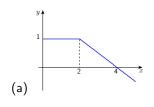
## Cálculo

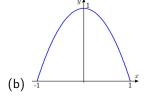
— folha 6 (nova) -

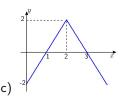
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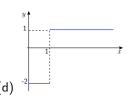
Primitivas.

1. Considere, em cada alínea, a função  $f:I:\longrightarrow \mathbb{R},\ I$  um intervalo, representada graficamente por









Esboce F, uma primitiva de f em I, sabendo que:

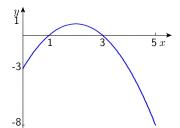
(a) 
$$I = [0, 5]$$

(c) 
$$I = [0, 4] e F(0) = -2$$

(b) 
$$I = [-1, 1], f(x) = 1 - x^2 \in F(0) = 0$$

(d) 
$$I = [0, 4] e F(0) = 1$$

**2.** Seja  $f:[0,5]\longrightarrow \mathbb{R}$  representada graficamente na figura ao lado. Considere uma função primitiva de  $f,\,F:[0,5]\longrightarrow \mathbb{R}$ .



- (a) Encontre os pontos críticos de F.
- (b) Classifique os pontos críticos de F.
- (c) Esboce graficamente F.
- 3. Calcule os seguintes integrais indefinidos

(a) 
$$\int (3x^2 - 2x^5) dx$$

(g) 
$$\int \frac{2x+1}{x^2+x+3} \, dx$$

(m) 
$$\int \frac{\sqrt{1+3 \ln a}}{a} da$$

(b) 
$$\int (\sqrt{x} + 2)^2 dx$$

(h) 
$$\int \frac{t}{3-t^2} dt$$

(n) 
$$\int z \sin z^2 dz$$

(c) 
$$\int (2\theta + 10)^{20} d\theta$$

$$(i) \int \frac{1}{4-3x} \, dx$$

(o) 
$$\int \frac{1}{x(\ln^2 x + 1)} \, dx$$

(d) 
$$\int x^4 (x^5 + 10)^9 dx$$

(j) 
$$\int \operatorname{th} x \, dx$$

(p) 
$$\int \left(\frac{2}{x} - 3\right)^2 \frac{1}{x^2} dx$$

(e) 
$$\int y^2 e^{y^3} \, dy$$

$$(k) \int \frac{1}{e^{3x}} \, dx$$

(q) 
$$\int \operatorname{sen}(\pi - 2x) \, dx.$$

(f) 
$$\int \sqrt{2x+1} \, dx$$

$$(1) \int \frac{-7}{\sqrt{1-5x}} \, dx$$

**4.** Sendo 
$$f: \mathbb{R} \longrightarrow \mathbb{R}$$
 definida por  $f(x) = x^2 \operatorname{sen} x$ , calcule a primitiva de  $f$  cujo gráfico passa pelo ponto  $(\frac{\pi}{2}, \pi)$ .

**5.** Encontre uma antiderivada F sabendo que F(1) = 0. A solução encontrada é única?

(a) 
$$f(x) = \operatorname{sen} x \cos x$$

(c) 
$$f(x) = \sin^2 x$$

(b) 
$$f(x) = \operatorname{sen}(2x) \cos x$$

(d) 
$$f(x) = \frac{1}{x}, x < 0$$

6. Usando primitivação por partes calcule:

(a) 
$$\int \ln x \, dx$$

(b) 
$$\int x \operatorname{sen}(2x) dx$$

(c) 
$$\int \operatorname{arctg} x \, dx$$

(d) 
$$\int x \cos x \, dx$$

(i) 
$$\int \ln^2 x \, dx$$

(n) 
$$\int x \arctan x \, dx$$

(e) 
$$\int \ln(1-x) dx$$

(j) 
$$\int e^x \cos x \, dx$$

(o) 
$$\int x^2 \ln x \, dx$$

(f) 
$$\int x \ln x \, dx$$

(k) 
$$\int \operatorname{arcsen} x \, dx$$

(p) 
$$\int \operatorname{sen}(\ln x) dx$$

(g) 
$$\int x^2 \sin x \, dx$$

(I) 
$$\int e^{\sin x} \sin x \cos x \, dx$$

(q) 
$$\int \operatorname{ch} x \operatorname{sen}(3x) dx$$

(h) 
$$\int x \sin x \cos x \, dx$$

(m) 
$$\int \frac{\arcsin\sqrt{x}}{\sqrt{x}} \, dx$$

(r) 
$$\int x^3 e^{x^2} dx$$
.

#### 7. Calcule os seguinte integrais indefinidos.

(a) 
$$\int \frac{3x^2 - 4x - 1}{(x^2 - 1)(x - 2)} dx$$
 (c)  $\int \frac{4x^2 + x + 1}{x^3 - x} dx$ 

(c) 
$$\int \frac{4x^2 + x + 1}{x^3 - x} dx$$

(e) 
$$\int \frac{x^4 - 8}{x^3 - 2x^2} dx$$

(b) 
$$\int \frac{2x^2 + x + 1}{(x - 1)(x + 1)^2} dx$$
 (d)  $\int \frac{27}{x^4 - 3x^3} dx$ 

(d) 
$$\int \frac{27}{x^4 - 3x^3} \, dx$$

(f) 
$$\int \frac{x+3}{(x-2)(x^2-2x+5)} dx$$

#### 8. Calcule as seguintes primitivas usando a substituição indicada.

(a) 
$$\int x\sqrt{x-1} \, dx$$
,  $x = t^2 + 1$ 

(c) 
$$\int \frac{e^{2x}}{1+e^x} dx, \quad x = \ln t$$

(b) 
$$\int \sqrt{1-x^2} \, dx, \quad x = \operatorname{sen} t$$

(d) 
$$\int \sqrt{1+x^2} \, dx, \quad x = \operatorname{sh} t$$

### 9. Calcule os seguintes integrais indefinidos

(a) 
$$\int \frac{x}{x^2 - 1} dx$$

(i) 
$$\int \frac{1}{x} (1 + \ln^2 x) dx$$

(p) 
$$\int \frac{x e^{\sqrt{1-x^2}}}{\sqrt{1-x^2}} dx$$

(b) 
$$\int \frac{x}{\sqrt{x^2 - 1}} \, dx$$

(j) 
$$\int \frac{2 + \sqrt{\arctan(2x)}}{1 + 4x^2} dx$$

(q) 
$$\int \frac{1}{\cos^2 x \, \sin^2 x} \, dx$$

(c) 
$$\int \frac{1}{x} \, \, \mathrm{sen}(\ln x) \, dx$$

$$\text{(k)} \int \frac{e^{\arctan x}}{1+x^2} \, dx$$

(d) 
$$\int \frac{-3}{x \left(\ln x\right)^3} \, dx$$

(I) 
$$\int \frac{\sin x}{\sqrt{1+\cos x}} \, dx$$

(r) 
$$\int \cos^2 x \, \sin^2 x \, dx$$

(e) 
$$\int \frac{e^x}{1 + e^{2x}} \, dx$$

(m) 
$$\int \frac{1}{(2+\sqrt{x})^7 \sqrt{x}} \, dx$$

(s) 
$$\int \frac{1}{1+e^x} dx$$

$$(f) \int \frac{e^x}{1 - 2e^x} \, dx$$

(n) 
$$\int tg^2 x dx$$

(n) 
$$\int \mathsf{tg}^2 \, x \, dx$$

(t) 
$$\int \frac{1}{x\sqrt{x^2-1}} dx$$

(g) 
$$\int \frac{1}{\cos^2(7x)} dx$$
(h) 
$$\int \left(\sqrt{2x-1} - \sqrt{1+3x}\right)$$

(h) 
$$\int \cos^2(7x)$$
 (o)  $\int \frac{x + [\arccos(3x)]^4}{\sqrt{1 - 9x^2}} dx$ 

(u) 
$$\int \frac{1}{x^2 \sqrt{4-x^2}} dx$$
.

# **10.** Em cada alínea, determine a única função $f: \mathbb{R} \longrightarrow \mathbb{R}$ , duas vezes derivável, tal que:

(a) 
$$f''(x) = 4x - 1$$
,  $x \in \mathbb{R}$ ,  $f(1) = 3$  e  $f'(2) = -2$ 

$$f(1) = 3$$

$$f'(2) = -2$$

(b) 
$$f''(x) = \sec x \cos x$$
,  $x \in \mathbb{R}$ ,  $f(0) = 0$  e  $f'(0) = 1$ .

$$f'(0) = 0$$
 e  $f'(0)$