Osztott rendszerek szintézise – 2. zárthelyi feladat

Legyen V $= vector([1..n], \mathbb{N}_0)$.
Tekintsük az alábbi, A állapottéren és B paramétertéren definiált S programot:

$$A = \underset{x}{V} \times \underset{a}{V} \times \underset{b}{V}$$
$$B = \underset{a'}{V} \times \underset{b'}{V}$$
$$S = \left(\underset{i=1}{\overset{n}{\|}} x_i := 0, \left\{ \underset{i=1}{\overset{n}{\Box}} x_i, b_i := x_i + a_i, b_i - 1, ha\ b_i > 0 \right\} \right)$$

Igazoljuk, hogy a program megfelel az alábbi specifikációnak.

$$Q = (a = a' \wedge b = b') \in init_h \qquad \text{(-)}$$
$$(\forall_i \in [1..n] : (x_i = a_i \cdot (b'_i - b_i))) \in inv_h \qquad \text{(6 pont)}$$
$$FP_h \Rightarrow (\forall_i \in [1..n] : (x_i = a_i \cdot b'_i)) \qquad \text{(6 pont)}$$
$$Q \hookrightarrow FP_h \qquad \text{(8 pont)}$$

Megoldás ?

1 $(\forall_i \in [1..n] : (x_i = a_i \cdot (b'_i - b_i))) \in inv_h$?

$P :=\ \forall_i \in [1..n] : x_i = a_i \cdot (b'_i - b_i))$
$K := \uparrow$

a) $a = a' \wedge b = b' \Rightarrow lf(s_0, P)$?
$a = a' \wedge b = b' \Rightarrow P^{x_i \leftarrow 0}$
$\forall_i \in [1..n] : x_i = a_i \cdot (b'_i - b_i))^{x_i \leftarrow 0}$
$0 = a'_i \cdot (b'_i - b'_i)$ ✓

b) $P \rhd_s \downarrow$?

$P \wedge b_i > 0 \Rightarrow P^{x_i \leftarrow x_i + a_i, b_i \leftarrow b_i - 1}$

$lf(S, P) \equiv$
$\equiv (b_i > 0 \to x_i + a_i = a_i \cdot (b'_i - b_i + 1)) \wedge (b_i \leq 0 \to (x_i = a_i \cdot (b'_i - b_i))$
$\equiv (b_i > 0 \to x_i + a_i = a_i \cdot b'_i - a_i \cdot b_i + a_i)) \wedge (b_i \leq 0 \to (x_i = a_i \cdot (b'_i - b_i))$
$\equiv (b_i > 0 \to x_i = a_i \cdot (b'_i - b_i)) \wedge (b_i \leq 0 \to (x_i = a_i \cdot (b'_i - b_i))$ ✓

2 $\varphi_S \wedge K \Rightarrow \underbrace{\forall_i \in [1..n] : (x_i = a_i \cdot b'_i)}_{R}$?

$K := \forall_i \in [1..n] : x_i = a_i \cdot (b'_i - b_i))$

$\varphi_S \equiv \forall_i \in [1..n] : b_i \leq 0 \vee (x_i = x_i + a_i \wedge \underbrace{b_i = b_i - 1}_{\downarrow}) \equiv b_i = 0$

$\varphi_S \wedge K \Rightarrow R \equiv b_i = 0 \wedge x_i = a_i \cdot (b'_i - b_i) \Rightarrow x_i = a_i \cdot b'_i$ ✓

3 $Q \hookrightarrow FP_h$?

$Q := (a = a' \wedge b = b')$
$K := \uparrow$
$\neg\varphi_S := b_i > 0$
$t := b_i$

a) $\neg\varphi_S \wedge K \Rightarrow t > 0$

$b_i > 0 \wedge \uparrow \Rightarrow b_i > 0$ ✓

b) $\forall t' \in \mathbb{N} : \underbrace{\neg\varphi \wedge b_i = t' \wedge K}_{P} \hookrightarrow \underbrace{(b_i < t' \wedge K) \vee \varphi_S}_{Q}$

a) $\triangleright_s$ ✓

$P \wedge \neg Q \Rightarrow lf(S, P \vee Q)$

$b_i > 0 \wedge b_i = t' \wedge K \Rightarrow lf(S, (b_i > 0 \wedge b_i = t') \vee (b_i < t' \vee b_i = 0))$

$lf(S, (b_i > 0 \wedge b_i = t') \vee (b_i < t' \vee b_i = 0)) \equiv$
$\equiv (b_i > 0) \rightarrow ((b_i - 1 = t') \wedge (b_i - 1 = t') \vee \underbrace{-1 < t}_{\checkmark} \vee b_i - 1 = 0)$

$\wedge \underbrace{(b_i = 0)}_{m>0\checkmark} \rightarrow ((b_i > 0 \wedge b_i = t') \vee b_i < t \vee b_i = 0)$

b) $\exists s \in S : (\neg\varphi_S \wedge b_i = t' \wedge K) \wedge \neg((b_i < t' \wedge K) \vee \varphi_S) \Rightarrow lf(S, (b_i < t' \wedge K) \vee \varphi_S) \equiv (b_i > 0 \wedge b_i = t' \wedge K) \wedge \neg((b_i < t' \wedge K) \vee b_i = 0) \Rightarrow lf(s, (b_i < t' \wedge K) \vee b_i = 0)$

$lf(s, (b_i < t' \wedge K) \vee b_i = 0) \equiv$
$b_i > 0 \rightarrow b_i - 1 < t' \vee b_i - 1 = 0$ ✓
$\wedge$
$b_i = 0 \rightarrow b_i < t' \vee b_i = 0$ ✓