

$$(\neg P \wedge \neg P) \vee (\neg P \wedge Q) \rightarrow \neg Q$$

∴ It is not a tautology.

PREDICATES AND QUANTIFIERS :-

Q) Let $P(x)$ be the statement $x > 3$ what are the truth values of $P(4)$ and $P(2)$

$$P(4) : \text{True}$$

$$P(2) : \text{False}$$

Q) Let $q(x, y)$ be the statement $x + y > 3$ what are the truth values of $q(1, 2)$ $q(3, 0)$

$$q(1, 2) \rightarrow \text{True}$$

$$q(3, 0) \rightarrow \text{False}$$

B) $q(x) : x > 0$ $\neg q(x)$ $\neg q(1)$ $\neg q(1)$

$$P(1) : \text{True}$$

Quantifiers

De Morgan's Law for Quantifiers:

$$① \neg(\forall x P(x)) \equiv \exists x \neg P(x)$$

$$② \neg(\exists x P(x)) \equiv \forall x \neg P(x)$$

① $\neg(\forall x P(x))$ is true $\Leftrightarrow \forall x P(x)$ is false.
 \Leftrightarrow There exist an element x in domain for which $P(x)$ is false.
 \Leftrightarrow There exist an element x in domain for which $\neg P(x)$ is true.
 $\Leftrightarrow \exists x \neg P(x)$ is true.

② $\neg(\exists x P(x))$ is true $\Leftrightarrow \exists x P(x)$ is false
 \Leftrightarrow For every x in domain, for which $P(x)$ is false.
 \Leftrightarrow For every x in domain for which $\neg P(x)$ is true.
 $\Leftrightarrow \forall x \neg P(x)$ is true.

show that $\neg(\forall x (P(x) \rightarrow Q(x)))$ is equivalent to $\exists x (P(x) \wedge \neg Q(x))$

Ans :-

$$\begin{aligned} \neg(\forall x (P(x) \rightarrow Q(x))) &\Rightarrow \exists x \neg(P(x) \rightarrow Q(x)) \\ &\Rightarrow \exists x \neg(P(x) \rightarrow Q(x)) \quad P \rightarrow Q \equiv \neg P \vee Q \\ &\Rightarrow \exists x (P(x) \wedge \neg Q(x)) \end{aligned}$$

What are the negation of the statement -

1) There is an honest politician. All Americans eat Burgers.

~~2) Every politician is dishonest.~~

There exists an Americans who doesn't eat Burgers.

Q) Let $P(x)$ is the statement. x spends more than 5 hours a week. Every week day in class. where the domain of x consists of all students express each of the following quantifications in English.

- 1) $\exists x P(x)$ 2) $\forall x P(x)$ 3) $\exists x \neg P(x)$ 4) $\forall x \neg P(x)$

- 1) There exist a student who spends more than 5 hours every week day in a class.
 2) All students spend more than 5 hours every week day in a class.
 3) There exists a student who doesn't spend more than 5 hours every week day in a class.
 4) Every student doesn't spend more than 5 hours every week day in a class.

Q) Let $P(x)$ be the statement $x = x^2$ if the domain consists of integers. What are the truth values

- 1) $P(0) \rightarrow \text{True}$ 2) $P(1) \rightarrow \text{True}$ 3) $P(2) \rightarrow \text{False}$ 4) $P(-1) \rightarrow \text{False}$
 5) $\exists x P(x) \rightarrow \text{True}$ 6) $\forall x P(x) \rightarrow \text{False}$

Q) Determine the truth values of each of the following statements if the domain consists of all integers.

- 1) $\forall n (n + 1 > n) \rightarrow \text{True}$ 2) $\exists n (2n = 3n) \rightarrow \text{True}$
 3) $\exists n (n = -n) \rightarrow \text{True}$ 4) $\forall n (n^2 \geq n) \rightarrow \text{True}$

Q) Suppose that the domain of $P(x)$ consists of the integers $\{1, 2, 3, 4, 5\}$

Express the following statements without using quantifiers. Instead, using only negations, \wedge , \vee .

$$\begin{aligned}\exists x P(x) &\equiv P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5) \\ \forall x P(x) &\equiv P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5) \\ \neg \exists x P(x) &\equiv \forall x \neg P(x) = \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4) \wedge \neg P(5) \\ \neg (\forall x P(x)) &\equiv \exists x \neg P(x) = \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4) \vee \neg P(5)\end{aligned}$$

* $\forall x (x \neq 3) \rightarrow P(x) \vee \exists x P(x)$ Represent using \forall, \wedge

Rules of inference:

An argument in propositional logic is a sequence of propositions. At least one final proposition in the argument is called premises and final proposition in the argument is called conclusion.

An argument is valid if the truth of all premises \rightarrow the truth of conclusion.

P_1, P_2, \dots, P_n \vdash Q
premises conclusion

Rules of inferences :-

P : "you score 40 marks in calculus".

q : "you will pass the calculus exam".

r : "you will successfully complete the course."

$P \rightarrow q$ $P \rightarrow$ premises $q \rightarrow$ conclusion.

Rules of inference

Tautology

Name

$$\begin{array}{l} P \\ P \rightarrow q \\ \hline \therefore q \end{array}$$

$$(P \wedge (P \rightarrow q)) \rightarrow q$$

Modus ponens

$$\begin{array}{l} \neg q \\ P \rightarrow q \\ \hline \neg P \end{array}$$

$$(\neg q \wedge (P \rightarrow q)) \rightarrow \neg P$$

Modus tollens

$$\begin{array}{l} P \rightarrow q \\ q \rightarrow r \\ \hline P \rightarrow r \end{array}$$

$$((P \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (P \rightarrow r)$$

Hypothetical syllogism

$$\begin{array}{l} P \vee q \\ \neg P \\ \hline q \end{array}$$

$$(P \vee q) \wedge (\neg P) \rightarrow q$$

Disjunctive syllogism

$\frac{P}{\therefore PVQ}$

$P \rightarrow (PVQ)$ addition

$(PVQ) \rightarrow P$

simplification

$\frac{PVQ}{P}$

$(P \wedge Q) \rightarrow P$

conjunction

$\frac{P}{\therefore PVQ}$

$(P \vee Q) \rightarrow P$

disjunction

$\frac{P}{\therefore PVQ}$

$(P \vee Q) \rightarrow P$

disjunction

$\frac{P}{\therefore PVQ}$

$(P \vee Q) \rightarrow P$

disjunction

$\frac{P}{\therefore PVQ}$

$(P \vee Q) \rightarrow P$

disjunction

$\frac{P}{\therefore PVQ}$

$(P \vee Q) \rightarrow P$

disjunction

$\frac{P}{\therefore PVQ}$

$(P \vee Q) \rightarrow P$

disjunction

$\frac{P}{\therefore PVQ}$

$(P \vee Q) \rightarrow P$

disjunction

$\frac{P}{\therefore PVQ}$

$(P \vee Q) \rightarrow P$

disjunction

$\frac{P}{\therefore PVQ}$

$(P \vee Q) \rightarrow P$

disjunction

* State which rule of inference is the basis of the following argument

It is below freezing now and raining now therefore it is freezing now

(Simplification)

* If it rains today then we will not have a bbs today. If we have a bbs today then we will have a bbs tomorrow. Therefore if it rains today then we will have a bbs tomorrow.

(Hypothetical syllogism)

1) Show that the hypothesis if it is not sunny this afternoon and it is colder than yesterday, we will go swimming only if it is sunny.

2) If we don't go to swimming then we will take a trip.

3) And if we take a trip, then we will be home by sunset.

Lead to the conclusion, we will be home by sunset.

A) All the above can be written as

P : It is sunny this afternoon

q : It is colder than yesterday

r : we will go swimming

s : we will take a trip

t : we will be home by sunset.

$\neg p \wedge q$ (H)

$\neg p$ (Simplification)

$r \rightarrow p$ (H)

$\neg r$ (Modus tollens)

$\neg r \rightarrow s$ (H)

s (Modus ponens)

$s \rightarrow t$ (H)

t (Modus ponens)

P only if q
 $P \rightarrow Q$

show that the hypothesis If you send me an msg message then I'll finish writing the program. If you don't send me an e-mail, then I'll go to sleep early.
 & I go to sleep early then I'll wake up feeling refreshed lead to the
 conclusion If I don't finish writing the program, then I will wake up
 feeling refreshed.

sol: p: you send me a email message
 q: I'll finish writing the program
 r: I'll go to sleep early
 s: I'll wake up feeling refreshed.

$$\begin{array}{l} P \rightarrow q \\ \neg P \rightarrow r \\ r \rightarrow s \end{array} \left. \vphantom{\begin{array}{l} P \rightarrow q \\ \neg P \rightarrow r \\ r \rightarrow s \end{array}} \right\} \text{premises}$$

$$\neg q \rightarrow s \rightarrow \text{conclusion}$$

take $P \rightarrow q$ (Hypothesis)
 $\neg q \rightarrow \neg P$ (contrapositive)
 $\neg P \rightarrow r$ (Hypothesis)
 $\neg q \rightarrow r$ (Hypothetical syllogism)
 $r \rightarrow s$
 $\neg q \rightarrow s$ (transitive)

check whether the $(P \rightarrow q) \wedge (q \rightarrow P)$ is a tautology or not.

truth table

P	q	$P \rightarrow q$	$q \rightarrow P$	$(P \rightarrow q) \wedge (q \rightarrow P)$
T	T	T	T	T
T	F	F	T	F
F	T	F	F	F
F	F	T	T	T

Not a tautology

$$(P \rightarrow q) \wedge P \rightarrow q$$

T	T	T
F	F	T
T	T	T
F	F	T

a tautology

Rules of Inference for Quantified Statements

Rules of Inference

$$\forall x p(x)$$

$$\therefore p(c) \text{ for any } c$$

Universal
Instantiation

$$p(c) \text{ for any arbitrary } c$$

$$\forall x p(x)$$

Universal generalization

$$\exists x p(x)$$

$$\therefore p(c) \text{ for some } c$$

Existential instantiation

$$p(c) \text{ for some } c$$

$$\therefore \exists x p(x)$$

Existential generalization

show that the predecessor "everyone in D.M class has taken a course in CS" and John is a student in the class.

Imply the conclusion, John has taken a course in CS.

f)

$$\forall x p(x) \rightarrow q(x)$$

John is a student in d.m class

$$p(\text{John})$$

$$\text{Conclusion} : q(\text{John})$$

$$\forall x (p(x) \rightarrow q(x)) \text{ (Hypothesis)}$$

$$p(\text{john}) \rightarrow q(\text{john}) \text{ (Universal Instantiation)}$$

$$p(\text{john}) \quad q(\text{john}) \quad \text{(Hypothesis)}$$

$$\quad \quad \quad \text{(Modus ponens)}$$

* show that the premises, "A student in the class has not read the book."
 "Everyone in the class passed the 1st exam." \rightarrow the conclusion someone
 who passed the first exam has not read the book.

$$\exists x p(x) \rightarrow q(x)$$

$$p(\text{student}) \rightarrow q(x) \text{ (Existential Instantiation)}$$

$$q(\text{student})$$

$$\begin{aligned} \exists x p(x) \wedge \neg q(x) \\ \forall x (p(x) \rightarrow R(x)) \end{aligned} \quad \left. \vphantom{\begin{aligned} \exists x p(x) \wedge \neg q(x) \\ \forall x (p(x) \rightarrow R(x)) \end{aligned}} \right\} \text{Premises}$$

$$\exists x (\neg q(x) \wedge R(x)) \text{ conclusion}$$

from \exists instantiation

$$\textcircled{2} p(c) \wedge \neg q(c) \text{ for some } c$$

$$\textcircled{3} \neg q(c) \text{ for some } c \text{ (simplification)}$$

$$\textcircled{4} \forall x (p(x) \rightarrow R(x)) \quad (H)$$

$$\textcircled{5} p(c) \rightarrow R(c) \text{ for any } c \text{ (Universal Instantiation)}$$

$$\textcircled{6} p(c) \quad \text{(simplification of 2)}$$

$$\textcircled{7} R(c) \quad \text{(Modus ponens)}$$

$$\textcircled{8} \neg q(c) \wedge R(c) \text{ for any } c \quad \text{(conjunction 3 \& 7)}$$

$$\textcircled{9} \exists x (\neg q(x) \wedge R(x)) \quad \text{(Ex generalis)}$$

Introduction to Theorem

Recall proof:
 O for any real no x and y , if $x > y$ then $x^2 > y^2$
 if $x < y$ and $x < y$ we have not then $x^2 < y^2$

* Give a direct proof of the theorem if n is an odd integer
 n is odd.

$$\forall n \in \mathbb{N} \rightarrow q(n)$$

$p(n)$: " n is odd" $q(n)$: " n^2 is odd".

$p(c) \rightarrow q(c)$ for arbitrary integer c .

* Proof: suppose n is odd

for integer $k \exists n = 2k + 1$

$$\begin{aligned}
 n^2 &= (2k + 1)^2 \\
 &= 2(2k^2 + 2k + 1) \\
 &= 2m + 1
 \end{aligned}$$

* Give a direct proof that if m and n are odd then mn is also a perfect square.

suppose m and n are perfect squares

we can write $m = k^2$

\exists an integer k and \exists an integer z

$$m = k^2 \times z^2$$

$$nm = k^2 \times z^2 \times k^2 \times z^2$$

$$nm = (kz)^4$$

$$nm = y^4$$

$$(kz)^4 = (y)^4$$

Prove that if n is an integer and $3n+2$ is odd then n is odd.
 $P(x)$: n is an integer ~~then~~ & $3n+2$.
 q : n is odd.

$$3n+2 = 2k+1$$

$$3n = 2k-1$$

$$3n = 2(k-1) + 1$$

$$3n = 2m+1$$

we can't prove this using this direct proof method.

Method of contradiction^{aposition}
suppose n is even

If n is even then $3n+2$ is even

$$n = 2k \text{ for some integer } k$$

$$3n = 6k$$

$$3n+2 = 6k+2$$

$$3n+2 = 2(\underbrace{3k+1}_{\text{integer}})$$

$$= 2m$$

even

Therefore by the method of contradiction if n is an integer and $3n+2$ is odd then n is odd. ^{aposition}

Method of contradiction:-

* prove that $\sqrt{2}$ is irrational.

consider: suppose $\sqrt{2}$ is rational.

$\sqrt{2} = \frac{p}{q}$ [where p and q have no common factors and $q \neq 0$]

$$2 = \frac{p^2}{q^2}$$

$$2q^2 = p^2$$

p^2 and q^2 are .

p^2 is a multiple of q^2 and 2

p^2 is an even no then p is an even number.

\exists an integer k $\ni p = 2k$

$$4k^2 = 2q^2$$

$$2k^2 = q^2$$

$\therefore q^2$ is a multiple of 2

q^2 is an even no then q is an even number.

$\therefore p$ and q are multiples of 2 and both have a common factor 2 .

Therefore our assumption contradicts with the standard fact.

Hence through method of contradiction $\sqrt{2}$ is an irrational.

show that \sqrt{x} need not to be an irrational number.

suppose \sqrt{x} is an irrational number where

$$= (\sqrt{x})^2$$

$$= x$$

$$= x$$

x is a rational.

$\therefore \sqrt{x}$ need not to be an irrational number.

FUNCTIONS

A function from a set x to another set y is a rule that assigns to each other element in x a unique element in y .

$f: x \rightarrow y$ (from x to y)
 \downarrow domain \downarrow codomain

Ex: $f(x) = x$; $f: \mathbb{R} \rightarrow \mathbb{R}$

$f(x) = x+1$

$f(x) = ax+b \rightarrow$ linear function

$f(x) = x^n \rightarrow$ power function

$f(x) = a^x \rightarrow$ exponential function

ONE TO ONE AND ONTO FUNCTIONS:-

ONE TO ONE (INJECTION):-

$f: x \rightarrow y$ let x_1 and x_2 be any 2 elements from x . if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$, then f is injective. (Horizontal line test)

ONTO OR SURJECTION:-

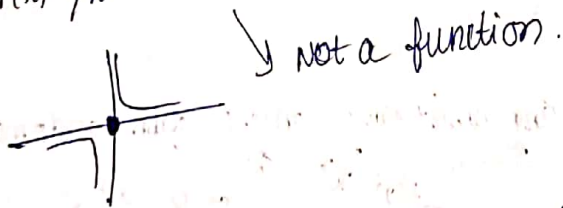
A function $f: x \rightarrow y$ is said to be onto (or surjective) if for every $y \in y$ there exist a $x \in x$ such that $f(x) = y$.

BISECTION (one-one correspondence):-

A function $f: x \rightarrow y$ is said to be a bijection if it is both one-on and onto.

Example:-

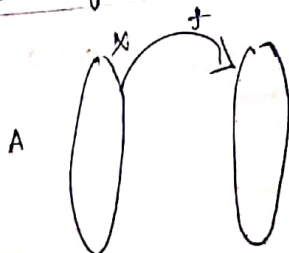
$f(x) = 1/x$ $\mathbb{R} \rightarrow \mathbb{R}$ at $x=0$ it's not defined.



$f(x) = \sqrt{x}$ $\mathbb{R} \rightarrow \mathbb{R} \rightarrow$ Not a function.

* Increasing fcn, Decreasing fcn, strictly increasing, strictly decreasing, periodic functions.

Note:-



Let x and y be two sets and let f be a fcn from x to y .
 Let $A \subseteq x$ then $\text{Image}(A) = \{ y \in y : y = f(x) \text{ for } x \in A \}$

Surjective function:

$$\forall y \exists x \in X$$

$$f(x) = y$$

$$f(f(x)) = y$$

Prove that $f: \mathbb{R} \rightarrow \mathbb{R}$ is a bijection.

one to one:

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

as it is one-one.

onto:

$$y = x+1 \quad \mathbb{R} \rightarrow \mathbb{R}$$

$$y-1 = x \in \mathbb{R} \rightarrow \mathbb{R}$$

$$f(y-1) = y-1+1 = y$$

* check whether $f(x) = x^2$ from $\mathbb{R} \rightarrow \mathbb{R}$ is one to one and onto.

$$f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2$$

$$x_1 = x_2 \text{ or } x_1 = -x_2 \in \text{codomain } \mathbb{R}^+ \setminus \{0\}$$

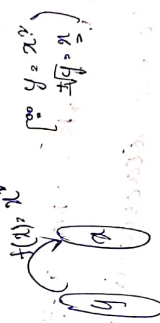
is not one to one.

onto:

Let $y \in \mathbb{R}^+ \setminus \{0\}$ (choose any arbitrary element from codomain)

$$x = \sqrt{y} \in \mathbb{R}$$

$$f(x) = (\sqrt{y})^2 = y$$



* Check whether $y = 1/x$ from $\mathbb{R} \rightarrow \mathbb{R}$ is one to one or onto.

is not a function on $\mathbb{R} \rightarrow \mathbb{R}$

$$f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$$

$$f_1: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} \setminus \{0\}$$

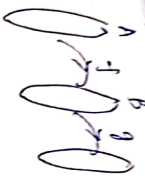
is one-one.

$$a^{1/y}$$

1st rule: as a is an irrational number and a has no pairing.

* We can only find inverse function if and only if its bijective.

composition of function:-



$$g \circ f: A \rightarrow C$$

$$g(f(x)) = g(e) = h$$

$$f: A \rightarrow B$$

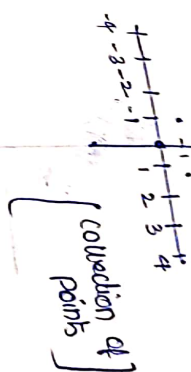
$$f(x) = \sin x$$

$$g: B \rightarrow C$$

$$g(x) = x^2$$

$$g(f(x)) = g(\sin x) = \sin^2 x$$

$$g \circ f(x) = f(g(x)) = f(\sin^2 x) = \sin^2 x$$



$$f(g(x)) = f(\sin^2 x) = \sin^2 x$$

ceiling function

$$\lceil x \rceil$$

$$\lceil -1.2 \rceil = -1$$

* floor function

$$\lfloor x \rfloor$$

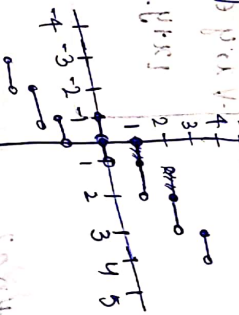
$$\lfloor 1.2 \rfloor = 1$$

$$\lfloor -1.2 \rfloor = -2$$

$$\text{floor } \lfloor x \rfloor = \lceil x \rceil - 1$$

draw the graph of ceiling function and floor function.

ceiling function



$$f(x) = \lceil x \rceil$$

$$f(x) = \lceil x \rceil$$

$$f(x) = \lceil x \rceil$$

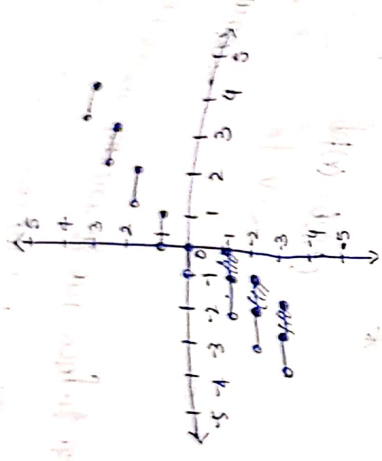
$$f(x) = \lceil x \rceil$$

$$f(x) = \lceil x \rceil$$

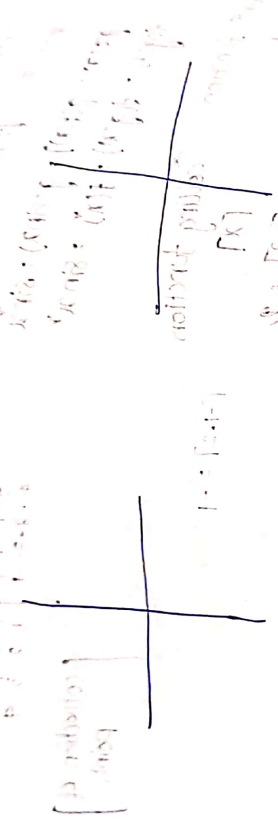
$$f(x) = \lceil x \rceil$$

$$f(x) = \lceil x \rceil$$

Ceiling Functions



*** Draw the graph of 1) $\lceil 3x+2 \rceil$ 2) $\lceil 2x+1 \rceil$



Properties of ceiling function and floor functions

- * $\lfloor x \rfloor = n \Leftrightarrow n \leq x < n+1$
- * $\lceil x \rceil = n \Leftrightarrow n-1 < x \leq n$

* floor of $-x = -\lceil x \rceil$ $\forall x \in \mathbb{R}$

$$\lfloor -x \rfloor + \lceil x \rceil = 0$$

$$\lceil x \rceil = -\lfloor -x \rfloor$$

floor: $\lfloor a+n \rfloor = \lfloor a \rfloor + n$ $\forall n \in \mathbb{Z}$

$$\lfloor n \rfloor = n$$

$$m \leq x < m+1$$

$$m+n \leq x+n < m+n+1$$

$$\lfloor x+n \rfloor = m+n$$

$\forall x, y \in \mathbb{R}$

$\lceil x+y \rceil = \lceil x \rceil + \lceil y \rceil$

\rightarrow False

* prove that if x is a real no then $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor$

* check whether $f(x) = \lfloor x \rfloor$ is a bijective and draw the graph $\mathbb{Z}^+ \rightarrow \mathbb{Z}$

(A) $\lfloor x \rfloor = n$
 $x = n + \varepsilon$ $0 \leq \varepsilon < 1$

Case 1:
 $0 \leq \varepsilon < \frac{1}{2}$
 $\lfloor x \rfloor = n$
 $x = n + \varepsilon$
 $x + \frac{1}{2} = n + \varepsilon + \frac{1}{2}$
 $< n + 1$
 $\lfloor x + \frac{1}{2} \rfloor = n$

Case 2: $(\frac{1}{2} \leq \varepsilon < 1)$
 $\lfloor x \rfloor = n$
 $x = n + \varepsilon$ $\frac{1}{2} \leq \varepsilon < 1$
 $x + \frac{1}{2} = n + \varepsilon + \frac{1}{2}$
 $\geq n + 1$
 $\lfloor x + \frac{1}{2} \rfloor = n + 1$
 $2x = 2n + 2\varepsilon$
 $\geq 2n + 1$
 $\lfloor 2x \rfloor = 2n + 1$

INDUCTION AND STRONG INDUCTION:-
 → usually we use this to prove for \mathbb{Z}^+

Basis step:-
 ① $P(1)$ is true

Induction step:-
 ② $P(k) \rightarrow P(k+1)$ is true
 [∵ Assume $P(k)$ is true and then prove $P(k+1)$ is true]

STRONG INDUCTION:-

Basis step
 ① $P(1)$ is true

Induction step
 ② $P(1) \wedge P(2) \wedge \dots \wedge P(k) \rightarrow P(k+1)$ is true.

Prove that the sum of first n positive integers is $\frac{n(n+1)}{2}$.

$P(n)$ = Sum of first n positive integer is $\frac{n(n+1)}{2}$

A) Basis step:-
 $P(1)$ = sum of 1st integer is $\frac{1(1+1)}{2} = 1$

Inductive Step
Assume $P(k)$ is true

i.e. $1+3+\dots+k = \frac{k(k+1)}{2}$ is true

we have to prove $P(k+1)$ is true.

$$1+3+\dots+k+(k+1) = \frac{(k+1)(k+2)}{2}$$

To prove

$1+3+\dots+k+(k+1)$

$$= \frac{k(k+1)}{2} + k+1$$

$$= \frac{k^2+k+2k+2}{2}$$

$$= \frac{(k+1)(k+2)}{2} \quad \text{R.H.S}$$

$$= \frac{(k+1)(k+2)}{2}$$

Hence by the method of induction, this P is true for all integers.

Prove that the sum of first n odd integers is n^2

$P(n)$: sum of first odd n is n^2 is true

$P(1)$ is true

Induction step:-

Assume $P(k)$ is true

i.e.

$$1+3+\dots+(2k-1) = k^2 \text{ is true.}$$

We have to prove $P(k+1)$ is true.

$$1+3+\dots+(2k+1) = (k+1)^2$$

$$\begin{aligned} & \left(\frac{k^2+2k+1}{k^2+2k+1} \right) = \frac{k^2+2k+1}{k^2+2k+1} \\ & = \frac{k^2+2k+1}{k^2+2k+1} \end{aligned}$$

$\therefore \text{L.H.S} = \text{R.H.S.}$

product of primes.

* use strong induction to prove that n is irrational.

1) For this n there is no $b \in \mathbb{Z}$ such that $n = m/b$.

$$n \neq 1/b \quad \forall b \in \mathbb{Z}$$

$$n > 1 \geq 1/b \quad \forall b \in \mathbb{Z}$$

$$n \geq 1/b$$

$$n \neq 1/b \quad \forall b \in \mathbb{Z}$$

$$n = \frac{m}{b}$$

$$n = \frac{(k+1)}{b}$$

* k is even

$k+1 = 2m$ for some $m \in \mathbb{Z}^+$

$$(2m)^2 = ab^2 \rightarrow b^2 \cdot 2m^2 \rightarrow b^2 \text{ is even}$$

$\Rightarrow b$ is even

$b = 2s$ for some $s \in \mathbb{Z}^+$

$$n = \frac{k+1}{b} = \frac{2m}{2s} = \frac{m}{s}$$

\downarrow
a contradiction.

then that every positive integer can be expressed as the sum of distinct powers of 2.

To prove some propositions are equal.

check whether the function $f(n) = n!$ one to one and onto.

At one-one

onto

$$f(x) = f(y)$$

$$x! = y!$$

$$\Rightarrow x = y$$

\therefore one to one

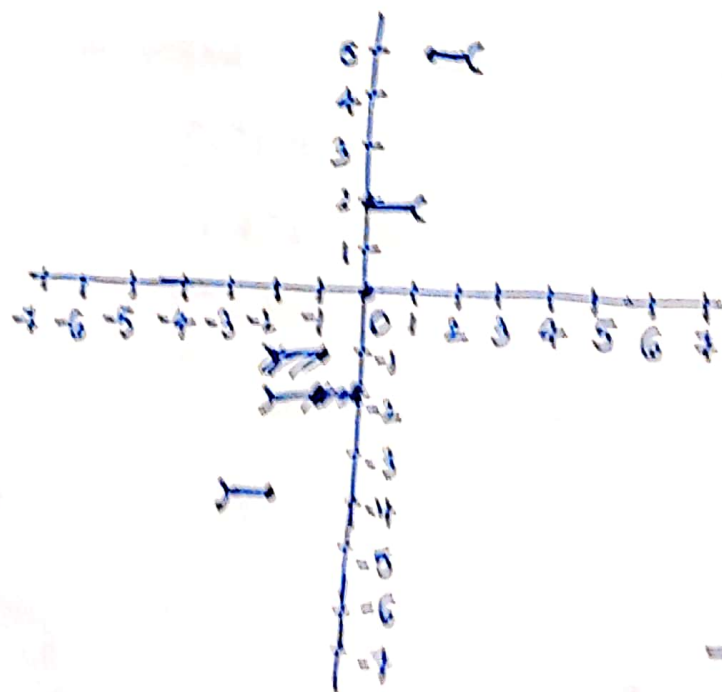
$$y = n!$$

$$f(4) = 4!$$

\therefore It is not onto.

$$f: \mathbb{N} \rightarrow \mathbb{N}$$

Draw the graph of $\lfloor 3x+2 \rfloor$



$$\begin{aligned} \lfloor 0.5 \rfloor &= 0 \\ \lfloor 1.5 \rfloor &= 1 \\ \lfloor 2.5 \rfloor &= 2 \end{aligned}$$

$$\begin{aligned} 3x+2 &= 0 \\ 3x &= -2 \\ x &= -2/3 \end{aligned}$$



$$\lfloor cx+d \rfloor$$

length of the interval = $1/c$
value at $x=0 = d$