Model for “Do Minimum Wages Increase Rents? Evidence from U.S. ZIP codes using High Frequency Data”

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In this note we attempt to develop an urban model for the project. The model should:

- Make more transparent the mechanisms behind our effect.
- Allow for welfare calculations
- Maybe, justify the experienced MW

1. Set-up

Consider a metropolitan area composed by Z ZIP codes, indexed by n and i. There are two types of workers $s \in \{\ell, h\}$, who maximize utility by chosing a residence-workplace pair (n, i). The exogenous measures of each type of worker are N_ℓ and N_h. They also consume a tradable good. The economy is populated by a representative firm at each ZIP code $i \in Z$ with monopsony power. The firm produces the tradable good using both types of labor as inputs.

1.1. Workers

We assume that preferences are Cobb-Douglas, so that the utility of residence-workplace pair (n, i) for worker ω of type S is given by

$$U_{ni\omega} = b_{ni\omega} \frac{B_i}{d_{ni}} \left(\frac{C_{n\omega}}{\alpha}\right)^{\alpha} \left(\frac{H_{n\omega}}{1-\alpha}\right)^{1-\alpha},$$

where $b_{ni\omega}$ is an idiosyncratic shock at the (n, i) level; B_i represents workplace amenities; d_{ni} is iceberg commuting cost between residence n and workplace i; $C_{n\omega}$ and $H_{n\omega}$ represent consumption and housing, respectively; and $1 - \alpha \in (0, 1)$ is the share spent on housing.

Letting the price of consumption be the numeraire, and Q_n denote the price of housing, the budget constraint of a worker of type $s \in \{\ell, h\}$ who works in i is

$$w_i^s = C_{nw} + Q_n H_{n\omega}$$

Note that the wage doesn’t depend on the worker’s identity. As a result, the solution of the maximization problem will be independent of it too.

Timinig is as follows. The worker being life in metropolitan area and observes draws $b_{ni\omega}$ for every workplace-residence pair. Conditional on these, workers maximize utililty with respect to the budget constraint for every (i, n). As shown in Appendix A, this results in indirect utility

$$u_{ni}^s = \frac{b_{ni} B_i w_i^s}{d_{ni} Q_n^{1-\alpha}} \tag{1}$$

where we dropped the ω sub-index because the solution is independent of the worker’s identity.

Finally, workers decide a workplace and residence location by selecting pair that yields a highest level of utility.

Following Eaton and Kortum (2002), we assume that the b_{ni} is drawn independently from a Fréchet distribution:

$$G^s_{ni}(b) = e^{-T^s_n X^s_i b^{-\epsilon}} \qquad T^s_n > 0, X^s_i > 0, \epsilon > 1 \ \forall s.$$

The variables T^s_n and X^s_i are parameters that reflect the average taste of worker type s for living in n and working in i, respectively. The dispersion parameter ϵ is common to all locations. As usual in the urban literature, larger ϵ implies less dispersion of preferences.

1.2. Firms

Each ZIP code i is endowed with a representative firm that produces a tradable good Y. Following Card, Cardoso, Heining and Klein (2016), production technology follows

$$Y_i = T_i f\left(L^\ell_i, L^h_i\right)$$

where T_i is a ZIP code-specific productivity shifter; $f(\cdot)$ is twice differentiable and strictly concave; and $L^s_i = \sum_n L^s_{ni}$ represents total employment of labor of type s, with L^s_{ni} being the measure of workers commuting from n to i.

Firm's i optimal wage choices solve the cost minimization problem

$$\min_{\{w^\ell_i, w^h_i\}} w^\ell_i L^\ell_i(w^\ell_i) + w^h_i L^h_i(w^h_i) \quad \text{s.t.} \quad T_i f\left(L^\ell_i(w^\ell_i), L^h_i(w^h_i)\right) \geq Y$$

As we show later, because the supply functions $L^\ell_i(\cdot)$ and $L^h_i(\cdot)$ are upward sloping, firms in each ZIP code will set a wage offer below the marginal product of labor. When the minimum wage is set at an appropriate level, the model can produce negative small, null, and even positive employment effects!

Note that, to simplify, we assume that firms don't use any land.

1.3. Housing

We make this market very simple –following much of the urban literature—. Land supply in location n is inelastic and given by D_n. Because of the Cobb-Douglas specification in the utility function, workers spend a fraction $1-\alpha$ of their income on housing, which is paid to absentee landlords determining the price of housing in each ZIP code.

2. Solving the model

2.1. Workers and labor supply

The distribution of indirect utility is

$$\begin{aligned}
\Pr\left(u^s_{ni} \leq u\right) =& \Pr\left(b_{ni} \frac{B_i w^s_i}{d_{ni} Q^{1-\alpha}_n} \leq u\right) \\
=& \Pr\left(b_{ni} \leq \frac{d_{ni} Q^{1-\alpha}_n}{B_i w^s_i} u\right) \\
=& e^{\Phi^s_{ni} u^{-\epsilon}}
\end{aligned}$$

where

$$\Phi_{ni}^s = -T_n^s X_i^s \left(d_{ni} Q_n^{1-\alpha}\right)^{-\epsilon} \left(B_i w_i^s\right)^{\epsilon}.$$

Per the properties of the Fréchet distribution, the joint probability a worker chooses to reside in n and work in i is

$$\lambda_{ni}^s = \frac{\Phi_{ni}^s}{\sum_{n \in Z} \sum_{i \in Z} \Phi_{ni}^s} = \frac{\Phi_{ni}^s}{\Phi^s}$$

The marginal probability of residing in n or working in i is simply

$$\lambda_n^s = \sum_{i \in Z} \lambda_{ni} \qquad \lambda_i^s = \sum_{n \in Z} \lambda_{ni}$$

The measure of workers residing in n and working in i is, respectively,

$$\tilde{L}_n^s = \lambda_n^s N^s \qquad \tilde{L}_i^s = \lambda_i^s N^s$$

2.2. Firms

Appendix

A. Derivation of indirect utility

For given (n, i), d_{ni}, and $b_{ni\omega}$, worker's solve

$$\max_{\{C_{n\omega}, H_{n\omega}\}} b_{ni\omega} \frac{B_i}{d_{ni}} \left(\frac{C_{n\omega}}{\alpha}\right)^{\alpha} \left(\frac{H_{n\omega}}{1-\alpha}\right)^{1-\alpha}$$
$$\text{s.t. } w_i^s = C_{nw} + Q_n H_{n\omega}$$

Standard arguments imply that optimal demands are

$$C_{n\omega}^* = \alpha w_i^s \qquad H_{n\omega}^* = (1-\alpha)\frac{w_i^s}{Q_n}$$

Note that they don't depend on the worker's identity, so we can safely drop the ω subindex. Replacing in the utility function we get indirect utility.