The *jet* colormap is a rainbow colormap containing all the color hues in the visible spectrum. This colormap can cause problems when reading data. For example, suppose a male doctor has trouble seeing the difference between red and green—a condition called deuteranomaly. He may be able to read the data effectively, but you don't want to have any doubts about the diagnosis. Your job is to find a colormap that is colorblind-friendly using the Calculus of Variations.

First, you need a model of color distance. Fortunately, the CIELab color space models the perceived distance between colors using three variables and a distance metric called δE, which is usually the Euclidean distance

$$\delta E = \sqrt{(L_2 - L_1)^2 + (a_2 - a_1)^2 + (b_2 - b_1)^2}$$

between two colors, (L_1, a_1, b_1) and (L_2, a_2, b_2). The axis L measures color lightness, taking values in $[0, 100]$. The variable a represents the green-to-red axis; negative values are green and positive values are red. The variable b represents the blue-to-yellow axis; negative values are blue and positive values are yellow. When both a and b are zero, the color is a shade of grey since it is completely described by the value of L. The values of a and b lie approximately in the range $[-45, 45]$.

Fix $L = 50$ and consider the a and b axes. Extreme values of a and b will have highly saturated colors and the middle around $a = b = 0$ is grey. It will be progressively harder to tell colors apart as the sequence of colors moves toward the origin because L is fixed. However, if the sequence of colors passes through lots of hues (like the rainbow), it will not be as colorblind friendly. The goal is to minimize the path through Lab space given the constraints that the path must be friendly to deuteranomalous viewers and avoid the grey zone.

Suppose the grey zone is described by a circle $a^2 + b^2 = r^2$ with radius $r = 10$. Now suppose that deuteranomaly forces $\tilde{a} = |\,a|$, meaning that the viewer can't see green. These two constraints can be described by a semi-circle about the origin: $\sqrt{100 - b^2} - a = 0$. Choose the starting color to be the saturated blue at the point $(b, a) = (-45, 0)$ and the end color to be the saturated yellow at $(b, a) = (45, 0)$.

The first minimization problem is to find the path from $(-45, 0)$ to the edge of the semi-circle, then from the edge to the peak at $(0, 10)$ where $h(b, a) = 0$. Treating a as a function of b, the functional is

$$J[a] = \int_{-45}^{b^*} \delta E(b, a, a')\, db + \int_{b^*}^{0} [\delta E(b, a, a') + \mu(b)h(b, a)]\, db$$

where $h(b, a) = \sqrt{100 - b^2} - a$ and μ is the Lagrange multiplier. Since this problem is symmetric, the path from the grey zone peak to yellow will be the solution to the above problem but reflected about the a axis.

NOTE Inspiration for this example came from https://bids.github.io/colormap/. This problem could be extended into a lab by using the colorspacious and viscm Python modules developed by the authors at the same website. For more information about colorblindness and the Lab color space, see the colorspacious documentation at colorspacious.readthedocs.io and the Lab color space Wikipedia page.