Benchmarking Computations

The MW-n-Rents Guys

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1. Static Model

Consider the following panel difference-in-differences model relating rents and the MW:

$$y_{it} = \alpha_i + \hat{\delta}_t + \beta \underline{w}_{it} + \epsilon_{it} \tag{1}$$

where:

- y_{it} is the log rent per square foot for the Zillow SFCC series,
- $\underline{w}$ is the log of the minimum wage, α_i is a zipcode fixed effect,
- α_i and α_t are fixed effects.

To remove the zipcode-specific interecept we estimate the model in in first-differences:

$$\Delta y_{it} = \delta_t + \beta \Delta \underline{w}_{it} + \Delta \epsilon_{it} \tag{2}$$

where $\delta_t = \hat{\delta}_t - \hat{\delta}_{t-1}$. Let $\Delta X_{it} = (D_t, \Delta \underline{w}_{it})$, where D_t is a time-period dummy common across all units. With this notation, the model becomes

$$\Delta y_{it} = \theta \Delta X_{it} + \Delta \epsilon_{it}$$

Alternative, we can write the model as deviations of time period means.[1]

Following Hansen (p. 621), we can write the estimate of equation (2) as follows:

$$\begin{aligned} \hat{\theta}_\Delta &= \left(\sum_i^N \sum_{t\geq 2} \Delta X_{it} \Delta X_{it}' \right)^{-1} \left(\sum_i^N \sum_{t\geq 2} \Delta X_{it} \Delta y_{it} \right) \\ &= \left(\sum_i^N \Delta \boldsymbol{X}_i \Delta \boldsymbol{X}_i' \right)^{-1} \left(\sum_i^N \Delta \boldsymbol{X}_i \Delta \boldsymbol{y}_i \right) \end{aligned} \tag{3}$$

where bold symbols are stacked observations for each individual –e.g., $\boldsymbol{X}_i = (X_{i2,..,X_{iT_i}})'$–, and N is the number of zipcodes. Replacing the model in the expression for the estimate we can write:

$$\hat{\theta}_\Delta - \theta = \left(\sum_i^N \Delta \boldsymbol{X}_i \Delta \boldsymbol{X}_i' \right)^{-1} \left(\sum_i^N \Delta \boldsymbol{X}_i \Delta \boldsymbol{\epsilon}_i \right)$$

[1] In particular, $\Delta \dot{X}_{it} = \Delta X_{it} - \frac{1}{N_t} \sum_{i \in S_t} \Delta X_i t$ denotes time-period means, where S_t is the set of zipcodes in the sample in period t, and N_t is the size of this set.

Therefore, for consistency of $\hat{\theta}_\Delta$ we need to assume *strict exogeneity* of covaraites:

$$E[\Delta \boldsymbol{X}_i \Delta\epsilon_{it}] = 0$$

This assumption corresponds to assumption 3 in Hansen (Assumption 7.2, p. 631). We also need the assumption that data $(\Delta\boldsymbol{\epsilon}_i, \Delta\underline{\boldsymbol{w}}_i)$, $i = 1, ..., N$ is i.i.d.

Hansen also makes an interesting argument for the case of unbalanced panels.

2. Dynamic Model

3. Partial Adjustment Model

Before we were assuming that curent values of $\underline{w}_{it}$ are not influenced by past values of y and ϵ (see Arellano and Honore 2001). We may want to relax that assumption by allowing for past rents or unobservables to (linearly) influence the MW. We say that MWs are *predetermined* if

$$E^*[\epsilon_{it}|\underline{w}_i^t, y_i^{t-1}] = 0 \quad (t = 2, ..., T)$$

where $x^t = (x_1, x_2, ..., x_t)$. This identifying assumption is less stringent than strict exogeneity because it implies that "current shocks are uncorrelated with past values of rents and current and past values of the MW", but "feedback effects from lagged dependent variables (or lagged errors) to current and future values of the explanatory variable are not ruled out". For example, ϵ_{it} is allowed to influence $\underline{w}_{i,t+1}$.

Arellano and Honore (2001) argue that this model, unlike the one with no dependent variable, restrict the serial correlation in the error term. Specifically, "errors in first differences exhibit first-order autocorrelation but are uncorrelated at all other lags."

Specify the model as

$$y_{it} = \gamma y_{i,t-1} + \alpha_i + \hat{\delta}_t + \beta \underline{w}_{it} + \epsilon_{it} \tag{4}$$

where γ is the auto-correlation coefficient in rents. The model in $t-1$ is

$$y_{i,t-1} = \gamma y_{i,t-2} + \alpha_i + \hat{\delta}_{t-1} + \beta \underline{w}_{i,t-1} + \epsilon_{i,t-1}.$$

Thus, the first-differenced model turns out to be

$$\Delta y_{it} = \gamma \Delta y_{i,t-1} + \delta_t + \beta \Delta \underline{w}_{it} + \Delta\epsilon_{it} \tag{5}$$

Note that $E[\Delta y_{i,t-1} \Delta\epsilon_{it}] \neq 0$. Therefore, estimation of this model requires IV.

An initial approach is to use $\Delta y_{i,t-2}$ as an intrument for $\Delta y_{i,t-1}$. This is called the "Anderson-Hsiao estimator." Arellano and Bond (1991) construct a GMM-like estimator for the above equation by exploiting the assumption that past values of the dependent variable should not predict the innovation $\Delta\epsilon_{it}$. (See Hansen sections 17.38 and 17.29, p. 670 onwards.)

References

- Arellano and Bond (1991). "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations". URL: https://www.jstor.org/stable/2297968
- Arellano and Honore (2001). Panel Data Models: Some Recent Developments. URL: https://econpapers.repec.org/bookchap/eeeecochp/5-53.htm

- Hansen (2020). *Econometrics.* URL: https://www.ssc.wisc.edu/~bhansen/econometrics/Econometrics.pdf