Raising the abstraction to separate concerns: enabling different physics for geophysical exploration

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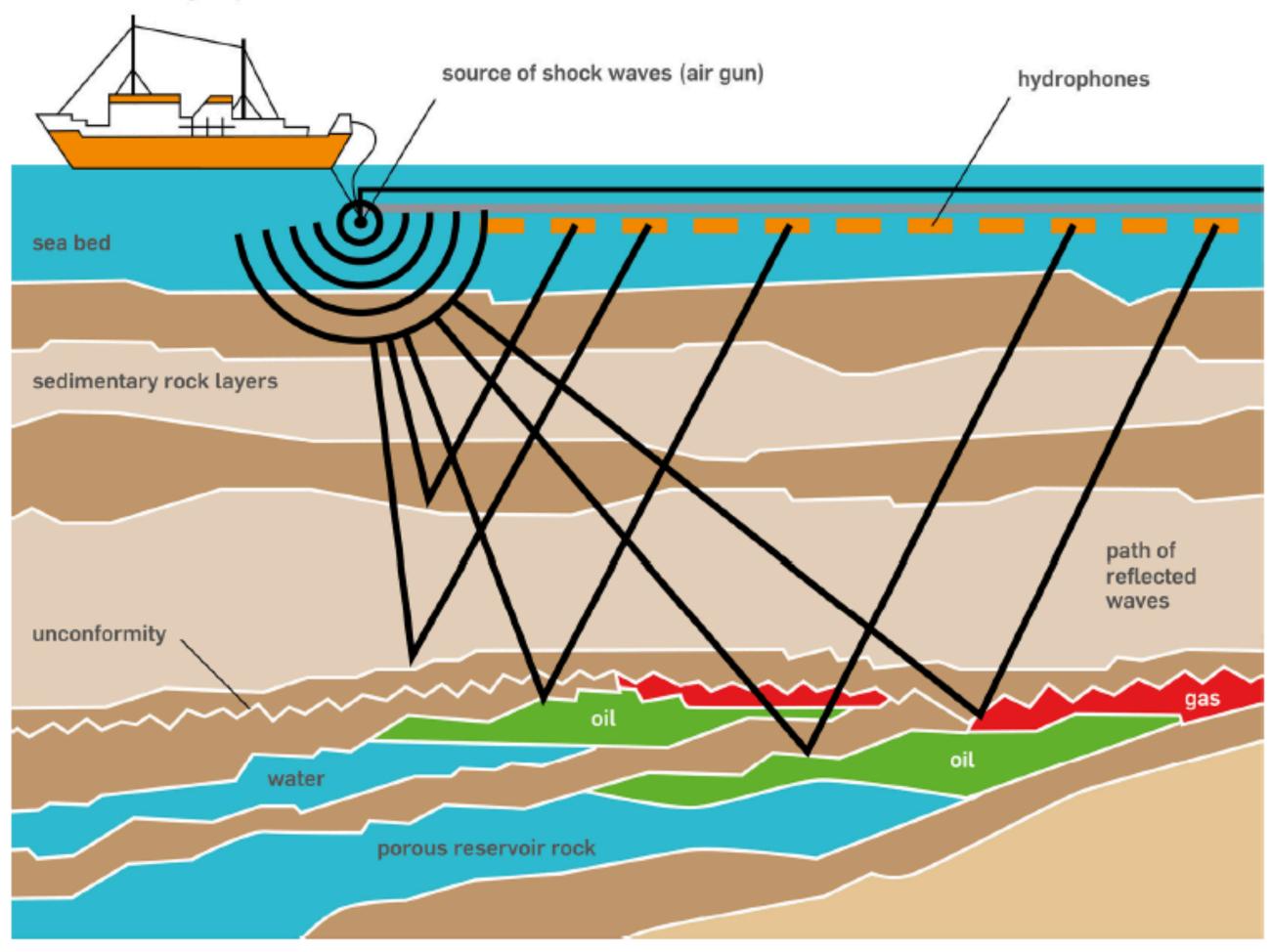




Wave-equation based geophysical exploration introduction



Physical problem survey ship



http://www.open.edu/openlearn/science-maths-technology/science/environmental-science/earths-physical-resources-petroleum/content-section-3.2.1



Mathematical problem

$$\underset{\mathbf{m}}{\text{minimize}} \quad \frac{1}{2}||\mathbf{A}^{-1}(\mathbf{m})\cdot\mathbf{q}-\mathbf{d}||_2^2 \qquad \text{(Virieux and Operto, 2009)}$$

m: squared slowness

d: field recorded data

A(m): discretized wave-equation

q: source term



Challenges

- Multiple representations of the physics
- Problem sizes are huge:
 - seismic surveys consist of tens of thousands of individual experiments
 - model wave propagation over thousands of time steps in large domains
 - typical size of modeling matrix \longrightarrow $\mathbf{A}(\mathbf{m}) \in \mathbb{R}^{n \times n}, n = 1e16$
- Least square problem requires adjoints of the system matrix
- Discrete wave-equation can not be formed as an explicit matrix
 - stencil-based implementation
- Needs scalable, flexible, performant and portable discretization



Scientific motivations

Different physics

- Isotropic acoustic
- Isotropic acoustic with density
- Anisotropic acoustic
- Isotropic elastic
- Anisotropic elastic
- **>**

Simulation for inversion

- Adjoint PDE
- Gradients
- **>**

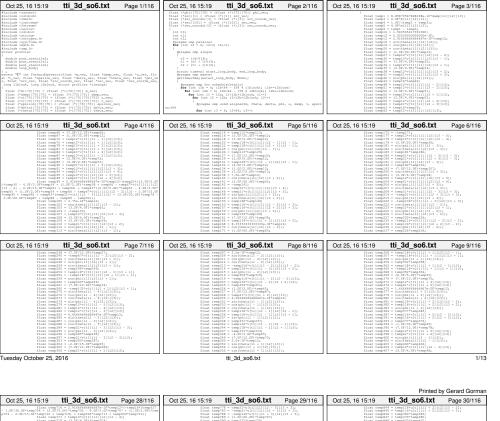


Design motivation

- Writing stencil codes is time consuming and hard for complicated equations. It is even harder because of the need for highly optimized implementations on a range of different computer architectures.
- Separation of Concerns with a finite-difference DSL
- Geophysicists need to be able to focus on the physics
- Computer scientists need to be able to focus on the software
- Mathematicians need to focus on numerical analysis



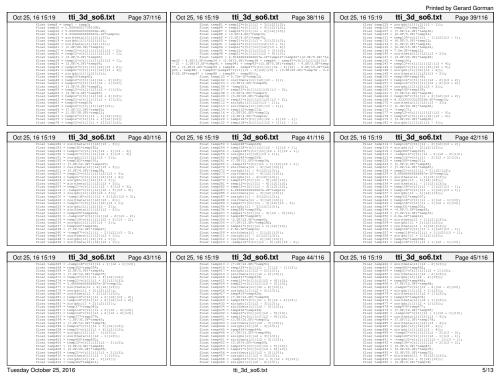
Design motivations



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s 3] - temp67*u[t1][11 - 2] O7*(temp607*u[t1][11 - 2] pp776)) - temp168*temp272*t - temp16*[t1][11 - 4][12] s*u[t1][11 - 4][12][13 + 1] - 4][12][13 + 3]) + temp27* s*temp274*(-temp272*temp604 s*temp274*(-temp272*temp604 334*u[t1][11 - 3][12][13 + 1] ill - 3][12][13 + 1] + temp27* ill - 3][12][13 + 1] ill - 3][12][13 + 1] ill - 3][12][13 + 1]		mp710 + temp788] - temp **temp738 + temp775 - te 1][i1 - 4][i2][i3 - 2] 4][i2][i3 - 1] - temp11 - 3] - temp22**[i1][i1 mp274*temp569] - temp18 temp28**temp288*(-temp2 - 3][i2][i3 + 1] + temp [i3 + 3] + temp22**[i1] [i3 - 2]) + temp22**tem	305*temp510*temp668 - temp50 [i1] [i2 - 4] [i3 + 1] + temp31 - 4] [i3 + 3] + temp21*u[t1] - 4] [i3 - 2]) + temp508*temp mp508*temp668) - temp182*tem 2.0F)*temp705 + (2.0F)*5.0F)*	3 - temp003*cmp039 - temp106 Transp128*u1111; 13 - 4 1 111[12 - 4] 13 - 4 temp003*u111; 13 - 4 1 111[12 - 4] 13 - 4 temp003*u11; 13 - 4 1 111[12 - 4] 13 - 4 temp003*u11; 13 - 4 1 111[12 - 4] 13 - 4 temp003*u11; 13 - 4 1 111[12 - 1] temp103*u11; 13 - 4 1 111[13 - 1] temp103*u11; 13 - 4 1 111[13 - 1] temp103*u11; 13 - 4 1 113[13 - 1] temp103*u11; 1	3 - 1] - temp132*u[t1] temp136*u[t1][i1][i2 - temp67*u[t1][i1][i2 *(temp505*temp666 - te /3.0F*temp704 + (1.0F/ temp769 + temp769) +	temp31:"(ctemp310") -23[12][13] + 1] + temp22*u 3 + 3] + temp632 + temp633) 13 (ctemp31:*temp635 + temp6110 15 (ctemp51:*temp635 + temp610 1[11][12] - 2][13 - 2][13 - 1] 1[11][12] - 2][13 + 2] - tem -2][13] + 4] + temp646 + to -21[13] + 4] + temp646 +		i] - tempili - 2[i2][sepi35] - tempi07*tempi sepi35] - tempi07*tempi tempi65*(-tempi62*tempi [i5 + 1] + tempi34*u[+ temp22*u[t1][i1][i pp677*temp554] + tempi sepi37*temp334*(-tempi 1][i2][i3 + 2] + tempi s + 3] + tempi36 - tempi
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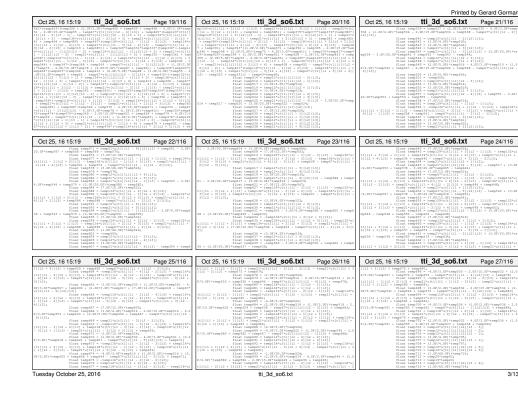
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emp21*v[t][i1 + 3]['(-2.0f'.50f'temp14 v[t][i1][i2][i3 - 1 [i3 + 1] + temp13*v + temp21*v[t][i1][2][i3 + 1] + temp] [i2][i3 + 1] + temp] [i2][i3 + 1] + temp] [i3 - 1] - temp13*v + temp29 + temp29 [i2 + 2][i3] - 2] + t [i3 - 1] - temp13*v + temp289 + temp29 [i2 + 2][i3] - temp1 [i3] - t temp21*v[t][i3]	12 - 11 - 12 - 12 - 12 - 12 - 12 - 12 -	11 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	**Composition** **Comp	16	1.00 1.00	a votal [13] - 40[22] - 10[23]	1,000 1,00	12
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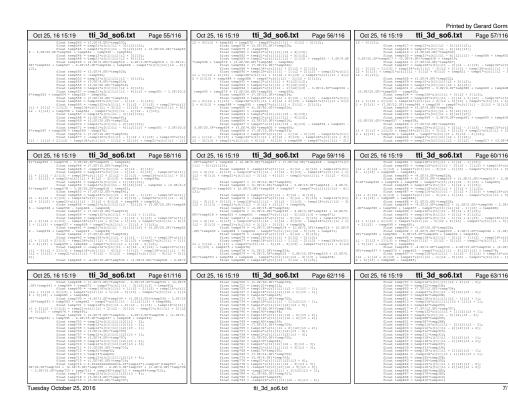


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raye /3/11	tti 3d so6.txt sanithesia (1911) sanithesia (1911	float temp270	rage /4/110	1 = cos(theta[i1][i2][i3 + 3]); i = (3.0F/4.0F)*temp38;		Fage 73/110	= (2.0F/15.0F)*temp12; = 7.5e-2F*temp12;	
	= cos(phi[i1 - 4][i2][i3]); = cos(theta[i1 - 4][i2][i3]);	float temp272 float temp273		1 = (3.0F/4.0F)*temp38; 2 = temp12*v[t1][i1][i2][i3 + 5].	float temp20 float temp20		= 7.5e-2F*temp12; = cos(theta[i1][i2][i3 + 2]);	float temp138 float temp140
,	= sin(phi[i1 - 4][i2][i3]); = temp12*v[t1][i1 - 5][i2][i3];	float temp274 float temp279		= temp12*v[t1][i1][i2][i3 + 5]; = temp21*v[t1][i1][i2][i3 + 6]; = sin(phi[i1][i2][i3 + 3]);	float temp20 float temp20		= (3.0F/20.0F)*temp65; = -temp141;	float temp141 float temp142
-	- temp12*v[t1][i1 - 4][i2][i3]; - temp12*v[t1][i1 - 3][i2][i3];	float temp281 float temp282	- 3];	7 = sin(theta[i1][i2][i3 + 3]); 8 = temp21*v[t1][i1][i2 - 3][i3 -	float temp20 float temp20	1;	= cos(theta[i1][i2][i3 + 2]); = (3.0F/20.0F)*temp65; = -temp141; = temp12*v[t1][i1][i2][i3 + 4] = temp21*v[t1][i1][i2][i3 + 5]	float temp143 float temp146
	= 6.6666666666667e-2F*temp12; = sin(theta[i1 - 3][i2][i3]);	float temp283 float temp285	+ 3];	<pre>i = -temp21*v[t1][i1][i2 + 3][i3 i = cos(phi[i1][i2][i3 + 3]);</pre>	float temp21 float temp21		= sin(phi[i1][i2][i3 + 2]); = sin(theta[i1][i2][i3 + 2]);	float temp145 float temp145
	= cos(phi[i1 - 3][i2][i3]); = cos(theta[i1 - 3][i2][i3]);	float temp287 float temp288	1 + 3];] = -temp136*v[t1][i1 + 3][i2][i: 2 = cos(theta[i1][i2][i3 - 2]);	float temp22 float temp22	3 + 2];	= tempis*(ri] 11 13 +3 = sin(phi[i] [2] [3+2] ; = sin(theta[i] [12] [3+2]); = tempis*(rempis*(ri] [12] [2-2] [12] = tempis*(ri] [13] [2-2] [13] [12-2] [13] = cos[phi[i] [12] [13+2]];	float temp150 float temp151
- 313	= sin(phi[i1 - 3][i2][i3]);	float temp295		= sin(phi[i1][i2][i3 - 2]);	float temp22	3 + 2];	= cos(phi[i1][i2][i3 + 2]);	float temp155
[13];	= temp203*temp295; = temp21*v[t1][i1 - 3][i2 - 3]	float temp290		1 = sin(theta[ii][i2][i3 - 2]); 7 = temp225*temp226;	float temp22	2 . 21.	= temp149*temp159; = (7.0F/12.0F)*temp38;	float tempidi
	2.0F/15.0F*temp65;	float temp304	- 212	= templ16*v[t1][11][12 + 2][13	float temp23	111	= 8.3333333333333=-3F*temp12;	float temp160
;	= temp21*v[t1][i1 + 1][i2][i3];	float temp303		= min [pai [11] [22] [13 - 3]); = min [base [11] [22] [14 - 3]); = min [base [11] [22] [14 - 3]); = min [base [11] [22] [14 - 3]); = min [base [11] [22] [15 - 3]); = min [base [11] [22] [15 - 3]); = min [base [11] [22] [15 - 2]);	float temp23		cos/phi[i][i2][i3 + 2]); temp149*cemp159; temp149*cemp159; 1.09/12.09 temp18; 1.09/12.09 temp18; 1.09/12.09 temp18; cos(theta[i1][i2][i3 - 3]); cos(theta[i1][i2][i3 - 3]); temp27*y[t1][i1][i2][i3 - 4] temp27*y[t1][i1][i2][i3 - 4] cos(d) temp18; co	float temp171
	= sin(theta[i1 - 2][i2][i3]); = cos(phi/i1 - 2)[i2][i3]);	float temp309 float temp311	- 2];	<pre>temp23*:temp236; 1</pre>	float temp24 float temp24	12	= temp12*v[t1][i1][i2][i3 - 5] = temp12*v[t1][i1][i2][i3 - 4]	float temp177 float temp178
3 - 21:	= cos(thets[i1 - 2][i2][i3]); = -templ16*v[t1][i1 - 2][i2][i]	float temp312 float temp313		= sin(phi[i1][i2][i3 + 1]); = sin(theta[i1][i2][i3 + 1]);	float temp24 float temp24	.,	= (3.0F/20.0F)*temp30; = min(phi[i][[i2][i3 - 3]);	float temp175 float temp181
+ 2];	= templ16*v[t1][i1 - 2][i2][i3 = sin(phi[i1 - 2][i2][i3]);	float temp314 float temp315	+ 11:	= temp240*temp249; = temp110*v[t]1[i]1[i] - 1[[i]	float temp25 float temp25		= sin(theta[i1][i2][i3 - 3]); = temp181*temp182;	float temp182 float temp182
2][13];	= temp309*temp315; = -temp116*v[t1][i1 - 2][i2 - :	float temp316 float temp319	1 + 1];	= -temp118*v[t1][i1][i2 + 1][i = cos(phi[i1][i2][i3 + 1]);	float temp25 float temp25	- 31; - 31;	= temp21*v[t1][i1][i2 - 3][i3 = temp21*v[t1][i1][i2 + 3][i3	float temp184 float temp190
([13])	= temp116*v[t1][i1 - 2][i2 + 2] = temp309*temp311;	float temp323 float temp325					= sin(phi[i1][12][13 - 3]); = sin(theta[i1][13][13 - 3]); = temp182*temp182; = temp21*v[t1][11][12 - 3][13 = temp21*v[t1][11][12 + 3][13 = temp182*temp191; = temp182*temp191;	float temp191 float temp192
	= (1.0F/2.0F)*temp65;	float temp326	[+ 1];	1 = -temp132*v[t1][i1 + 1][i2][i	float temp26		= (7.0F/12.0F)*temp107;	float temp191
Page 78/11	tti 3d so6.txt	Oct 25, 16 15:19	Page 77/116	tti_3d_so6.txt	Oct 25, 16 15:19	Page 76/116	tti 3d so6.txt	Oct 25, 16 15:19
	tti 3d so6.txt mainteinii 2, 71123/1,	float temp425		7 = sin(theta[i1 + 2][i2][i3]); 8 = cos(phi[i1 + 2][i2][i3]); 1 = cos(theta[i1 + 2][i2][i3]); 1 = cos(theta[i1 + 2][i2][i3]); 1 = templi6*v[t1][i1 + 2][i2][i3]; 2 = templi6*v[t1][i1 + 2][i2][i3]; 3 = sin(phi[i1 + 2][i2][i3]);	float temp37	1-	= (7.05/12.07) temp8; = temp21*v[t1][11 + 2][12][13] = 6.65656666666667-1F*temp12; = sin(theta[i1 - 1][12][13]); = cos(phi[i1 - 1][12][13]); = cos(theta[i1 - 1][12][13]);	float temp327
	= temp419*temp426; = (7.0F/12.0F)*temp59;	float temp427 float temp428	- 21:	= cos(theta[i1 + 2][i2][i3]); = temp116*v[t1][i1 + 2][i2][i3]	float temp38 float temp38	;	= 6.65656666666667e-1F*temp12; = min(theta[i1 - 1][i2][i3]);	float temp325 float temp331
1(43);	= temp134*v[t1][i1 + 2][i2 + 2] = sin(phi[i1][i2 - 3][i3]);	float temp429 float temp432	+ 213	= templ16*v[t1][i1 + 2][i2][i3 = sin(phi/i1 + 2)[i2][i3]);	float temp38 float temp38		= cos(phi[i1 - 1][i2][i3]); = cos(theta[i1 - 1][i2][i3]);	float temp333 float temp334
	= sin(theta[i1][i2 - 3][i3]); = temp432*temp434;	float temp434 float temp435	[13];	= temp377*temp383; 7 = temp116*v[t1][i1 + 2][i2 - 2	float templE float templE	3 - 11; 3 + 11;	= comptneta[1 - 1][12][13][1 = templ18*v[t][11 - 1][12][13 = templ18*v[t][11 - 1][12][13 = sin(phi[i1 - 1][12][13]); = templ18*v[t][11 - 1][12 - 1 = templ18*v[t][11 - 1][12 - 1	float temp335 float temp336
;	= temp12*v[t1][i1][i2 - 5][i3]; = temp12*v[t1][i1][i2 - 4][i3];	float temp440 float temp441	[13];	1 = templ16*v[t1][i1 + 2][i2 + 2 1 = temp377*temp379;	float temp39 float temp39		= sin(phi[i1 - 1][i2][i3]); = temp331*temp337;	float temp337 float temp338
	= (3.0F/20.0F)*temp51; = cos(theta[i1][i2 - 3][i3]);	float temp442 float temp443		<pre>a min*[phi[i1 + 2][i2][i3]); temp371*cmp383; temp314*v[i3][i1 + 2][i2 - 2] temp314*v[i3][i1 + 2][i2 + 2] temp314*v[i3][i1] + 2][i2 + 2] temp314*v[i3][i1] + 2][i2 + 2] temp314*v[i3][i1] + 2][i2] + 2] temp314*v[i3][i1] + 3[[i3][i3]] temp314*v[i3][i1] + 3[[i3][i3]] temp314*v[i3][i1][i2 - 1][i3][i]; min*[phi[i1][i2 - 1][i3]]; smitheta([i3][i2 - 1][i3]); temp404*vemp402;</pre>	float temp39 float temp39	1][i3]; 1][i3];	= temp118*v[t1][i1 - 1][i2 - 1 = temp118*v[t1][i1 - 1][i2 + 1	float temp342 float temp344
	= cos(phi[i1][i2 - 3][i3]); = temp434*temp444;	float temp444 float temp445		<pre>i = temp12*v[t1][i1 + 5][i2][i3], i = temp21*v[t1][i1 + 6][i2][i3],</pre>	float temp39 float temp39		= temp331*temp333; = -4.0F/3.0F*temp65;	float temp347 float temp348
	= (7.0F/12.0F)*temp406; = sim(phi[i1][i2 + 3][i3]);	float temp440 float temp442		= sin(phi[i1][i2 - 1][i3]); = sin(theta[i1][i2 - 1][i3]);	float temp40 float temp40		= (7.0F/12.0F)*temp72; = 2.0e-1F*temp12;	float temp350 float temp350
	= (3.0F/4.0F) *temp59;	float temp452		= temp400*temp402; i = temp12*v[t1][i1][i2 - 3][i3].	float temp40		= cos(phi[i1 + 1][i2][i3]);	float temp354
;	= temp21*v[t1][i1][i2 + 6][i3];	float temp455		1 = temp400*temp402; = (3.59/4.07)*temp403; = (3.59/4.07)*temp403; = (3.59/4.07)*temp403; = temp21*y(13[14][12]-21[13]); = cons(theta[13][12]-21[13]); = cons(theta[13][12]-21[13]); = (7.59/42.07)*temp41; = temp21*y(13[11]-11[12]-1 = sin(theta[13][12]-21[13]); = sin(theta[13][12]-21[13]); = sin(theta[13][12]-21[13]);	float temp40	3 - 11;	= temp118*v[t1][i1 + 1][i2][i1	float temp350
3111311	= cos(phi[i1][i2 + 3][i3]); = cos(phi[i1][i2 + 3][i3]);	float temp457		= cos(theta[i1][i2 - 1][i3]);	float temp41	13 + 11,	= sin(phi[i1 + 1][i2][i3]);	float temp350
	= sin(phi[i1][i2 - 2][i3]); = sin(theta[i1][i2 - 2][i3]);	float temp460 float temp462		= temp402*temp411; 1 = (7.0F/12.0F)*temp51;	float temp41 float temp41	1][13];	= temp118*v[t1][i1 + 1][i2 - 1 = -temp118*v[t1][i1 + 1][i2 +	float temp361 float temp361
	= temp460*temp462; = temp21*v(r11()11()2 + 11()31	float temp463	[13];	= temp129*v[t1][i1 - 1][i2 - 1	float temp41		= temp352*temp354; = (2.0E/5.0E)*temp65*	float temp368
	= cos(theta[i1][i2 - 2][i3]); = cos(phi[i1][i2 - 2][i3]);	float temp465 float temp465		= sin(theta[i1][i2 + 2][i3]); = temp417*temp419; = temp12*v[t1][i1][i2 + 4][i3].	float temp41 float temp42	1:	= (7.0F/12.0F)*temp75; = temp12*v[t1][i1 + 4][i2][i3]	float temp370 float temp371
	= temp462*temp466; = (7.0F/12.0F)*temp48;	float temp467 float temp468		l = temp12*v[t1][i1][i2 + 4][i3] l = temp21*v[t1][i1][i2 + 5][i3]	float temp42 float temp42	1)	Lemp138**(1) 1 1 1 2 1 1 1 1	float temp374 float temp375
Page 81/11	tti_3d_so6.txt	Oct 25, 16 15:19	Page 80/116	tti_3d_so6.txt	Oct 25, 16 15:19	Page 79/116	tti_3d_so6.txt	Oct 25, 16 15:19
- 1][i3])) - temp1	7*(-temp332*v[t]][s1 + 1][s2 - 1] 36*v[t]][s1 + 3][s2 - 1][s3] 9644 - temp67*v[t][s1 - 2][s2 - 1] 9644 - temp67*v[t][s1 - 2][s2 - 1] 1][s2 - temp18*v[t][s2 - temp12*v[t] 1][s3 - 2] + temp12*v[temp21*v[t] 1][s3 - 2] + temp12*v[temp21*v[t] 1][s3 - 2] + temp12*v[temp18*v[t]] 1][s2 - 1][s2 - temp18*v[t] 1][s3 - 2] + temp18*v[t] 1][s2 - 1][s3 - temp18*v[t] 1][s2 - 1][s3 - temp18*v[t] 1][s2 - 1][s3 - temp18*v[t] 1][s2 - 1][s3 - temp18*v[t] 1][s3 - temp18*v[t][s4 - temp18*v[t]] 1][s4 - temp18*v[t][s4 - temp18*v[t]] 1][s5 - temp18*v[t][s1 - temp18*v[t]] 1][s6 - temp18*v[t][s1 - temp18*v[t]] 1][s6 - temp18*v[t][s1 - temp18*v[t]] 1][s6 - temp18*v[t][s1 - temp18*v[t]] 1][s6 - temp18*v[t][s1 - temp18*v[t]] 1][s7 - temp18*v[t][s1 - temp18*v[t]] 1][s8 - temp18*v[t][s1 - temp18*v[t]] 2][s8 - temp18*v[t][s1 - temp18*v[12 - 1][13] + temp413 + tem amp477***********************************	(3 - 4]) + temp114*(-t	<pre>j = comptostamplo2*(temploa*(templ</pre>	p107 + temp108 + temp109 -		= sin(theta[i1][i2 + 1][i3]); = temp472*temp474;	float temp474
t1][11][12 + 1][13 [t1][11][12 + 1][13	temp98) + temp476*(-temp116*v[t	452 + (3.0F/20.0F)*temp95 - 1 + temp116*v[t]1[1]1777 +	[11][12 + 3][13 - 1]) 5 - 1] + temp134*v[+1]	[[12 - 3][13 - 1] - temp21*v[t1] 11 - temp132*v[t1][11 +][147][4	temp122 + temp21*v[t1][i1 + temp126*(temp127 + ******)		= cos(theta[i1][i2 + 1][i3]); = cos(phi[i1][i2 + 1][i3]);	float temp476
'(temp129'v[t1][i1 6'v[t1][i1 + 31/i2	i][i3 + 3] + temp256) + temp478* i][i1 + 2][i2 + 1][i3] - temp13	3] - temp21*v[t1][i1][i2 + [[i2 + 1][i3] + temp134*v[t	temp21*v[t1][i1 + 4][temp246*(temp246*(tem	0136*v[t1][i1 + 3][i2][i3 - 1] + [i1 - 2][i2][i3 - 1])) - temp100	[11 + 2][12][13 - 1] - tem 12][13 - 1] - temp67*v[t1]		= temp474*temp477; = (7.0F/12.0F)*temp55;	float temp478 float temp479
60 - temp67*v[t1][: .0F/5.0F)*temp27 +	[i2 + 1][i3] + temp479 + temp48 *temp329*(temp102*(temp127 + (2	[[i3] + temp21*v[t1][i1 + 4 2][i2 + 1][i3])) + temp102	(3.0F/20.0F)*temp91 - sp116*v[t1][i1][i2 + 2	temp21*v[t1][i1][i2][i3 - 2] + I*v[t1][i1][i2 - 2][i3 + 1] + ter	p103 - temp179 - temp201 + temp94) + temp250*(-temp11	1[[13]]	= -temp132*v[t1][i1 + 1][i2 + = cos(theta[i1][i2][i3 - 4]);	float temp480 float temp480
emp67*v[t1][i1][i2 1] + temp134*v[t1	2.0F/15.0F*temp38 + temp41 - te temp132*v[t1][i1][i2 + 1][i3 -	OF/2.OF)*temp34 + temp348 - - 3]) + temp114*(temp127 -	[i1][i2 + 3][i3 + 1] i3 + 1] + temp134*v[t1	l][i2 - 3][i3 + 1] - temp21*v[t1 :260*(temp129*v[t1][i1 - 1][i2][][i3 + 1] + temp21*v[t1][i + temp254 + temp256) + tem		= (7.0F/12.0F)*temp178; = sin(theta[i1][i2][i3 - 4]);	float temp484 float temp485
ap21*v[t1][i1][i2 - amp126*(-temp116*v	Twiti[[1][[2] 3][[3 - 1] + ter "v[t1][[1][[2 - 2][[3 - 1]) + ter "v[t1][[1][[1 - 2][[13 - 1]) + ter 116"v[t1][[1] + 2][[13 - 1] + terp335 - "v[t1][[1][[1][[13][[13 - 3] - 3.0F/20][i2 + 2][i3 - 1] - temp136 [i3 - 1] + temp490 - temp67	temp21*v[t1][i1 + 4] [i3 + 1])) + temp100*	11 - tempi32*v[t] [11 *] [3] [1] 13[-v[t] [1] *] [3] [3] *] 13[-v[t] [1] *] [3] [3] *] 13[-v[t] [1] *] [3] [3] *] 13[-v[t] [1] *] [3] [3] 13[-v[t] [1] *] [4] 13[-v[t] [1] *] [4] 13[-v[t] [1] *] [4] 13[-v[t] [1] *] [4] 13[-v[t] [1] [1] 13[-v[t] [1] [1] [1] 13[-v[t] [1] [1] [1] [1] 13[-v[t] [1] [1] [1] [1] 13[-v[t] [1] [1] [1] [1] [1] 13[-v[t] [1] [1] [1] [1] [1] [1] 13[-v[t] [1] [1] [1] [1] [1] [1] [1] [1] [1] [1	[12][13 + 1] + temp261 + t		-maniphese(13)12 - 51(23)2 -maniphese(13)12 - 51(23)2 -maniphese(13)2 - 5	float temp487 float temp488
temp316)) - temp1:	.16*v[t1][i1 + 2][i2][i3 - 1] + i1 + 3][i2][i3 - 1] + temp335 -	[11 - 2][12][13 - 1] + temp 12][13 - 1] - temp21*v[t1][2][i3 + 1] + temp134* 3] + temp21*v[t1][i1	temp131 - temp132*v[t1][i1 - 1][- temp136*v[t1][i1 - 1][i2][i3 +	temp331*temp333*(temp334*(v[t1][i1 - 1][i2][i3 + 2]	3 - 2]; 3 - 1];	= -temp67*v[t1][i1][i2 - 2][i3 = temp129*v[t1][i1][i2 - 1][i3	float temp480 float temp490
7.uv*temp27 + temp [t1][i1][i2 - 3][i	V[t1][11][12][13 - 3] - 3.0F/20 58 - temp41) + temp43*(temp21*v)	mpio*temp19*(temp20*(temp21 temp35 + (3.0F/20.0F)*temp	2) + temp338*(-temp1 [i3] - temp136*v[t1]	- temps/*v[t1][i1 - 1][i2][i3 - i] + temp134*v[t1][i1 - 1][i2 + :	- 1][12][13 + 4] + temp349 32*v[t1][11 - 1][12 + 1][1	13 + 1]; 3 + 2];	= -temp132*v[t1][i1][i2 + 1][i = temp134*v[t1][i1][i2 + 2][i3	float temp491 float temp491
- 4.0F/3.0F*temp75	"V[1][11][13] 3 - 3 - 3.07/20 36 - temp51) + temp43*(temp52**v] - temp56 + (3.0F/20.0F)*temp59 [12][13] (13) + (2.0F/5.0F)*temp59 [12][13] + (2.0F/5.0F)*temp53* - 0F*temp61 + temp64) + temp138* - temp146 + temp21**v[t1][11][12] (temp118*v[t1][11][12 - 1][13]	emp66 - temp67*v[t1][i1 - 2	*v[t1][i1 - 4][i2][i3	- temps/*v[t][i] - i][2/[i]] temp]34*v[t][i] - 1][2/ + 221*v[t][i] - 1][12 + 4][i]] + (i]) + temp[47*(temp]04 + temp2 smp328 + temp493 + temp494)) - t [+ 1][i2][i] - 1] + temp134*v[t] [i] + 1] + temp22*v[t][i] + 1	mp47*v[t1][i1 - 1][i2 - 2]		= (3.0F/20.0F)*temp75;	float temp491
2][i3 - 1] + temp3	- temp146 + temp21*v[t1][i1][i2	p142 + (3.0F/20.0F)*temp143][i1 + 1][i2][i3 + 2]	1 + 1][i2][i3 - 1] + temp134*v[t	*(temp355*(temp129*v[t1][i		= sin(theta[i1 + 3][i2][i3]);	float temp491
12 - 3[[13 + 2] -	3 + rembis; + rembis, A[ri][ii][i	11[15 + 1][13 + 5] - rembig	[temp129*v[t1][i1 + 1]	[11 + 1][12][13 - 2]] + temp359*	5 + temp370 - temp67*v[t1]		= (3.0F/4.0F)*temp78; = cos(theta[i1 + 3][i2][+3]].	float temp500
[13 + 2] + temp161 1 - 21[12][13 + 2]	2] - temp136*v[t1][i1 + 3][i2] 4][i2][i3 + 2] - temp67*v[t1][i	p132*v[t1][i1 + 1][i2][i3 + emp166 + temp21*v[t1][i1 +	- temp67*v[t1][i1 + 21[i3] - temp495 - te	[[12 + 4][13] + temp370 + temp48 (temp103 + temp21*v[t1][11 - 2][[i3] + temp21*v[t1][i1 + 1 11[i2 - 21[i3]] + temp368*		= (7.0F/12.0F)*temp81; = min(phi/i1 + 31/121/i31);	float temp502 float temp502
3.0F/20.0F*temp17 *(temp118*v[t11711	+ templ36*v[t] [11 + 3][12] 2] - templ36*v[t] [11 + 3][12] 4][12][13 + 2] - temp67*v[t][1] 3.007/4.00] *temp107 + temp11] - temp233 - temp31) + temp227* 1][11][12 + 1][13 - 2] + temp230 + 3][13 - 2] + temp230 + temp234	temp138*temp222*(temp222*(temp21*v[t1][i1][i2][i3 - 5	mp402*(temp403*(temp1 mp407 + temp408 - tem	- temp84)) + temp100*temp400*to 4][i3] - 3.0F/20.0F*temp406 + to	mp500 + (3.0F/20.0F)*temp8 04 + temp21*v[t1][i11fi2 -		= sin(phi[i1][i2 - 4][i3]); = sin(theta[i1][i2 - 4][i3]);	float temp505 float temp507
	[][11][12 + 1][13 - 2] + temp21	- 1][i3 - 2] - temp118*v[t - 2] - temp21*v[t1][i1][i2	16*v[t1][11][12 - 1][121*v[t1][11][12 - 1][\[13 + 3\] + temp22*v[t][11 + 1 [11 + 1][42][15 - 2\]) + temp35* \[111 + 1][42 + 2][43\] + temp35* \[121 + 4][43\] + temp375* \[124 + 4][43\] + temp375 + temp46* \[(temp103 + temp21*v[t1][41 - 2][4 - temp40] + temp20*v[t1][41 - 2][4 - 1][43\] - 3.0F/20.0F*temp406 + t \[41[43] - 3.0F/20.0F*temp406 + t \[41[43] - 3.0F/20.1F*temp406 + t \[41[43] - 3.0F/20.2F*temp406 + t \[41[43] - 3.0F*temp406 + t \[41[43] - 3.0F*te	p409) + temp410*(-temp116* 13 + 2] + temp120 + temp21		= cos(phi[i1][i2 - 4][i3]); = (7.0F/12.0F)*temp441;	float temp508 float temp508
V[t1][11][12 - 3] 4) + temp237(temp								

Oct 25, 16 15:19	tti_3d_so6.txt	Page 10/116	Oct 25, 16 15:19	tti_3d_so6.txt	Page 11/116	Oct 25, 16 15:19	tti_3d_so6.txt	Page 12/116
float too float cut float	Control Cont	12	Close Clos	March	13); 2); 2); 2); 2); 2); 3); 4); 4); 4); 4); 4); 4); 4	Finest tend	SAGE = 11.0772.07 tempty: 10.0712.07 tempty:	13 31); 12); 13); 14); 15); 16); 17); 18); 18); 18); 18); 18); 18); 18); 18
Oct 25, 16 15:19	tti 3d so6.txt	Page 13/116	Oct 25, 16 15:19	tti 3d so6.txt	Page 14/116	Oct 25, 16 15:19	tti 3d so6.txt	Page 15/116
3-6 (Long)230 (L	1,000 1,00	4-0-111 1122 123 1	Compile C - C0772-0079 - compile C - C0772-0		12 21 23 23 23 23 23 23 23 23	12 23 - 3 23 - comp3245	A A A A A A A A A A	22247611[11 + 4][22 - 4] 22247611[11 + 4][22 - 4] 2347611[12][22 - 2] 2347611[12][22 - 2][23 - 4] 2347611[12][22 - 2][23 - 4] 2347611[12][23 - 2][23 - 4] 2347611[12][23 - 2][23 - 4] 2347611[12][23 - 2][23 - 4] 2347611[23][23 - 2][23 - 4] 2347611[23][23 - 2][23 - 4] 2347611[23][23 - 2][23 - 4] 2347611[23][23 - 2][23 - 4] 2347611[23][23 - 2][23 - 4] 2347611[23][23 - 2][23 - 4] 2347611[23][23 - 2][23 - 4] 2347611[23][23 - 2][23 - 4] 2347611[23][23 - 2][23 - 4] 2347611[23][23 - 2][23 - 4] 2347611[23 - 2][23 -
[13] - [sempile v[t]] [1] - [1] [12] - 3] sempile v[t] [11] - 4] [12] - 3] sempile v[t] [11] - 4] [12] - 3] v[t] [11] - 4] [12] [13] - 4] v[t] [11] - 4] [12] [13] - 4] v[t] [11] - 4] [12] [13] - 4] v[t] [11] - 13] [12] - 4 v[t] [11] - 13] [12] - 13] v[t] - 13] - 13] - 13]	tti_3d_so6.tvt	11 - 4 [12 - 3 [13] - t] - 1 [12 - 3 [13] - t] - 1 [12 - 3] - temp118 - 3 - temp21*v[t1] [11 - 3] - temp21*v[t1] [11 - 3] - temp21*v[t1] [11 - 3] [12] [13 + 2] - 4 [13] + temp30 - 4 [13] + temp30 - 21*v[t1] [11 - 6] [12] [13 - temp495]) + temp108*tem [13] - temp495] + temp108*tem [13] - temp495] + temp108*tem [13] - temp495]	124(V[1], [1], [1], [1], [1], [1], [1], [1],	tti 3d_so6.txt	13 - 4 - temp132 v[t] + temp64 v[t] + temp64 v[t] + temp64 v[t] + temp67 v[t] + temp67 v[t] + temp67 v[t] + temp67 v[t] + temp130 v[t] + t	13 - 2 * temp313 * temp3 + temp146 * temp261 * temp3 + (1.07/2.07)*(emp92) * temp (1.07/2.07)*(emp92) * temp (1.07/2.07)*(emp92) * temp (-temp16**v[t1] (1.27) * (1.27)	tti_3d_so6.txt [1 - 3][12 - 2] = mag12***[13] [2 - 3][12 - 2] = mag12***[13] [3 - 4][12 - 2][12][13 - 3][12][13 - 3] [3 - 4][12 - 2][12][13 - 3][12][13 - 3] [4 - 4][12 - 2][13 - 2][13 - 3] [5 - 4][13 - 2][13 - 3][13 - 3][13 - 3] [5 - 4][13 - 2][13 - 3][13 - 3] [6 - 4][13 - 2][13 - 3][13 - 3][13 - 3] [7 - 4][13 - 2][13 - 3][13 - 3][13 - 3] [7 - 4][13 - 2][13 - 3][13 - 3][13 - 3][13 - 3] [7 - 4][13 - 2][13 - 2][13 - 3][13 - 3][13 - 3] [7 - 4][13 - 2][13 - 2][13 - 3][13 -	*(-2.09/15.09 *cmpla*) "(13) 1] 1 1 1 1 1 1 1 1
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$$m\frac{d^{2}p(x,t)}{dt^{2}} - (1+2\epsilon)(G_{\bar{x}\bar{x}} + G_{\bar{y}\bar{y}})p(x,t) - \sqrt{(1+2\delta)}G_{\bar{z}\bar{z}}r(x,t) = q,$$

$$m\frac{d^{2}r(x,t)}{dt^{2}} - \sqrt{(1+2\delta)}(G_{\bar{x}\bar{x}} + G_{\bar{y}\bar{y}})p(x,t) - G_{\bar{z}\bar{z}}r(x,t) = q,$$

$$m\frac{d^2r(x,t)}{dt^2} - \sqrt{(1+2\delta)}(G_{\bar{x}\bar{x}} + G_{\bar{y}\bar{y}})p(x,t) - G_{\bar{z}\bar{z}}r(x,t) = q,$$

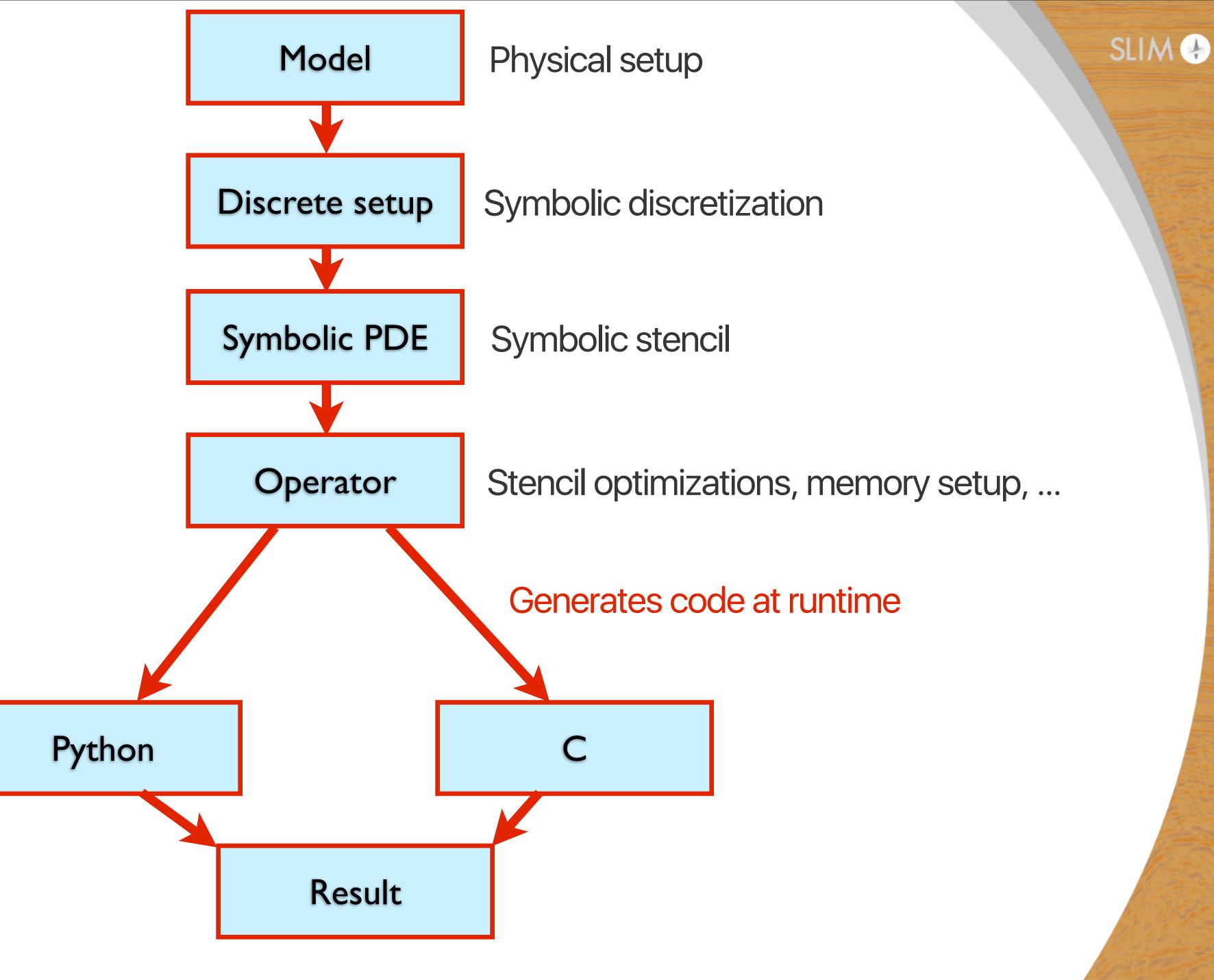
70% of the code (81/116 pages) anisotropic modelling



Objectives

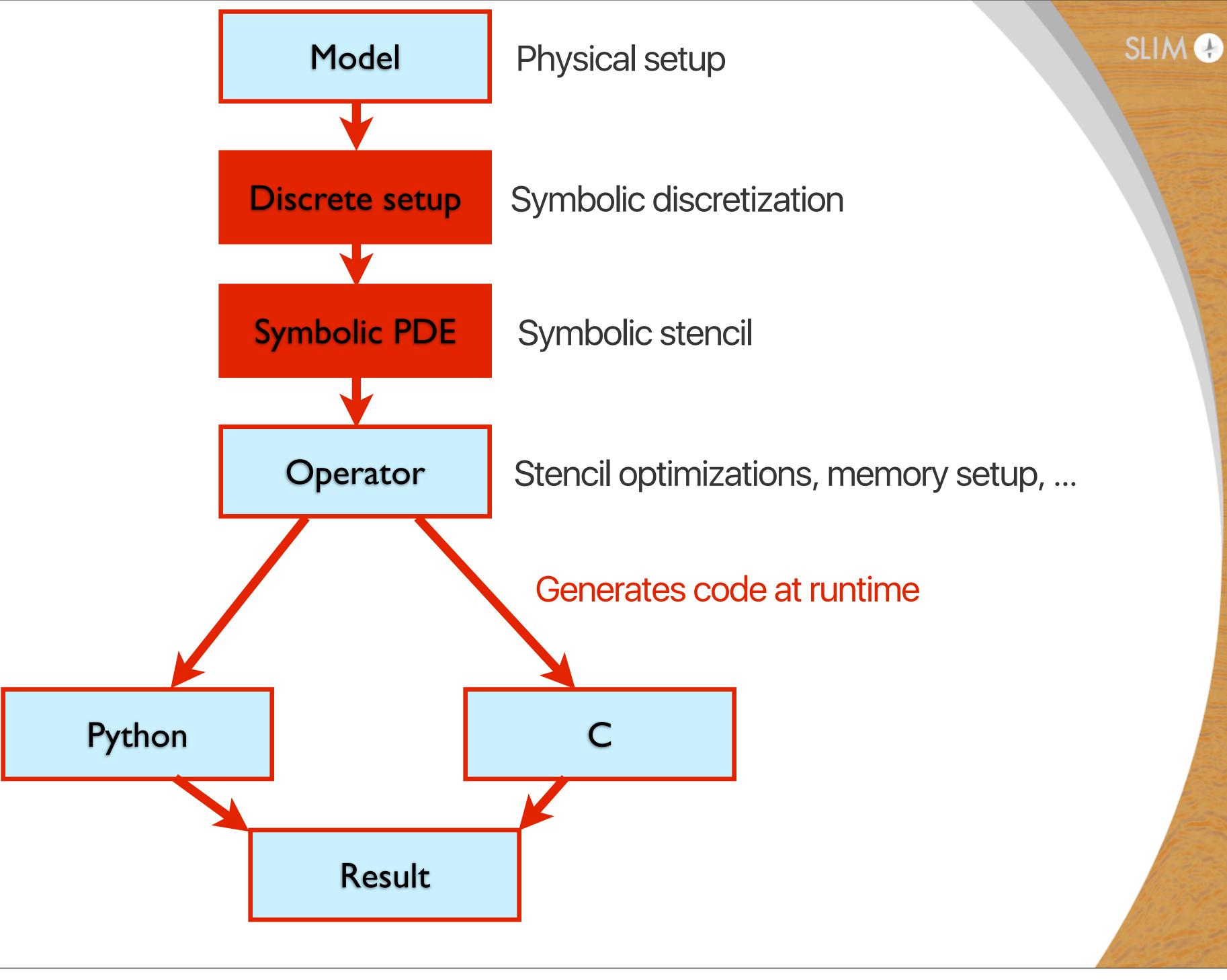
- Flexible finite-differences discretization
- One framework for all equations
- Symbolic interface
 - easy to use
 - simple introduction of new PDEs

Summary





Summary





Enabling different physics



Wave-equations

$$\frac{1}{c^2} \frac{d^2 p(x,t)}{dt^2} - \Delta p(x,t) = 0$$

$$\frac{1}{\rho c^2} \frac{d^2 p(x,t)}{dt^2} - \nabla \cdot \left(\frac{1}{\rho} \nabla p(x,t)\right) = 0$$

$$m\frac{d^2p(x,t)}{dt^2} - (1+2\epsilon)(G_{\bar{x}\bar{x}} + G_{\bar{y}\bar{y}})p(x,t) - \sqrt{(1+2\delta)}G_{\bar{z}\bar{z}}r(x,t) = q,$$

 $m\frac{d^2p(x,t)}{dt^2} - (1+2\epsilon)(D_{xx} + D_{yy})p(x,t) - \sqrt{(1+2\delta)}D_{zz}r(x,t) = q,$

$$m\frac{d^2r(x,t)}{dt^2} - \sqrt{(1+2\delta)}(G_{\bar{x}\bar{x}} + G_{\bar{y}\bar{y}})p(x,t) - G_{\bar{z}\bar{z}}r(x,t) = q,$$

Acoustic isotropic

$$m\frac{d^2r(x,t)}{dt^2} - \sqrt{(1+2\delta)}(D_{xx} + D_{yy})p(x,t) - D_{zz}r(x,t) = q,$$

Zhang, Y., Zhang, H., and Zhang, G., 2011, A stable TTI reverse time migration and its implementation: Geophysics, 76



Symbolic discretization

Symbolic object with finite-difference discretization as a property

is a symbolic object with derivatives properties u.dx, u.dy, u.dz, u.dx2, ..., u.laplace,

```
 ln[69]: u.dx \\ Out[69]: -u(t-s, x-3*h, y, z)/(60*h) + 3*u(t-s, x-2*h, y, z)/(20*h) - 3*u(t-s, x-h, y, z)/(4*h) + 3*u(t-s, x+h, y, z)/(4*h) - 3*u(t-s, x+2*h, y, z)/(20*h) + u(t-s, x+3*h, y, z)/(60*h)
```



Symbolic wave-equations

Acoustic

$$0 = m \frac{d^2 u(x,t)}{dt^2} - \Delta u(x,t) + damp \frac{du(x,t)}{dt}$$
 eqn = m * u.dt2 - u.laplace + damp * u.dt

Acoustic 4th order in time

$$0 = m \frac{d^2 u(x,t)}{dt^2} - \Delta u(x,t) - \frac{dt^2}{12} \Delta \left(\frac{1}{m} \Delta u(x,t)\right) + \operatorname{damp} \frac{du(x,t)}{dt}$$

eqn = m * u.dt2 - u.laplace - (s**2)/12 * u.laplace2(1/m)+ damp * u.dt



Worked example

Acoustic modelling



Wave-equation setup

$$\mathbf{m}(\mathbf{x}) \frac{\mathbf{u}(\mathbf{x}, \mathbf{t} + dt) - 2\mathbf{u}(\mathbf{x}, t) + \mathbf{x}, \mathbf{t} - dt)}{dt^2} - \Delta \mathbf{u}(\mathbf{x}, \mathbf{t}) = 0$$

Absorbing boundary condition

u: discretized wavefield

 Δ : discretized Laplacian

```
u = TimeData(name="u", shape=model.get_shape_comp(),
             time_dim=nt, time_order=time_order,
             space_order=spc_order,
             save=save, dtype=damp.dtype)
```

m: discretized square slowness m = DenseData(name="m", shape=model.get_shape_comp(), dtype=damp.dtype)

Lap = u.laplace



Stencil

$$\mathbf{u}(\mathbf{x}, \mathbf{t} + dt) = 2\mathbf{u}(\mathbf{x}, t) - \mathbf{u}(\mathbf{x}, \mathbf{t} - dt) + \frac{dt^2}{\mathbf{m}(\mathbf{x})} \Delta \mathbf{u}(\mathbf{x}, \mathbf{t})$$

u.forward

solve(equation, u.forward)

stencil = Eq(u.forward, solve(equation, u.forward))



Forward operator



Generate code at runtime

#include <cassert> #include <cstdlib> #include <cmath> #include <iostream> #include <fstream> #include <vector> #include <cstdio> #include <string>

#include <inttypes.h>

Application developer

(rec, u) = Acoustic.Forward()

```
long loop_stencils_a;
long loop_body;
long long kernel;
```

Generated code

```
#include <sys/time.h>
#include <math.h>
struct profiler
 double loop_stencils_a;
 double loop_body;
 double kernel;
struct flops
extern "C" int ForwardOperator(double *m_vec, double *u_vec, double *damp_vec, double *src_vec, float
*src_coords_vec, double *rec_vec, float *rec_coords_vec, long i1block, struct profiler *timings, struct flops *flops)
 double (*m)[280] = (double (*)[280]) m_vec;
  double (*u)[280][280] = (double (*)[280][280]) u_vec;
 double (*damp)[280] = (double (*)[280]) damp_vec;
  double (*src)[2] = (double (*)[2]) src_vec;
  float (*src_coords)[2] = (float (*)[2]) src_coords_vec;
  double (*rec)[101] = (double (*)[101]) rec_vec;
  float (*rec_coords)[2] = (float (*)[2]) rec_coords_vec;
   struct timeval start_kernel, end_kernel;
    gettimeofday(&start_kernel, NULL);
    int t0;
    int t1;
    int t2;
    for (int i3 = 0; i3<500; i3+=1)
      flops->kernel += 2.000000;
         t0 = (i3)\%(3);
         t1 = (t0 + 1)%(3);
         t2 = (t1 + 1)%(3);
        struct timeval start_loop_body, end_loop_body;
        gettimeofday(&start_loop_body, NULL);
         for (int i1b = 1; i1b<279 - (278 % i1block); i1b+=i1block)</pre>
           for (int i1 = i1b; i1<i1b+i1block; i1++)</pre>
```



Scientific motivations

Multi-physics

- Isotropic acoustic
- Isotropic acoustic with density
- Anisotropic acoustic
- Isotropic elastic
- Anisotropic elastic
- •

Simulation for inversion

- Adjoint PDE
- Gradients
- •



Adjoint-state

$$\underset{\mathbf{m}}{\text{minimize}} \quad \frac{1}{2}||\mathbf{A}^{-1}(\mathbf{m})\cdot\mathbf{q}-\mathbf{d}||_2^2 \qquad \text{(Virieux and Operto, 2009)}$$

Gradient with respect tom requires adjoint

$$\mathbf{g} = -\left(\frac{\partial \mathbf{u}}{\partial m}\right)^{\top} \mathbf{A}^{-\top}(\mathbf{m})(\mathbf{A}^{-1}(\mathbf{m}).\mathbf{q} - \mathbf{d})$$



Discretization for inversion

Extend symbolic discretization to adjoints

first_derivative(u, dim=x, side=centered, order=spc_order, matvec=transpose)

CRITICAL for odd order derivatives (anti-symmetric stencil)

Not required for acoustic (self-adjoint equation)



Acoustic adjoint operator

Self-adjoint => Same stencil
Backward in time
Data as source



Generate code at runtime

Application developer

Generated code

srca = Acoustic.Adjoint(dSyn - d0bs)

```
#include <cassert>
#include <cstdlib>
#include <cmath>
#include <iostream>
#include <fstream>
#include <vector>
#include <cstdio>
#include <string>
#include <inttypes.h>
#include <sys/time.h>
#include <math.h>
struct profiler
 double loop_stencils_a;
 double loop_body;
 double kernel;
struct flops
 long loop_stencils_a;
 long loop_body;
  long long kernel;
extern "C" int AdjointOperator(double *m_vec, double *u_vec, double *damp_vec, double *src_vec, float
*src_coords_vec, double *rec_vec, float *rec_coords_vec, long i1block, struct profiler *timings, struct flops *flops)
 double (*m)[280] = (double (*)[280]) m_vec;
  double (*u)[280][280] = (double (*)[280][280]) u_vec;
  double (*damp)[280] = (double (*)[280]) damp_vec;
  double (*src)[2] = (double (*)[2]) src_vec;
  float (*src_coords)[2] = (float (*)[2]) src_coords_vec;
  double (*rec)[101] = (double (*)[101]) rec_vec;
  float (*rec_coords)[2] = (float (*)[2]) rec_coords_vec;
   struct timeval start_kernel, end_kernel;
    gettimeofday(&start_kernel, NULL);
    int t0;
    int t1;
    int t2;
    for (int i3 = 500; i3>0; i3-=1)
      flops->kernel += 2.000000;
         t0 = (i3)\%(3);
         t1 = (t0 + 1)%(3);
         t2 = (t1 + 1)%(3);
        struct timeval start_loop_body, end_loop_body;
        gettimeofday(&start_loop_body, NULL);
         for (int i1b = 1; i1b<279 - (278 % i1block); i1b+=i1block)</pre>
           for (int i1 = i1b; i1<i1b+i1block; i1++)</pre>
```

25



Standardized verification for optimization

Rigorousness tests used as unit tests

Verify implementation

Allows stable continuous software integration with automated testing (TRAVIS)



Testing framework

Forward-adjoint test:

for any random $\mathbf{x} \in \text{span}(\mathbf{P}_s \mathbf{A}^T \mathbf{P}_r^T), \mathbf{y} \in \text{span}(\mathbf{P}_r \mathbf{A} \mathbf{P}_s^T)$ $< \mathbf{P}_r \mathbf{A} \mathbf{P}_s^T \mathbf{x}, \mathbf{y} > - < \mathbf{x}, \mathbf{P}_s \mathbf{A}^T \mathbf{P}_r^T \mathbf{y} > = 0$ $< \mathbf{P}_r \mathbf{A} \mathbf{P}_s^T \mathbf{x}, \mathbf{y} > - < \mathbf{x}, \mathbf{P}_s \mathbf{A}^T \mathbf{P}_r^T \mathbf{y} > = 1$

Gradient test:

for any small model perturbation dm

$$\Phi_s(\mathbf{m} + h\mathbf{dm}) = \Phi_s(\mathbf{m}) + \mathcal{O}(h)$$

$$\Phi_s(\mathbf{m} + h\mathbf{dm}) = \Phi_s(\mathbf{m}) + h(\mathbf{J}[\mathbf{m}]^T \delta \mathbf{d}) \mathbf{dm} + \mathcal{O}(h^2)$$



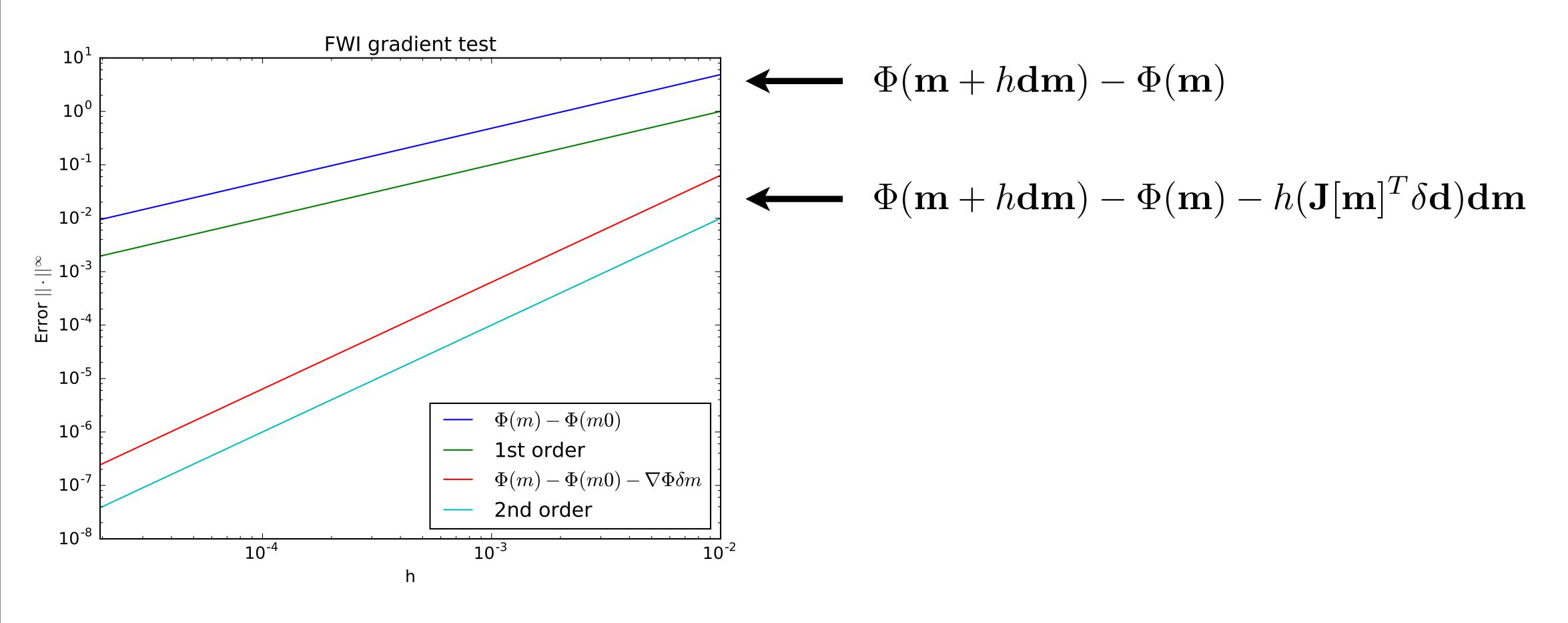
Adjoint test

Order	Dimension	$<\mathbf{F}\vec{x},\vec{y}>$	$<\vec{x},\mathbf{F}^t\vec{y}>$	Difference	ratio
2nd order	2D:	373323.7976042	373323.7975435	6.07169350e-05	1.0
4th order	2D:	340158.1486528	340158.1485253	0.00012756	1.0
6th order	2D:	341557.3948828	341557.3947399	0.00014287	1.0
8th order	2D:	358240.8513606	358240.8511932	0.00016741	1.0
10th order	2D:	393488.5561654	393488.5559270	0.00023841	1.0
12th order	2D:	439561.4005613	439561.4002034	0.00035794	1.0
2nd order	3D:	2.17496552	2.17496553	-1.23030883e-08	0.99999999
4th order	3D:	3.64447937	3.64447939	-2.13132316e-08	0.99999999
6th order	3D:	3.78730372	3.78730375	-2.22477072e-08	0.99999999
8th order	3D:	3.80286229	3.80286231	-2.23545817e-08	0.99999999
10th order	3D:	3.80557957	3.80557959	-2.23736993e-08	0.9999999
12th order	3D:	3.80318675	3.80318677	-2.23587757e-08	0.99999999



Gradient test

Check correct gradient implementation of FWI objective: $\Phi(\mathbf{m}) = \frac{1}{2}||\mathbf{d}_{obs} - \mathbf{PA}(\mathbf{m})^{-1}\mathbf{q}||_2^2$





Future tests

Discretization tests

 test the finite-difference scheme generated against polynomials

Accuracy test

- test against analytical solution
- method of manufactured solutions (MMS)

• • •



Computational performance and optimization

MS84

Domain-Specific Abstractions for Full-Waveform Inversion

Symbolic Math for Automated Fast Finite Difference Computations

Navjot Kukreja, Imperial College London, United Kingdom

MS44

Efficiency of High-Order Methods on the 2nd Generation Intel Xeon Phi Processor

Vectorization and Locality Optimizations for Seismic Imagining Methods Through Automated Code Generation Fabio Luporini, Imperial College, United Kingdom; Gerard J Gorman, Paul Kelly, and Michael Lange, Imperial College London, United Kingdom

Slides available upon request



Conclusions

Flexible physics with a simple finite-difference interface

- weeks, months, ... of development time saved
- write your own problem

Minimal coding required for geophysicists/mathematicians

- domain specialists only focus on their own problem
- improves collaborations with a high-level common ground

Simulation for inversion with adjoint-aware discretization

- not only restricted to modelling
- adjoints are inherently hard, specially for complicated physics

And all advantages of code generation with Devito (performance, architecture portability, ...)



References and link

OPESCI: http://www.opesci.org/

- documentation
- list of publications
- examples
- link to DEVITO source code

SLIM: https://www.slim.eos.ubc.ca/

- examples
- documentation
- list of publications



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