}{2h} = f'(x) + \frac{h^2}{3!} f'''(x) + O(h^4) = f'(x) + O(h^2)$

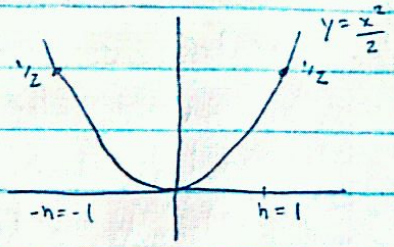
exact error term: $\frac{2f'''(\xi) h^2}{3!}$

Backwards difference: Same as forward difference

$$f''(x) = \frac{f(x+h) + f(x-h) - 2f(x)}{h^2}$$

$$= \frac{0 + 0 + h^2 f''(x) + 0 + O(h^4)}{h^2}$$

$$= f''(x) + O(h^2)$$



$f''(x) = 1$

$f''(x) = \frac{f(x+h) + f(x-h) - 2f(x}{h^2}$