Identification of MW effects with spillovers across units for "Minimum Wage as a Place-Based Policy"

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Consider the causal model for rents given by

$$r_{it} = r_{it}(\{\underline{w}_{zt}\}_{z\in\mathcal{Z}})$$

where $\underline{w}_{zt}$ represents the log of the statutory MW, which we also refer to as the "dose" of treatment received by unit i from ZIP code z in period t. We think of $\mathcal{Z}$ as the ZIP codes in a closed metropolitan area. Also, for simplicity we let $t \in \{1, 2\}$.

Definition 1 (Policy)**.** *In the first period, all locations are bound by a common MW level, $\underline{w}_{i1} = \underline{w}$ for all i. In the second period, the MW increases to $\underline{w}' > \underline{w}$ in some (directly treated) ZIP codes $i \in \mathcal{Z}_0 \subset \mathcal{Z}$ for non-empty $\mathcal{Z}_0$. The MW levels in the second period are then $\underline{w}_{i2} = \underline{w}$ for $i \notin \mathcal{Z}_0$ and $\underline{w}_{i2} = \underline{w}'$ for $i \in \mathcal{Z}_0$. We define the vectors $\underline{\mathbf{w}}_1$ and $\underline{\mathbf{w}}_2$ that collect treatment status for all units in each period.*

We want to learn the average effect of the policy, both directly and indirectly (through other units).

Definition 2 (Average Treatment Effects)**.** *Consider the policy in Definition 1. For any group of directly untreated ZIP codes $\mathcal{Z}_u \subseteq \mathcal{Z} \setminus \mathcal{Z}_0$, the indirect effect of the policy is given by*

$$\tau^{wkp} = \frac{E\left[r_{i2}(\underline{\mathbf{w}}_2)|i \in \mathcal{Z}_u\right] - E\left[r_{i2}(\underline{\mathbf{w}}_1)|i \in \mathcal{Z}_u\right]}{\underline{w}' - \underline{w}}. \tag{1}$$

The direct effect of the policy is given by

$$\tau^{res} = \frac{E\left[r_{i2}(\underline{\mathbf{w}}_2) - r_{i2}(\underline{\mathbf{w}}_1)|i \in \mathcal{Z}_0\right]}{\underline{w}' - \underline{w}}. \tag{2}$$

Following Section 2, we assume the following functional form for the causal model:

$$r_{it}(\underline{\mathbf{w}}_t) = \alpha_i + \hat{\delta}_t + \gamma \underline{w}_{it} + \beta \sum_{z\in\mathcal{Z}} \pi_{iz}\underline{w}_{zt} + \delta_t + \varepsilon_{it} \tag{3}$$

where α_i is a unit fixed effect, $\hat{\delta}_t$ is a time fixed effect, and ε_{it} is an error term. We show that the treatment effects of interest are identified given this functional form.

Proposition 1 (Identification)**.** *Consider the policy in Definition 1 and the functional form in equation* (3)*. Assume (1) there exist at least two not directly treated ZIP codes with differential exposure to the policy, (2) parallel trends among two non-empty subgroups of not directly treated ZIP codes (specified below), (3) parallel trends across directly treated ZIP codes and not directly treated ZIP codes. Then, the average treatment effects in Definition 2 are identified.*

Proof. Rank ZIP codes $i \in \mathcal{Z} \setminus \mathcal{Z}_0$ based on their exposure to the policy $\Pi_i = \sum_{z \in \mathcal{Z}_0} \pi_{iz}$. Let ZIP codes with $\Pi_i > \bar{\Pi}$ belong to a "high exposure" group $\mathcal{Z}_h \subset \mathcal{Z} \setminus \mathcal{Z}_0$, and the rest to a "low exposure" group $\mathcal{Z}_l = \{\mathcal{Z} \setminus \mathcal{Z}_0\} \setminus \mathcal{Z}_h$, where $\bar{\Pi} \in (\min \Pi_i, \max \Pi_i)$. Given Assumption (1), such partition is always possible. We can write (1) as

$$(\underline{w}' - \underline{w})\tau^{\text{wkp}} = E\left[r_{i2}(\underline{\mathbf{w}}_2) - r_{i2}(\underline{\mathbf{w}}_1) | i \in \mathcal{Z}_h\right] - E\left[r_{i1}(\underline{\mathbf{w}}_1) | i \in \mathcal{Z}_l\right] + E\left[r_{i1}(\underline{\mathbf{w}}_1) | i \in \mathcal{Z}_h\right]$$

The expected of rents in these groups is:

$$r_{it}(\underline{\mathbf{w}}_t) = \alpha_i + \hat{\delta}_t + \gamma \underline{w}_{it} + \beta \sum_{z \in \mathcal{Z}} \pi_{iz} \underline{w}_{zt} + \delta_t + \varepsilon_{it}$$

Using (**??**) we can compute, for $t \neq \bar{t}$,

$$E[\Delta r_{it} | i \in \mathcal{Z}_h] - E[\Delta r_{it} | i \in \mathcal{Z}_l] = E[\Delta \varepsilon_{it} | i \in \mathcal{Z}_h] - E[\Delta \varepsilon_{it} | i \in \mathcal{Z}_l],$$

and for $t = \bar{t}$,

$$\begin{aligned} E[\Delta r_{it} | i \in \mathcal{Z}_h] - E[\Delta r_{it} | i \in \mathcal{Z}_l] = \beta \left(E\left[\sum_{z \in \mathcal{Z}_0} \pi_{iz} \middle| i \in \mathcal{Z}_h \right] - E\left[\sum_{z \in \mathcal{Z}_0} \pi_{iz} \middle| i \in \mathcal{Z}_l \right] \right) \Delta \underline{w} \\ + E[\Delta \varepsilon_{it} | i \in \mathcal{Z}_h] - E[\Delta \varepsilon_{it} | i \in \mathcal{Z}_l]. \end{aligned}$$

Under assumption (2), namely that $E[\Delta \varepsilon_{it} | i \in \mathcal{Z}_h] - E[\Delta \varepsilon_{it} | i \in \mathcal{Z}_l] = 0$, we can re-arrange the previous equation to obtain

$$\beta = \frac{E[\Delta r_{it} | i \in \mathcal{Z}_h] - E[\Delta r_{it} | i \in \mathcal{Z}_l]}{\left(E[\sum_{z \in \mathcal{Z}_0} \pi_{iz} | i \in \mathcal{Z}_h] - E[\sum_{z \in \mathcal{Z}_0} \pi_{iz} | i \in \mathcal{Z}_l] \right) \Delta \underline{w}}. \tag{4}$$

Assumption (1) guarantees that $\left(E[\sum_{z \in \mathcal{Z}_0} \pi_{iz} | i \in \mathcal{Z}_h] - E[\sum_{z \in \mathcal{Z}_0} \pi_{iz} | i \in \mathcal{Z}_l] \right) \neq 0$, thus β is identified.

For those ZIP codes that are treated directly, the expected evolution of rents is

$$E[\Delta r_{it} | i \in \mathcal{Z}_0] = \begin{cases} \delta_t + E[\Delta \varepsilon_{it} | z \notin \mathcal{Z}_0] & \text{if } t \neq \bar{t} \\ \gamma \Delta \underline{w} + \beta \sum_{z \in \mathcal{Z}_0} \pi_{iz} \Delta \underline{w} + \delta_t + E[\Delta \varepsilon_{it} | z \notin \mathcal{Z}_0] & \text{if } t = \bar{t}. \end{cases}$$

Differencing with respect to (**??**) obtains, for $t = \bar{t}$,

$$\begin{aligned} E[\Delta r_{it}|i \in \mathcal{Z}_0] - E[\Delta r_{it}|i \notin \mathcal{Z}_0] = \gamma \Delta \underline{w} \\ + \beta \left(E\left[\sum_{z \in \mathcal{Z}_0} \pi_{iz} \middle| i \in \mathcal{Z}_0 \right] - E\left[\sum_{z \in \mathcal{Z}_0} \pi_{iz} \middle| i \notin \mathcal{Z}_0 \right] \right) \Delta \underline{w} \\ + E[\Delta \varepsilon_{it}|i \in \mathcal{Z}_0] - E[\Delta \varepsilon_{it}|i \notin \mathcal{Z}_0]. \end{aligned}$$

Assumption (3) means that $E[\Delta \varepsilon_{it}|i \in \mathcal{Z}_0] - E[\Delta \varepsilon_{it}|i \notin \mathcal{Z}_0] = 0$. Substituting in the previous equation yields

$$\gamma = \frac{E[\Delta r_{it}|i \in \mathcal{Z}_0] - E[\Delta r_{it}|i \notin \mathcal{Z}_0]}{\Delta \underline{w}} - \beta \left(E\left[\sum_{z \in \mathcal{Z}_0} \pi_{iz} \middle| i \in \mathcal{Z}_0 \right] - E\left[\sum_{z \in \mathcal{Z}_0} \pi_{iz} \middle| i \notin \mathcal{Z}_0 \right] \right). \tag{5}$$

Since β is known per equation (4), γ is identified as well. $\square$