Artificial Neural Networks

Inteligencia Artificial en los Sistemas de Control Autónomo
Máster Universitario en Ingeniería Industrial

Departamento de Automática

Universidad de Alcalá

ISG

Objectives

1. Describe biological neurons and networks
2. Basics of artifical neurons and networks
3. Understand the role of trainning in ANNs
4. Strengths and weaknesses of ANNs

Bibliography

- A. Tettamanzi, M. Tomassini. *Soft Computing. Integrating Evolutionary, Neural, and Fuzzy Systems.* Springer-Verlag. 2001
- McCulloch, W. and Pitts, W. (1943). *A logical calculus of the ideas immanent in nervous activity.* Bulletin of Mathematical Biophysics, 7:115 - 133.
- Rosenblatt, Frank. (1958). *The Perceptron: A Probabilistic Model for Information Storage and Organization in the Brain.* Psychological Review, 65:386-408

Table of Contents

1. Introduction
   - History
   - Structure of neurons
2. Artificial neurons
   - Definition
   - Logical gates with a neuron
   - Activation functions
   - Learning limits
3. Artificial Neural Networks
   - Definition
   - Application examples
4. Feedforward networks
   - Definition
   - Separability
   - Bias in a MLP
5. Training algorithms
   - Problem statement
   - Gradient descent algorithm
   - Stochastic Gradient Descent Algorithm
   - Other optimization algorithms
   - Backpropagation algorithm
   - Learning problems

Introduction

History

1888 Ramón y Cajal. Discovery of biological neurons

1943 McCulloch & Pitts. First neural network designers

1949 Hebb. First learning rule

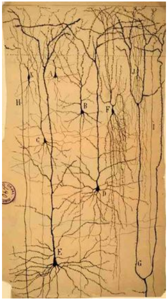
1958 Rosenblatt. Perceptron

1969 Minsky & Papert. Perceptron limitation - Death of ANN

1986 Rumelhart et al. Re-emergence of ANN: Backpropagation

201x Convolutional Neural Networks (CNNs) - Deep learning

2014 Goodfellow et al. Generative Adversarial Networks (GANs)



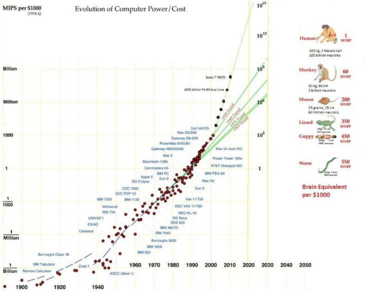
Introduction

Structure of neurons (I)

Animal	Neurons
Sponge	0
Roundworm	302
Jellyfish	800
Ant	250,000
Cockroach	1,000,000
Frog	16,000,000
Mouse	71,000,000
Cat	760,000,000
Macaque	6,376,000,000
Human	86,000,000,000
Elephant	267,000,000,000

Human brain

Neuron switching time: 0.001 s
Synapsis: 10-100 thousand
Scene recognition time: 0.1 s



(Source)

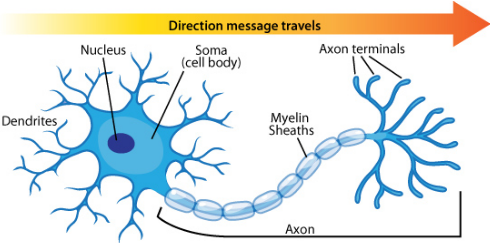
Introduction

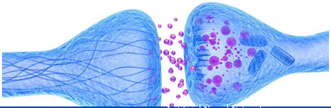
Structure of neurons (II)

A neuron has a cell body ...

- ... a branching input structure (dendrite) and
- ... a branching output structure (axon)

Axons connect to dendrites via synapses





Introduction

Structure of neurons (III)

A neuron only fires if its input signal exceeds a threshold

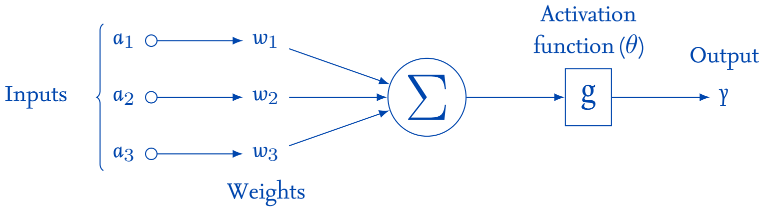
- Good connections allowing a large signal
- Slight connections allowing a weak signal
- Synapses may be either excitatory or inhibitory

Synapses vary in strength

- Biological learning involves setting that strength

Artificial neurons

Definition (I)



- a_i Normalized input ($0 \leq a_i \leq 1$)
- w_i Weight of input j
- θ Threshold
- g Activation function

Neuron model (perceptron)

$$y = g\left(\sum_{i=1}^{n} w_i a_i\right)$$

Artificial neurons

Definition (II)

- Each neuron has a threshold value
- Each neuron has weighted inputs
- The input signals form a weighted sum
- If the activation level exceeds the threshold, the neuron activates

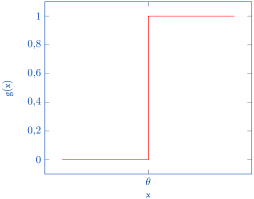
Artificial neurons

Definition (III)

The idealized activation function is a step function

$$g(x) = \begin{cases} 1 & \text{if } \sum_{i=1}^{N} w_i x_i > \theta \\ 0 & \text{otherwise} \end{cases}$$

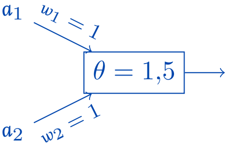
The step function is rarely used in practice



Artificial neurons

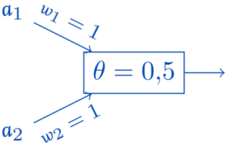
Logical gates with a neuron

AND



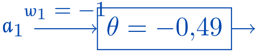
a_1	a_2	y
0	0	0
0	1	0
1	0	0
1	1	1

OR



a_1	a_2	y
0	0	0
0	1	1
1	0	0
1	1	1

NOT

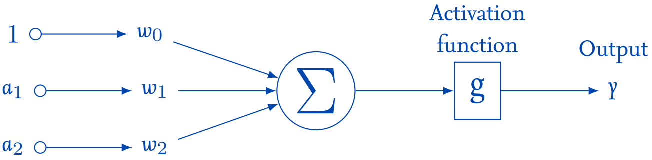


a_1	y
0	1
1	0

(A neuron in Excel)

Artificial neurons

Definition of neuron (alternative version)



a_i Normalized input ($0 \leq a_i \leq 1$)

w_i Weight of input j

w_0 Bias

g Activation function

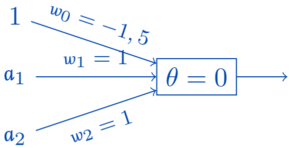
Neuron model

$$y = g\left(\sum_{i=0}^{n} w_i a_i\right)$$

Artificial neurons

Example of biased neuron

AND logical gate with a biased input

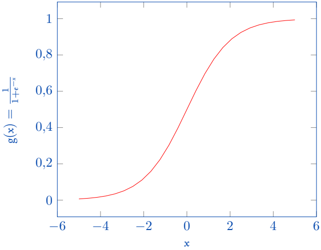


a_0	a_1	a_2	Output
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Artificial neurons

Activation functions: Sigmoid function

- S-shaped, continuous and everywhere differentiable
- Asymptotically approach saturation points
- Derivative fast computation
- Range $\in [0, 1]$



Sigmoid function

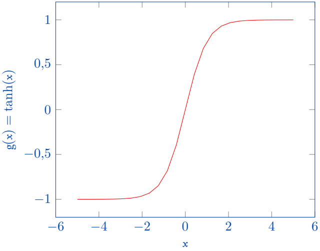
$$g(x) = \frac{1}{1 + e^{-x}}$$

$$g'(x) = g(x)(1 - g(x))$$

Artificial neurons

Activation functions: Tanh function

- Asymptotically approach saturation points
- Range $\in [-1, 1]$
- Bigger derivative than sigmoid (faster training)



Tanh function

$$g(x) = \tanh(x) = \frac{2}{1+e^{-2x}} - 1$$

$$g'(x) = 1 - g(x)^2$$

Artificial neurons

Activation functions: Softmax function

- Generalization of the logistic function
- Usually used in the output layer in classification problems
- Asymptotically approach saturation points

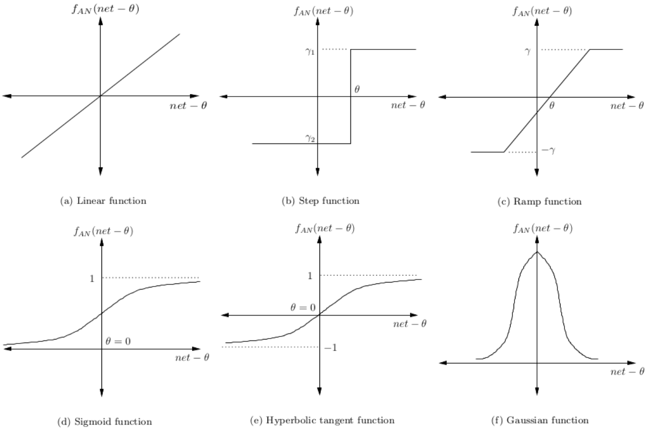
Softmax function

$$g(\boldsymbol{z})_j = \frac{e^{z_j}}{\sum_{k=1}^{K} e^{z_k}} \text{ for } j = 1, \ldots, K$$

with $\mathbf{z}$ a K-dimensional vector

Artificial neurons

Other activation functions



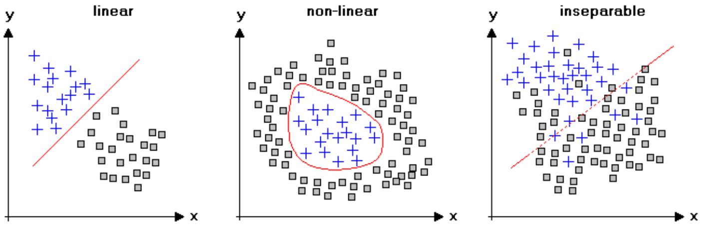
(a) Linear function (b) Step function (c) Ramp function

(d) Sigmoid function (e) Hyperbolic tangent function (f) Gaussian function

(Source)

Artificial neurons

Learning limits (I)

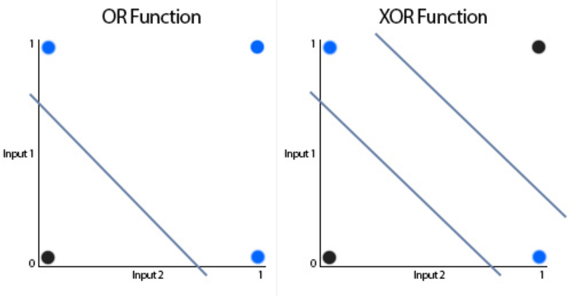


Problem: A single neuron only can solve linearly separable problems

Artificial neurons

Learning limits (II)

XOR cannot be implemented with a neuron



Solution: Neuronal networks

Artificial Neural Networks

Definition (I)

- A very much simplified version of biological nerve systems
- A set of nodes (neurons)
    - Each node has input and output
    - Each node performs a simple computation
- Weighted connections between nodes
    - Connectivity gives the structure of the net
    - What can be computed by an ANN is primarily determined by the connections and their weights
- It can recognize patterns, learn and generalize

Artificial Neural Networks

Definition (II)

ANN properties

- Noise tolerance
- General function approximator

Machine Learning tasks

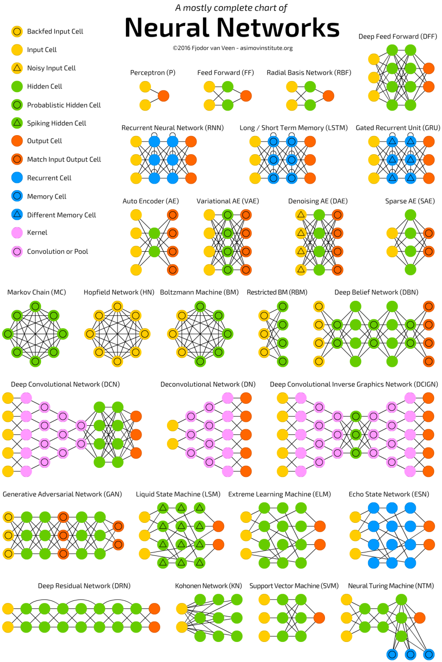
- Supervised learning (classification and regression)
- Unsupervised learning (known as self-organizing maps in ANN terminology)

Many topologies

- Acyclic, recurrent (cyclic), modular, etc
- Feed Forward networks (MLPs)

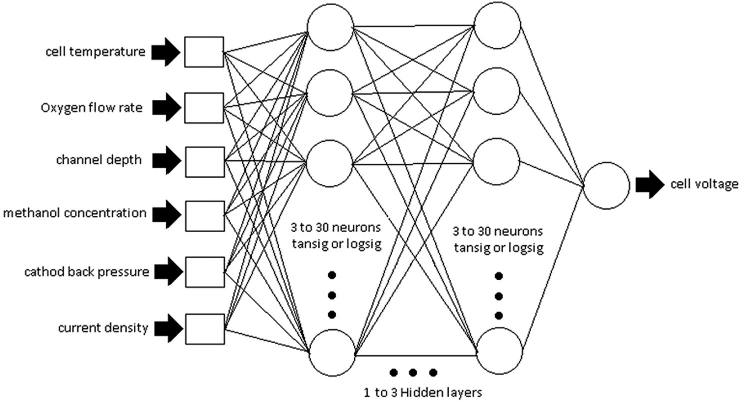
Human readability less important than performance

(More info)


A mostly complete chart of
Neural Networks
©2016 Fjodor van Veen - asimovinstitute.org
Backfed Input Cell
Input Cell
Noisy Input Cell
Hidden Cell
Probablistic Hidden Cell
Spiking Hidden Cell
Output Cell
Match Input Output Cell
Recurrent Cell
Memory Cell
Different Memory Cell
Kernel
Convolution or Pool
Perceptron (P)
Feed Forward (FF)
Radial Basis Network (RBF)
Deep Feed Forward (DFF)
Recurrent Neural Network (RNN)
Long / Short Term Memory (LSTM)
Gated Recurrent Unit (GRU)
Auto Encoder (AE)
Variational AE (VAE)
Denoising AE (DAE)
Sparse AE (SAE)
Markov Chain (MC)
Hopfield Network (HN)
Boltzmann Machine (BM)
Restricted BM (RBM)
Deep Belief Network (DBN)
Deep Convolutional Network (DCN)
Deconvolutional Network (DN)
Deep Convolutional Inverse Graphics Network (DCIGN)
Generative Adversarial Network (GAN)
Liquid State Machine (LSM)
Extreme Learning Machine (ELM)
Echo State Network (ESN)
Deep Residual Network (DRN)
Kohonen Network (KN)
Support Vector Machine (SVM)
Neural Turing Machine (NTM)

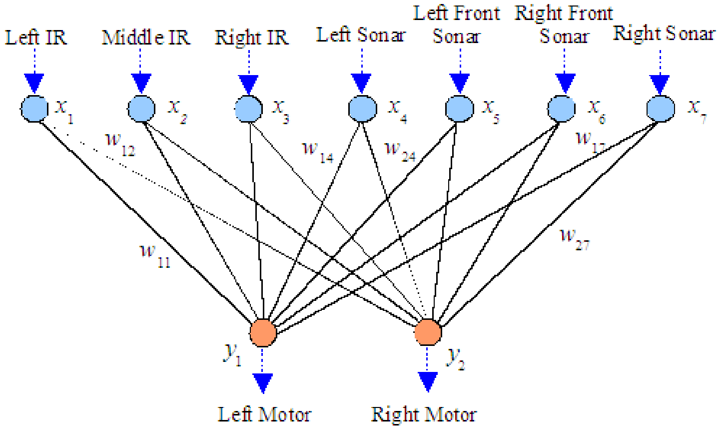
Artificial Neural Networks

Application examples (I)



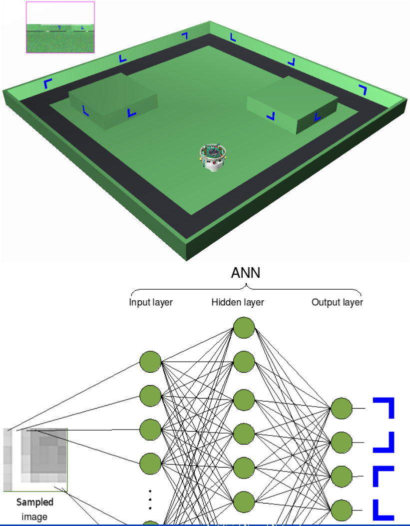
Artificial Neural Networks

Application examples (II)



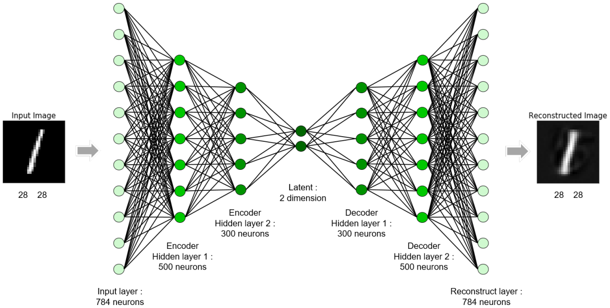
Artificial Neural Networks

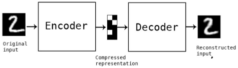
Application examples (III)



Artificial Neural Networks

Application examples (III)





(Source)

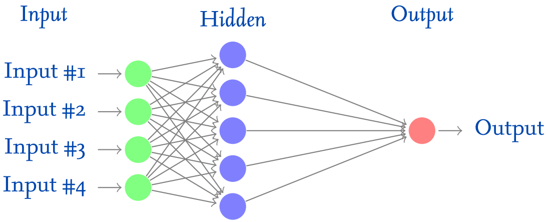
Feedforward networks

Definition (I)

Neurons are arranged in layers

- *Input* Which consists of any normalized data
- *Output* Which are the net outcome
- *Hidden* (Optional) No direct interaction

Also known as **multilayer perceptron** (MLP)



Feedforward networks

Definition (II)

The input layer

- Introduces input values into the network
- No activation function or other processing

The hidden layer(s)

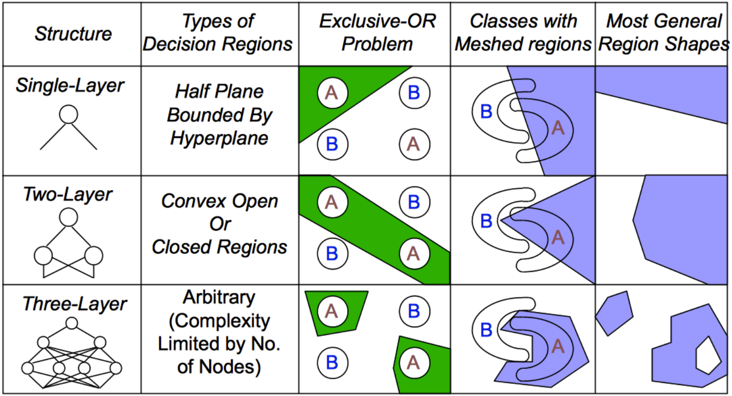
- Perform classification of features
- Two hidden layers are sufficient to solve any problem
- Features imply more layers may be better

The output layer

- Functionally just like the hidden layers
- Outputs are passed on to the world outside the neural network

Feedforward Networks

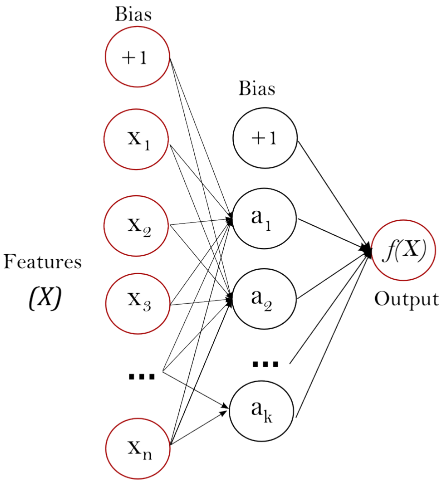
Separability



(Online demo)

Feedforward Networks

Bias in a MLP



(Source)

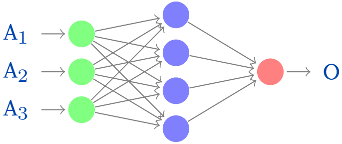
Training algorithms

Problem statement (I)

ANN can perform different tasks

- Classification, regression, others

Classification (or supervised learning) uses a *training set*



A_1	A_2	A_3	O	Y
1,1	2,5	4,5	0,2	−0,1
0,9	2,4	1,2	0,5	0,4
1,0	2,0	9,9	0,4	1,2

Toss function: Measure of the error

- Usually mean squared error (mse): $E = \frac{1}{2}(y - o)^2 = f(w)$
- Y and O are the desired and observed outputs

Training algorithms

Problem statement (II)

$$E = \frac{1}{2}Err^2 = \frac{1}{2}\left[y - g\left(\sum_{j=0}^{n} w_j x_j\right)\right]^2$$

where

- y Desired output
- w_j Weight connection j
- x_j Input j

Problem: Determine w that minimize $f(w)$

- This is a classical optimization problem
- Any optimization algorithm can be used
- ... in AI, optimization means search

Training algorithms

Gradient Descent Algorithm (I)

Given the error

$$E = \frac{1}{2}Err^2$$

Take partial derivatives

$$\begin{aligned}\frac{\partial E}{\partial w_j} &= Err\frac{\partial Err}{\partial w_j}\\ &= Err\frac{\partial}{\partial w_j}g\left(y - \sum_{j=0}^{n} w_j x_j\right)\\ &= -Err \times g'(w) \times x_j\end{aligned}$$

Training algorithms

Gradient Descent Algorithm (II)

Weight update

$$w_j^{k+1} = w_j^k + \alpha \times \mathrm{Err} \times g'(w) \times x_j$$

with

- α Learning rate ($|\alpha| < 1$)
- *err* Difference desired and current output
- g' Derivate of activation function
- x_j Input j

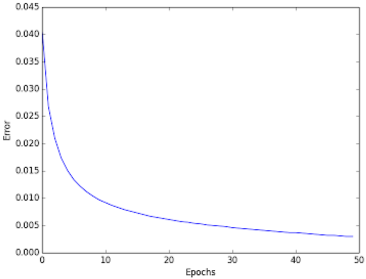
Each iteration is named epoch

Learning algorithm (single neuron)

1. Apply input signal and compute outout
2. If output $==$ desired output, do nothing
3. If output $<$ desired output, increase weights
4. If output $>$ desired output, decrease weights

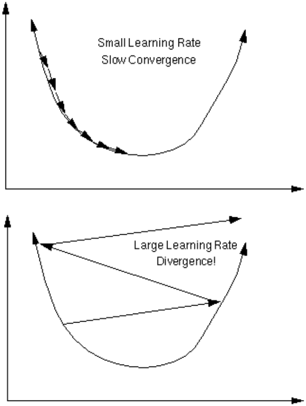
Training algorithms

Gradient Descent Algorithm (III)



(Source)

Learning rate



(Source)

Training algorithms

Stochastic Gradient Descent (I)

SDG approximates the gradient sampling the dataset

On-line	One sample
Mini-batch	Several samples
Batch	All the samples (Gradient Descent)

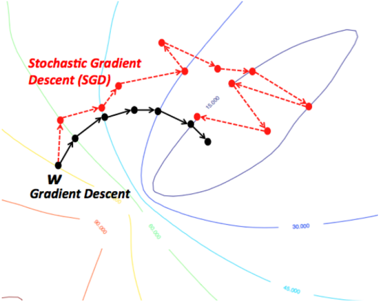
Weights update rule: $w^{k+1} = w^k - \alpha \nabla g(in)$

- where α is the learning rate

SGD is slow and prone to local minima

Training algorithms

Stochastic Gradient Descent (II)



(Source)

Usually, a momentum is introduced: $w^{k+1} = w^k - \alpha z^{k+1}$, where $z^{k+1} = \beta z^k + \nabla g(in)$

- α is the learning rate
- β is the momentum strength
- If $\beta = 0$ then gradient descent

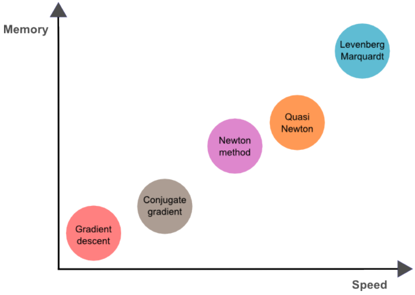
(On-line demo)

Training algorithms

Other optimization algorithms

Other optimization algorithms

- Newton's method
- Quasi-Newton's method
- Levenberg-Marquardt method
- Conjugate Gradient



(Source)

Training algorithms

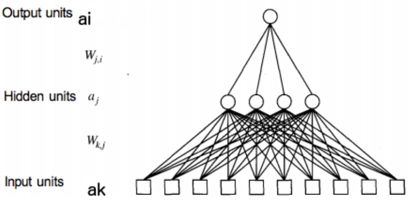
Backpropagation algorithm (I)

Efficient learning algorithm for multilayer perceptrons. Three steps

1. **Feed-forward step**. Feed input, compute output and error
2. **Feed-backward step**. Compute individual contribution to error
3. **Adjust weights.** Modify weights to minimize error: Input, output and hidden layers

Training algorithms

Backpropagation algorithm (II)



Output layer: Same as single neuron

$$W_j \leftarrow W_j + \alpha \times Err \times g'(in) \times x_j$$

Define modified error as
$\Delta_i = Err_i \times g'(in_i)$, then

$$W_{j,i} \leftarrow W_{j,i} + \alpha \times a_j \times \Delta_i$$

Training algorithms

Backpropagation algorithm (III)

Hidden layer: Propagate error

$$W_{k,j} \leftarrow W_{k,j} + \alpha \times a_k \times \Delta_j$$

where

$$\Delta_j = g'(in) \times \sum_j W_{j,i} \Delta_i$$

Backpropagation algorithm

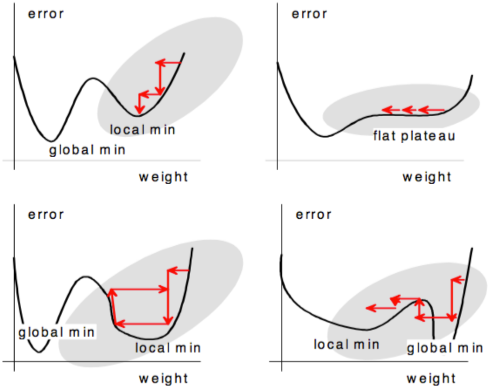
1. Compute output
2. Compute output error Δ
3. For each layer, repeat the following steps
   3.1 Propagate *Delta* backwards
   3.2 Update weights between two layers

Training algorithms

Learning problems

Potential problems

- Local minima
- Flat plateau
- Oscillation
- Missing good minima



Training algorithms

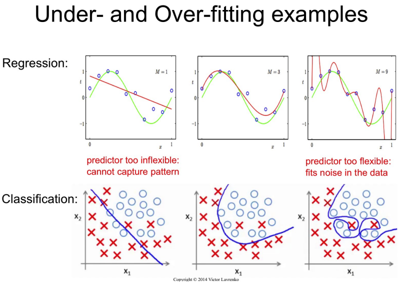
Learning problems: Under and overfitting (I)

Underfitting: Does not learn

- Topology too simple

Overfitting: Memorizes samples

- Topology too complex
- Perhaps, the most serious concern in ML
- The net fails when exposed to new data



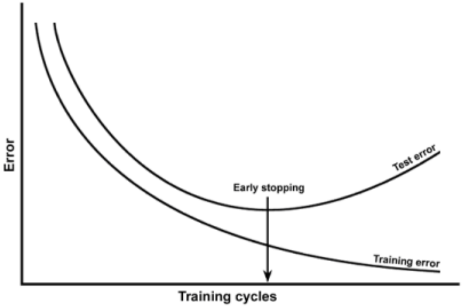
(Source)

Training algorithms

Learning problems: Under and overfitting (II)

Solution: Evaluate generalization capabilities

- Split training and validation sets and measure errors



(Source)

Acknowlegements

- Jesus Aguilar Ruiz, Pablo de Olavide, Seville, Spain
- Daniel Rodríguez, Universidad de Alcalá, Alcalá de Henares, Spain