Benchmarking Computations

The MW-n-Rents Guys

9/10/2020

Framework

For simplicity, focus on the supply and demand of housing in a given zipcode. Assume that two groups exist within each zipcode, MW and non-MW households (HH). The former are fully affected by the MW, whereas the latter are not affected at all.

Notation:

- H represents the number of homogeneous housing units
- r is the rent, $\underline{w}$ and w represent the monthly income of MW HH and non-MW HH, respectively
- $\underline{H}(r, \underline{w})$ is the demand of housing units of MW HH
- $\overline{H}(r, w)$ is the demand of housing units of non-MW HH
- $H(r)$ is the supply of housing

Derivaties:

- $\underline{H}_r(r, \underline{w}) < 0$ and $\overline{H}_r(r, w) < 0$, i.e., the demand for housing is downward sloping
- $\underline{H}_w(r, \underline{w}) > 0$ and $\overline{H}_w(r, w) > 0$, i.e., housing demand is increasing in income
- $H'(r) > 0$, meaning that housing supply is upward sloping

Equilibrium

The local housing market is in equilibrium if

$$H(r) = \underline{H}(r, \underline{w}) + \overline{H}(r, w)$$

Write this condition as $H(e^{\ln r}) = \underline{H}(e^{\ln r}, e^{\ln \underline{w}}) + \overline{H}(e^{\ln r}, e^{\ln w})$ and fully differentiate with respect to $\ln r$ and $\ln \underline{w}$ and reorder to get the elasticity of rents to the minimum wage:

$$\frac{d \ln r}{d \ln \underline{w}} = -\frac{\underline{w}\underline{H}_w}{r\underline{H}_r + r\overline{H}_r - rH'(r)} \tag{1}$$

CES functions

Assume that all functions are CES, namely:

- $\underline{H}(r, \underline{w}) = Ae^{-\underline{\gamma}r + \underline{\beta}\underline{w}}$
- $\overline{H}(r, w) = Be^{-\overline{\gamma}r + \overline{\beta}w}$
- $H(r) = Ce^{kr}$

where $A, B, C > 0$ are constants, and $\underline{\gamma}, \underline{\beta}, \overline{\gamma}, \overline{\beta}, k > 0$ are elasticities. These functional forms imply that

- $\underline{H}_r = -\underline{\gamma}\underline{H}$ and $\overline{H}_r = -\overline{\gamma}\overline{H}$
- $\underline{H}_w = \underline{\beta}\underline{H}$
- $H'(r) = kH$

Replacing the previous expression, we can rewrite (1) as

$$\frac{d \ln r}{d \ln \underline{w}} = \frac{\underline{\beta}\underline{w}\underline{H}}{\underline{\gamma} r \underline{H} + \overline{\gamma} r \overline{H} + k r H}$$

Now define the following parameters:

- $\alpha = \frac{\underline{w}\underline{H}}{r\underline{H}} = \frac{\underline{w}}{r}$, or the minimum wage to rent ratio
- $\underline{s} = \frac{\underline{H}}{H}$, or the share of housing occupied by MW HH
- $\overline{s} = \frac{\overline{H}}{H}$, or the share of housing occupied by non-MW HH. Note that $\overline{s} = 1 - \underline{s}$

Then, we reach to the following expression of interest:

$$\frac{d \ln r}{d \ln \underline{w}} = \frac{\underline{\beta}\alpha\underline{s}}{\underline{\gamma}\underline{s} + \overline{\gamma}\overline{s} + k} \tag{2}$$