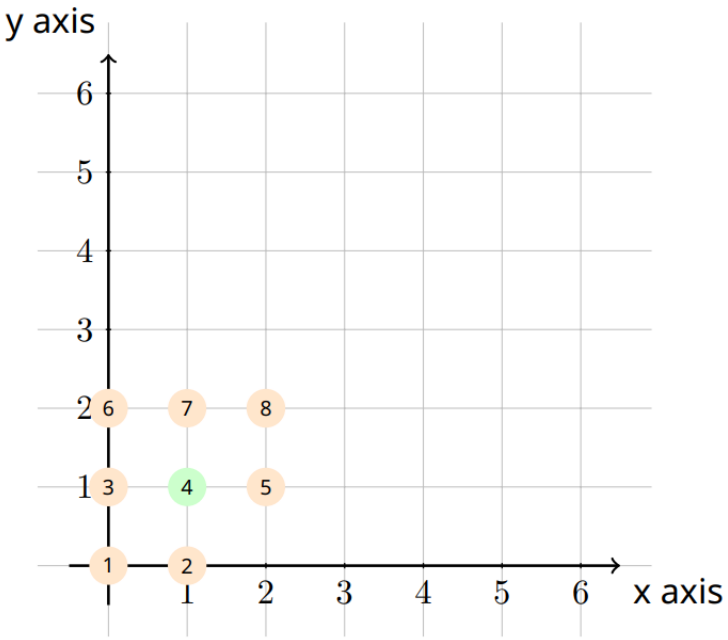
Данные задачи (Простой тест):

$$\begin{cases} u(x,y) = 2x + 4y \\ f(x,y) = 6x + 12y \\ \lambda = 2 \\ \gamma = 3 \end{cases}$$



Краевые условия:

Первого рода на всех ребрах

Содержимое json-файла:

```
{
    "CountX": 3,
    "CountY": 3,
    "X"     : [0, 1, 2],
    "Y"     : [0, 1, 2],
    "GX"    : 2,
    "GY"    : 2
}
```

CountX -> Количество точек на оси X

CountY -> Количество точек на оси Y

X -> Значения X-ов

Y -> Значения Y-ов

GX -> К-во точек на нижней границе области «Г»

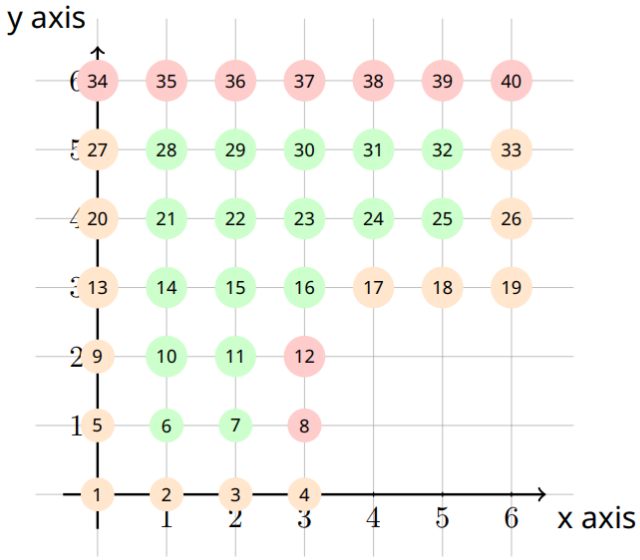
GY -> К-во точек на правой границе области «Г»

Табличка с решением:

X	Y	U	U`	|U` - U|
0	0	0,000000E+000	0,000000E+000	0,000000E+000
1	0	2,000000E+000	2,000000E+000	0,000000E+000
0	1	4,000000E+000	4,000000E+000	0,000000E+000
1	1	6,000000E+000	6,000000E+000	0,000000E+000
2	1	8,000000E+000	8,000000E+000	0,000000E+000
0	2	8,000000E+000	8,000000E+000	0,000000E+000
1	2	1,000000E+001	1,000000E+001	0,000000E+000
2	2	1,200000E+001	1,200000E+001	0,000000E+000

Данные задачи (Тест на вторые краевые):

$$\begin{cases} u(x,y) = 2x + 4y \\ f(x,y) = 6x + 12y \\ \lambda = 2 \\ \gamma = 3 \end{cases}$$



Краевые условия:

Первого рода – оранжевый цвет

Второго рода – красный цвет

Содержимое json-файла:

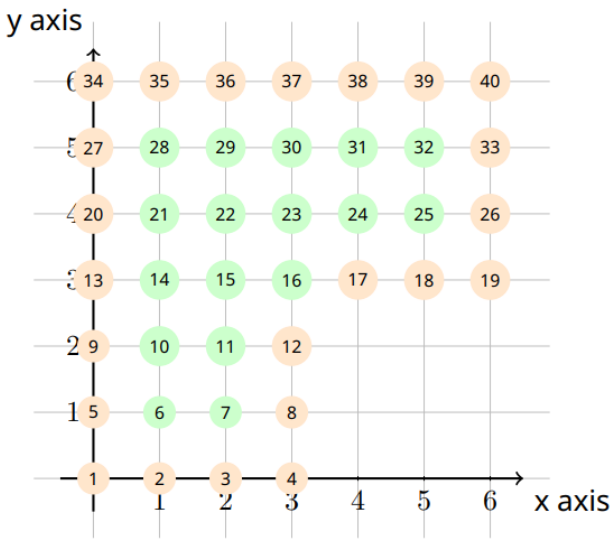
```
{
    "CountX": 7,
    "CountY": 7,
    "X"     : [0, 1, 2, 3, 4, 5, 6],
    "Y"     : [0, 1, 2, 3, 4, 5, 6],
    "GX"    : 4,
    "GY"    : 4
}
```

Табличка с решением:

X	Y	U	U`	|U` - U|
0	0	0,000000E+000	0,000000E+000	0,000000E+000
1	0	2,000000E+000	2,000000E+000	0,000000E+000
2	0	4,000000E+000	4,000000E+000	0,000000E+000
3	0	6,000000E+000	6,000000E+000	0,000000E+000
0	1	4,000000E+000	4,000000E+000	0,000000E+000
1	1	6,000000E+000	6,000000E+000	3,375078E-014
2	1	8,000000E+000	8,000000E+000	4,529710E-014
3	1	1,000000E+001	1,000000E+001	4,618528E-014
0	2	8,000000E+000	8,000000E+000	0,000000E+000
1	2	1,000000E+001	1,000000E+001	4,440892E-014
2	2	1,200000E+001	1,200000E+001	5,684342E-014
3	2	1,400000E+001	1,400000E+001	5,684342E-014
0	3	1,200000E+001	1,200000E+001	0,000000E+000
1	3	1,400000E+001	1,400000E+001	4,263256E-014
2	3	1,600000E+001	1,600000E+001	4,796163E-014
3	3	1,800000E+001	1,800000E+001	3,197442E-014
4	3	2,000000E+001	2,000000E+001	0,000000E+000
5	3	2,200000E+001	2,200000E+001	0,000000E+000
6	3	2,400000E+001	2,400000E+001	0,000000E+000
0	4	1,600000E+001	1,600000E+001	0,000000E+000
1	4	1,800000E+001	1,800000E+001	4,263256E-014
2	4	2,000000E+001	2,000000E+001	4,973799E-014
3	4	2,200000E+001	2,200000E+001	3,907985E-014
4	4	2,400000E+001	2,400000E+001	1,776357E-014
5	4	2,600000E+001	2,600000E+001	7,105427E-015
6	4	2,800000E+001	2,800000E+001	0,000000E+000
0	5	2,000000E+001	2,000000E+001	0,000000E+000
1	5	2,200000E+001	2,200000E+001	4,618528E-014
2	5	2,400000E+001	2,400000E+001	4,973799E-014
3	5	2,600000E+001	2,600000E+001	3,907985E-014
4	5	2,800000E+001	2,800000E+001	2,131628E-014
5	5	3,000000E+001	3,000000E+001	1,065814E-014
6	5	3,200000E+001	3,200000E+001	0,000000E+000
0	6	2,400000E+001	2,400000E+001	0,000000E+000
1	6	2,600000E+001	2,600000E+001	4,618528E-014
2	6	2,800000E+001	2,800000E+001	4,973799E-014
3	6	3,000000E+001	3,000000E+001	3,907985E-014
4	6	3,200000E+001	3,200000E+001	2,131628E-014
5	6	3,400000E+001	3,400000E+001	1,421085E-014
6	6	3,600000E+001	3,600000E+001	0,000000E+000

Данные задачи (Полином второй степени):

$$\begin{cases} u(x,y) = 2x^2 + 4y^2 \\ f(x,y) = 6x^2 + 12y^2 - 24 \\ \lambda = 2 \\ \gamma = 3 \end{cases}$$



Краевые условия:

Первого рода на всех ребрах

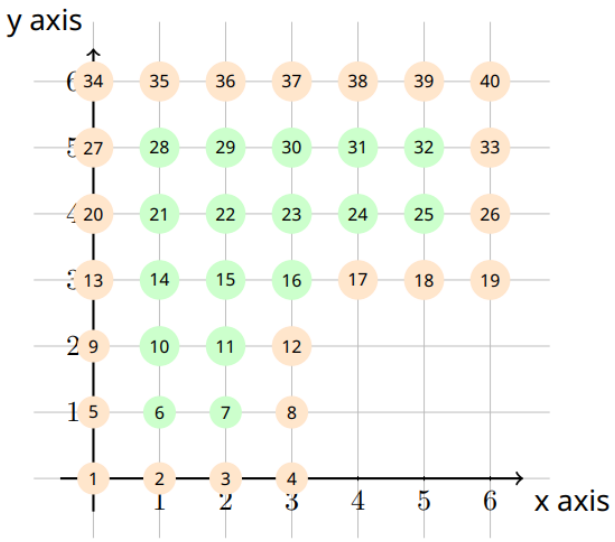
Содержимое json-файла: как в прошлом тесте

Табличка с решением:

X	Y	U	U`	|U`- U|
0	0	0,000000E+000	0,000000E+000	0,000000E+000
1	0	2,000000E+000	2,000000E+000	0,000000E+000
2	0	8,000000E+000	8,000000E+000	0,000000E+000
3	0	1,800000E+001	1,800000E+001	0,000000E+000
0	1	4,000000E+000	4,000000E+000	0,000000E+000
1	1	6,000000E+000	6,000000E+000	1,803002E-013
2	1	1,200000E+001	1,200000E+001	1,083578E-013
3	1	2,200000E+001	2,200000E+001	0,000000E+000
0	2	1,600000E+001	1,600000E+001	0,000000E+000
1	2	1,800000E+001	1,800000E+001	2,060574E-013
2	2	2,400000E+001	2,400000E+001	1,278977E-013
3	2	3,400000E+001	3,400000E+001	0,000000E+000
0	3	3,600000E+001	3,600000E+001	0,000000E+000
1	3	3,800000E+001	3,800000E+001	1,705303E-013
2	3	4,400000E+001	4,400000E+001	1,278977E-013
3	3	5,400000E+001	5,400000E+001	3,552714E-014
4	3	6,800000E+001	6,800000E+001	0,000000E+000
5	3	8,600000E+001	8,600000E+001	0,000000E+000
6	3	1,080000E+002	1,080000E+002	0,000000E+000
0	4	6,400000E+001	6,400000E+001	0,000000E+000
1	4	6,600000E+001	6,600000E+001	9,947598E-014
2	4	7,200000E+001	7,200000E+001	8,526513E-014
3	4	8,200000E+001	8,200000E+001	4,263256E-014
4	4	9,600000E+001	9,600000E+001	1,421085E-014
5	4	1,140000E+002	1,140000E+002	0,000000E+000
6	4	1,360000E+002	1,360000E+002	0,000000E+000
0	5	1,000000E+002	1,000000E+002	0,000000E+000
1	5	1,020000E+002	1,020000E+002	4,263256E-014
2	5	1,080000E+002	1,080000E+002	2,842171E-014
3	5	1,180000E+002	1,180000E+002	0,000000E+000
4	5	1,320000E+002	1,320000E+002	0,000000E+000
5	5	1,500000E+002	1,500000E+002	0,000000E+000
6	5	1,720000E+002	1,720000E+002	0,000000E+000
0	6	1,440000E+002	1,440000E+002	0,000000E+000
1	6	1,460000E+002	1,460000E+002	0,000000E+000
2	6	1,520000E+002	1,520000E+002	0,000000E+000
3	6	1,620000E+002	1,620000E+002	0,000000E+000
4	6	1,760000E+002	1,760000E+002	0,000000E+000
5	6	1,940000E+002	1,940000E+002	0,000000E+000
6	6	2,160000E+002	2,160000E+002	0,000000E+000

Данные задачи (Полином третьей степени):

$$\begin{cases} u(x,y) = 2x^3 + 4y^3 \\ f(x,y) = -24x - 48y + 6x^3 + 12y^3 \\ \lambda = 2 \\ \gamma = 3 \end{cases}$$



Краевые условия:

Первого рода на всех ребрах

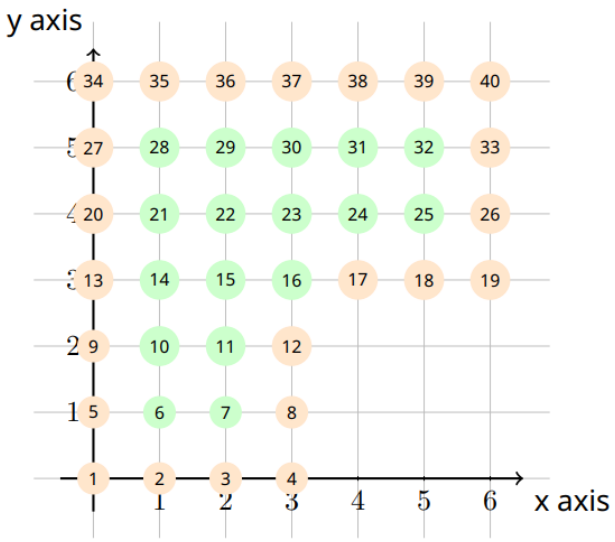
Содержимое json-файла: как в прошлом тесте

Табличка с решением:

X	Y	U	U`	|U` - U|
0	0	0,000000E+000	0,000000E+000	0,000000E+000
1	0	2,000000E+000	2,000000E+000	0,000000E+000
2	0	1,600000E+001	1,600000E+001	0,000000E+000
3	0	5,400000E+001	5,400000E+001	0,000000E+000
0	1	4,000000E+000	4,000000E+000	0,000000E+000
1	1	6,000000E+000	6,000000E+000	2,859935E-013
2	1	2,000000E+001	2,000000E+001	1,740830E-013
3	1	5,800000E+001	5,800000E+001	0,000000E+000
0	2	3,200000E+001	3,200000E+001	0,000000E+000
1	2	3,400000E+001	3,400000E+001	3,268497E-013
2	2	4,800000E+001	4,800000E+001	2,131628E-013
3	2	8,600000E+001	8,600000E+001	0,000000E+000
0	3	1,080000E+002	1,080000E+002	0,000000E+000
1	3	1,100000E+002	1,100000E+002	2,842171E-013
2	3	1,240000E+002	1,240000E+002	2,131628E-013
3	3	1,620000E+002	1,620000E+002	8,526513E-014
4	3	2,360000E+002	2,360000E+002	0,000000E+000
5	3	3,580000E+002	3,580000E+002	0,000000E+000
6	3	5,400000E+002	5,400000E+002	0,000000E+000
0	4	2,560000E+002	2,560000E+002	0,000000E+000
1	4	2,580000E+002	2,580000E+002	1,705303E-013
2	4	2,720000E+002	2,720000E+002	1,705303E-013
3	4	3,100000E+002	3,100000E+002	5,684342E-014
4	4	3,840000E+002	3,840000E+002	0,000000E+000
5	4	5,060000E+002	5,060000E+002	0,000000E+000
6	4	6,880000E+002	6,880000E+002	0,000000E+000
0	5	5,000000E+002	5,000000E+002	0,000000E+000
1	5	5,020000E+002	5,020000E+002	5,684342E-014
2	5	5,160000E+002	5,160000E+002	0,000000E+000
3	5	5,540000E+002	5,540000E+002	0,000000E+000
4	5	6,280000E+002	6,280000E+002	0,000000E+000
5	5	7,500000E+002	7,500000E+002	0,000000E+000
6	5	9,320000E+002	9,320000E+002	0,000000E+000
0	6	8,640000E+002	8,640000E+002	0,000000E+000
1	6	8,660000E+002	8,660000E+002	0,000000E+000
2	6	8,800000E+002	8,800000E+002	0,000000E+000
3	6	9,180000E+002	9,180000E+002	0,000000E+000
4	6	9,920000E+002	9,920000E+002	0,000000E+000
5	6	1,114000E+003	1,114000E+003	0,000000E+000
6	6	1,296000E+003	1,296000E+003	0,000000E+000

Данные задачи (Полином четвертой степени):

$$\begin{cases} u(x,y) = 2x^4 + 4y^4 \\ f(x,y) = -48x^2 - 96y^2 + 6x^4 + 12y^4 \\ \lambda = 2 \\ \gamma = 3 \end{cases}$$



Краевые условия:

Первого рода на всех ребрах

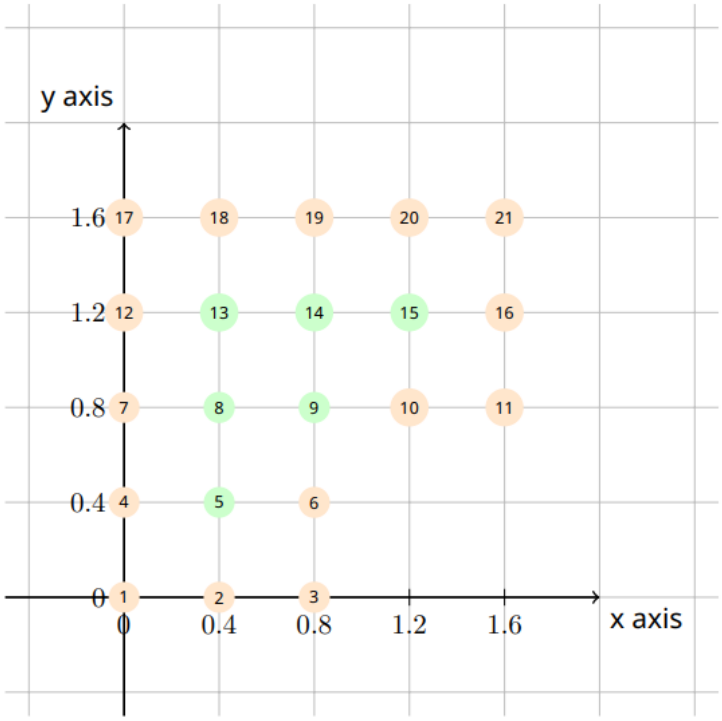
Содержимое json-файла: как в прошлом тесте

Табличка с решением:

X	Y	U	U`	|U` - U|
0	0	0,000000E+000	0,000000E+000	0,000000E+000
1	0	2,000000E+000	2,000000E+000	0,000000E+000
2	0	3,200000E+001	3,200000E+001	0,000000E+000
3	0	1,620000E+002	1,620000E+002	0,000000E+000
0	1	4,000000E+000	4,000000E+000	0,000000E+000
1	1	9,699224E+000	6,000000E+000	3,699224E+000
2	1	3,972019E+001	3,600000E+001	3,720188E+000
3	1	1,660000E+002	1,660000E+002	0,000000E+000
0	2	6,400000E+001	6,400000E+001	0,000000E+000
1	2	7,062554E+001	6,600000E+001	4,625542E+000
2	2	1,007618E+002	9,600000E+001	4,761812E+000
3	2	2,260000E+002	2,260000E+002	0,000000E+000
0	3	3,240000E+002	3,240000E+002	0,000000E+000
1	3	3,309794E+002	3,260000E+002	4,979443E+000
2	3	3,618442E+002	3,560000E+002	5,844237E+000
3	3	4,903070E+002	4,860000E+002	4,306995E+000
4	3	8,360000E+002	8,360000E+002	0,000000E+000
5	3	1,574000E+003	1,574000E+003	0,000000E+000
6	3	2,916000E+003	2,916000E+003	0,000000E+000
0	4	1,024000E+003	1,024000E+003	0,000000E+000
1	4	1,030917E+003	1,026000E+003	4,917159E+000
2	4	1,062095E+003	1,056000E+003	6,095053E+000
3	4	1,191844E+003	1,186000E+003	5,844237E+000
4	4	1,540762E+003	1,536000E+003	4,761812E+000
5	4	2,277720E+003	2,274000E+003	3,720188E+000
6	4	3,616000E+003	3,616000E+003	0,000000E+000
0	5	2,500000E+003	2,500000E+003	0,000000E+000
1	5	2,505970E+003	2,502000E+003	3,969876E+000
2	5	2,536917E+003	2,532000E+003	4,917159E+000
3	5	2,666979E+003	2,662000E+003	4,979443E+000
4	5	3,016626E+003	3,012000E+003	4,625542E+000
5	5	3,753699E+003	3,750000E+003	3,699224E+000
6	5	5,092000E+003	5,092000E+003	0,000000E+000
0	6	5,184000E+003	5,184000E+003	0,000000E+000
1	6	5,186000E+003	5,186000E+003	0,000000E+000
2	6	5,216000E+003	5,216000E+003	0,000000E+000
3	6	5,346000E+003	5,346000E+003	0,000000E+000
4	6	5,696000E+003	5,696000E+003	0,000000E+000
5	6	6,434000E+003	6,434000E+003	0,000000E+000
6	6	7,776000E+003	7,776000E+003	0,000000E+000

Данные задачи (Не полиноминальная функция):

$$\begin{cases} u(x,y) = \sin(x+y) \\ f(x,y) = 3\sin(x+y) \\ \lambda = 1 \\ \gamma = 1 \end{cases}$$



Краевые условия:

Первого рода на всех ребрах

Содержимое json-файлов

h:

```
{
    "CountX": 5,
    "CountY": 5,
    "X"     : [0, 0.4, 0.8, 1.2, 1.6],
    "Y"     : [0, 0.4, 0.8, 1.2, 1.6],
    "GX"    : 3,
    "GY"    : 3
}
```

h/2:

```
{
    "CountX": 9,
    "CountY": 9,
    "X"     : [0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6],
    "Y"     : [0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6],
    "GX"    : 5,
    "GY"    : 5
}
```

h/4:

```
{
    "CountX": 17,
    "CountY": 17,
    "X"     : [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6],
    "Y"     : [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6],
    "GX"    : 9,
    "GY"    : 9
}
```

X	Y	U`	U(h)	U(h/2)	U(h/4)	|U` - U(h)|	|U` - U(h/2)|	|U` - U(h/4)|
0	0	0,000000E+000	0,000000E+000	0,000000E+000	0,000000E+000	0,000000E+000	0,000000E+000	0,000000E+000
0,4	0	3,894183E-001	3,894183E-001	3,894183E-001	3,894183E-001	0,000000E+000	0,000000E+000	0,000000E+000
0,8	0	7,173561E-001	7,173561E-001	7,173561E-001	7,173561E-001	0,000000E+000	0,000000E+000	0,000000E+000
0	0,4	3,894183E-001	3,894183E-001	3,894183E-001	3,894183E-001	0,000000E+000	0,000000E+000	0,000000E+000
0,4	0,4	7,173561E-001	7,186332E-001	7,176716E-001	7,174342E-001	1,277102E-003	3,155432E-004	7,814708E-005
0,8	0,4	9,320391E-001	9,320391E-001	9,320391E-001	9,320391E-001	0,000000E+000	0,000000E+000	0,000000E+000
0	0,8	7,173561E-001	7,173561E-001	7,173561E-001	7,173561E-001	0,000000E+000	0,000000E+000	0,000000E+000
0,4	0,8	9,320391E-001	9,343074E-001	9,325508E-001	9,321622E-001	2,268302E-003	5,117270E-004	1,230924E-004
0,8	0,8	9,995736E-001	1,001675E+000	9,998866E-001	9,996215E-001	2,101741E-003	3,129873E-004	4,790169E-005
1,2	0,8	9,092974E-001	9,092974E-001	9,092974E-001	9,092974E-001	0,000000E+000	0,000000E+000	0,000000E+000
1,6	0,8	6,754632E-001	6,754632E-001	6,754632E-001	6,754632E-001	0,000000E+000	0,000000E+000	0,000000E+000
0	1,2	9,320391E-001	9,320391E-001	9,320391E-001	9,320391E-001	0,000000E+000	0,000000E+000	0,000000E+000
0,4	1,2	9,995736E-001	1,001675E+000	1,000079E+000	9,996976E-001	2,101741E-003	5,056700E-004	1,239681E-004
0,8	1,2	9,092974E-001	9,115302E-001	9,098002E-001	9,094183E-001	2,232774E-003	5,027895E-004	1,208528E-004
1,2	1,2	6,754632E-001	6,766890E-001	6,757650E-001	6,755378E-001	1,225823E-003	3,017819E-004	7,463351E-005
1,6	1,2	3,349882E-001	3,349882E-001	3,349882E-001	3,349882E-001	0,000000E+000	0,000000E+000	0,000000E+000
0	1,6	9,995736E-001	9,995736E-001	9,995736E-001	9,995736E-001	0,000000E+000	0,000000E+000	0,000000E+000
0,4	1,6	9,092974E-001	9,092974E-001	9,092974E-001	9,092974E-001	0,000000E+000	0,000000E+000	0,000000E+000
0,8	1,6	6,754632E-001	6,754632E-001	6,754632E-001	6,754632E-001	0,000000E+000	0,000000E+000	0,000000E+000
1,2	1,6	3,349882E-001	3,349882E-001	3,349882E-001	3,349882E-001	0,000000E+000	0,000000E+000	0,000000E+000
1,6	1,6	-5,837414E-002	-5,837414E-002	-5,837414E-002	-5,837414E-002	0,000000E+000	0,000000E+000	0,000000E+000

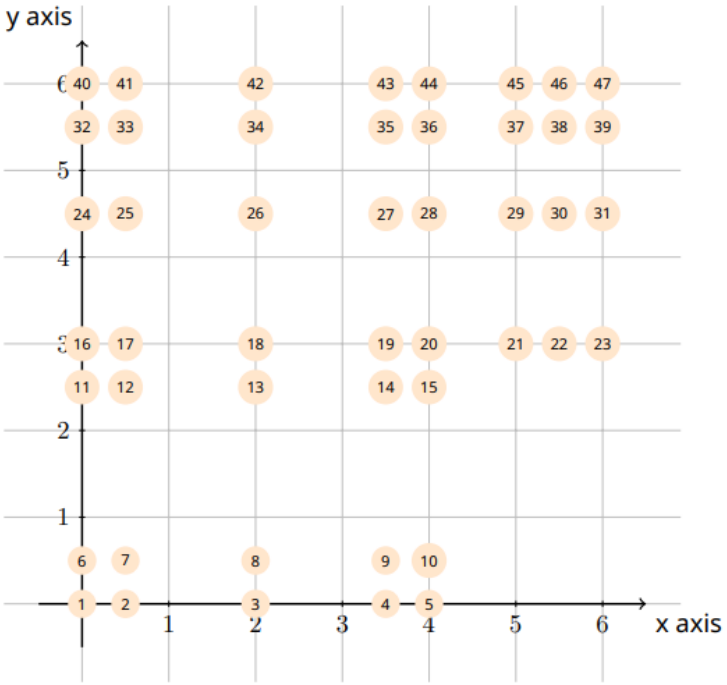
$$k = \log_2 \frac{\left\| u^* - u_h \right\|}{\left\| u^* - u_{h/2} \right\|} \approx 2{,}1916$$

$$k = \log_2 \frac{\left\| u^* - u_{h/2} \right\|}{\left\| u^* - u_{h/4} \right\|} \approx 2{,}0817$$

Данные задачи (Полином третьей степени):

$$\begin{cases} u(x,y) = 2x + 4y \\ f(x,y) = 6x + 12y \\ \lambda = 2 \\ \gamma = 3 \end{cases}$$

Краевые условия:
Первого рода на всех ребрах



Содержимое json-файла:

```
{
    "CountX": 8,
    "CountY": 7,
    "X"     : [0, 0.5, 2, 3.5, 4, 5, 5.5, 6],
    "Y"     : [0, 0.5, 2.5, 3, 4.5, 5.5, 6],
    "GX"    : 5,
    "GY"    : 4
}
```

Табличка с решением:

X	Y	U	U`	|U` - U|
0	0	0,000000E+000	0,000000E+000	0,000000E+000
0,5	0	1,000000E+000	1,000000E+000	0,000000E+000
2	0	4,000000E+000	4,000000E+000	0,000000E+000
3,5	0	7,000000E+000	7,000000E+000	0,000000E+000
4	0	8,000000E+000	8,000000E+000	0,000000E+000
0	0,5	2,000000E+000	2,000000E+000	0,000000E+000
0,5	0,5	3,000000E+000	3,000000E+000	7,105427E-015
2	0,5	6,000000E+000	6,000000E+000	1,243450E-014
3,5	0,5	9,000000E+000	9,000000E+000	3,552714E-015
4	0,5	1,000000E+001	1,000000E+001	0,000000E+000
0	2,5	1,000000E+001	1,000000E+001	0,000000E+000
0,5	2,5	1,100000E+001	1,100000E+001	2,131628E-014
2	2,5	1,400000E+001	1,400000E+001	3,552714E-014
3,5	2,5	1,700000E+001	1,700000E+001	1,776357E-014
4	2,5	1,800000E+001	1,800000E+001	0,000000E+000
0	3	1,200000E+001	1,200000E+001	0,000000E+000
0,5	3	1,300000E+001	1,300000E+001	1,598721E-014
2	3	1,600000E+001	1,600000E+001	2,664535E-014
3,5	3	1,900000E+001	1,900000E+001	2,131628E-014
4	3	2,000000E+001	2,000000E+001	7,105427E-015
5	3	2,200000E+001	2,200000E+001	0,000000E+000
5,5	3	2,300000E+001	2,300000E+001	0,000000E+000
6	3	2,400000E+001	2,400000E+001	0,000000E+000
0	4,5	1,800000E+001	1,800000E+001	0,000000E+000
0,5	4,5	1,900000E+001	1,900000E+001	7,105427E-015
2	4,5	2,200000E+001	2,200000E+001	1,421085E-014
3,5	4,5	2,500000E+001	2,500000E+001	2,131628E-014
4	4,5	2,600000E+001	2,600000E+001	1,421085E-014
5	4,5	2,800000E+001	2,800000E+001	7,105427E-015
5,5	4,5	2,900000E+001	2,900000E+001	7,105427E-015

6	4,5	3,000000E+001	3,000000E+001	0,000000E+000
0	5,5	2,200000E+001	2,200000E+001	0,000000E+000
0,5	5,5	2,300000E+001	2,300000E+001	3,552714E-015
2	5,5	2,600000E+001	2,600000E+001	3,552714E-015
3,5	5,5	2,900000E+001	2,900000E+001	1,065814E-014
4	5,5	3,000000E+001	3,000000E+001	1,065814E-014
5	5,5	3,200000E+001	3,200000E+001	7,105427E-015
5,5	5,5	3,300000E+001	3,300000E+001	0,000000E+000
6	5,5	3,400000E+001	3,400000E+001	0,000000E+000
0	6	2,400000E+001	2,400000E+001	0,000000E+000
0,5	6	2,500000E+001	2,500000E+001	0,000000E+000
2	6	2,800000E+001	2,800000E+001	0,000000E+000
3,5	6	3,100000E+001	3,100000E+001	0,000000E+000
4	6	3,200000E+001	3,200000E+001	0,000000E+000
5	6	3,400000E+001	3,400000E+001	0,000000E+000
5,5	6	3,500000E+001	3,500000E+001	0,000000E+000
6	6	3,600000E+001	3,600000E+001	0,000000E+000

Вывод:

В результате всех исследований можем сказать, что:

- Метод конечных разностей отлично справляется с уравнениями с полиномиальным решением до 3 степени включительно. На полиномах более высокого уровня решение несколько хуже.
- Порядок аппроксимации соответствует теоретическому, на равномерных сетках с краевыми условиями первого рода МКР имеет второй порядок аппроксимации.
- На неравномерных сетках может привести к понижению порядка.