Long-run effect

MW-rents

12/14/2020

Simplifying notation a bit and neglecting controls and time-period effects, the model is

$$y_t = \alpha + \gamma y_{t-1} + \sum_{r=0}^{s} \beta_r w_t + \epsilon_t$$

for some window s. First-differencing, we get

$$\Delta y_t = \gamma \Delta y_{t-1} + \sum_{r=0}^{s} \beta_r \Delta w_t + \Delta \epsilon_t$$

Initial equilibrium

Consider a long-run equilibrium with some exogenous level $w_{it} = \bar{w}$ and no shocks. The long-run outcome variable will be

$$\bar{y} = \sum_{r=0}^{s} \beta_r \bar{w} + \gamma \bar{y} \implies \bar{y} = \frac{\sum_{r=0}^{s} \beta_r}{1-\gamma} \bar{w}$$

To make the math easy, we consider $\bar{w} = 1$, so that

$$\bar{y} = \frac{\sum_{r=0}^{s} \beta_r}{1-\gamma}$$

We let $y_0 = \bar{y}$ in the exploration below.

Simulation: model in levels

Long run equilibrium and parameters are below

```
gamma <- 0.4
b0 <- 0.3
b1 <- 0.1
b2 <- 0.1
b3 <- 0

w_bar <- 1

y_bar <- (b0 + b1 + b2 + b3)/(1-gamma)
```

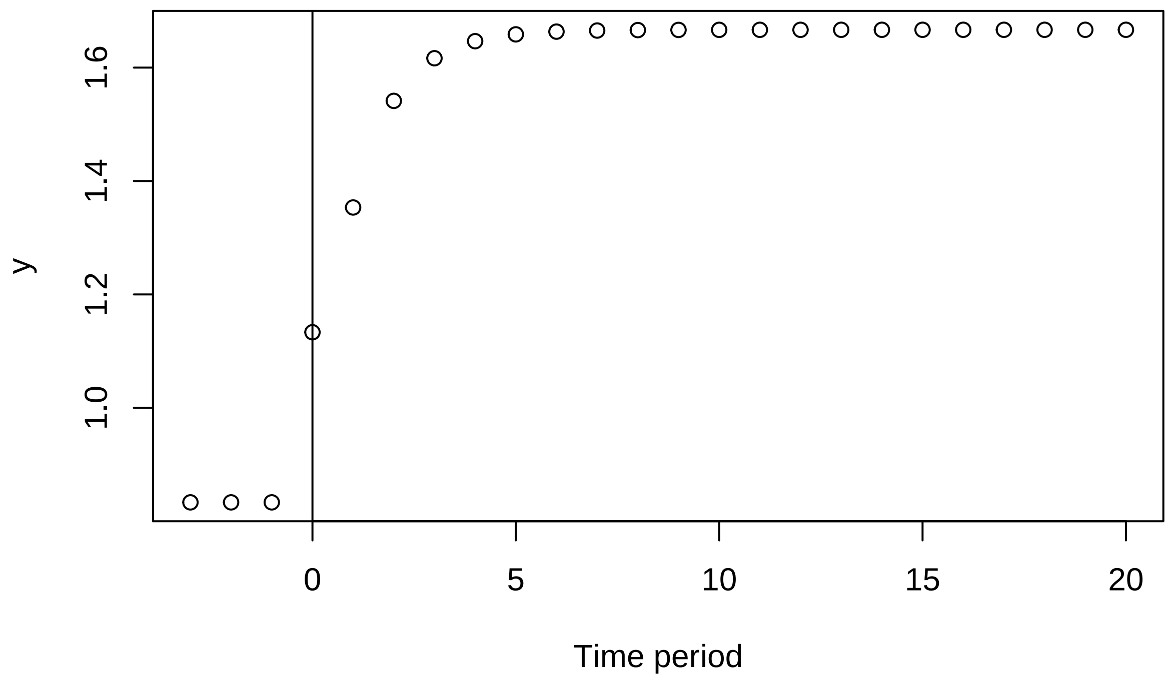
We now increase the MW by 1 unit and simulate the trajectory of y under the assumption of no shocks.

```r
n = 20

y <- rep(y_bar, n + 4)
w <- rep(w_bar, n + 4)

for (i in seq(4, n + 4)) { # index 4 is period zero
  w[i] <- w_bar + 1
  y[i] <- gamma*y[i-1] + b0*w[i] + b1*w[i-1] + b2*w[i-2] + b3*w[i-3]
}

plot(seq(-3, n), y, xlab = "Time period")
abline(v = 0)
```



The long run effect is

```r
y[length(y)] - y_bar
```

```
## [1] 0.8333333
```

which corresponds to $\frac{\sum_{r=0}^{s} \beta_r}{1-\gamma}$

```r
(b0 + b1 + b2)/(1-gamma)
```

```
## [1] 0.8333333
```

Simulation: model in first-differences

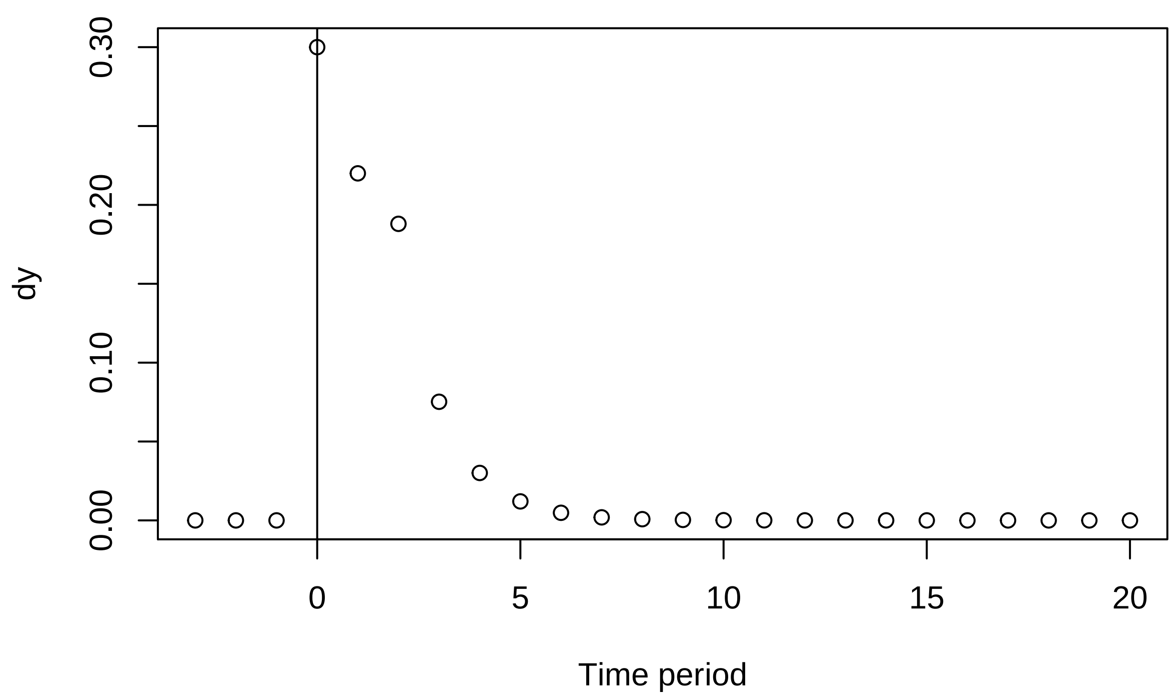
```
n = 20

dy <- rep(0, n + 4)  # Initially the df is zero
dw <- rep(0, n + 4)

for (i in seq(4, n + 4)) { # index 4 is period zero
  if (i == 4) dw[i] <- 1    # Increase in first period
  else        dw[i] <- 0

  dy[i] <- gamma*dy[i-1] + b0*dw[i] + b1*dw[i-1] + b2*dw[i-2] + b3*dw[i-3]
}

plot(seq(-3, n), dy, xlab = "Time period")
abline(v = 0)
```



The long run effect on y is given by the sum of the differences:

```
sum(dy)
```

```
## [1] 0.8333333
```

which is again equal to $\frac{\sum_{r=0}^{s} \beta_r}{1-\gamma}$.