

# Identification of MW effects with spillovers across units for “Minimum Wage as a Place-Based Policy”

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Consider the causal model for rents given by

$$r_{it} = r_{it}(\{\underline{w}_{zt}\}_{z \in \mathcal{Z}})$$

where  $\underline{w}_{zt}$  represents the log of the statutory MW, which we also refer to as the “dose” of treatment received by unit  $i$  from ZIP code  $z$  in period  $t$ . The set  $\mathcal{Z}$  contains all ZIP codes in a closed metropolitan area.

We assume that the effects of MW across locations can be summarized in the residence and workplace MW measures, so that

$$r_{it} = r_{it}(\underline{w}_{it}^{\text{res}}, \underline{w}_{it}^{\text{wkp}}). \quad (1)$$

We follow Angrist and Imbens (1995) and define the treatment effects of interest as follows. Focus on the workplace MW conditioning on a level of the residence MW. A unit’s causal response is  $\partial r_{it}(\underline{w}_{it}^{\text{res}}, \underline{w}_{it}^{\text{wkp}}) / \partial \underline{w}_{it}^{\text{wkp}}$ . The average causal response for a given dose  $w$  of the workplace MW is

$$ACRT^{\text{wkp}}(d|\underline{w}^{\text{res}}, w) = \frac{\partial \mathbb{E}[r_{it}(\underline{w}^{\text{res}}, l) | \underline{w}^{\text{res}}, w]}{\partial l} \Big|_{l=w}$$

and the average causal response for any level of the workplace MW is

$$ACR^{\text{wkp}}(d|\underline{w}^{\text{res}})^{\text{wkp}} = \frac{\partial \mathbb{E}[r_{it}(\underline{w}^{\text{res}}, w)]}{\partial w}.$$

We are also interested in analogous parameters for the residence MW.

We study the effects of the following policy.

**Definition 1** (Policy change). *In period  $t - 1$ , all locations are bound by a common MW level,  $\underline{W}_{i,t-1} = \underline{W}_0$  for all  $i$ . In period  $t$ , the MW increases to  $\underline{W}_1 > \underline{W}_0$  in some (directly treated) ZIP codes  $i \in \mathcal{Z}_1 \subset \mathcal{Z}$  for non-empty  $\mathcal{Z}_1$ . The MW levels in  $t$  are then  $\underline{W}_{i,t} = \underline{W}_0$  for  $i \notin \mathcal{Z}_1$  and  $\underline{W}_{i,t} = \underline{W}_1$  for  $i \in \mathcal{Z}_1$ . Thus, for each  $i$  the residence MW levels are*

$$\underline{w}_{i,t}^{\text{res}} = \begin{cases} \ln \underline{W}_1 & \text{if } i \in \mathcal{Z}_1 \\ \ln \underline{W}_0 & \text{if } i \notin \mathcal{Z}_1 \end{cases} \quad \text{and} \quad \underline{w}_{i,t-1}^{\text{res}} = \ln \underline{W}_0,$$

and the workplace MW levels are

$$\underline{w}_{i,t}^{wkp} = \sum_{z \in \mathcal{Z}} \pi_{iz} \ln W_{i,t}$$

where  $\{\pi_{iz}\}$  are (unchanging) commuting shares.

We are envisioning a policy change in which a subset of ZIP codes in a metropolitan area raise the level of the MW from a previously uniform level.

**Assumption 1** (Parallel trends). *Consider the policy change in Definition 1. We assume that, for all dose levels  $w > \underline{w}_0 = \ln \underline{W}_0$ ,*

$$\mathbb{E} \left[ r_{it}(\underline{w}_0, \underline{w}_0) - r_{i,t-1}(\underline{w}_0, \underline{w}_0) | \underline{w}_{it}^{wkp} = w \right] = \mathbb{E} \left[ r_{it}(\underline{w}_0, \underline{w}_0) - r_{i,t-1}(\underline{w}_0, \underline{w}_0) | \underline{w}_{it}^{wkp} = \underline{w}_0 \right] \quad (2)$$

and that

$$\mathbb{E} \left[ r_{it}(\underline{w}_0, w) - r_{i,t-1}(\underline{w}_0, w) | \underline{w}_{it}^{res} = \underline{w}_1 \right] = \mathbb{E} \left[ r_{it}(\underline{w}_0, w) - r_{i,t-1}(\underline{w}_0, w) | \underline{w}_{it}^{res} = \underline{w}_0 \right]. \quad (3)$$

Equation (2) imposes that, conditional on not having changed the residence MW, the counterfactual evolution of rents is the same in ZIP codes that received any dose of the workplace MW  $w$ . Equation (3) imposes that, conditional on a given value of the workplace MW, the counterfactual evolution of rents across those ZIP codes directly treated and those not directly treated is the same.

Focusing on the workplace MW, under Assumption 1 Proposition 3 in Callaway, Goodman-Bacon, and Sant'Anna (2021) implies that

$$\frac{\partial \mathbb{E} \left[ r_{it}(\underline{w}_0, w) | \underline{w}_{it}^{res} = \underline{w}_0, \underline{w}_{it}^{wkp} = w \right]}{\partial w} = ACRT^{wkp}(w | \underline{w}_0, w) + \frac{\partial ATT^{wkp}(w | \underline{w}_0, l)}{\partial w} \Big|_{l=w}$$

where  $ATT^{wkp}(w | \underline{w}_0, w) = \mathbb{E} \left[ r_{it}(\underline{w}_0, w) - r_{it}(\underline{w}_0, \underline{w}_0) | \underline{w}_{it}^{res} = \underline{w}_0, \underline{w}_{it}^{wkp} = w \right]$ . The slope of average rents with respect to the minimum wage identifies the average causal response of interest plus a selection bias term that arises due to differences in treatment effects across groups.

We have a discrete jump in the residence MW. Then, under Assumption 1 as well, Proposition 3 in Callaway, Goodman-Bacon, and Sant'Anna (2021) implies that

$$\begin{aligned} & \mathbb{E} \left[ \Delta r_{it}(\underline{w}_1, w) | \underline{w}_{it}^{res} = \underline{w}_1, \underline{w}_{it}^{wkp} = w \right] - \mathbb{E} \left[ \Delta r_{it}(\underline{w}_0, w) | \underline{w}_{it}^{res} = \underline{w}_0, \underline{w}_{it}^{wkp} = w \right] \\ &= ACRT^{res}(\underline{w}_1 | \underline{w}_1, w) + ATT^{res}(\underline{w}_0 | \underline{w}_1, w) - ATT^{res}(\underline{w}_0 | \underline{w}_0, w) \end{aligned}$$

where  $ATT^{res}(\underline{w}_1 | \underline{w}_1, w) = \mathbb{E} \left[ r_{it}(\underline{w}_1, w) - r_{it}(\underline{w}_0, w) | \underline{w}_{it}^{wkp} = w \right]$ . For a fixed level of the workplace MW, a difference in differences between those directly treated (with  $\underline{w}_{i,t}^{res} = \underline{w}_1$ ) and those indirectly treated (with  $\underline{w}_{i,t-1}^{res} = \underline{w}_0$ ) identifies the causal response of interest plus a selection bias term.

As Callaway, Goodman-Bacon, and Sant'Anna (2021) point out, the usual parallel trends assumption is not enough to identify the treatment effects because there may be selection of units to a different dosage level. To remove these selection bias terms we could rely on a stronger version of the parallel trends assumption (see Assumption 5 in Callaway, Goodman-Bacon, and Sant'Anna 2021).

However, as the authors explain in Remark 5, an alternative is to directly assume that the selection bias terms are zero. In this setting, we are willing to maintain this assumption. The reason is that we are assuming an unchanging commuting structure, and so units cannot endogenously affect the amount of dose they receive. We think that conditioning on the level of the other MW measure makes this assumption plausible.

What comparisons are we making to get these estimates? To obtain the workplace MW we exploit the slope of the relationship between the change in the workplace MW and the change in rents, conditioning on units that were not directly treated. For the residence MW we compare directly treated and directly untreated units that have a similar level of the workplace MW.