

# Benchmarking Computations

The MW-n-Rents Guys

9/10/2020

## 1. Theoretical Framework

### Set-up

For simplicity, focus on the supply and demand of housing in a given zipcode. Assume that two groups exist within each zipcode, MW and non-MW households (HH). The former are fully affected by the MW, whereas the latter are not affected at all.

Notation:

- $H$  represents a continuous measure of housing units available in the zipcode
- $r$  is the rent,  $\underline{w}$  and  $w$  represent the *monthly income* of MW HH and non-MW HH, respectively
- $\underline{H}(r, \underline{w})$  is the demand of housing units of MW HH
- $\bar{H}(r, w)$  is the demand of housing units of non-MW HH
- $H(r)$  is the supply of housing

Derivatives:

- $\underline{H}_r(r, \underline{w}) < 0$  and  $\bar{H}_r(r, w) < 0$ , i.e., the demand for housing is downward sloping
- $\underline{H}_w(r, \underline{w}) > 0$  and  $\bar{H}_w(r, w) > 0$ , i.e., housing demand is increasing in income
- $H'(r) > 0$ , meaning that housing supply is upward sloping

### Equilibrium

The local housing market is in equilibrium if

$$H(r) = \underline{H}(r, \underline{w}) + \bar{H}(r, w)$$

Write this condition as  $H(e^{\ln r}) = \underline{H}(e^{\ln r}, e^{\ln \underline{w}}) + \bar{H}(e^{\ln r}, e^{\ln w})$  and fully differentiate with respect to  $\ln r$  and  $\ln \underline{w}$  to obtain.

$$rH'(r)d\ln r = (r\underline{H}_r + r\bar{H}_r) d\ln r + \underline{w}\underline{H}_w d\ln \underline{w}$$

Reorder to get the elasticity of rents to the minimum wage:

$$\frac{d\ln r}{d\ln \underline{w}} = -\frac{\underline{w}\underline{H}_w}{r\underline{H}_r + r\bar{H}_r - rH'(r)} \quad (1)$$

Note that, since  $\underline{H}_r < 0$  and  $\bar{H}_r < 0$ , the above expression is always positive. When the MW increases the local housing market moves to a new equilibrium with higher rents.

## Constant Elasticity functions

Assume that all functions are constant elasticity, namely:

- $\underline{H}(r, \underline{w}) = A e^{-\underline{\gamma} \ln r + \underline{\beta} \ln \underline{w}}$
- $\overline{H}(r, w) = B e^{-\overline{\gamma} \ln r + \overline{\beta} \ln w}$
- $H(r) = C e^{k \ln r}$

where  $A, B, C > 0$  are constants, and  $\underline{\gamma}, \underline{\beta}, \overline{\gamma}, \overline{\beta}, k > 0$  are elasticities.<sup>1</sup> These functional forms imply that

- $\underline{H}_r = -\underline{\gamma} \frac{\underline{H}}{r}$  and  $\overline{H}_r = -\overline{\gamma} \frac{\overline{H}}{r}$
- $\underline{H}_w = \underline{\beta} \frac{\underline{H}}{\underline{w}}$
- $H'(r) = k \frac{H}{r}$

Replacing the previous expression, we can rewrite (1) as

$$\frac{d \ln r}{d \ln \underline{w}} = \frac{\underline{w} \underline{\beta} \frac{\underline{H}}{\underline{w}}}{-r \underline{\gamma} \frac{\underline{H}}{r} - r \overline{\gamma} \frac{\overline{H}}{r} + r k \frac{H}{r}}$$

Simplifying

$$\frac{d \ln r}{d \ln \underline{w}} = \frac{\underline{\beta} \underline{H}}{-\underline{\gamma} \underline{H} - \overline{\gamma} \overline{H} + k H}$$

Now define the following parameters:

- $\underline{s} = \frac{\underline{H}}{\underline{H}} \cdot$  share of housing occupied by MW HH
- $\overline{s} = \frac{\overline{H}}{\overline{H}} \cdot$  share of housing occupied by non-MW HH.

Note that  $\overline{s} = 1 - \underline{s}$

Then, we reach to the following expression of interest:

$$\frac{d \ln r}{d \ln \underline{w}} = \frac{\underline{\beta} \underline{s}}{-\underline{\gamma} \underline{s} - \overline{\gamma} (1 - \underline{s}) + k} \quad (2)$$

## 2. Simulation

Define the following function

```
elasticity_rent_MW <- function(beta_low, s_low,
                                gamma_low, gamma_high, k) {
  numerator    = beta_low*s_low
  denominator  = - gamma_low*s_low - gamma_high*(1-s_low) + k
  return(numerator/denominator)
}
```

Suppose that the share of MW is  $\underline{s} = 0.3$ , so that  $\overline{s} = 0.7$ .

Finally, take the following values for the elasticities:

- $\underline{\beta} = 0.1$ , so housing demand of the MW HH is inelastic to income
- $\underline{\gamma} = -0.7$  and  $\overline{\gamma} = -0.5$ , so MW HH are more sensible to rents

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<sup>1</sup>Take  $H(r)$  as example to show that these functions are constant elasticity. Taking natural logarithms yields

$$\ln H(r) = \ln C + k \ln r.$$

Thus, the elasticity of supply is  $\frac{d \ln H(r)}{d \ln r} = k$ ,

- $k = 0.1$ , similar to estimate of Diamond (2016, Table 5)

```
elasticity_rent_MW(0.1, 0.3, - 0.7, - 0.5, 0.1)
```

```
## [1] 0.04545455
```

Our estimate is right in the ballpark of this estimate