Assignment 6（Writeup）

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COMP 540 – Statistical Machine Learning
Rice University

Problem 1)

a) O={low,medium,high}, S={healthy,unhealthy}, λ ={ π ,a,b}, π =[0.5,0.5]

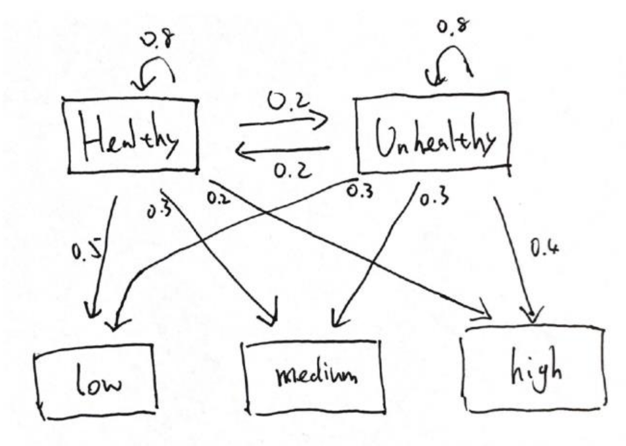
a: $\begin{array}{r} healthy \\ unhealthy \end{array}\begin{pmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{pmatrix}$

healthy unhealthy

b: $\begin{array}{r} healthy \\ unhealthy \end{array}\begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{pmatrix}$

low medium high

$$\begin{array}{lccccccccc} \text{Hidden stats:} & X_0 & \rightarrow & X_1 & \rightarrow & X_1 & \rightarrow & \cdots & \rightarrow & X_t \\ & & & \downarrow & & \downarrow & & & & \downarrow \\ \text{Observations:} & & & e_1 & & e_2 & & & & e_t \end{array}$$



b) $S_1: healthy, S_2: unhealthy, e_1 = e_2 = low$

$$\alpha_t(i) = P(e_1, \dots, e_t, x_t = S_i)$$
$$\alpha_0(1) = \pi_1 = 0.5$$
$$\alpha_0(2) = \pi_2 = 0.5$$
$$\alpha_1(1) = 0.5 * (0.5 * 0.8 + 0.5 * 0.2) = 0.25$$
$$\alpha_1(2) = 0.3 * (0.5 * 0.2 + 0.5 * 0.8) = 0.15$$
$$\alpha_2(1) = 0.5 * (0.25 * 0.8 + 0.15 * 0.2) = 0.115$$
$$\alpha_2(2) = 0.3 * (0.25 * 0.2 + 0.15 * 0.8) = 0.051$$

Thus, $P(x_2 = S_1 | e_1, e_2) = \frac{0.115}{0.115+0.051} = 0.693$

c) $\delta_t(i) = max_{x_1,\dots,x_{t-1}} P(x_1, \dots, x_{t-1}, x_t = S_i, e_1, \dots, e_t)$

$$\delta_0(1) = 0.5$$
$$\delta_0(2) = 0.5$$
$$For\ \delta_1(1), for\ i = 1, it\ equals\ 0.5 * 0.8 * 0.5 = 0.2,$$
$$for\ i = 2, it\ equals\ to\ \ 0.5 * 0.2 * 0.5 = 0.05.$$
$$Thus, by\ taking\ the\ max,\ \delta_1(1) = 0.2$$

$$For\ \delta_1(2), for\ i = 1, it\ equals\ 0.5 * 0.2 * 0.3 = 0.03,$$
$$for\ i = 2, it\ equals\ to\ \ 0.5 * 0.8 * 0.3 = 0.12.$$
$$Thus, by\ taking\ the\ max,\ \ \delta_1(1) = 0.12$$
$$For\ \delta_2(1), for\ i = 1, it\ equals\ 0.2 * 0.8 * 0.5 = 0.08,$$
$$for\ i = 2, it\ equals\ to\ \ 0.12 * 0.2 * 0.5 = 0.012.$$
$$Thus, by\ taking\ the\ max,\ \ \delta_1(1) = 0.08$$
$$For\ \delta_2(2), for\ i = 1, it\ equals\ 0.2 * 0.2 * 0.3 = 0.012,$$
$$for\ i = 2, it\ equals\ to\ \ 0.12 * 0.8 * 0.3 = 0.0288.$$
Thus, $\delta_2(2) = 0.0288 < \ \delta_2(1). Therefore, x_2 = S_1.$

$$Since\ \delta_1(1)a_{11} > \delta_1(2)a_{21}, x_1 = S_1$$

Since $\delta_0(1)a_{11} > \delta_0(2)a_{21}. Therefore, x_0 = S_1$

Given that $e_1 = e_2 = low$, the most likely state sequence is $x_0 \to x_1 \to x_2\ which\ is\ healthy\ \to healthy \to healthy$

Problem 2)

a) Likelihood:

$$L(D;\pi,\mu) = \prod_{i=1}^{m}\prod_{k=1}^{k}\left[P\left(z^{(i)} = k,\pi\right)P\left(x^{(i)}\middle|z^{(i)} = k;\mu\right)\right]^{I\left(z^{(i)}=k\right)}$$
$$= \prod_{i=1}^{m}\prod_{k=1}^{k}[\pi_k Bernoulli(x^{(i)},\mu_k)]^{I(z^{(i)}=k)}$$

Thus, the log likelihood:

$$l(D;\pi,\mu) = \sum_{i=1}^{m}\sum_{k=1}^{k} I(z^{(i)} = k)[\pi_k Bernoulli(x^{(i)},\mu_k)]$$

Then the expected log likelihood:

$$E[l(D;\pi,\mu)] = \sum_{i=1}^{m}\sum_{k=1}^{K}\gamma_k^{(i)}\{log\pi_k + \log[\mu_k^{x^{(i)}}(1-\mu_k)^{1-x^{(i)}}]\}$$
$$= \sum_{i=1}^{m}\sum_{k=1}^{K}\gamma_k^{(i)}\{log\pi_k + x^{(i)}\log\mu_k + (1-x^{(i)})\log(1-\mu_k)\}$$

For MLE, for each μ_{kj}

We let $\frac{\partial E[l(D;\pi,\mu)]}{\partial\mu_{kj}} = 0$

Then $\sum_{i=1}^{m}\gamma_k^{(i)}\left(\frac{x_j^{(i)}}{\mu_{kj}} - \frac{1-x_j^{(i)}}{1-\mu_{kj}}\right) = 0$

Then $\mu_{kj} = \frac{\sum_{i=1}^{m}\gamma_k^{(i)}x_j^{(i)}}{\sum_{i=1}^{m}\gamma_k^{(i)}}$

b) $P(\mu|D) \propto P(D|\mu)P(\mu)$ $where\ \mu \sim beta(\alpha,\beta)$

Thus, $P(\mu|D) \propto \prod_{i=1}^{m}\prod_{k=1}^{k}\left[\pi_k Bernoulli\left(x^{(i)},\mu_k\right)\right]^{I\left(z^{(i)}=k\right)} \times \prod_{k=1}^{K} beta(\mu_k;\alpha,\beta)$

$$Thus, the\ operation\ is:$$

$E = \sum_{i=1}^{m}\sum_{k=1}^{K} I\left(z^{(i)}=k\right)\left[log\pi_k + logBernoulli\left(x^{(i)},\mu_k\right)\right] + \sum_{k=1}^{K} log\ beta(\mu_k;\alpha,\beta)$

$$= \sum_{i=1}^{m}\sum_{k=1}^{K}\gamma_k^{(i)}\{log\pi_k + x^{(i)}\log\mu_k + (1-x^{(i)})\log(1-\mu_k)\}$$

$$+ \sum_{k=1}^{K}[(\alpha-1)log\ \mu_k + (\beta-1)log\ (1-\mu_k)]$$

For each μ_{kj}, do MAP estimation,

Let $\frac{\partial E}{\partial \mu_{kj}} = 0,$

Then $\sum_{i=1}^{m}\gamma_k^{(i)}\left(\frac{x_j^{(i)}}{\mu_{kj}} - \frac{1-x_j^{(i)}}{1-\mu_{kj}}\right) + (\alpha-1)/log\ \mu_k - (\beta-1)/log\ (1-\mu_k) = 0$

Then $\mu_{kj} = \frac{\sum_{i=1}^{m}\gamma_k^{(i)}x_j^{(i)}+\alpha-1}{\sum_{i=1}^{m}\gamma_k^{(i)}+\alpha+\beta-2}$

Problem 3)

$$V = \{au: a \in R\}$$

$$then\ f_u(x) = argmin_{v\in V}||x-v||^2 = \left(argmin_a\left||x-au|\right|^2\right) * u$$

$$= [argmin_a(x^Tx + 2au^Tx + a^2u^Tu)] * u$$

since u s a unit vector, then $u^Tu = 1$

let $\frac{\partial(x^Tx+2au^Tx+a^2u^Tu)}{\partial a} = 0,$

we get $a = \frac{-2u^Tx}{-2u^Tu} = u^Tx$

Then $f_u(x) = u^Txu$

Then

$$argmin_{u=u^Tu=1}\sum_{i=1}^{m}\left|\left|x^{(i)} - f_u\left(x^{(i)}\right)\right|\right|^2$$

$$= argmin_u\sum_{i=1}^{m}\left|\left|x^{(i)} - u^Tx^{(i)}u\right|\right|^2$$

$$= argmin_u \sum_{i=1}^{m} \left(x^{(i)} - u^T x^{(i)} u\right)^T \left(x^{(i)} - u^T x^{(i)} u\right)$$

$$= argmin_u \sum_{i=1}^{m} \left(x^{(i)^T} x^{(i)} - 2(u^T x^{(i)})^2 + u^T u (u^T x^{(i)})^2\right)$$

$$= argmin_u \sum_{i=1}^{m} -\left(u^T x^{(i)}\right)^2$$

$$= argmin_u u^T \sum_{i=1}^{m} (x^{(i)^T} x^{(i)})\, u$$

Thus, we say that $x^{(i)}$ have zero mean and unit variance. So, this is the first principal component.

Problem 4)

Problem 4.1: Finding closest centroids

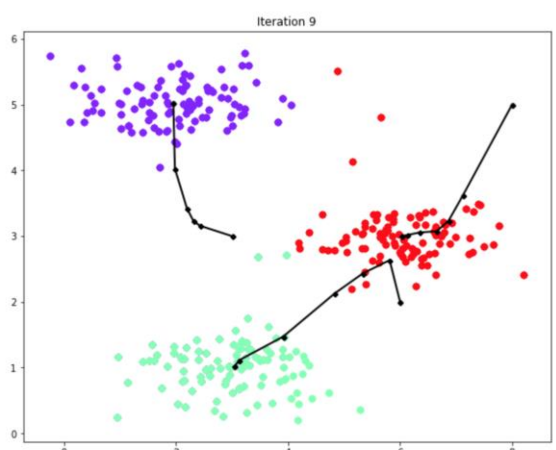
```
Finding closest centroids.
Closest centroids for the first 3 examples: (should be [0 2 1]):  [0 2 1]
```

Problem 4.2: Computing centroid means

```
Computing centroids means.
Centroids computed after initial finding of closest centroids:
[[ 2.42830111  3.15792418]
 [ 5.81350331  2.63365645]
 [ 7.11938687  3.6166844 ]]
(the centroids should be
   [ 2.428301 3.157924 ],  [ 5.813503 2.633656 ], [ 7.119387 3.616684 ]
```

K-means on example dataset

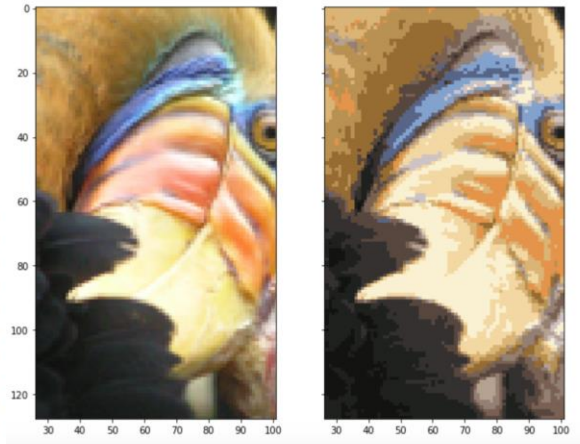
```
Running k-means clustering on example dataset.
K-Means iteration  0 10
K-Means iteration  1 10
K-Means iteration  2 10
K-Means iteration  3 10
K-Means iteration  4 10
K-Means iteration  5 10
K-Means iteration  6 10
K-Means iteration  7 10
K-Means iteration  8 10
K-Means iteration  9 10
k-means Done.
```



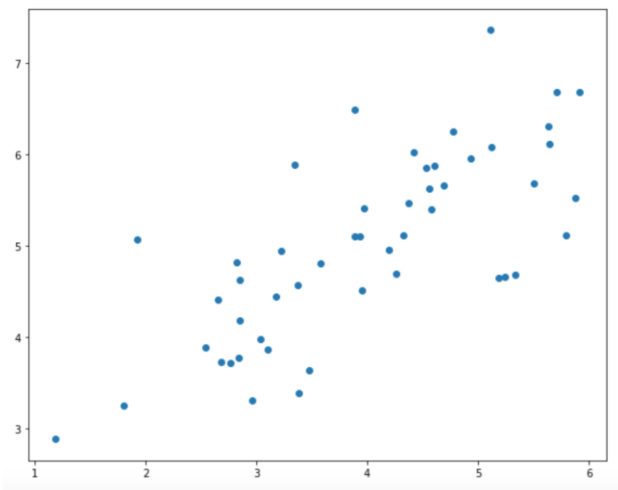
Problem 4.3: Random initialization

Image compression with k-means

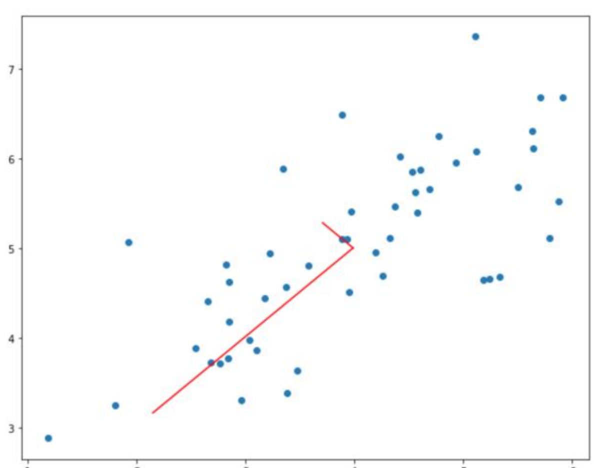
```
K-Means iteration  0 10
K-Means iteration  1 10
K-Means iteration  2 10
K-Means iteration  3 10
K-Means iteration  4 10
K-Means iteration  5 10
K-Means iteration  6 10
K-Means iteration  7 10
K-Means iteration  8 10
K-Means iteration  9 10
```



Problem 5)
PCA on a simple 2D example



Problem 5.1: Implementing PCA



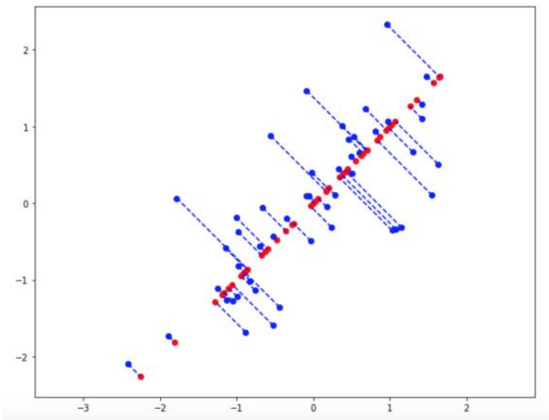
Dimensionality reduction with PCA

Problem 5.2: Projecting the data onto the principal components

Problem: 5.3 Reconstructing an approximation of the data

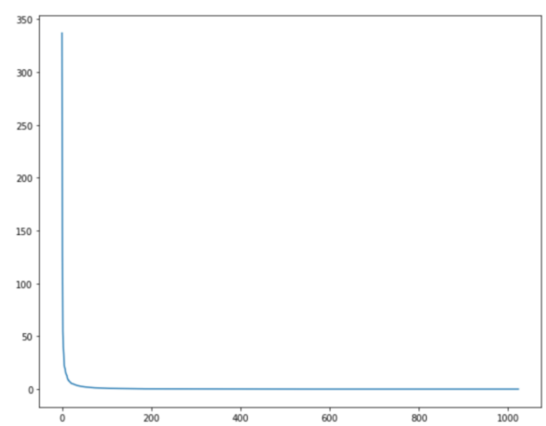
```
The projection of the first example (should be about 1.496)  [ 1.49631261]
Approximation of the first example (should be about [-1.058 -1.058])  [-1.05805279 -1.05805279]
```

Visualizing the projections



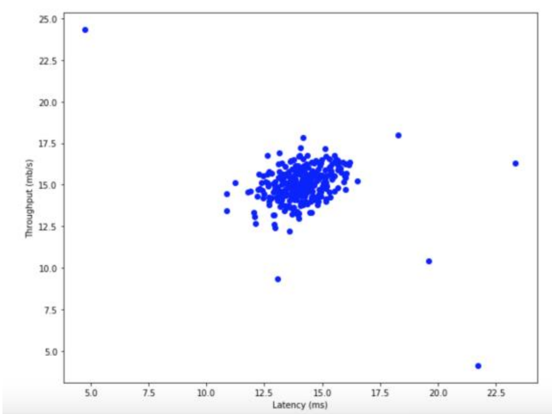
Face image dataset

PCA on faces

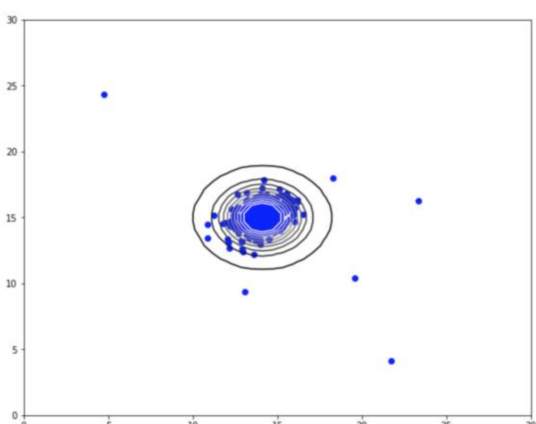


dimensionality reduction

Problem 6)
Anomaly detection on a simple 2D data set

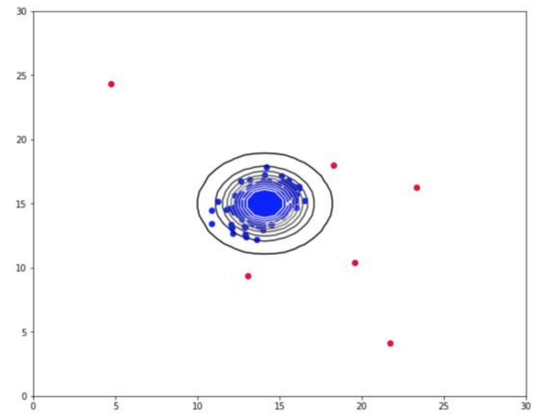


gaussian distributions
Problem 6.1: Estimating parameters of a Gaussian distribution



Problem 6.2: Selecting the threshold ϵ

```
8.99085277927e-05 1.55555555556
```

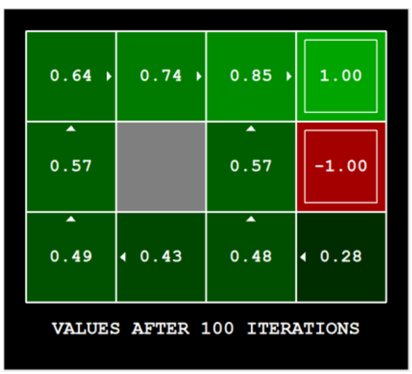


High dimensional dataset

```
1.37722889076e-18 0.615384615385
(117L,)
```

Problem 7)

7.1 Value Iteration




```
EPISODE 10 COMPLETE: RETURN WAS 0.531441

AVERAGE RETURNS FROM START STATE: 0.4847450508
```

```
Question q1
===========

*** PASS: test_cases\q1\1-tinygrid.test
*** PASS: test_cases\q1\2-tinygrid-noisy.test
*** PASS: test_cases\q1\3-bridge.test
*** PASS: test_cases\q1\4-discountgrid.test

### Question q1: 6/6 ###


Finished at 1:35:21

Provisional grades
==================
Question q1: 6/6
------------------
Total: 6/6

Your grades are NOT yet registered.  To register your grades, make sure
to follow your instructor's guidelines to receive credit on your project.
```

7.2 Bridge Crossing Analysis


```python
def question2():
    answerDiscount = 0.9
    #answerNoise = 0.2
    answerNoise = 0
    return answerDiscount, answerNoise
```

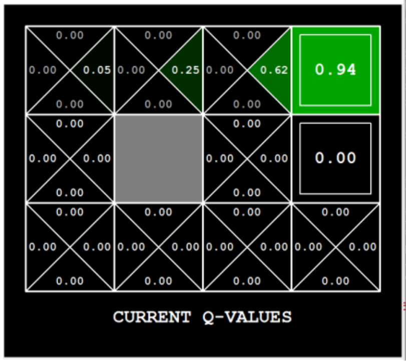
```
Question q2
===========

*** PASS: test_cases\q2\1-bridge-grid.test

### Question q2: 1/1 ###


Finished at 1:34:14

Provisional grades
==================
Question q2: 1/1
------------------
Total: 1/1

Your grades are NOT yet registered.  To register your grades, make sure
to follow your instructor's guidelines to receive credit on your project.
```

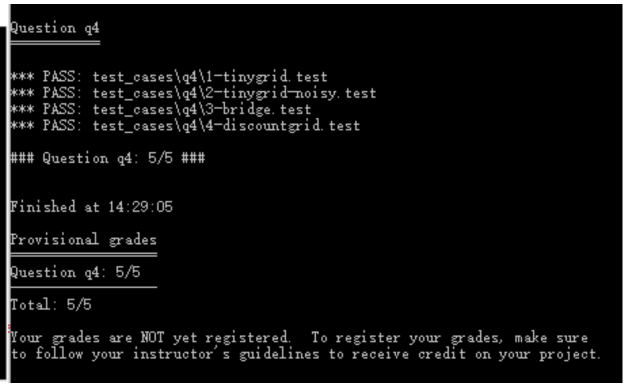
7.3 Policies

```
Question q3
===========

*** PASS: test_cases\q3\1-question-3.1.test
*** PASS: test_cases\q3\2-question-3.2.test
*** PASS: test_cases\q3\3-question-3.3.test
*** PASS: test_cases\q3\4-question-3.4.test
*** PASS: test_cases\q3\5-question-3.5.test

### Question q3: 5/5 ###


Finished at 1:48:59

Provisional grades
==================
Question q3: 5/5
------------------
Total: 5/5

Your grades are NOT yet registered.  To register your grades, make sure
to follow your instructor's guidelines to receive credit on your project.
```

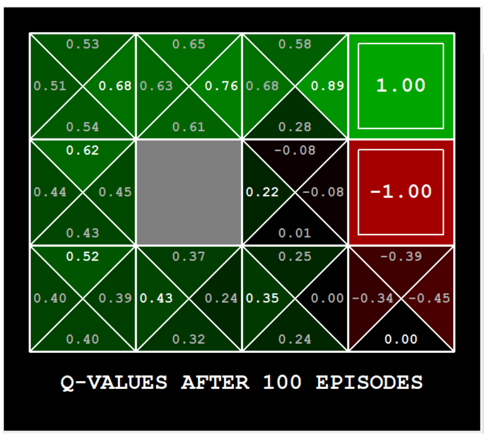
7.4 Q-learning

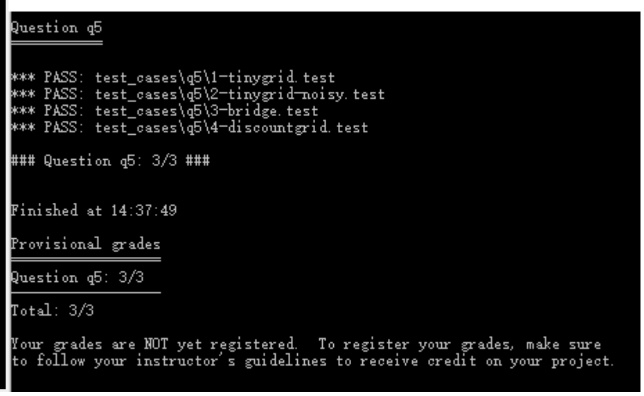
Steer Pacman north and then east along the optimal path for four episodes:





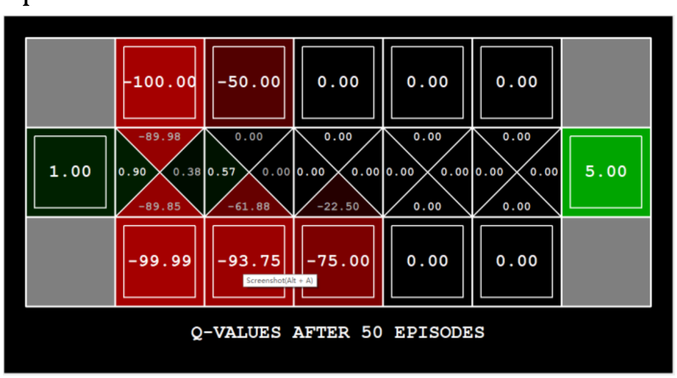
7.5 Epsilon-greedy policies





7.6 Bridge Crossing Revisited

Epsilon = 1:



Epsilon = 0:


```
Question q6
===========
*** PASS: test_cases\q6\grade-agent.test

### Question q6: 1/1 ###

Finished at 14:59:52

Provisional grades
==================
Question q6: 1/1
------------------
Total: 1/1

Your grades are NOT yet registered.  To register your grades, make sure
to follow your instructor's guidelines to receive credit on your project.
```

7.7 Q-learning and Pacman

```
Reinforcement Learning Status:
        Completed 2000 out of 2000 training episodes
        Average Rewards over all training: -55.07
        Average Rewards for last 100 episodes: 156.43
        Episode took 1.76 seconds
Training Done (turning off epsilon and alpha)
---------------------------------------------
Pacman emerges victorious! Score: 502
Pacman emerges victorious! Score: 502
Pacman emerges victorious! Score: 495
Pacman emerges victorious! Score: 495
Pacman emerges victorious! Score: 503
Pacman emerges victorious! Score: 502
Pacman emerges victorious! Score: 499
Pacman emerges victorious! Score: 495
Pacman emerges victorious! Score: 495
Pacman emerges victorious! Score: 503
Average Score: 499.1
Scores:        502.0, 502.0, 495.0, 495.0, 503.0, 502.0, 499.0, 495.0, 495.0, 503.0
Win Rate:      10/10 (1.00)
Record:        Win, Win, Win, Win, Win, Win, Win, Win, Win, Win
```

```
Win Rate:      100/100 (1.00)
Record:        Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Wi
in, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win,
n, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win,
, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win, Win
*** PASS: test_cases\q7\grade-agent.test (1 of 1 points)
***     Grading agent using command:  python pacman.py -p PacmanQAgent -x 2000 -n 2100 -l smallGrid -q -f --fixRandomSeed
***     100 wins (1 of 1 points)
***         Grading scheme:
***          < 70:  0 points
***         >= 70:  1 points

### Question q7: 1/1 ###


Finished at 15:20:11

Provisional grades
==================
Question q7: 1/1
------------------
Total: 1/1

Your grades are NOT yet registered.  To register your grades, make sure
to follow your instructor's guidelines to receive credit on your project.
```

7.8 Approximate Q-learning

smallGrid:

```
Training Done (turning off epsilon and alpha)
---------------------------------------------
Pacman emerges victorious! Score: 495
Pacman emerges victorious! Score: 495
Pacman emerges victorious! Score: 503
Pacman emerges victorious! Score: 499
Pacman emerges victorious! Score: 499
Pacman emerges victorious! Score: 503
Pacman emerges victorious! Score: 495
Pacman emerges victorious! Score: 503
Pacman emerges victorious! Score: 503
Pacman emerges victorious! Score: 495
Average Score: 499.0
Scores:        495.0, 495.0, 503.0, 499.0, 499.0, 503.0, 495.0, 503.0, 503.0, 495.0
Win Rate:      10/10 (1.00)
Record:        Win, Win, Win, Win, Win, Win, Win, Win, Win, Win
```

mediumGrid:

```
Beginning 50 episodes of Training
Training Done (turning off epsilon and alpha)
---------------------------------------------
Pacman emerges victorious! Score: 529
Pacman emerges victorious! Score: 529
Pacman emerges victorious! Score: 527
Pacman emerges victorious! Score: 527
Pacman emerges victorious! Score: 527
Pacman emerges victorious! Score: 525
Pacman emerges victorious! Score: 529
Pacman emerges victorious! Score: 529
Pacman emerges victorious! Score: 525
Pacman emerges victorious! Score: 529
Average Score: 527.6
Scores:        529.0, 529.0, 527.0, 527.0, 527.0, 525.0, 529.0, 529.0, 525.0, 529.0
Win Rate:      10/10 (1.00)
Record:        Win, Win, Win, Win, Win, Win, Win, Win, Win, Win
```

mediumClassic:

```
Beginning 50 episodes of Training
Training Done (turning off epsilon and alpha)
---------------------------------------------
Pacman emerges victorious! Score: 1312
Pacman emerges victorious! Score: 1308
Pacman emerges victorious! Score: 1318
Pacman emerges victorious! Score: 1318
Pacman emerges victorious! Score: 1307
Pacman emerges victorious! Score: 1323
Pacman emerges victorious! Score: 1340
Pacman emerges victorious! Score: 1321
Pacman emerges victorious! Score: 1315
Pacman emerges victorious! Score: 1332
Average Score: 1319.4
Scores:        1312.0, 1308.0, 1318.0, 1318.0, 1307.0, 1323.0, 1340.0, 1321.0, 1315.0, 1332.0
Win Rate:      10/10 (1.00)
Record:        Win, Win, Win, Win, Win, Win, Win, Win, Win, Win
```

```
Question q8
===========

*** PASS: test_cases\q8\1-tinygrid.test
*** PASS: test_cases\q8\2-tinygrid-noisy.test
*** PASS: test_cases\q8\3-bridge.test
*** PASS: test_cases\q8\4-discountgrid.test
*** PASS: test_cases\q8\5-coord-extractor.test

### Question q8: 3/3 ###


Finished at 16:11:16

Provisional grades
==================
Question q8: 3/3
------------------
Total: 3/3

Your grades are NOT yet registered.  To register your grades, make sure
to follow your instructor's guidelines to receive credit on your project.
```

Reference:

http://ai.berkeley.edu/reinforcement.html
https://github.com/shiro873/pacman-projects