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Secularity condition of the Broadwell kinetic system

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Abstract. In this paper we study the kinetic system of Broadwell equations of 4 groups of particles with periodic initial data in weight space. The solution is sought in the vicinity of the equilibrium state. The perturbation is expanded in a Fourier series. The conditions of local equilibrium for solutions of the Cauchy problem are found.

Keywords: kinetic model, Fourier series, Knudsen parameter, secular terms.

1 Introduction

In this article we will continue the study of stabilization (asymptotic stability) of solutions of nonlinear hyperbolic partial differential equations using the example of the so-called discrete models of the kinetic Boltzmann equation. Namely, the study of stabilization of solutions of the Cauchy problem for the two-dimensional Broadwell system [1, 7, 13]:

$$
\begin{aligned}
\partial_t n_1 + \partial_x n_1 &= \frac{1}{\varepsilon}(n_3 n_4 - n_1 n_2), \\
\partial_t n_2 - \partial_x n_2 &= \frac{1}{\varepsilon}(n_3 n_4 - n_1 n_2), \\
\partial_t n_3 + \partial_y n_3 &= \frac{1}{\varepsilon}(n_1 n_2 - n_3 n_4), \\
\partial_t n_4 - \partial_y n_4 &= \frac{1}{\varepsilon}(n_1 n_2 - n_3 n_4),
\end{aligned} \tag{1}
$$

$$
n_1|_{t=0} = n_1^0, n_2|_{t=0} = n_2^0, n_3|_{t=0} = n_3^0, n_4|_{t=0} = n_4^0. \tag{2}
$$

The system (1)-(2) is the Boltzmann kinetic equation of a model two-dimensional gas of particles moving on a two-dimensional plane, the velocities of which $((1,0),\ (-1,0),\ (0,1),\ (0,-1))$ will be assumed to be directed along the coordinate axes. Here $u = n_1(x,y,t),\ v = n_2(x,y,t),\ w = n_3(x,y,t),\ z = n_4(x,y,t)-$ is the density (number of particles per unit area) of particles of the corresponding four groups. All particles are distributed into four groups with velocities $(1,0),\ (-1,0),\ (0,1),\ (0,-1)$. The first group of particles, interacting with the second, passes into the third and fourth, respectively. Similarly, the third group, interacting with the fourth, passes into the first and second. We assume that the initial data are periodic.

Many works are devoted to the study of kinetic equations. In recent works, solutions of kinetic systems were found using the Painlevé expansion [4, 8, 13], various analytical methods [3, 14]. Works on the study of solution stabilization are given in [6, 9, 10, 11, 12]. A numerical proof of stabilization is presented in [15, 16]. In this paper, basing on the technique presented in [2, 5] we found the secularity conditions for the system (1).

1.1 Small perturbations

We study the Cauchy problem (1) for small perturbations of the equilibrium state $w_e z_e = u_e v_e$. We set

$$
n_1 = u_e + \varepsilon^2 u_e^{1/2}\widehat{u},\ n_2 = v_e + \varepsilon^2 v_e^{1/2}\widehat{v},\ n_3 = w_e + \varepsilon^2 w_e^{1/2}\widehat{w},\ n_4 = z_e + \varepsilon^2 z_e^{1/2}\widehat{z}.
$$

We consider periodic disturbances of an equilibrium state with limited energy

$$
\widehat{u} = \sum_{k,l\in Z} u_{k,l} e^{ikx} e^{ily},\ \widehat{v} = \sum_{k,l\in Z} v_{k,l} e^{ikx} e^{ily},
$$

$$\widehat{w}=\sum_{k,l\in Z}w_{k,l}e^{ikx}e^{ily},\ \widehat{z}=\sum_{k,l\in Z}z_{k,l}e^{ikx}e^{ily}.$$

Let's introduce into weight spaces $L_{2,\gamma}(\mathbb{R}_+;\mathcal{H}_\sigma),\mathcal{H}_\sigma$:

$$\|\widehat{u}\|^2_{L_{2,\gamma}(\mathbb{R}_+;\mathcal{H}_\sigma)}=\int_0^\infty\ e^{2\gamma t}\mid u_0(t)\mid^2\ \ dt+$$

$$+\int_0^\infty\ e^{2\gamma t}\sum_{k\in\mathbb{Z}_0}\mid k\mid^{2\sigma}\mid u_k(t)\mid^2\ \ dt,\quad \mid\mid\mid\widehat{u}\mid_{t=0}\mid\mid\mid^2_{\mathcal{H}_\sigma}=\mid u_0^0\mid^2+\sum_{k\in\mathbb{Z}_0}\mid k\mid^{2\sigma}\mid u_k^0\mid^2,$$

where $\gamma>0,\sigma=const$. The system will look like this:

$$\begin{gathered}
\partial_t\widehat{u}+\partial_x\widehat{u}-\frac{1}{\varepsilon}v_e^{1/2}\Big(w_e^{1/2}\widehat{z}+z_e^{1/2}\widehat{w}-u_e^{1/2}\widehat{v}-v_e^{1/2}\widehat{u}\Big)=\\
=v_e^{1/2}\varepsilon\Big(\widehat{w}\widehat{z}-\widehat{u}\widehat{v}\Big), \qquad (3)\\
\partial_t\widehat{v}-\partial_x\widehat{v}-\frac{1}{\varepsilon}u_e^{1/2}\Big(w_e^{1/2}\widehat{z}+z_e^{1/2}\widehat{w}-u_e^{1/2}\widehat{v}-v_e^{1/2}\widehat{u}\Big)=\\
=u_e^{1/2}\varepsilon\Big(\widehat{w}\widehat{z}-\widehat{u}\widehat{v}\Big),\\
\partial_t\widehat{w}+\partial_y\widehat{w}+\frac{1}{\varepsilon}z_e^{1/2}\Big(w_e^{1/2}\widehat{z}+z_e^{1/2}\widehat{w}-u_e^{1/2}\widehat{v}-v_e^{1/2}\widehat{u}\Big)=\\
=-z_e^{1/2}\varepsilon\Big(\widehat{w}\widehat{z}-\widehat{u}\widehat{v}\Big),\\
\partial_t\widehat{z}-\partial_y\widehat{z}+\frac{1}{\varepsilon}w_e^{1/2}\Big(w_e^{1/2}\widehat{z}+z_e^{1/2}\widehat{w}-u_e^{1/2}\widehat{v}-v_e^{1/2}\widehat{u}\Big)=\\
=-w_e^{1/2}\varepsilon\Big(\widehat{w}\widehat{z}-\widehat{u}\widehat{v}\Big),\\
\widehat{u}|_{t=0}=\widehat{u}^0,\widehat{v}|_{t=0}=\widehat{v}^0,\widehat{w}|_{t=0}=\widehat{w}^0,\widehat{z}|_{t=0}=\widehat{z}^0. \qquad (4)
\end{gathered}$$

Hence the conservation laws are

$$u_e^{1/2}(\partial_t\widehat{u}+\partial_x\widehat{u})=v_e^{1/2}(\partial_t\widehat{v}-\partial_x\widehat{v}),$$

$$w_e^{1/2}(\partial_t\widehat{w}+\partial_y\widehat{w})=z_e^{1/2}(\partial_t\widehat{z}-\partial_y\widehat{z}).$$

In Fourier's images we have

$$u_{k,l}=(u^0_{k,l}-\frac{v_e^{1/2}}{u_e^{1/2}}v^0_{k,l})e^{-ikt}+\frac{v_e^{1/2}}{u_e^{1/2}}v_{k,l}-2ik\frac{v_e^{1/2}}{u_e^{1/2}}\int_0^t e^{ik(s-t)}v_{k,l}ds.$$

We will also receive

$$w_{k,l}=(w^0_{k,l}-\frac{z_e^{1/2}}{w_e^{1/2}}z^0_{k,l})e^{-ilt}+\frac{z_e^{1/2}}{w_e^{1/2}}z_{k,l}-2il\frac{z_e^{1/2}}{w_e^{1/2}}\int_0^t e^{il(s-t)}z_{k,l}ds.$$

Next we obtain another conservation law from

$$\frac{d}{dt}v_{k,l} - ikv_{k,l} - \frac{1}{\varepsilon}u_e^{1/2}\Big(w_e^{1/2}z_{k,l} + z_e^{1/2}w_{k,l} - u_e^{1/2}v_{k,l} - v_e^{1/2}u_{k,l}\Big) =$$
$$= u_e^{1/2}\varepsilon\Big(\widehat{w}\widehat{z} - \widehat{u}\widehat{v}\Big)_{k,l},$$

$$\frac{d}{dt}z_{k,l} - ilz_{k,l} + \frac{1}{\varepsilon}w_e^{1/2}\Big(w_e^{1/2}z_{k,l} + z_e^{1/2}w_{k,l} - u_e^{1/2}v_{k,l} - v_e^{1/2}u_{k,l}\Big) =$$
$$= -w_e^{1/2}\varepsilon\Big(\widehat{w}\widehat{z} - \widehat{u}\widehat{v}\Big)_{k,l}.$$

Then

$$\frac{d}{dt}z_{k,l} - ilz_{k,l} = -\frac{w_e^{1/2}}{u_e^{1/2}}(\frac{d}{dt}v_{k,l} - ikv_{k,l})$$

or

$$z_{k,l} = z_{k,l}^0 e^{ilt} - e^{ilt}\int_0^t e^{-ils}\frac{w_e^{1/2}}{u_e^{1/2}}(\frac{d}{ds}v_{k,l} - ikv_{k,l})ds =$$
$$= (z_{k,l}^0 + \frac{w_e^{1/2}}{u_e^{1/2}}v_{k,l}^0)e^{ilt} - \frac{w_e^{1/2}}{u_e^{1/2}}v_{k,l} + i(k-l)\frac{w_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il(s-t)}v_{k,l}ds.$$

We have

$$w_{k,l} = -\frac{z_e^{1/2}}{u_e^{1/2}}v_{k,l} + (w_{k,l}^0 + \frac{z_e^{1/2}}{u_e^{1/2}}v_{k,l}^0)e^{-ilt} + i(k-l)\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il(s-t)}v_{k,l}ds +$$
$$+2il\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{il(s-t)}v_{k,l}ds - 2ili(k-l)\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{il(s-t)}\int_0^s e^{-il(p-s)}v_{k,l}(p)dpds =$$
$$= -\frac{z_e^{1/2}}{u_e^{1/2}}v_{k,l} + (w_{k,l}^0 + \frac{z_e^{1/2}}{u_e^{1/2}}v_{k,l}^0)e^{-ilt} + i(k+l)\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{il(s-t)}v_{k,l}ds.$$

1.2 Reduction to one equation

We have the following substitution of transition to one equation

$$u_{k,l} = \frac{v_e^{1/2}}{u_e^{1/2}}v_{k,l} + (u_{k,l}^0 - \frac{v_e^{1/2}}{u_e^{1/2}}v_{k,l}^0)e^{-ikt} - 2ik\frac{v_e^{1/2}}{u_e^{1/2}}\int_0^t e^{ik(s-t)}v_{k,l}ds, \qquad (5)$$

$$w_{k,l} = -\frac{z_e^{1/2}}{u_e^{1/2}}v_{k,l} + (w_{k,l}^0 + \frac{z_e^{1/2}}{u_e^{1/2}}v_{k,l}^0)e^{-ilt} + i(k+l)\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{il(s-t)}v_{k,l}ds, \qquad (6)$$

$$z_{k,l} = -\frac{w_e^{1/2}}{u_e^{1/2}} v_{k,l} + (z_{k,l}^0 + \frac{w_e^{1/2}}{u_e^{1/2}} v_{k,l}^0) e^{ilt} + i(k-l)\frac{w_e^{1/2}}{u_e^{1/2}} \int_0^t e^{-il(s-t)} v_{k,l} ds.$$

For $k = 0, l = 0$ we have

$$u_{0,0} = \frac{v_e^{1/2}}{u_e^{1/2}} v_{0,0} + (u_{0,0}^0 - \frac{v_e^{1/2}}{u_e^{1/2}} v_{0,0}^0),$$

$$w_{0,0} = -\frac{z_e^{1/2}}{u_e^{1/2}} v_{0,0} + (w_{0,0}^0 + \frac{z_e^{1/2}}{u_e^{1/2}} v_{0,0}^0),$$

$$z_{0,0} = -\frac{w_e^{1/2}}{u_e^{1/2}} v_{0,0} + (z_{0,0}^0 + \frac{w_e^{1/2}}{u_e^{1/2}} v_{0,0}^0).$$

We demand that

$$u_{0,0}^0 = w_{0,0}^0 = z_{0,0}^0 = 0.$$

Then

$$u_{0,0} = \frac{v_e^{1/2}}{u_e^{1/2}} v_{0,0},$$

$$w_{0,0} = -\frac{z_e^{1/2}}{u_e^{1/2}} v_{0,0},$$

$$z_{0,0} = -\frac{w_e^{1/2}}{u_e^{1/2}} v_{0,0}.$$

Now let us note that

$$u_e^{1/2}(w_e^{1/2} z_{k,l} + z_e^{1/2} w_{k,l} - u_e^{1/2} v_{k,l} - v_e^{1/2} u_{k,l}) =$$

$$= -\Big(u_e + v_e + u_e^{1/2} z_e^{1/2} \frac{z_e^{1/2}}{u_e^{1/2}} + u_e^{1/2} w_e^{1/2} \frac{w_e^{1/2}}{u_e^{1/2}}\Big) v_{k,l} +$$

$$+ u_e^{1/2} w_e^{1/2} (z_{k,l}^0 + \frac{w_e^{1/2}}{u_e^{1/2}} v_{k,l}^0) e^{ilt} + u_e^{1/2} z_e^{1/2} (w_{k,l}^0 + \frac{z_e^{1/2}}{u_e^{1/2}} v_{k,l}^0) e^{-ilt} -$$

$$- v_e^{1/2} (u_e^{1/2} u_{k,l}^0 - v_e^{1/2} v_{k,l}^0) e^{-ikt} +$$

$$+ i(k+l) u_e^{1/2} z_e^{1/2} \frac{z_e^{1/2}}{u_e^{1/2}} \int_0^t e^{il(s-t)} v_{k,l} ds + 2ikv_e \int_0^t e^{ik(s-t)} v_{k,l} ds$$

$$+ i(k-l) u_e^{1/2} w_e^{1/2} \frac{w_e^{1/2}}{u_e^{1/2}} \int_0^t e^{-il(s-t)} v_{k,l} ds.$$

Let be

$$L_e = u_e + v_e + w_e + z_e > 0.$$

Then

$$\frac{d}{dt}v_{k,l} - ikv_{k,l} + \frac{1}{\varepsilon}L_e v_{k,l} - \tag{7}$$
$$-\frac{1}{\varepsilon}\Big(i(k+l)z_e\int_0^t e^{il(s-t)}v_{k,l}ds + 2ikv_e\int_0^t e^{ik(s-t)}v_{k,l}ds +$$
$$+i(k-l)w_e\int_0^t e^{-il(s-t)}v_{k,l}ds\Big) =$$
$$=\frac{1}{\varepsilon}\Big(u_e^{1/2}w_e^{1/2}(z^0_{k,l} + \frac{w_e^{1/2}}{u_e^{1/2}}v^0_{k,l})\Big)e^{ilt} + \frac{1}{\varepsilon}\Big(u_e^{1/2}z_e^{1/2}(w^0_{k,l} + \frac{z_e^{1/2}}{u_e^{1/2}}v^0_{k,l})\Big)e^{-ilt} +$$
$$+\frac{1}{\varepsilon}\Big(-v_e^{1/2}(u_e^{1/2}u^0_{k,l} - v_e^{1/2}v^0_{k,l})\Big)e^{-ikt} + u_e^{1/2}\varepsilon\Big(\widehat{w}\widehat{z} - \widehat{u}\widehat{v}\Big)_{k,l}.$$

1.3 Complexification

Let us rewrite the system (7) as

$$\frac{d}{dt}v_{k,l} - ikv_{k,l} + \frac{1}{\varepsilon}L_e v_{k,l} - \tag{8}$$
$$-\frac{1}{\varepsilon}\Big(i(k+l)z_e\int_0^t e^{il(s-t)}v_{k,l}ds + 2ikv_e\int_0^t e^{ik(s-t)}v_{k,l}ds$$
$$+i(k-l)w_e\int_0^t e^{-il(s-t)}v_{k,l}ds\Big) =$$
$$=\frac{1}{\varepsilon}\Big(u_e^{1/2}w_e^{1/2}(z^0_{k,l} + \frac{w_e^{1/2}}{u_e^{1/2}}v^0_{k,l})\Big)e^{ilt} + \frac{1}{\varepsilon}\Big(u_e^{1/2}z_e^{1/2}(w^0_{k,l} + \frac{z_e^{1/2}}{u_e^{1/2}}v^0_{k,l})\Big)e^{-ilt} +$$
$$+\frac{1}{\varepsilon}\Big(-v_e^{1/2}(u_e^{1/2}u^0_{k,l} - v_e^{1/2}v^0_{k,l})\Big)e^{-ikt} + u_e^{1/2}\varepsilon\frac{1}{2}\Big(\widehat{w}\widehat{\overline{z}} + \widehat{z}\widehat{\overline{w}} - \widehat{u}\widehat{\overline{v}} - \widehat{v}\widehat{\overline{u}}\Big)_{k,l}.$$

Then

$$\frac{d}{dt}v_{k,l} - ikv_{k,l} + \frac{1}{\varepsilon}L_e v_{k,l} - \tag{9}$$
$$-\frac{1}{\varepsilon}\Big(i(k+l)z_e\int_0^t e^{il(s-t)}v_{k,l}ds + 2ikv_e\int_0^t e^{ik(s-t)}v_{k,l}ds +$$

$$+i(k-l)w_e\int_0^t e^{-il(s-t)}v_{k,l}ds\Big)=$$
$$=\frac{1}{\varepsilon}u_e^{1/2}D_l^+e^{ilt}+\frac{1}{\varepsilon}u_e^{1/2}D_l^-e^{-ilt}+\frac{1}{\varepsilon}u_e^{1/2}D_k^-e^{-ikt}+\frac{1}{2}\varepsilon u_e^{1/2}T_{k,l}^{add}(v)+$$
$$+\frac{1}{2}\varepsilon u_e^{1/2}\Big(L_{k,l}(v)+B_{k,l}(v,v)\Big),$$
$$v_k\big|_{t=0}=v_k^0, \tag{10}$$

where

$$D_l^+=w_e^{1/2}(z_{k,l}^0+\frac{w_e^{1/2}}{u_e^{1/2}}v_{k,l}^0)+$$
$$+\frac{1}{2}\varepsilon^2\sum_{k_1+k_2=k,l_1+l_2=l,k_1,k_2,l_1,l_2\neq 0}(z_{k_2,l_2}^0+\frac{w_e^{1/2}}{u_e^{1/2}}v_{k_2,l_2}^0)(\overline{w_{k_1,l_1}^0}+\frac{z_e^{1/2}}{u_e^{1/2}}\overline{v_{k_1,l_1}^0}),$$
$$D_l^-=z_e^{1/2}(w_{k,l}^0+\frac{z_e^{1/2}}{u_e^{1/2}}v_{k,l}^0)+$$
$$+\frac{1}{2}\varepsilon^2\sum_{k_1+k_2=k,l_1+l_2=l,k_1,k_2,l_1,l_2\neq 0}(w_{k_1,l_1}^0+\frac{z_e^{1/2}}{u_e^{1/2}}v_{k_1,l_1}^0)(\overline{z_{k_2,l_2}^0}+\frac{w_e^{1/2}}{u_e^{1/2}}\overline{v_{k_2,l_2}^0}),$$
$$D_k^-=-v_e^{1/2}(u_{k,l}^0-\frac{v_e^{1/2}}{u_e^{1/2}}v_{k,l}^0),$$

$$L_{k,l}(v)=\sum_{k_1+k_2=k,l_1+l_2=l,k_1,k_2\neq 0}\Big(-\frac{z_e^{1/2}}{u_e^{1/2}}v_{k_1,l_1}(\overline{z_{k_2,l_2}^0}+\frac{w_e^{1/2}}{u_e^{1/2}}\overline{v_{k_2,l_2}^0})e^{-il_2t}-$$
$$-(w_{k_1,l_1}^0+\frac{z_e^{1/2}}{u_e^{1/2}}v_{k_1,l_1}^0)e^{-il_1t}\frac{w_e^{1/2}}{u_e^{1/2}}\overline{v_{k_2,l_2}}-(w_{k_1,l_1}^0+\frac{z_e^{1/2}}{u_e^{1/2}}v_{k_1,l_1}^0)e^{-il_1t}\times$$
$$\times i(k_2-l_2)\frac{w_e^{1/2}}{u_e^{1/2}}\int_0^t e^{il_2(s-t)}\overline{v_{k_2,l_2}}ds-$$
$$+i(k_1+l_1)\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{il_1(s-t)}v_{k_1,l_1}ds(\overline{z_{k_2,l_2}^0}+\frac{w_e^{1/2}}{u_e^{1/2}}\overline{v_{k_2,l_2}^0})e^{-il_2t}-$$
$$-\frac{w_e^{1/2}}{u_e^{1/2}}v_{k_2,l_2}(\overline{w_{k_1,l_1}^0}+\frac{z_e^{1/2}}{u_e^{1/2}}\overline{v_{k_1,l_1}^0})e^{il_1t}-(z_{k_2,l_2}^0+\frac{w_e^{1/2}}{u_e^{1/2}}v_{k_2,l_2}^0)e^{il_2t}\frac{z_e^{1/2}}{u_e^{1/2}}\overline{v_{k_1,l_1}}-$$
$$-(z_{k_2,l_2}^0+\frac{w_e^{1/2}}{u_e^{1/2}}v_{k_2,l_2}^0)e^{il_2t}i(k_1+l_1)\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il_1(s-t)}\overline{v_{k_1,l_1}}ds+$$

$$+i(k_2-l_2)\frac{w_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il_2(s-t)}v_{k_2,l_2}ds(\overline{w^0_{k_1,l_1}}+\frac{z_e^{1/2}}{u_e^{1/2}}\overline{v^0_{k_1,l_1}})e^{il_1t}$$

$$-(u^0_{k_1,l_1}-\frac{v_e^{1/2}}{u_e^{1/2}}v^0_{k_1,l_1})e^{-ik_1t}\overline{v_{k_2,l_2}}-v_{k_2,l_2}(\overline{u^0_{k_1,l_1}}-\frac{v_e^{1/2}}{u_e^{1/2}}\overline{v^0_{k_1,l_1}})e^{ik_1t}\Big),$$

$$B_{k,l}(v,v)=\sum_{k_1+k_2=k,l_1+l_2=l,k_1,k_2\neq 0}\Big\{\frac{z_e^{1/2}}{u_e^{1/2}}v_{k_1,l_1}\frac{w_e^{1/2}}{u_e^{1/2}}\overline{v_{k_2,l_2}}-\frac{z_e^{1/2}}{u_e^{1/2}}v_{k_1,l_1}\times$$

$$\times\overline{i(k_2-l_2)\frac{w_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il_2(s-t)}v_{k_2,l_2}ds}-i(k_1-l_1)\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il_1(s+t)}v_{k_1,l_1}ds\frac{w_e^{1/2}}{u_e^{1/2}}\overline{v_{k_2,l_2}}-$$

$$+i(k_1-l_1)\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il_1(s+t)}v_{k_1,l_1}ds\overline{i(k_2-l_2)\frac{w_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il_2(s-t)}v_{k_2,l_2}ds}+$$

$$+\frac{w_e^{1/2}}{u_e^{1/2}}v_{k_2,l_2}\frac{z_e^{1/2}}{u_e^{1/2}}\overline{v_{k_1,l_1}}-\frac{w_e^{1/2}}{u_e^{1/2}}v_{k_2,l_2}\overline{i(k_1-l_1)\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il_1(s+t)}v_{k_1,l_1}ds}-$$

$$-i(k_2-l_2)\frac{w_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il_2(s-t)}v_{k_2,l_2}ds\frac{z_e^{1/2}}{u_e^{1/2}}\overline{v_{k_1,l_1}}-$$

$$+i(k_2-l_2)\frac{w_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il_2(s-t)}v_{k_2,l_2}ds\overline{i(k_1-l_1)\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il_1(s+t)}v_{k_1,l_1}ds}-$$

$$-\Big(\frac{v_e^{1/2}}{u_e^{1/2}}v_{k_1,l_1}-2ik_1\frac{v_e^{1/2}}{u_e^{1/2}}\int_0^t e^{ik_1(s-t)}v_{k_1,l_1}ds\Big)\overline{v_{k_2,l_2}}-$$

$$-v_{k_2,l_2}\overline{\Big(\frac{v_e^{1/2}}{u_e^{1/2}}v_{k_1,l_1}-2ik_1\frac{v_e^{1/2}}{u_e^{1/2}}\int_0^t e^{ik_1(s-t)}v_{k_1,l_1}ds\Big)}\Big\},$$

$$T^{add}_{k,l}(v)=-\frac{z_e^{1/2}}{u_e^{1/2}}v_{0,0}(-\frac{w_e^{1/2}}{u_e^{1/2}}\overline{v_{k,l}}+(\overline{z^0_{k,l}}+\frac{w_e^{1/2}}{u_e^{1/2}}\overline{v^0_{k,l}})e^{-ilt}-$$

$$-i(k-l)\frac{w_e^{1/2}}{u_e^{1/2}}\int_0^t e^{il(s-t)}\overline{v_{k,l}}ds)+$$

$$+(-\frac{z_e^{1/2}}{u_e^{1/2}}v_{k,l}+(w^0_{k,l}+\frac{z_e^{1/2}}{u_e^{1/2}}v^0_{k,l})e^{-ilt}+i(k+l)\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{il(s-t)}v_{k,l}ds)(-\frac{w_e^{1/2}}{u_e^{1/2}}\overline{v_{0,0}})-$$

$$-\frac{w_e^{1/2}}{u_e^{1/2}}v_{0,0}(-\frac{z_e^{1/2}}{u_e^{1/2}}\overline{v_{k,l}}+(\overline{w_{k,l}^0}+\frac{z_e^{1/2}}{u_e^{1/2}}\overline{v_{k,l}^0})e^{ilt}-i(k+l)\frac{z_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il(s-t)}\overline{v_{k,l}}ds)+$$

$$+(-\frac{w_e^{1/2}}{u_e^{1/2}}v_{k,l}+(z_{k,l}^0+\frac{w_e^{1/2}}{u_e^{1/2}}v_{k,l}^0)e^{ilt}+i(k-l)\frac{w_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-il(s-t)}v_{k,l}ds)(-\frac{z_e^{1/2}}{u_e^{1/2}}\overline{v_{0,0}})-$$

$$-\frac{v_e^{1/2}}{u_e^{1/2}}v_{0,0}\overline{v_{k,l}}-(\frac{v_e^{1/2}}{u_e^{1/2}}v_{k,l}+(u_{k,l}^0-\frac{v_e^{1/2}}{u_e^{1/2}}v_{k,l}^0)e^{-ikct}-2ik\frac{v_e^{1/2}}{u_e^{1/2}}\int_0^t e^{ikc(s-t)}v_{k,l}ds)\overline{v_{0,0}}-$$

$$-v_{0,0}(\frac{v_e^{1/2}}{u_e^{1/2}}\overline{v_{k,l}}+(\overline{u_{k,l}^0}-\frac{v_e^{1/2}}{u_e^{1/2}}\overline{v_{k,l}^0})e^{ikt}+2ik\frac{v_e^{1/2}}{u_e^{1/2}}\int_0^t e^{-ik(s-t)}\overline{v_{k,l}}ds)-v_{k,l}\frac{v_e^{1/2}}{u_e^{1/2}}\overline{v_{0,0}}.$$

1.4 Reduction to homogeneous data of the Cauchy problem

We make the following substitution

$$v_{k,l}=v_{k,l}^0e^{t(ik-\frac{1}{\varepsilon}L_e)}+y_{k,l}(t).$$

Then

$$\begin{gathered}\frac{d}{dt}y_{k,l}-iky_{k,l}+\frac{1}{\varepsilon}L_ey_{k,l}- \qquad (11)\\ -\frac{1}{\varepsilon}\Big(i(k+l)z_e\int_0^t e^{il(s-t)}y_{k,l}ds+2ikv_e\int_0^t e^{ik(s-t)}y_{k,l}ds\\ +i(k-l)w_e\int_0^t e^{-il(s-t)}y_{k,l}ds\Big)=\\ =\frac{1}{\varepsilon}u_e^{1/2}\mathcal{D}_l^+e^{ilt}+\frac{1}{\varepsilon}u_e^{1/2}\mathcal{D}_l^-e^{-ilt}+\frac{1}{\varepsilon}u_e^{1/2}\mathcal{D}_k^-e^{-ikt}+\\ +e^{-\frac{1}{\varepsilon}L_et}(\frac{1}{2}\varepsilon u_e^{1/2}\Big(f_{k,l}^L(t)+f_{k,l}^B(t))+f_{k,l}^{int}\Big)+\\ +\frac{1}{2}\varepsilon u_e^{1/2}T_{k,l}^{add}(y)+\frac{1}{2}\varepsilon u_e^{1/2}\Big(\mathcal{L}_{k,l}(y)+B_{k,l}(y,y)\Big),\\ y_{k,l}|_{t=0}=0. \qquad (12)\end{gathered}$$

Here

$$\begin{gathered}\mathcal{L}_{k,l}(y)=L_{k,l}(y)+B_{k,l}(v^0e^{t(ik-\frac{1}{\varepsilon}L_e)},y)+B_{k,l}(y,v^0e^{t(ik-\frac{1}{\varepsilon}L_e)}),\\ f_{k,l}^{int}=\frac{1}{\varepsilon}\Big(v_{k,l}^0z_e\frac{i(k+l)}{i(k+l)-\frac{1}{\varepsilon}L_e}+v_{k,l}^0v_e\frac{2ik}{2ik-\frac{1}{\varepsilon}L_e}+\\ +v_{k,l}^0w_e\frac{i(k-l)}{i(k-l)-\frac{1}{\varepsilon}L_e}\Big)e^{ikt},\end{gathered}$$

$$\frac{1}{\varepsilon}u_e^{1/2}\mathcal{D}_l^- = \frac{1}{\varepsilon}u_e^{1/2}z_e^{1/2}\Big(w_{k,l}^0 - \frac{z_e^{1/2}}{u_e^{1/2}}\frac{v_{k,l}^0\frac{1}{\varepsilon}L_e}{i(k+l)-\frac{1}{\varepsilon}L_e}\Big)+$$
$$+\frac{1}{2}\varepsilon u_e^{1/2}\sum_{k_1+k_2=k,l_1+l_2=l,k_1,k_2,l_1,l_2\neq 0}\overline{\Big(z_{k_2,l_2}^0 - \frac{w_e^{1/2}}{u_e^{1/2}}\frac{v_{k_2,l_2}^0\frac{1}{\varepsilon}L_e}{i(k_2-l_2)-\frac{1}{\varepsilon}L_e}\Big)}\times$$
$$\times\Big(w_{k_1,l_1}^0 - \frac{z_e^{1/2}}{u_e^{1/2}}\frac{v_{k_1,l_1}^0\frac{1}{\varepsilon}L_e}{i(k_1+l_1)-\frac{1}{\varepsilon}L_e}\Big),$$

$$\frac{1}{\varepsilon}u_e^{1/2}\mathcal{D}_l^+ = \frac{1}{\varepsilon}u_e^{1/2}w_e^{1/2}\Big(z_{k,l}^0 - \frac{w_e^{1/2}}{u_e^{1/2}}\frac{v_{k,l}^0\frac{1}{\varepsilon}L_e}{i(k-l)-\frac{1}{\varepsilon}L_e}\Big)+$$
$$+\frac{1}{2}\varepsilon u_e^{1/2}\sum_{k_1+k_2=k,l_1+l_2=l,k_1,k_2,l_1,l_2\neq 0}\Big(z_{k_2,l_2}^0 - \frac{w_e^{1/2}}{u_e^{1/2}}\frac{v_{k_2,l_2}^0\frac{1}{\varepsilon}L_e}{i(k_2-l_2)-\frac{1}{\varepsilon}L_e}\Big)\times$$
$$\times\overline{\Big(w_{k_1,l_1}^0 - \frac{z_e^{1/2}}{u_e^{1/2}}\frac{v_{k_1,l_1}^0\frac{1}{\varepsilon}L_e}{i(k_1+l_1)-\frac{1}{\varepsilon}L_e}\Big)},$$
$$\frac{1}{\varepsilon}u_e^{1/2}\mathcal{D}_k^- = -\frac{1}{\varepsilon}u_e^{1/2}v_e^{1/2}\Big(u_{k,l}^0 + \frac{v_e^{1/2}}{u_e^{1/2}}\frac{v_{k,l}^0\frac{1}{\varepsilon}L_e}{2ik-\frac{1}{\varepsilon}L_e}\Big).$$

The Fourier solution of the Cauchy problem for the system (1) will be called the system of absolutely continuous Fourier coefficients $U_k = (u_k, v_k, w_k, z_k),\ k \in Z$, satisfying (5), (7) for almost all $t \in R_+$

1.5 Elimination of secular members

Let's move on to finite-dimensional approximation

$$T_{k,l}(y^{(m)}) = \frac{d}{dt}y_{k,l}^{(m)} - iky_{k,l}^{(m)} + \frac{1}{\varepsilon}L_e y_{k,l}^{(m)} - \tag{13}$$
$$-\frac{1}{\varepsilon}\Big(i(k+l)z_e\int_0^t e^{il(s-t)}y_{k,l}^{(m)}ds + 2ikv_e\int_0^t e^{ik(s-t)}y_{k,l}^{(m)}ds$$
$$+i(k-l)w_e\int_0^t e^{-il(s-t)}y_{k,l}^{(m)}ds\Big) =$$

$$= \frac{1}{\varepsilon} u_e^{1/2} \mathcal{D}^+_{l,m} e^{ilt} + \frac{1}{\varepsilon} u_e^{1/2} \mathcal{D}^-_{l,m} e^{-ilt} + \frac{1}{\varepsilon} u_e^{1/2} \mathcal{D}^-_{k,m} e^{-ikt} +$$
$$e^{-\frac{1}{\varepsilon} L_e t} \Big(\frac{1}{2} \varepsilon u_e^{1/2} (f^{(m)}_{k,l,L}(t) + f^{(m)}_{k,l,B}(t)) +$$
$$+ f^{(m)}_{k,l,int} \Big) + \frac{1}{2} \varepsilon u_e^{1/2} T^{add}_{k,l}(y^{(m)}) + \frac{1}{2} \varepsilon u_e^{1/2} \Big(\mathcal{L}^{(m)}_{k,l}(y^{(m)}) + B^{(m)}_{k,l}(y^{(m)}, y^{(m)}) \Big),$$
$$y^{(m)}_{k,l}|_{t=0} = 0. \tag{14}$$

We set

$$y_{k,l} = Q_{k,-l} T^{-1}_{k,l}(e^{-ilt}) + Q_{k,l} T^{-1}_{k,l}(e^{ilt}) + Q_{-k,l} T^{-1}_{k,l}(e^{-ikt}) + T^{-1}_{k,l}(z_{k,l}), \quad k, l \in Z_0.$$

If we substitute $y_{k,l}(t) = Q_{k,-l} T^{-1}_{k,l}(e^{-ilt}) + Q_{k,l} T^{-1}_{k,l}(e^{ilt}) + Q_{-k,l} T^{-1}_{k,l}(e^{-ikt}) + T^{-1}_{k,l}(z_{k,l}), \quad k, l \in Z_0$ into $\mathcal{L}_{k,l}(y), B_{k,l}(y, y)$ and make the appropriate transformations that isolate the secular terms, then we can find the secular equation. We isolate the terms in these integrals that are not integrable in R_+ using our operator

$$i(k+l) \int_0^t e^{il(s-t)} Q_{k,-l} T^{-1}_{k,l}(e^{-ils}) ds = -\frac{\varepsilon}{z_e} Q_{k,-l} e^{-ilt} +$$
$$+ \frac{\varepsilon}{z_e} \Big(\frac{d}{dt} Q_{k,-l} T^{-1}_{k,l}(e^{-ilt}) - ik Q_{k,-l} T^{-1}_{k,l}(e^{-ilt}) + \frac{1}{\varepsilon} L_e Q_{k,-l} T^{-1}_{k,l}(e^{-ilt}) \Big) -$$
$$- \frac{1}{z_e} \Big(2ikv_e \int_0^t e^{ik(s-t)} Q_{k,-l} T^{-1}_{k,l}(e^{-ils}) ds + i(k-l) w_e \int_0^t e^{-il(s-t)} Q_{k,-l} T^{-1}_{k,l}(e^{-ils}) ds \Big).$$

The same is true for the others. Summing up, we obtain a reduction of the Cauchy problem (13)-(14) to an integral equation in the Hilbert space $L_{2,\gamma}(R_+)$ of the form

$$z^{(m)}_{k,l} + Q_{k,l}\ e^{ilt} + Q_{k,-l}\ e^{-ilt} + Q_{-k,l}\ e^{-ikt} = \frac{1}{2} \varepsilon u_e^{1/2} \Big(S_{k,-l}(Q_{k,-l}, Q_{k,l}) e^{-ilt} + \tag{15}$$
$$+ S_{k,l}(Q_{k,-l}, Q_{k,l}) e^{ilt} \Big) + \frac{1}{\varepsilon} u_e^{1/2} \mathcal{D}^+_{l,m} e^{ilt} + \frac{1}{\varepsilon} u_e^{1/2} \mathcal{D}^-_{l,m} e^{-ilt} +$$
$$+ \frac{1}{\varepsilon} u_e^{1/2} \mathcal{D}^-_{k,m} e^{-ikt} + e^{-\frac{1}{\varepsilon} L_e t} \Big(\frac{1}{2} \varepsilon u_e^{1/2} (F^{(m)}_{k,l,L}(t) + F^{(m)}_{k,l,B}(t)) + f^{(m)}_{k,l,int} \Big) +$$
$$+ \frac{1}{2} \varepsilon u_e^{1/2} \Big(H^{(m)}_{k,l,L}(t) + H^{(m)}_{k,l,B}(t) + \mathcal{U}^{(m)}_{k,l}(T^{-1}_{k,l}(z^{(m)})) + B^{(m)}_{k,l}(T^{-1}_{k,l}(z^{(m)}), T^{-1}_{k,l}(z^{(m)})) +$$
$$+ B_{k,l} \Big(Q_{k,-l} T^{-1}_{k,l}(e^{-ilt}) + Q_{k,l} T^{-1}_{k,l}(e^{ilt}) + Q_{-k,l} T^{-1}_{k,l}(e^{-ikt}), T^{-1}_{k,l}(z^{(m)}_{k,l}) \Big) +$$
$$+ B_{k,l} \Big(T^{-1}_{k,l}(z^{(m)}_{k,l}), Q_{k,-l} T^{-1}_{k,l}(e^{-ilt}) + Q_{k,l} T^{-1}_{k,l}(e^{ilt}) + Q_{-k,l} T^{-1}_{k,l}(e^{-ikt})) \Big) \Big).$$

where

$$F^{(m)}_{k,l,L}(t)+F^{(m)}_{k,l,B}(t)=F^{(m)}_{k,l,L,2}(t)+F^{(m)}_{k,l,B,2}(t)+F^{(m)}_{k,l,L,1}(t)+F^{(m)}_{k,l,B,1}(t),$$

$$\mathcal{U}^{(m)}_{k,l}(T^{-1}_{k,l}(z^{(m)}))=\mathcal{L}^{(m)}_{k,l}(T^{-1}_{k,l}(z^{(m)}))+B^{(m)}_{k,l}(v^0e^{t(ik-\frac{1}{\varepsilon}L_e)},T^{-1}_{k,l}(z^{(m)}))+$$
$$+B^{(m)}_{k,l}(T^{-1}_{k,l}(z^{(m)}),v^0e^{t(ik-\frac{1}{\varepsilon}L_e)})).$$

To reduce the Cauchy problem (13) to an integral equation in the Hilbert space $L_{2,\gamma}(R_+)$, we must cancel the secular terms by requiring that

$$Q_{k,l}\ e^{ilt}+Q_{k,-l}\ e^{-ilt}+Q_{-k,l}\ e^{-ikt}=\frac{1}{2}\varepsilon u_e^{1/2}\Big(S_{k,-l}(Q_{k,-l},Q_{k,l})e^{-ilt}+ \tag{16}$$
$$+S_{k,l}(Q_{k,-l},Q_{k,l})e^{ilt}\Big)+\frac{1}{\varepsilon}u_e^{1/2}\mathcal{D}^+_{l,m}e^{ilt}+\frac{1}{\varepsilon}u_e^{1/2}\mathcal{D}^-_{l,m}e^{-ilt}+\frac{1}{\varepsilon}u_e^{1/2}\mathcal{D}^-_{k,m}e^{-ikt}.$$

Then we finally get

$$z^{(m)}_{k,l}=e^{-\frac{1}{\varepsilon}L_et}\Big(\frac{1}{2}\varepsilon u_e^{1/2}(F^{(m)}_{k,l,L}(t)+F^{(m)}_{k,l,B}(t))+f^{(m)}_{k,l,int}\Big)+$$
$$+\frac{1}{2}\varepsilon u_e^{1/2}\Big(H^{(m)}_{k,l,L}(t)+H^{(m)}_{k,l,B}(t)+$$
$$+\mathcal{U}^{(m)}_{k,l}(T^{-1}_{k,l}(z^{(m)}))+B^{(m)}_{k,l}(T^{-1}_{k,l}(z^{(m)}),T^{-1}_{k,l}(z^{(m)}))+$$
$$+B_{k,l}\Big(Q_{k,-l}T^{-1}_{k,l}(e^{-ilt})+Q_{k,l}T^{-1}_{k,l}(e^{ilt})+Q_{-k,l}T^{-1}_{k,l}(e^{-ikt}),T^{-1}_{k,l}(z^{(m)}_{k,l})\Big)+$$
$$+B_{k,l}\Big(T^{-1}_{k,l}(z^{(m)}_{k,l}),Q_{k,-l}T^{-1}_{k,l}(e^{-ilt})+Q_{k,l}T^{-1}_{k,l}(e^{ilt})+Q_{-k,l}T^{-1}_{k,l}(e^{-ikt}))\Big)\Big). \tag{17}$$

The condition on the secular terms can be rewritten as an algebraic system for $(Q_{k,l},Q_{k,-l},Q_{-k,l})$:

$$Q_{k,l}=\frac{1}{\varepsilon}u_e^{1/2}\mathcal{D}^+_{l,m}+\frac{1}{2}\varepsilon u_e^{1/2}S_{k,l}(Q_{k,-l},Q_{k,l}), \tag{18}$$
$$Q_{k,-l}=\frac{1}{\varepsilon}u_e^{1/2}\mathcal{D}^-_{l,m}+\frac{1}{2}\varepsilon u_e^{1/2}S_{k,-l}(Q_{k,-l},Q_{k,l}),$$
$$Q_{-k,l}=\frac{1}{\varepsilon}u_e^{1/2}\mathcal{D}^-_{k,m},|k|=1,\ldots,m,$$

where

$$S_{k,-l}(Q_{k,-l},Q_{k,l})e^{-ilt}+S_{k,l}(Q_{k,-l},Q_{k,l})e^{ilt}=\sum_{k_1+k_2=k,l_1+l_2=l,|k_1|,|k_2|,|l_1|,|l_2|=1,\ldots,m}\Big\{$$

$$\frac{\varepsilon}{z_e}\frac{\varepsilon}{w_e}\frac{z_e^{1/2}}{u_e^{1/2}}\frac{w_e^{1/2}}{u_e^{1/2}}\Big(Q_{k_1,-l_1}\overline{Q_{k_2,l_2}}e^{-ilt}+Q_{k_2,l_2}\overline{Q_{k_1,-l_1}}e^{ilt}\Big)+$$

$$+\Big(\frac{z_e^{1/2}}{u_e^{1/2}}\frac{v^0_{k_1,l_1}i(k_1+l_1)}{i(k_1+l_1)-\frac{1}{\varepsilon}L_e}\frac{w_e^{1/2}}{u_e^{1/2}}\frac{\varepsilon}{w_e}\overline{Q_{k_2,l_2}}+\frac{z_e^{1/2}}{u_e^{1/2}}\frac{\overline{v^0_{k_2,l_2}}i(k_2-l_2)}{i(k_2-l_2)+\frac{1}{\varepsilon}L_e}\frac{w_e^{1/2}}{u_e^{1/2}}\frac{\varepsilon}{z_e}Q_{k_1,-l_1}\Big)e^{-ilt}+$$

$$+\Big(\frac{z_e^{1/2}}{u_e^{1/2}}\frac{\overline{v^0_{k_1,l_1}}i(k_1+l_1)}{i(k_1+l_1)+\frac{1}{\varepsilon}L_e}\frac{w_e^{1/2}}{u_e^{1/2}}\frac{\varepsilon}{w_e}Q_{k_2,l_2}+\frac{z_e^{1/2}}{u_e^{1/2}}\frac{v^0_{k_2,l_2}i(k_2-l_2)}{i(k_2-l_2)+\frac{1}{\varepsilon}L_e}\frac{w_e^{1/2}}{u_e^{1/2}}\frac{\varepsilon}{w_e}\overline{Q_{k_1,-l_1}}\Big)e^{ilt}-$$

$$-\Big((w^0_{k_1,l_1}+\frac{z_e^{1/2}}{u_e^{1/2}}v^0_{k_1,l_1})\frac{w_e^{1/2}}{u_e^{1/2}}\frac{\varepsilon}{w_e}\overline{Q_{k_2,l_2}}+\frac{z_e^{1/2}}{u_e^{1/2}}(\overline{z^0_{k_2,l_2}}+\frac{w_e^{1/2}}{u_e^{1/2}}\overline{v^0_{k_2,l_2}})\frac{\varepsilon}{z_e}Q_{k_1,-l_1}\Big)e^{-ilt}-$$

$$-\Big((z^0_{k_2,l_2}+\frac{w_e^{1/2}}{u_e^{1/2}}v^0_{k_2,l_2})\frac{\varepsilon}{z_e}\overline{Q_{k_1,-l_1}}\frac{z_e^{1/2}}{u_e^{1/2}}+(\overline{w^0_{k_1,l_1}}+\frac{z_e^{1/2}}{u_e^{1/2}}\overline{v^0_{k_1,l_1}})\frac{w_e^{1/2}}{u_e^{1/2}}\frac{\varepsilon}{w_e}Q_{k_2,l_2}\Big)e^{ilt}\Big\}.$$

1.6 Conclusion

A two-dimensional discrete kinetic system of Broadwell is considered. This system is reduced to a nonlinear equation in weight space, provided that the secularity conditions are satisfied. These conditions allow us to study the nonlinear equation and prove exponential stability to the equilibrium state in the future.

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