

ASSIGNMENT COVER SHEET

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Course: Computer Science

Year: 4

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The material contained in this assignment is the authors original work, except where work quoted is duly acknowledged in the text. No aspect of this assignment has been previously submitted for assessment in any other unit or course.

Signed: Derek McCarthy

Date: 25/10/2018

1. Formal Specification

[Con N: int {N > 0}

Var

f: array [0..N) of char;

{ $\forall j: 0 \leq j < N: f.j = 'A' \vee f.j = 'B' \vee f.j = 'C'$ }

k: int;

t: int;

S

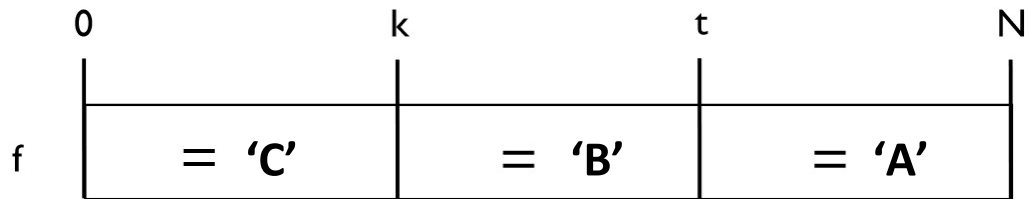
{ $\exists k, t: 0 \leq k \leq t \leq N: \forall j: 0 \leq j < k: f.j = 'C' \wedge$

$\forall j: k \leq j < t: f.j = 'B' \wedge$

$\forall j: t \leq j < N: f.j = 'A'$ }

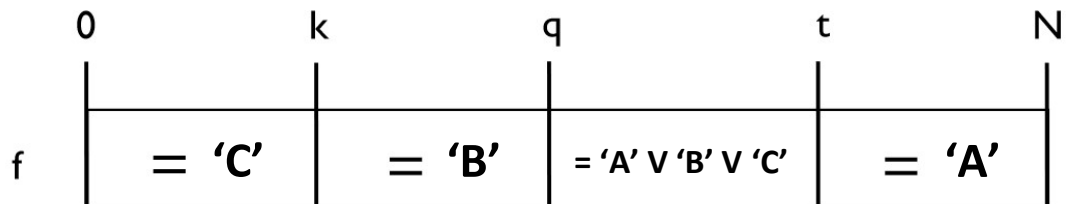
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2. Diagram representation of post Condition



{ $\exists k, t: 0 \leq k \leq t \leq N: \forall j: 0 \leq j < k: f.j = 'C' \wedge \forall j: k \leq j < t: f.j = 'B' \wedge \forall j: t \leq j < N: f.j = 'A'$ }

3. Invariant Diagram showing snapshot during processing

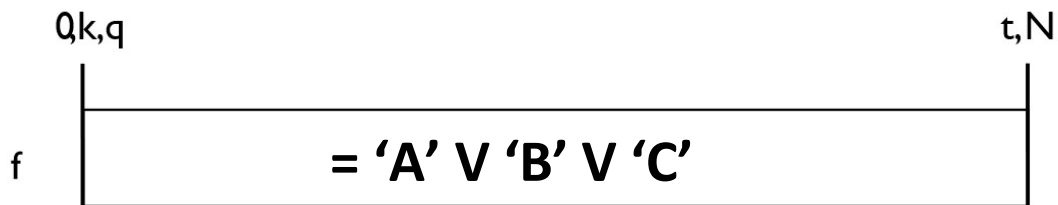


{ $\exists k, t, q: 0 \leq k \leq q \leq t \leq N: \forall j: 0 \leq j < k: f.j = 'C' \wedge \forall j: k \leq j < q: f.j = 'B' \wedge$

$\forall j: q \leq j < t: f.j = 'C' \vee f.j = 'B' \vee f.j = 'A' \wedge \forall j: t \leq j < N: f.j = 'A'$ }

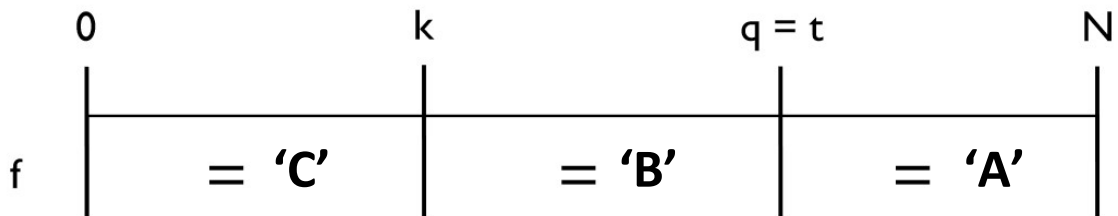
4. Values for variables at BEGINNING of processing

$k, q, t := 0, 0, N$



5. Values of variables at the END of processing

As $f[q..t]$ will be empty ($q = t$), the guard on the loop will be $q < t$.



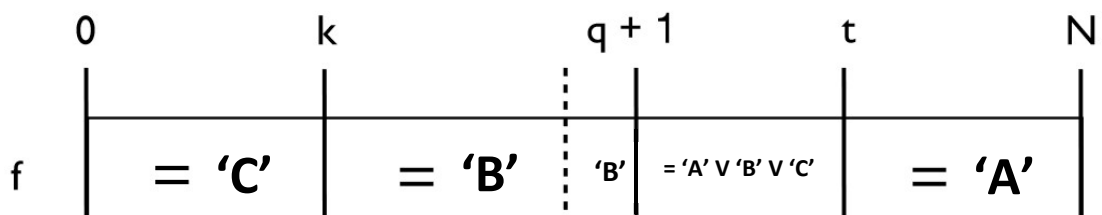
6. Assignment for variables during the MIDDLE of processing

To process the middle of the solution (loop body), we focus and consider the possible cases for $f.q$. There are three possibilities for $f.q$ they are, $f.q = 'A' \vee f.q = 'B' \vee f.q = 'C'$.

Step 1,

$f.q = 'B' \Rightarrow$ Increment q by 1

$q := q + 1$

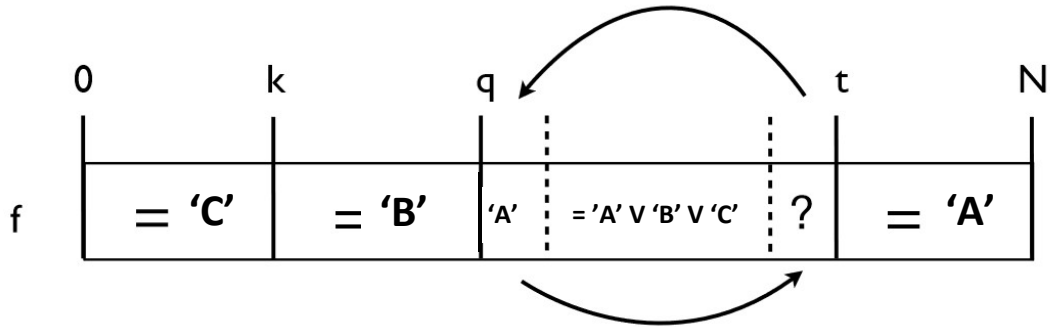


Step 2,

f.q = 'A' => Swap f.q with f.t - 1 and decrement t

f.q, f.t - 1 := f.t - 1, f.q;

t := t - 1;

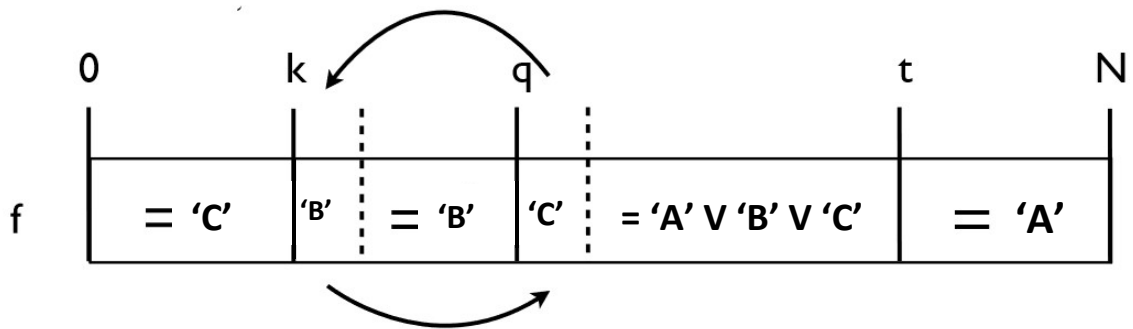


Step 3,

f.q = 0 => Swap f.q with f.k and increment k and q.

f.q, f.k := f.k, f.q;

q, k := q + 1, k + 1;



7. Termination proof

Initialisation

$t - q > 0$ ($q \ t := 0, N$)

\equiv [Substitution]

$N - 0 > 0$

\equiv [Arithmetic]

$N > 0$

\equiv [\Leftarrow Given $\{N > 0\}$]

TRUE

q := q + 1

$(t - q) (q := q + 1)$

\equiv [Substitution]

$t - (q + 1)$

\equiv [Arithmetic]

$t - q - 1$

$<$

$t - q$

q := q + 1

$(t - q) (q := q + 1)$

\equiv [Substitution]

$t - (q + 1)$

\equiv [Arithmetic]

$t - q - 1$

$<$

$t - q$

t := t - 1

$(t - q) (t := t - 1)$

\equiv [Substitution]

$(t - 1) - q$

\equiv [Arithmetic]

$t - q - 1$

$<$

$t - q$

8. Complete Solution

```
|| [ Con N: int {N > 0}
  Var
  f: array [0..N) of char;
    { $\forall j: 0 \leq j < N: f.j = 'A' \vee f.j = 'B' \vee f.j = 'C'$ }
    k,q,t: int;

  k,q,t := 0,0,N;
  do q < t  $\rightarrow$ 
    if f.q = 'C'  $\rightarrow$ 
      f.q, f.k := f.k, f.q;
      q,k := q+1, k+1;
    [] f.q = 'B'  $\rightarrow$ 
      q := q+1;
    [] f.q = 'A'  $\rightarrow$ 
      f.q, f.t-1 := f.t-1, f.q;
      t := t-1;
    fi
  od
  { $\exists k,t: 0 \leq k \leq t \leq N: \forall j: 0 \leq j < k: f.j = 'C' \wedge$ 
     $\forall j: k \leq j < t: f.j = 'B' \wedge$ 
     $\forall j: t \leq j < N: f.j = 'A'$ }
  ]|
```