Benchmarking Advanced

The MW-n-Rents Guys

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1. Theoretical Framework

1.1 Set-up

Imagine there are a set of zipcodes $\mathcal{Z}$. There is an exogenously given distribution of people with differing residence n and workplace i locations. Denote by L_{ni} the number of people from n who work in i; by $L_n = \sum_{i\in\mathcal{Z}} L_{ni}$ and $L_i = \sum_{n\in\mathcal{Z}} L_{ni}$ the number of residents in n and workers in i, respectively; and by $\mathcal{L} = \sum_{n\in\mathcal{Z}} L_n = \sum_{i\in\mathcal{Z}} L_i$ the total number of workers.

Furthermore, let:

- $\underline{w}_i$ be the wage paid in location i;
- $H_n\left(r_n, y_n\right)$ be demand for housing in n, where r_n represents rents, and y_n is disposable income per capita available in n (defined below);
- $D_n(r_n)$ be the exogenously given measure of housing units in n.

The derivatives of housing demand are

$$\frac{dH_n(r_n, y_n)}{dr} < 0, \qquad \frac{dH_n(r_n, y_n)}{dY_n} > 0.$$

whereas housing supply is increasing in rents, so $\frac{dD_n(r_n)}{dr_n} > 0$.

Per capita disposable income is defined as a weighted average of income generated in workplace locations of residents of n:

$$y_n = \sum_{i\in\mathcal{Z}} \pi_{ni} y_{ni}, \qquad \pi_{ni} := \frac{L_{ni}}{L_n}$$

where y_{ni} is disposable income of workers residing in n and working in i. This, in turn, depends on the minimum wage paid in the workplace i and the one paid in n, so that $y_{ni} = y(\underline{w}^L, \underline{w}^R)$ and

$$\frac{dy}{d\underline{w}^L} > 0, \qquad \frac{dy}{d\underline{w}^R} < 0,$$

where we use $(\underline{w}^L, \underline{w}^R)$ to avoid potentially confusing indexes in the defintion. We also, for simplicity, let $y(\cdot)$ be the the same across places.

Note that we let the effect of MWs on incomes unspecificed. We only ask that workplace minimum wages increase incomes, whereas residence ones decrease them. An increase in the wage in some workplace location increases disposable income because workers earn a higher wage. However, if there is a minimum wage in the residence location disposable income goes down. The reason is that local prices go up in that case.

Note that, for those who work and reside in n with minimum wage $\underline{w}$, we have

$$y_{nn} = y(\underline{w}_n, \underline{w}_n).$$

1.2. Equilibrium

The local housing market for ZIP code n is in equilibrium if

$$H_n(r_n, y_n) = D_n(r_n).$$

The question we want to ask is the following: what is the effect of increasing the wage in location $j \neq n$ (by imposing a MW)? What if there is an increase in the MW in n? What if the minimum wage increases in a set of locations $\mathcal{J} \subset \mathcal{Z}$?

1.2.1. Increase in minimum wage in a different location

Write the condition as

$$H_n\left(e^{\ln r_n}, \sum_{i\in\mathcal{Z}} \pi_{ni} y(e^{\ln \underline{w}_i}, e^{\ln \underline{w}_n})\right) = D_n(e^{\ln r_n})$$

and fully differentiate wrt $\ln r_n$, and $\ln w_j$ for some $j \neq n$ to get

$$\frac{dH_n}{dr_n} r_n d\ln r_n + \frac{dH_n}{dy_n}\frac{dy_{nj}}{d\underline{w}^L}\underline{w}_j \pi_{nj} d\ln \underline{w}_j = \frac{D_n}{dr_n} r_n d\ln r_n$$

Re-arranging, we get

$$\frac{d\ln r_n}{d\ln \underline{w}_j} = \frac{\pi_{nj}\frac{dH_n}{dy_n}\frac{dy_{nj}}{d\underline{w}^L}}{\left(\frac{D_n}{dr_n} - \frac{dH_n}{dr_n}\right)}\frac{\underline{w}_j}{r_n} \tag{1}$$

Since $\frac{dH_n}{dr_n} < 0$, we have that $\frac{d\ln r_n}{d\ln \underline{w}_j} < 0$.

1.2.2. Increase in wages in residence

Now, we assume that the MW increases marginally in n from an initial level of $\underline{w}_n$. This, in turn, affects disposable income of those living there through both arguments. Fully differentiate the market-clearing condition wrt to $\ln r_n$ and $\ln \underline{w}_n$ to get

$$\frac{dH_n}{dr_n} r_n d\ln r_n + \frac{dH_n}{dy_n}\underline{w}\left(\frac{dy_{nn}}{d\underline{w}^L}\pi_{nn} + \frac{dy_{nn}}{d\underline{w}^R}\right) d\ln \underline{w} = \frac{D_n}{dr_n} r_n d\ln r_n$$

We are interested in the elasticity

$$\frac{d\ln r_n}{d\ln \underline{w}} = \frac{\frac{dH_n}{dy_n}\left(\frac{dy_{nn}}{d\underline{w}^L}\pi_{nn} + \frac{dy_{nn}}{d\underline{w}^R}\right)}{\left(\frac{D_n}{dr_n} - \frac{dH_n}{dr_n}\right)}\frac{\underline{w}_n}{r_n} \tag{2}$$

It is easy to see that, in this case, the sign of the derivative is ambigous. The term $\left(\frac{dy_{nn}}{d\underline{w}^L}\pi_{nn} + \frac{dy_{nn}}{d\underline{w}^R}\right)$ represents the change in per capita disposable income in ZIP code n. If it's positive, so that the increase in wages of local workers is larger than the negative effect through raising local prices, then $\frac{d\ln r_n}{d\ln \underline{w}_n} > 0$. On the other hand, if it's negative and local prices increase a lot, lowering disposable income, we will have $\frac{d\ln r_n}{d\ln \underline{w}_n} < 0$.

1.2.3. Increase in wages in multiple workplace locations

We assume that the MW increases marginall in a subset of locations $\mathcal{J}$, where $n \notin \mathcal{J}$, from the level $\underline{w}$ common to all of them. Fully differentiate the market-clearing condition wrt to $\ln r_n$ and $\ln \underline{w}_j$, $j \in \mathcal{J}$, to get

$$\frac{dH_n}{dr_n} r_n d \ln r_n + \frac{dH_n}{dy_n} \sum_{j \in \mathcal{J}} \frac{dy_{nj}}{d\underline{w}^L} \underline{w} \pi_{nj} d \ln \underline{w} = \frac{D_n}{dr_n} r_n d \ln r_n.$$

The elasticity of rents to the MW is

$$\frac{d \ln r_n}{d \ln \underline{w}} = \frac{\frac{dH_n}{dy_n} \sum_{j \in \mathcal{J}} \frac{dy_{nj}}{d\underline{w}^L} \pi_{nj}}{\left(\frac{D_n}{dr_n} - \frac{dH_n}{dr_n} \right)} \frac{\underline{\tilde{w}}}{r_n} \tag{3}$$

The effect is unambigously positive. Furthermore, the more workers from n reside in other locations $j \in \mathcal{J}$, the stronger the effect will be.

1.2.4. Increase in wages in multiple workplace locations and residence

What if there is a MW increase in locations $j \in \tilde{\mathcal{J}} = \mathcal{J} \cup \{n\}$ from a common level $\underline{w}$. In that case, the derivative becomes

$$\frac{dH_n}{dr_n} r_n d \ln r_n + \frac{dH_n}{dy_n} \left(\sum_{j \in \mathcal{J}} \frac{dy_{nj}}{d\underline{w}^L} \pi_{nj} + \frac{dy_{nn}}{d\underline{w}^R} \right) \underline{w} d \ln \underline{w} = \frac{D_n}{dr_n} r_n d \ln r_n.$$

Then, our elasticity is

$$\frac{d \ln r_n}{d \ln \underline{w}} = \frac{\frac{dH_n}{dy_n} \left(\sum_{j \in \mathcal{J}} \frac{dy_{nj}}{d\underline{w}^L} \pi_{nj} + \frac{dy_{nn}}{d\underline{w}^R} \right)}{\left(\frac{D_n}{dr_n} - \frac{dH_n}{dr_n} \right)} \frac{\underline{\tilde{w}}}{r_n} \tag{4}$$

The sign of the effect will depend on the sign of $\left(\sum_{j \in \mathcal{J}} \frac{dy_{nj}}{d\underline{w}^L} \pi_{nj} + \frac{dy_{nn}}{d\underline{w}^R} \right)$.

If we relax the assumption that the MW is the same across places, we can write this expression in terms of the MW and the experienced MW.

1.3. Constant Elasticity functions

Assuming constant elasticity functions would simplify the parameters we are interested in.

2. Simulation

Simulate here.