ODEs Boot Camp

Zombies invasion

Welcome to the ODEs Bootcamp!

The aim of this project will be to simulate a Zombie invasion. It could be based on a movie, a book, a videogame or whatever you want, just be creative.

Table of Contents

Example

The following is an example based 100% on Robert Smith? (the question mark is actually part of his lastname) paper called: **WHEN ZOMBIES ATTACK!: MATHEMATICAL MODELLING OF AN OUTBREAK OF ZOMBIE INFECTION.** So, I cheated but you can't, your model cannot be any of the ones mentioned in that article.

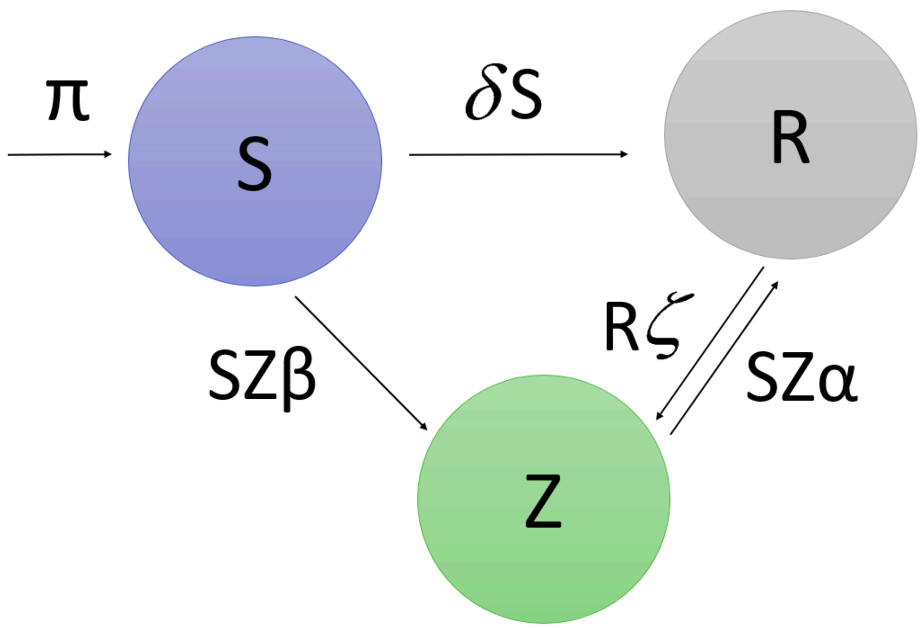
Model

Let's assume we have 3 basic classes:

- **Susceptibles (S):** They can become zombies (parameter β) or die of "natural" causes (parameter δ).
- **Zombies (Z):** They can also die (parameter α).
- **Removed (R):** Individuals who have died through attack or "natural" causes.

Assumptions:

- Removed humans can resurrect or become zombies.
- Susceptibles can become zombies through transmission via an encounter with a zombie.
- There is a constant birth rate Π.
- There are just 2 ways of becoming a zombie: they resurrect from the newly deceased or they are susceptibles who have lost an encounter with a zombie.



ODEs system

$$\frac{\mathrm{dS}}{\mathrm{dt}} = \Pi - \beta \mathrm{SZ} - \delta S\,,$$

$$\frac{\mathrm{dZ}}{\mathrm{dt}} = \beta \mathrm{SZ} + \zeta R - \alpha \mathrm{SZ}\,,$$

$$\frac{\mathrm{dR}}{\mathrm{dt}} = \delta S + \alpha \mathrm{SZ} - \zeta R\,.$$

The **mass-action incidence** specifies that an average member of the population makes contact sufficient time to transmit infection with βN others per unit time, where *N* is the total population without infection. Then, the probability that a random contact by a zombie is made with a susceptible is *S/N*. So, the total number of new zombies through this transmision process in unit time per zombie is,

$$(\beta N)\left(\frac{S}{N}\right) Z = \beta \mathrm{SZ}.$$

The ODEs satisfy

$$S' + Z' + R' = \Pi\,,$$

and hence as $t \to \infty$ if $\Pi \neq 0$

$$S + Z + R \to \infty\,,$$

since $S \neq \infty$ an outbreak of zombies will lead to the collapse of a civilisation as a large number of people are zombified or dead.

Equilibrium

If an outbreak happens over a short period we can ignore birth and background rates so, we set $\Pi = \delta = 0$. Setting the differential equations to 0 gives,

$$-\beta \mathrm{SZ} = 0\,,$$

$$\beta \mathrm{SZ} + \zeta R - \alpha \mathrm{SZ} = 0\,,$$

$$\alpha \mathrm{SZ} - \zeta R = 0\,.$$

This means that either $S = 0$ (doomsday equilibrium) or $Z = 0$ (disease-free equilibrium).

Doomsday equilibrium

$(\bar{S}, \bar{Z}, \bar{R}) = (0, \bar{Z}, 0\,)$

Disease-free equilibrium

$(\bar{S}, \bar{Z}, \bar{R}) = (N, 0, 0)$

which means human-zombie coexistance is impossible.

The Jacobian is

$$J = \begin{bmatrix} -\beta Z & -\beta S & 0 \\ \beta Z - \alpha Z & \beta S - \alpha S & \zeta \\ \alpha Z & \alpha S & -\zeta \end{bmatrix}.$$

The Jacobian for the **disease-free equilibrium** is,

$$J = \begin{bmatrix} 0 & -\beta N & 0 \\ 0 & \beta N - \alpha N & \zeta \\ 0 & \alpha N & -\zeta \end{bmatrix}.$$

We have,

$$\det(J - \lambda I) = -\lambda\{\lambda^2 + [\zeta - (\beta - \alpha)N]\lambda - \beta\zeta N\}.$$

The characteristic equation always has a root with positive real part so equilibrium is always unstable.

The Jacobian for **doomsday equilibrium** is,

$$J = \begin{bmatrix} -\beta\bar{Z} & 0 & 0 \\ \beta\bar{Z} - \alpha\bar{Z} & 0 & \zeta \\ \alpha\bar{Z} & 0 & -\zeta \end{bmatrix}.$$

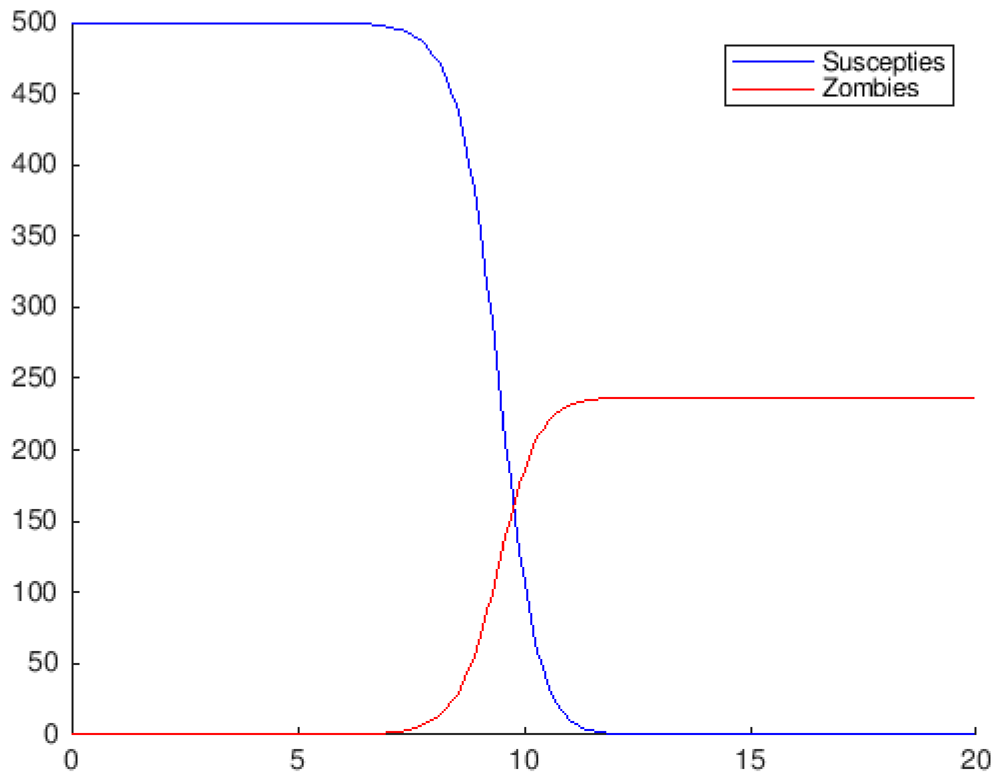
We have,

$$\det(J - \lambda I) = -\lambda(-\beta\bar{Z} - \lambda)(-\zeta - \lambda).$$

Since all the eigenvalues of the doomsday equilibrium are negative, it is asymptotically stable which means that, in a short outbreak zombies will likely infect everyone.

Simulation

```
zombies(0.005,0.0095,0.0001,0.0001,20,0.1)
```



Function

```
function [ ] = zombies(a,b,ze,d,T,dt)
% This function will solve the system of ODE’s for the basic model used in
% the Zombie Dynamics project for MAT 5187. It will then plot the curve of
% the zombie population based on time.
% Function Inputs:
% a - alpha value in model: "zombie destruction" rate
% b - beta value in model: "new zombie" rate
% ze - zeta value in model: zombie resurrection rate
% d - delta value in model: background death rate
% T - Stopping time
% dt - time step for numerical solutions
% Created by Philip Munz, November 12, 2008
%Initial set up of solution vectors and an initial condition
N = 500; %N is the population
n = T/dt;
t = zeros(1,n+1);
s = zeros(1,n+1);
z = zeros(1,n+1);
r = zeros(1,n+1);
s(1) = N;
z(1) = 0;
r(1) = 0;
t = 0:dt:T;
```

```
% Define the ODE’s of the model and solve numerically by Euler’s method:
for i = 1:n
    s(i+1) = s(i) + dt*(-b*s(i)*z(i)); %here we assume birth rate = background deathrat
    z(i+1) = z(i) + dt*(b*s(i)*z(i) -a*s(i)*z(i) +ze*r(i));
    r(i+1) = r(i) + dt*(a*s(i)*z(i) +d*s(i) - ze*r(i));
    if s(i)<0 || s(i) >N
    break
    end
    if z(i) > N || z(i) < 0
    break
    end
    if r(i) <0 || r(i) >N
    break
    end
end
hold on
plot(t,s,'b');
plot(t,z,'r');
legend('Suscepties','Zombies')

end
```

References

P. Munz, I. Hudea, J. Imad and R.J. Smith? When zombies attack!: Mathematical modelling of an outbreak of zombie infection (Infectious Disease Modelling Research Progress 2009, in: J.M. Tchuenche and C. Chiyaka, eds, pp133-150).

And just for fun:

http://mattbierbaum.github.io/zombies-usa/