Model proposal

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In this note I lay down a simply model that exemplifies the potential opposing effects of the experienced versus the statutory MW and makes clear the mechanism behind "transfers of income across ZIP codes" that we have in mind.

A simple model of ZIP code's housing market

Set-up

Imagine there are a set of zipcodes $\mathcal{Z}$. There is an exogenously given distribution of people with differing residence n and workplace i locations. Denote by L_{ni} the number of people from n who work in i; by $L_n = \sum_{i\in\mathcal{Z}} L_{ni}$ and $L_i = \sum_{n\in\mathcal{Z}} L_{ni}$ the number of residents in n and workers in i, respectively; and by $\mathcal{L} = \sum_{n\in\mathcal{Z}} L_n = \sum_{i\in\mathcal{Z}} L_i$ the total number of workers.

Workers are characterized by residence and workplace location, but are otherwise identical to each other. Let $y_{ni} = y_{ni}(\underline{w}^L, \underline{w}^R)$ be disposable income of workers in the (n,i) cell, which depends on the minimum wage in the workplace $\underline{w}^L$ and the minimum wage in the residence $\underline{w}^R$. We assume that

$$\frac{dy_{ni}}{d\underline{w}^L} > 0, \quad \frac{dy_{ni}}{d\underline{w}^R} < 0 \qquad \forall (n,i),$$

so that workplace minimum wages increase disposable income, whereas residence ones lower them.

We denote housing demand by that group as $H_{ni}(r_n, y_{ni})$, where r_n represents rents. The derivatives of housing demand are

$$\frac{dH_n(r_n,\bar{y}_n)}{dr} < 0, \qquad \frac{dH_n(r_n,\bar{y}_n)}{d\bar{y}_n} > 0.$$

Finally, $D_n(r_n)$ is a measure of housing units supplied in n, with $\frac{dD_n(r_n)}{dr_n} > 0$.

Note that we let the effect of MWs on incomes unspecificed. We only ask that workplace minimum wages increase incomes, whereas residence ones decrease them. An increase in the wage in some workplace location increases disposable income because some workers commuting there earn a higher wage. However, if there is a minimum wage in the residence location disposable income goes down. The reason is that local prices go up in that case.

Note that, for those who work and reside in n with minimum wage $\underline{w}$, we have

$$y_{nn} = y(\underline{w}_n, \underline{w}_n).$$

Equilibrium and Comparative Statics

The local housing market for ZIP code n is in equilibrium if total demand for housing equals total supply. Formally,

$$\sum_{i \in \mathcal{Z}} L_{ni} H_{ni}(r_n, y_{ni}) = D_n(r_n).$$

Here is a proposition.

Proposition 1 *Under the assumptions of (i) fixed distribution of workers across residence and workplace ZIP codes, (ii) disposable income in a ZIP code is increasing in workplace MW, and (iii) disposable income in a ZIP code is decreasing in residence MW. Then,*

1. *conditional on changes in the statutory MW, changes in the experienced MW increase rents.*
2. *conditional on changes in the experienced MW, changes in the statutory MW decrease rents.*

Proof: Fully diferentiate the market clering condition with respect to $\ln r_n$, $\ln \underline{w}_i$ for all i and $\ln \underline{w}_n$ to get

$$\sum_i L_{ni} \frac{dH_{ni}}{dr} r_n d\ln r_n + \sum_i L_{ni} \frac{dH_{ni}}{dy} y_{ni} \left(\frac{dy_{ni}}{d\underline{w}^L} \underline{w}_i d\ln \underline{w}_i + \frac{dy_{ni}}{d\underline{w}^R} \underline{w}_n d\ln \underline{w}_n \right) = \frac{dD_n}{dr_n} r_n d\ln r_n$$

We can transform this expression into elasticities as follows. First, divide on both sides by the equlibrium condition $\sum_i L_{ni} H_{ni} = D_n$ and multiply and divide in the first two terms by L_n. This leads to

$$\sum_i \pi_{ni} \xi^r_{ni} d\ln r_n + \sum_i \pi_{ni} \xi^y_{ni} \epsilon^L_{ni} d\ln \underline{w}_i + \sum_i \pi_{ni} \xi^y_{ni} \epsilon^R_{ni} d\ln \underline{w}_n = \eta_n d\ln r_n$$

where $\pi_{ni} = \frac{L_{ni}}{L_n}$ represent the share of workers from n working in i; $\xi^r_{ni} = \frac{dH_{ni}}{dr} \frac{r_n}{\sum_i \pi_{ni} H_{ni}}$ and $\xi^y_{ni} = \frac{dH_{ni}}{dy} \frac{y_{ni}}{\sum_i \pi_{ni} H_{ni}}$ are the elasticities of housing demand to rents and income evaludated at the weighted average housing demand of zipcode n; and $\epsilon^L_{ni} = \frac{dy_{ni}}{d\underline{w}^L} \frac{\underline{w}_i}{y_{ni}}$ and $\epsilon^R_{ni} = \frac{dy_{ni}}{d\underline{w}^R} \frac{\underline{w}_n}{y_{ni}}$ are the elasticities of disposable income of group (n, i) to changes in either workplace or residence minimum wage; and $\eta_n = \frac{dD_n}{dr_n} \frac{r_n}{D_n}$ is the elasticity of housing supply in ZIP code n.

Solving for $d\ln r_n$ in the above expression we can already see that a workplace MW increase unambiguosly increases rents (as $\epsilon^L_{ni} > 0$), whereas a residence MW increase will have a muted effect.[1] From the expression it is clear that, conditional on workplace MWs, residence MWs (i.e., statutory MWs) lower rents.

To simplify further, assume that elasticities of disposable income don't vary by workplace location, so that $\epsilon^x_{ni} = \epsilon_n \ \forall i \in \mathcal{Z}$ and $x \in \{L, R\}$. Then, we can write

$$d\ln r_n = \frac{\epsilon^L_n}{\eta_n - \sum_i \pi_{ni} \xi^r_{ni}} \sum_i \pi_{ni} d\ln \underline{w}_i + \frac{\epsilon^R_n}{\eta_n - \sum_i \pi_{ni} \xi^r_{ni}} d\ln \underline{w}_n.$$

The first term represents the effect of experienced (log) MW changes, whereas the second one represents statutory MW changes.

$\square$

[1] The sign of the effect depends on the sign of $\pi_{nn} \epsilon^L_{nn} + \sum_i \pi_{ni} \epsilon^R_{ni}$. It the decrease in disposable income in n is strong enough (second term), then rents will go down.