MATRIX

Let a matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots\cdots & a_{mn} \end{bmatrix}$$

1) Matrix is enclosed by [ ] or ( ) or | | | |

2) Representation of matrix, $[a_{ij}]_{m \times n}$ or $A = [a_{ij}]$

3) Element of a Matrix, The numbers a_{11}, a_{12} … etc.
Elements of matrix is represented as a_{ij}, which denotes element in ith row and jth column.

4) Order of a Matrix has m rows and n columns, then order = m x n.

5) Number of elements = mn

Types of Matrices

1. Row Matrix : A matrix having only one row and any number of columns

2. Column Matrix : A matrix having only one column and any number of rows

3. Rectangular Matrix : A matrix of order m x n, such that $m \neq n$

4. Horizontal Matrix : A matrix in which the number of rows is less than the number of columns

5. Vertical Matrix : A matrix in which the number of rows is greater than the number of columns

6. Null/Zero Matrix : A matrix of any order, having all its elements are zero

7. Square Matrix : A matrix of order m x n, such that m = n

8. Diagonal Matrix : A square matrix $A = [a_{ij}]_{m \times n}$, is called a diagonal matrix, if all the elements except those in the leading diagonals are zero, i.e., $a_{ij} = 0$ for $i \neq j$. It can be represented as
   $A = diag[a_{11}\ a_{22} \ldots\ a_{nn}]$

9. Scalar Matrix : A square matrix in which every non-diagonal element is zero and all diagonal elements are equal, i.e., in scalar matrix
   $a_{ij} = 0$, for $i \neq j$ and $a_{ij} = k$, for $i = j$

10. Unit/Identity Matrix : A square matrix, in which every non-diagonal element is zero and every diagonal element is 1

11. Upper Triangular Matrix : A square matrix $A = a[_{ij}]_{n \times n}$ is called a upper triangular matrix, if $a[_{ij}], = 0, \forall\ i > j$.

12. Lower Triangular Matrix : A square matrix $A = a[_{ij}]_{n \times n}$ is called a lower triangular matrix, if $a[_{ij}], = 0, \forall\ i < j$.

13. Submatrix : A matrix which is obtained from a given matrix by deleting any number of rows or columns or both is called a submatrix of the given matrix.

14. Equal Matrices : Two matrices A and B are said to be equal, if both having same order and corresponding elements of the matrices are equal.

15. Principal Diagonal of a Matrix In a square matrix, the diagonal from the first element of the first row to the last element of the last row is called the principal diagonal of a matrix.

    NOTE : $a_{11}, a_{22}, a_{33}, \ldots\ldots a_{mn}$ are principal diagonal elements.

16. Singular Matrix : A square matrix A is said to be singular matrix, if determinant of A denoted by det (A) or $|A|$ is zero, i.e., $|A| = 0$, otherwise it is a non-singular matrix.

Algebra of Matrices

1. Addition of Matrices

Let A and B be two matrices each of order m x n. Then, the sum of matrices A + B is defined only if matrices A and B are of same order.

If $A = [a_{ij}]_{m \times n}$, $A = [a_{ij}]_{m \times n}$

Then, $A + B = [a_{ij} + b_{ij}]_{m \times n}$

Properties of Addition of Matrices If A, B and C are three matrices of order m x n, then

1. **Commutative Law** A + B = B + A
2. **Associative Law** (A + B) + C = A + (B + C)
3. **Existence of Additive Identity** A zero matrix (0) of order m x n (same as of A), is additive identity, if
   A + 0 = A = 0 + A
4. **Existence of Additive Inverse** If A is a square matrix, then the matrix (- A) is called additive inverse, if
   A + (– A) = 0 = (- A) + A
5. **Cancellation Law**
   A + B = A + C ⇒ B = C (left cancellation law) B + A = C + A ⇒ B = C (right cancellation law)

2. Subtraction of Matrices

Let A and B be two matrices of the same order, then subtraction of matrices, A – B, is defined as

$A - B = [a_{ij} - b_{ij}]_{n \times n}$,

where $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{m \times n}$

3. Multiplication of a Matrix by a Scalar

Let $A = [a_{ij}]_{m \times n}$ be a matrix and k be any scalar. Then, the matrix obtained by multiplying each element of A by k is called the scalar multiple of A by k and is denoted by kA, given as $kA= [ka_{ij}]_{m \times n}$

Properties of Scalar Multiplication If A and B are matrices of order m x n, then

1. $k(A + B) = kA + kB$
2. $(k_1 + k_2)A = k_1A + k_2A$
3. $k_1k_2A = k_1(k_2A) = k_2(k_1A)$
4. $(-k)A = -(kA) = k(-A)$

4. Multiplication of Matrices

Let $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{n \times p}$ are two matrices such that the number of columns of A is equal to the number of rows of B, then multiplication of A and B is denoted by AB, where c_{ij} is the element of matrix C and $C = AB$

Properties of Multiplication of Matrices

1. Commutative Law Generally $AB \neq BA$
2. Associative Law $(AB)C = A(BC)$
3. Existence of multiplicative Identity $A.I = A = I.A$,
   I is called multiplicative Identity.
4. Distributive Law $A(B + C) = AB + AC$
5. Cancellation Law If A is non-singular matrix,
   then $AB = AC \Rightarrow B = C$ (left cancellation law)
   $BA = CA \Rightarrow B = C$ (right cancellation law)
6. $AB = 0$, does not necessarily imply that $A = 0$ or $B = 0$ or both A and $B = 0$

Important Points to be Remembered

(i) If A and B are square matrices of the same order, say n, then both the product AB and BA are defined and each is a square matrix of order n.

(ii) In the matrix product AB, the matrix A is called premultiplier (prefactor) and B is called postmultiplier (postfactor).

(iii) The rule of multiplication of matrices is row column wise (or $\rightarrow \downarrow$ wise) the first row of AB is obtained by multiplying the first row of A with first, second, third,... columns of B respectively; similarly second row of A with first, second, third, ... columns of B, respectively and so on.

Positive Integral Powers of a Square Matrix

Let A be a square matrix. Then, we can define

1. $A^{n+1} = A^n . A$, where $n \in N$.
2. $A^m . A^n = A^{m+n}$
3. $(A^m)^n = A^{mn}$, where $m, n \in N$

Transpose of a Matrix

Let $A = [a_{ij}]_{m \times n}$, be a matrix of order m x n. Then, the n x m matrix obtained by interchanging the rows and columns of A is called the transpose of A and is denoted by A' or A^T.
$A' = A^T = [a_{ij}]_{n \times m}$

Properties of Transpose

1. $(A')' = A$
2. $(A + B)' = A' + B'$
3. $(AB)' = B'A'$
4. $(KA)' = kA'$
5. $(A^N)' = (A')^N$
6. $(ABC)' = C' B' A'$

Symmetric and Skew-Symmetric Matrices

1. A square matrix $A = [a_{ij}]_{n \times n}$, is said to be symmetric, if $A' = A$.
   i.e., $a_{ij} = a_{ji}$
2. A square matrix A is said to be skew-symmetric matrices, if i.e., $aij = — aji$

Properties of Symmetric and Skew-Symmetric Matrices

1. Elements of principal diagonals of a skew-symmetric matrix are all zero. i.e., $a_{ii} = — a_{ii}$ $2_{ii} = 0$ or $a_{ii} = 0$, for all values of i.
2. If A is a square matrix, then
   (a) $A + A'$ is symmetric.
   (b) $A — A'$ is skew-symmetric matrix.
3. If A and B are two symmetric (or skew-symmetric) matrices of same order, then A + B is also symmetric (or skew-symmetric).
4. If A is symmetric (or skew-symmetric), then kA (k is a scalar) is also symmetric for skew-symmetric matrix.

5. If A and B are symmetric matrices of the same order, then the product AB is symmetric, if BA = AB.
6. Every square matrix can be expressed uniquely as the sum of a symmetric and a skew- symmetric matrix.
7. The matrix B' AB is symmetric or skew-symmetric according as A is symmetric or skew-symmetric matrix.
8. All positive integral powers of a symmetric matrix are symmetric.
9. All positive odd integral powers of a skew-symmetric matrix are skew-symmetric and positive even integral powers of a skew-symmetric are symmetric matrix.
10. If A and B are symmetric matrices of the same order, then
    (a) AB – BA is a skew-symmetric and
    (b) AB + BA is symmetric.
11. For a square matrix A, AA' and A' A are symmetric matrix.

Equivalent Matrix

- Two matrices A and B are said to be equivalent, if one can be obtained from the other by a sequence of elementary transformation.
- The symbol ≈ is used for equivalence.

Thank You!!!