Benchmarking Computations

The MW-n-Rents Guys

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1. Static Model

Consider the following panel difference-in-differences model relating rents and the MW:

$$y_{it} = \alpha_i + \hat{\delta}_t + \beta \underline{w}_{it} + \epsilon_{it} \tag{1}$$

where:

- y_{it} is the log rent per square foot for the Zillow SFCC series,
- $\underline{w}$ is the log of the minimum wage, α_i is a zipcode fixed effect,
- α_i and α_t are fixed effects.

To remove the zipcode-specific interecept we estimate the model in in first-differences:

$$\Delta y_{it} = \delta_t + \beta \Delta \underline{w}_{it} + \Delta \epsilon_{it} \tag{2}$$

where $\delta_t = \hat{\delta}_t - \hat{\delta}_{t-1}$. Let $\Delta X_{it} = (D_t, \Delta \underline{w}_{it})$, where D_t is a time-period dummy common across all units. With this notation, the model becomes

$$\Delta y_{it} = \theta \Delta X_{it} + \Delta \epsilon_{it}$$

Alternative, we can write the model as deviations of time period means.[1]

Following Hansen (p. 621), we can write the estimate of equation (2) as follows:

$$\begin{aligned} \hat{\theta}_\Delta &= \left(\sum_i^N \sum_{t \geq 2} \Delta X_{it} \Delta X_{it}' \right)^{-1} \left(\sum_i^N \sum_{t \geq 2} \Delta X_{it} \Delta y_{it} \right) \\ &= \left(\sum_i^N \Delta \boldsymbol{X}_i \Delta \boldsymbol{X}_i' \right)^{-1} \left(\sum_i^N \Delta \boldsymbol{X}_i \Delta \boldsymbol{y}_i \right) \end{aligned} \tag{3}$$

where bold symbols are stacked observations for each individual –e.g., $\boldsymbol{X}_i = (X_{i2,..,X_{iT_i}})'$–, and N is the number of zipcodes. Replacing the model in the expression for the estimate we can write:

$$\hat{\theta}_\Delta - \theta = \left(\sum_i^N \Delta \boldsymbol{X}_i \Delta \boldsymbol{X}_i' \right)^{-1} \left(\sum_i^N \Delta \boldsymbol{X}_i \Delta \boldsymbol{\epsilon}_i \right)$$

[1] In particular, $\Delta \dot{X}_{it} = \Delta X_{it} - \frac{1}{N_t} \sum_{i \in S_t} \Delta X_i t$ denotes time-period means, where S_t is the set of zipcodes in the sample in period t, and N_t is the size of this set.

Therefore, for consistency of $\hat{\theta}_\Delta$ we need to assume *strict exogeneity* of covaraites:

$$E[\Delta \boldsymbol{X}_i \Delta \epsilon_{it}] = 0$$

This assumption corresponds to assumption 3 in Hansen (Assumption 7.2, p. 631). Hansen also makes an interesting argument for the case of unbalanced panels.