Benchmarking Computations

The MW-n-Rents Guys

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1. Theoretical Framework

Set-up

For simplicity, focus on the supply and demand of housing in a given zipcode. Assume that two groups exist within each zipcode, MW and non-MW households (HH). The former are fully affected by the MW, whereas the latter are not affected at all.

Notation:

- H represents a continuous measure of housing units available in the zipcode
- r is the rent, $\underline{w}$ and w represent the *monthly income* of MW HH and non-MW HH, respectively
- $\underline{H}(r, \underline{w})$ is the demand of housing units of MW HH
- $\overline{H}(r, w)$ is the demand of housing units of non-MW HH
- $H(r)$ is the supply of housing

Derivatives:

- $\underline{H}_r(r, \underline{w}) < 0$ and $\overline{H}_r(r, w) < 0$, i.e., the demand for housing is downward sloping
- $\underline{H}_w(r, \underline{w}) > 0$ and $\overline{H}_w(r, w) > 0$, i.e., housing demand is increasing in income
- $H'(r) > 0$, meaning that housing supply is upward sloping

Equilibrium

The local housing market is in equilibrium if

$$H(r) = \underline{H}(r, \underline{w}) + \overline{H}(r, w)$$

Write this condition as $H(e^{\ln r}) = \underline{H}(e^{\ln r}, e^{\ln \underline{w}}) + \overline{H}(e^{\ln r}, e^{\ln w})$ and fully differentiate with respect to $\ln r$ and $\ln \underline{w}$ to obtain.

$$rH'(r) d \ln r = \left(r\underline{H}_r + r\overline{H}_r\right) d \ln r + \underline{w} H_w d \ln \underline{w}$$

Reorder to get the elasticity of rents to the minimum wage:

$$\frac{d \ln r}{d \ln \underline{w}} = -\frac{\underline{w}\ \underline{H}_w}{r\underline{H}_r + r\overline{H}_r - rH'(r)} \tag{1}$$

Note that, since $\underline{H}_r < 0$ and $\overline{H}_r < 0$, the above expression is always positive. When the MW increases the local housing market moves to a new equilibrium with higher rents.

Constant Elasticity functions

Assume that all functions are constant elasticity, namely:

- $\underline{H}(r, \underline{w}) = Ae^{-\underline{\gamma} \ln r + \underline{\beta} \ln \underline{w}}$
- $\overline{H}(r, w) = Be^{-\overline{\gamma} \ln r + \overline{\beta} \ln w}$
- $H(r) = Ce^{k \ln r}$

where $A, B, C > 0$ are constants, and $\underline{\gamma}, \underline{\beta}, \overline{\gamma}, \overline{\beta}, k > 0$ are elasticities.[1] These functional forms imply that

- $\underline{H}_r = -\underline{\gamma}\frac{\underline{H}}{r}$ and $\overline{H}_r = -\overline{\gamma}\frac{\overline{H}}{r}$
- $\underline{H}_w = \underline{\beta}\frac{\underline{H}}{\underline{w}}$
- $H'(r) = k\frac{H}{r}$

Replacing the previous expression, we can rewrite (1) as

$$\frac{d \ln r}{d \ln \underline{w}} = \frac{\underline{w}\underline{\beta}\frac{\underline{H}}{\underline{w}}}{-r\underline{\gamma}\frac{\underline{H}}{r} - r\overline{\gamma}\frac{\overline{H}}{r} + rk\frac{H}{r}}$$

Simplifying

$$\frac{d \ln r}{d \ln \underline{w}} = \frac{\underline{\beta}\ \underline{H}}{-\underline{\gamma}\ \underline{H} - \overline{\gamma}\overline{H} + kH}$$

Now define the following parameters:

- $\underline{s} = \frac{\underline{H}}{H}$: share of housing occupied by MW HH
- $\overline{s} = \frac{\overline{H}}{H}$: share of housing occupied by non-MW HH.

Note that $\overline{s} = 1 - \underline{s}$

Then, we reach to the following expression of interest:

$$\frac{d \ln r}{d \ln \underline{w}} = \frac{\underline{\beta}\ \underline{s}}{-\underline{\gamma}\ \underline{s} - \overline{\gamma}(1 - \underline{s}) + k} \tag{2}$$

2. Simulation

Define the following function

```
elasticity_rent_MW <- function(beta_low, s_low,
                               gamma_low, gamma_high, k) {
  numerator   = beta_low*s_low
  denominator = gamma_low*s_low + gamma_high*(1-s_low) + k

  return(numerator/denominator)
}
```

Suppose that the share of MW is $\underline{s} = 0.3$, so that $\overline{s} = 0.7$.

Finally, take the following values for the elasticities:

- $\underline{\beta} = 0.1$, so housing demand is inelastic to income
- $\underline{\gamma} = 0.7$ and $\overline{\gamma} = 0.5$, so MW HH are more sensible to rents

[1] Take $H(r)$ as example to show that these functions are constant elasticity. Taking natural logarithms yields

$$\ln H(r) = \ln C + k \ln r.$$

Thus, the elasticity of supply is $\frac{d \ln H(r)}{d \ln r} = k$,

- $k = 0.1$, similar to estimate of Diamond (2016, Table 5)

```r
elasticity_rent_MW(0.1, 0.3, 0.7, 0.5, 0.1)
```

```
## [1] 0.04545455
```

Our estimate is right in the ballpark of this estimate