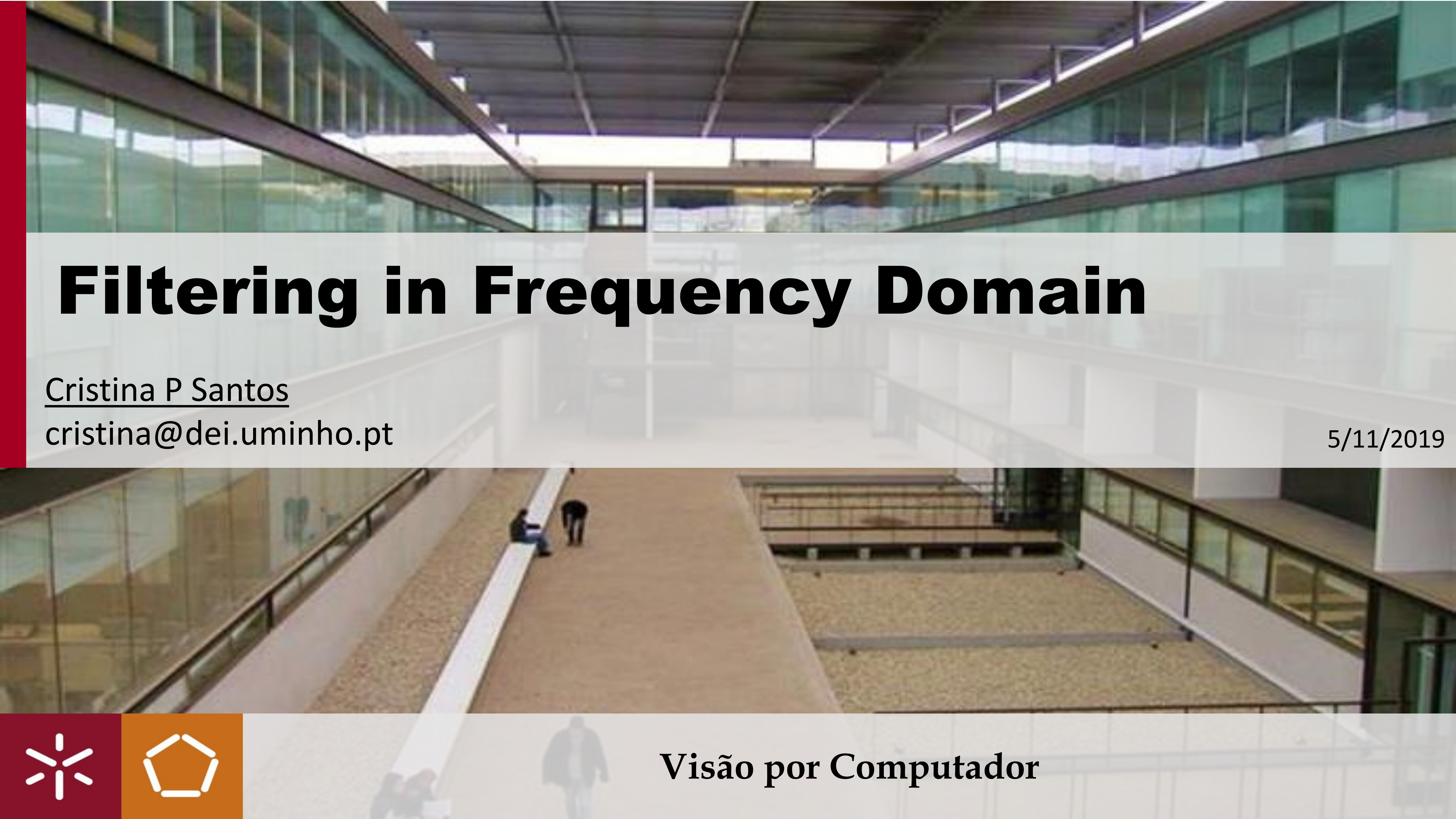

Filtering in Frequency Domain

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Visão por Computador

Contents

This lecture will cover:

- Filtering: Spatial domain *vs* Frequency domain
- Image filtering in frequency domain
    - Low-Pass filtering
    - High-Pass filtering
    - Band-reject filtering
    - Applications

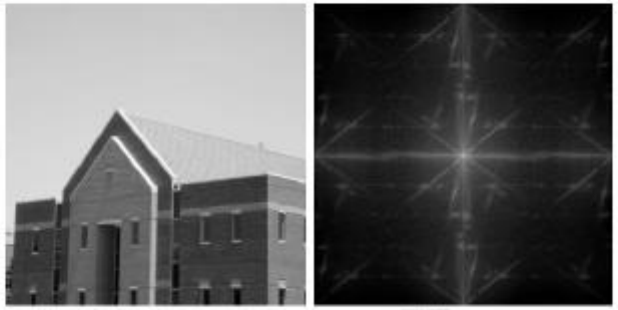
Filtering:
Spatial domain *vs* Frequency domain

Image Filtering

- **Filtering Domain:**
  - **Spatial domain:** acting directly on the pixels of the image
  - **Frequency domain:** acting on Fourier transform of an image and not directly on the image, and then computing the inverse transform to obtain the processed result.

Frequency Domain vs Spatial Domain

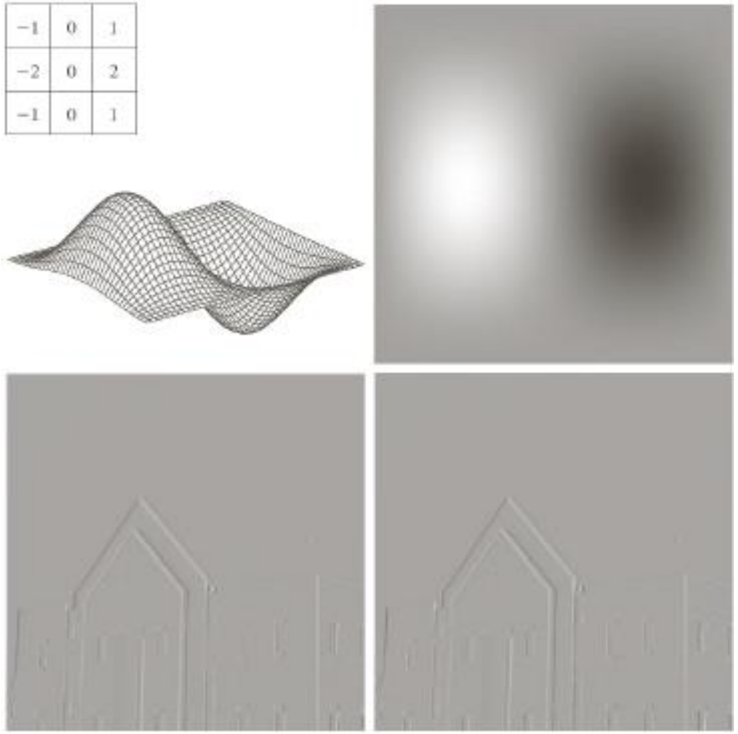
Comparison using Sobel filter



a b

FIGURE 4.38 (a) Image of a building, and (b) its spectrum.

- Similar results among frequency domain and spatial domain.
- The relation between both images is **convolution theorem**.



a b
c d

FIGURE 4.39 (a) A spatial mask and perspective plot of its corresponding frequency domain filter. (b) Filter shown as an image. (c) Result of filtering Fig. 4.38(a) in the frequency domain with the filter in (b). (d) Result of filtering the same image with the spatial filter in (a). The results are identical.

Frequency domain Spatial domain

Frequency Domain vs Spatial Domain

- **In practice**, we prefer to implement c**onvolution filtering with small filter masks** because of **speed** and **ease of implementation** in hardware and/or firmware.

- Moreover, **images do not have their information encoded in the frequency domain**, making these techniques much less useful

- Fourier transform do not help you to understand the information encoded in images.

Frequency Domain vs Spatial Domain

- However, **filtering concepts are more intuitive** in the frequency domain.
  - The **high frequencies** correspond to **more abrupt transitions and faster and faster intensity changes** in the image. E.g., edges of the image, and other abrupt changes in intensity.
  - The **low frequencies** correspond to the **slowly varying intensity components of an image.**
  - Furthermore, **noise is usually embedded in the high end of the spectrum**, so low-pass filtering can be used for noise reduction.

Frequency Domain vs Spatial Domain

- One way to **take advantage of the properties of both domains**
  - to **specify a filter in the frequency domain**, compute its IDFT
  - Then, use the resulting **full-size spatial filter for constructing smaller spatial filter masks**
  - Finally, do the **filtering in spatial domain through convolution.**

Treating Images in Frequency Domain

$$f(x,y) \longrightarrow \boxed{h(x,y)} \longrightarrow g(x,y)$$

Pixel Domain	$g(x,y) = f(x,y) * h(x,y)$	Convolution
Freq. Domain	$G(\omega_x,\omega_y) = F(\omega_x,\omega_y) \times H(\omega_x,\omega_y)$	Multiplication

The Convolution Property of the Fourier Transform

Treating Images in Frequency Domain

Fourier transform of an image has two components:

- **Magnitude (spectrum):** usually provides prominent components, e.g., edges
- **Phase angle**: visual analysis of phase component it is not useful

$$I(x,y) \xrightarrow{\text{DFT}} \left|\hat{I}(x,y)\right| \quad \angle\hat{I}(x,y)$$

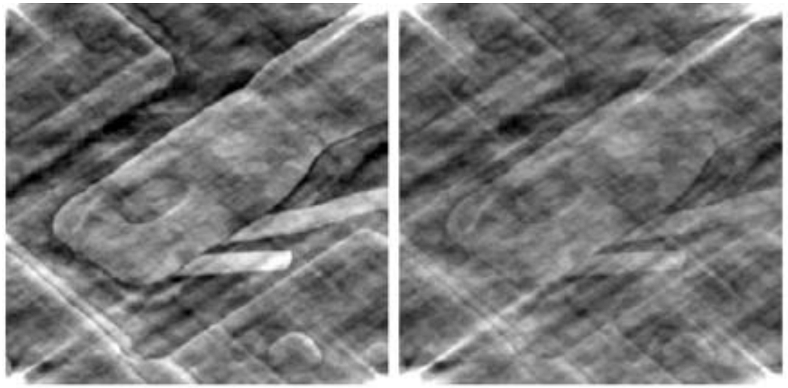
- Phase simply reflects the (group) delay for each of the frequency components.
- The (group) delay = $-\frac{d(phase)}{d(w)}$, where $w = 2\pi f$.
- An interesting kind of system is that for which the **phase is linear** => delay is flat

Treating Images in Frequency Domain

The **Fourier transform of an image has two components**:

- **Magnitude (spectrum):** usually provides prominent components, e.g., edges
- **Phase angle**: visual analysis of phase component it is not useful

$$I(x,y) \xrightarrow{\text{DFT}} \left|\hat{I}(x,y)\right| \;\; \angle \hat{I}(x,y)$$



a b

FIGURE 4.35 (a) Image resulting from multiplying by 0.5 the phase angle in Eq. (4.6-15) and then computing the IDFT. (b) The result of multiplying the phase by 0.25. The spectrum was not changed in either of the two cases.

Phase angle can have **undesired effects** on the filtered output, since it can **distort the intensity distribution**.

Treating Images in Frequency Domain

Fourier transform :

Let $f(x, y)$, for $x = 0, 1, 2, \ldots, M-1$ and $y = 0, 1, 2, \ldots, N-1$, denote an $M \times N$ image. The 2-D, *discrete Fourier transform* (DFT) of f, denoted by $F(u, v)$, is given by the equation

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$$

for $u = 0, 1, 2, \ldots, M-1$ and $v = 0, 1, 2, \ldots, N-1$. We could expand the exponential into sines and cosines with the variables u and v determining their frequencies (x and y are summed out). The *frequency domain* is simply the

Even if $f(x, y)$ is real, its transform in general is complex. The principal method of visually analyzing a transform is to compute its *spectrum* [i.e., the magnitude of $F(u, v)$] and display it as an image. Letting $R(u, v)$ and $I(u, v)$ represent the real and imaginary components of $F(u, v)$, the Fourier spectrum is defined as

$$|F(u, v)| = \left[R^2(u, v) + I^2(u, v)\right]^{1/2}$$

The *phase angle* of the transform is defined as

$$\phi(u, v) = \tan^{-1}\left[\frac{I(u, v)}{R(u, v)}\right]$$

The preceding two functions can be used to represent $F(u, v)$ in the familiar polar representation of a complex quantity:

$$F(u, v) = |F(u, v)|e^{-j\phi(u, v)}$$

The *power spectrum* is defined as the square of the magnitude:

$$\begin{aligned} P(u, v) &= |F(u, v)|^2 \\ &= R^2(u, v) + I^2(u, v) \end{aligned}$$

For purposes of visualization it typically is immaterial whether we view $|F(u, v)|$ or $P(u, v)$.

Filtering in Frequency Domain

Main Steps

Main steps for the filtering in frequency domain

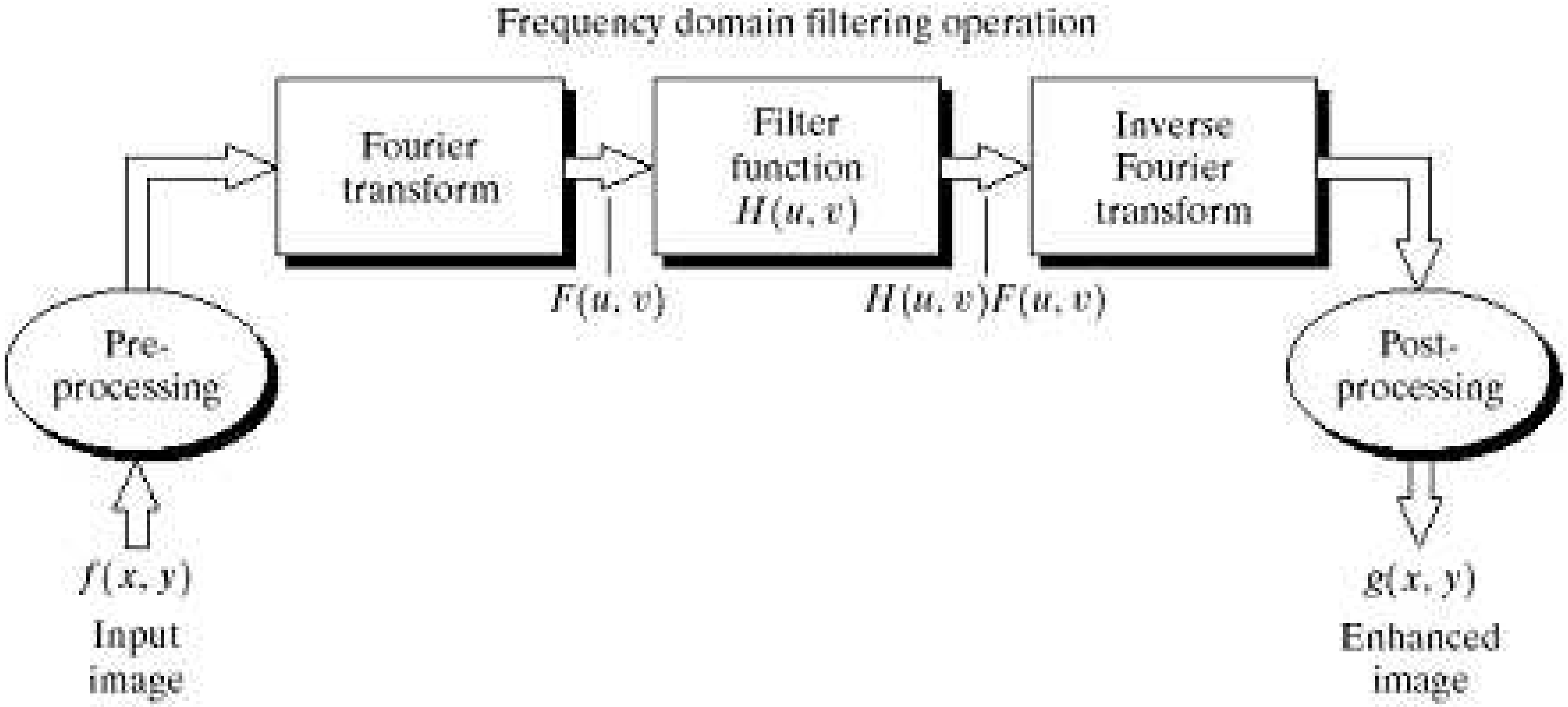


FIGURE 4.5 Basic steps for filtering in the frequency domain.

***H(u, v) a* filter,** also called filter transfer function

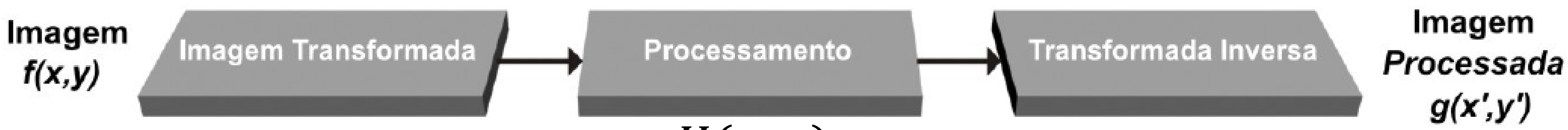
H multiplies real and imaginary parts of *F*

The phase does not change, if *H* is real

Main Steps

Main steps for the filtering in frequency domain

- A imagem *f(x,y)* (size *M x N*) é **transformada do domínio espacial para o da frequência** (transformada de Fourier).
- Operações de filtragem são realizadas na imagem transformada *F(u,v)*.
- Ao produto *F(u,v).H(u,v)* é aplicada a **inversa da transformada de Fourier** para retornar ao **domínio espacial**, onde se tem a imagem processada *f'(x,y).*

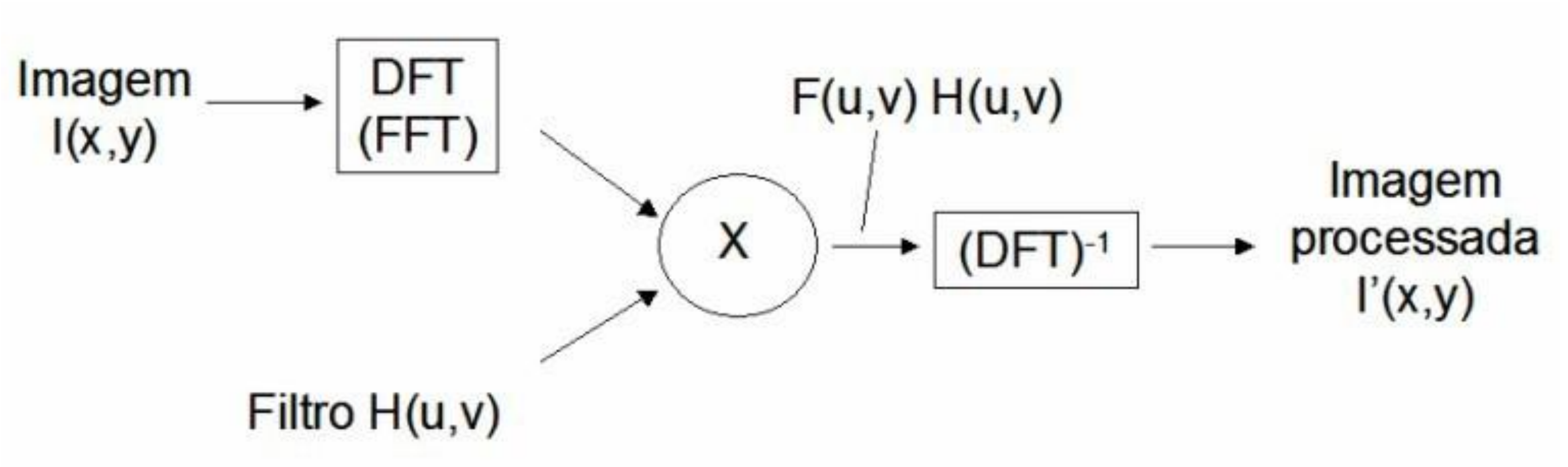


$$H(u,v)$$

$$g(x,y) = T^{-1}\{H(u,v).F(u,v)\} \quad ,T^{-1} = \ IDFT$$

Main Steps

Main steps for the filtering in frequency domain



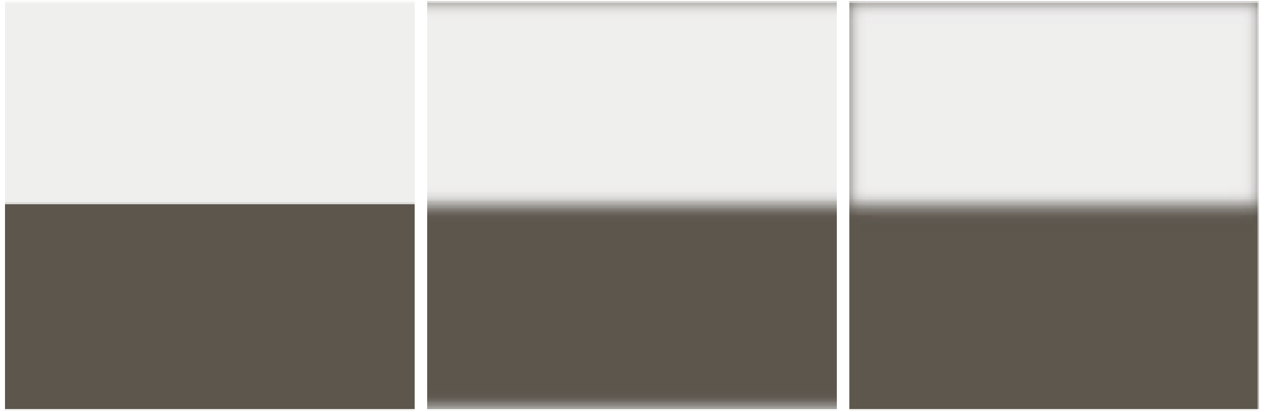
- **In general, *H(u; v)* is real => zero-phase shift filters**: filters that affects the real and imaginary parts equally, and thus have no effect on the phase.

Image Padding

- **What are the diferences between image b) and c) ?**

Gaussian lowpass filter **Without** padding

Gaussian lowpass filter **With** padding



a b c

FIGURE 4.32 (a) A simple image. (b) Result of blurring with a Gaussian lowpass filter without padding. (c) Result of lowpass filtering with padding. Compare the light area of the vertical edges in (b) and (c).

- The images were filtered using the same low-pass filter.
- **Image b)** => **blurring is not uniform**: the top white edge is blurred, but the side white edges are not.
- **Image c) =>** padding the input image

$$P \geq 2M - 1 \qquad Q \geq 2N - 1$$

Image Padding

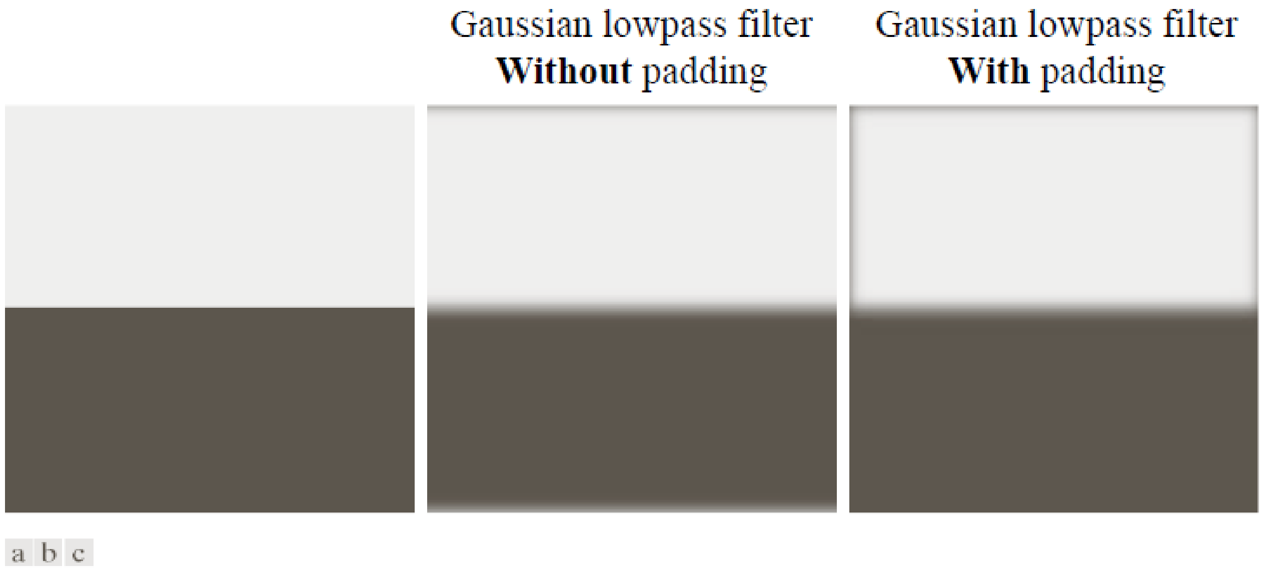


FIGURE 4.32 (a) A simple image. (b) Result of blurring with a Gaussian lowpass filter without padding. (c) Result of lowpass filtering with padding. Compare the light area of the vertical edges in (b) and (c).

DFT representation

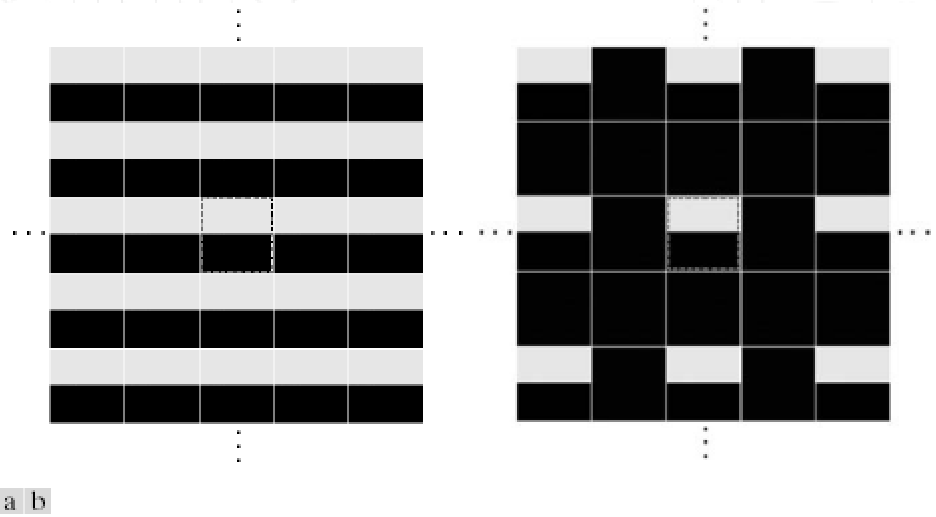


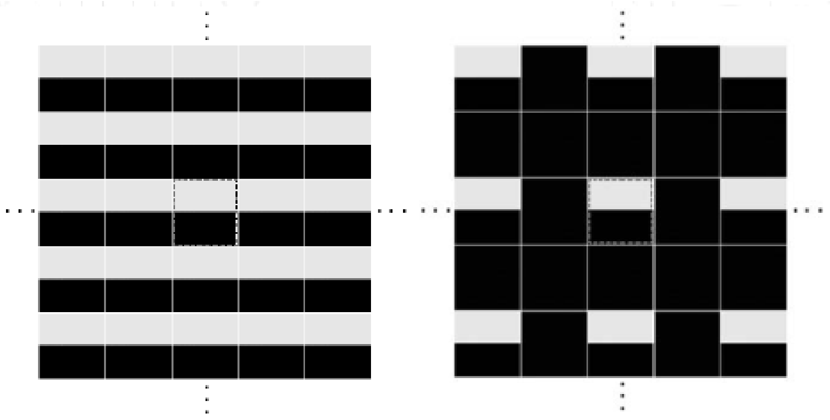
FIGURE 4.33 2-D image periodicity inherent in using the DFT. (a) Periodicity without image padding. (b) Periodicity after padding with 0s (black). The dashed areas in the center correspond to the image in Fig. 4.32(a). (The thin white lines in both images are superimposed for clarity; they are not part of the data.)

Note

The padding introduces **changes in DFT representation of the image**.

Image Padding

- The **dashed areas** corresponds to the image.
- When the filter is passing through the top of the dashed image, it will encompass **part of the image** and also part of the **bottom of the periodic image** right above it.
  - **Filter has a dark and a light region => Result is a mid-gray, blurred output**.
- When the filter is passing through the top right side of the image, the **filter will encompass only light areas in the image and its right neighbor**. The average of a constant, is constant, **so filtering will have no effect in this area (resulting image b).**

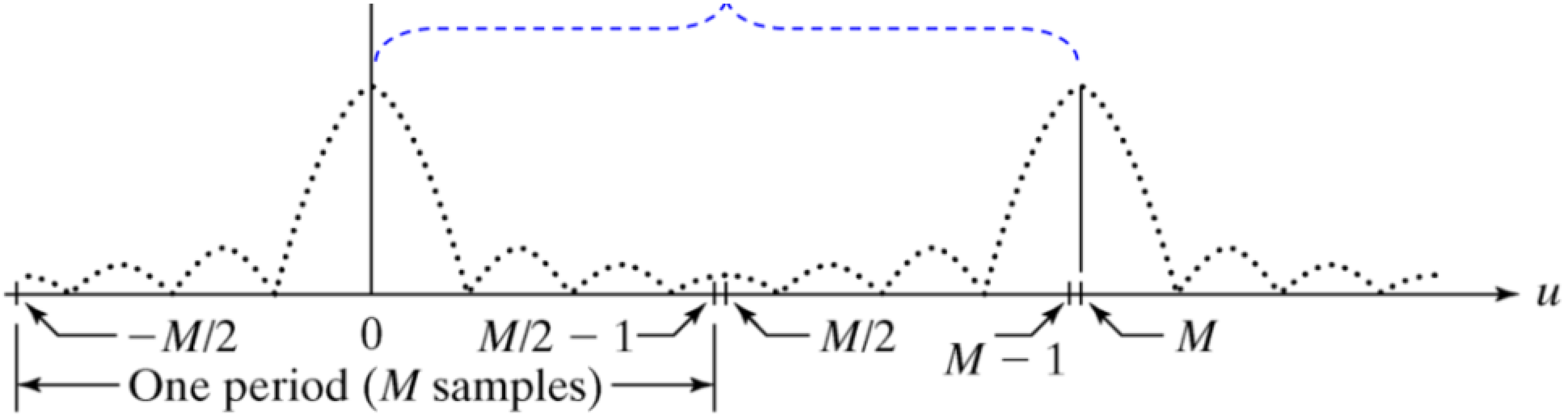


a b

FIGURE 4.33 2-D image periodicity inherent in using the DFT. (a) Periodicity without image padding. (b) Periodicity after padding with 0s (black). The dashed areas in the center correspond to the image in Fig. 4.32(a). (The thin white lines in both images are superimposed for clarity; they are not part of the data.)

Image Padding

Fourier transform is periodic in nature. Periodic function can cause interference between adjacent periods and this will lead to wraparound error.

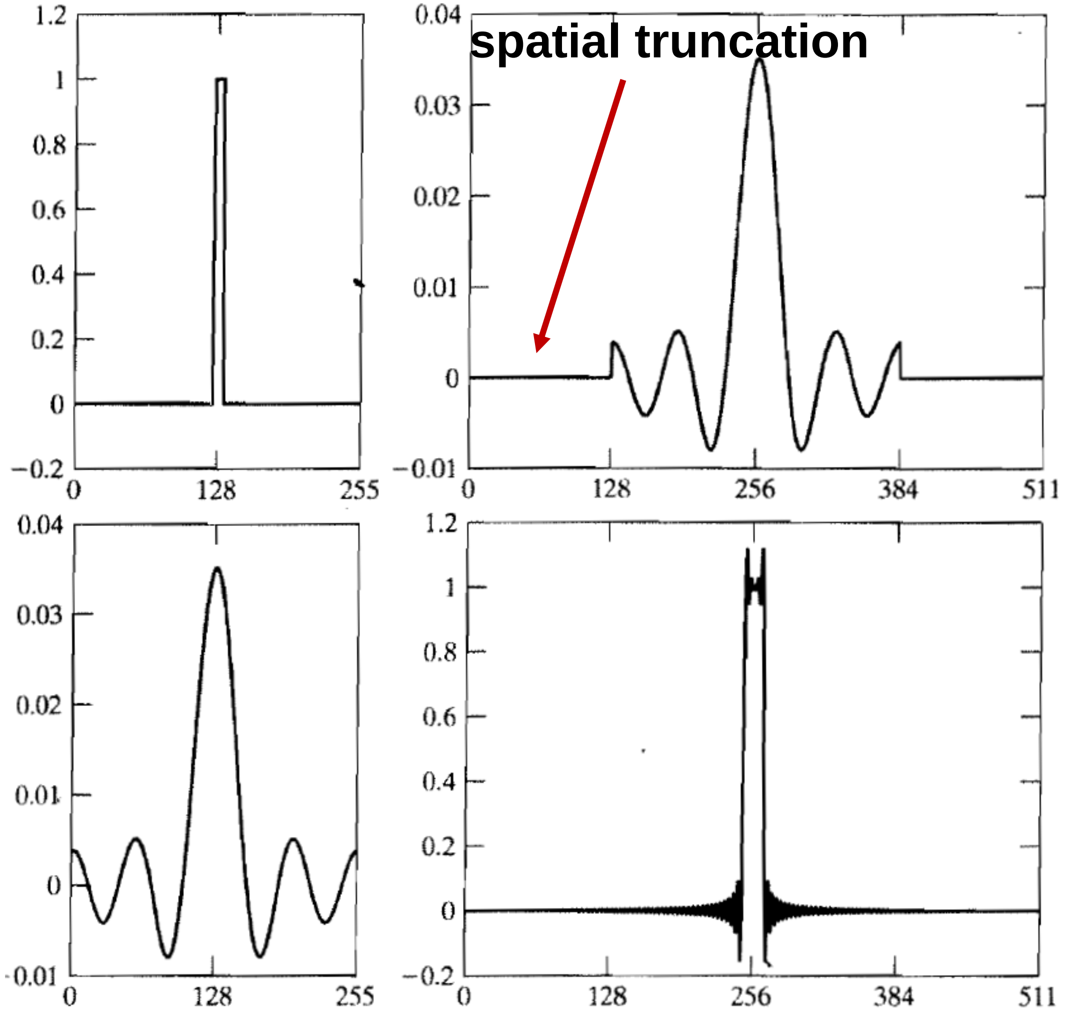


To overcome this issue: Padding the image with 0 **creates a uniform border** around the periodic sequence. **Convolving the blurring function with the padded**, gives the correct result in image c).

Image Padding

- The extremes of the spatial function are not zero so, as shows figure (4.34 (c)), **zero-padding introduces a spatial truncation** at the *sinc* function (infinite), and consequently it **introduces discontinuities.**

- The **discontinuities in spatial domain** created **ringing in its frequency domain.**



a c
b d

FIGURE 4.34
(a) Original filter specified in the (centered) frequency domain. (b) Spatial representation obtained by computing the IDFT of (a). (c) Result of padding (b) to twic its length (note the discontinuities). (d) Corresponding filter in the frequency domain obtained by computing the DFT of (c). Note the ringing caused by the discontinuities in (c). (The curves appear continuous because the points were joined to simplify visual analysis.)

Image Padding

To avoid discontinuities, and the ringing effect:

- **One approach**: **zero-pad image** and the **created filters in the frequency domain** to be of the same size as the padded images

Note Filters and images must have the same size. **Why??**

- This will result in **wraparound error** because **no padding is used for the filter**, but in practice **this error is mitigated significantly** by the separation provided by the padding of the image and it **is preferable to ringing effect**.

Procedure for frequency domain

1. Given an input image $f(x,y)$ of size MxN, obtain padding parameters P and Q. Typically, $P=2M$ and $Q=2N$.
2. Form a padded image $f_p(x,y)$ of size PxQ by appending the necessary number of zeros to f(x,y).
3. Multiply $f_p(x,y)$ by $(-1)^{x+y}$ to centre its transform.
4. Compute the DFT, $F(u,v)$, of the image from step 3.
5. Generate a real, symmetric filter function, $H(u,v)$, of size PxQ with centre at coordinates $(P/2,Q/2)$. Form the product $G(u,v)=H(u,v)F(u,v)$ using array multiplication.
6. Obtain the processed image: $g_p(x,y) = \text{real}\left[IDFT\left[G(u,v)\right]\right](-1)^{x+y}$
7. Obtain the final processed result, $g(x,y)$, by extracting the MxN region from the top, left quadrant of $g_p(x,y)$

Image padding

$H(u,v)$ is simplified by using functions that are **symmetric** about their center, which requires that $\boldsymbol{F(u,v)}$ **be centered** also.

Procedure for frequency domain

1. Given an input image $f(x,y)$ of size MxN, obtain padding parameters P and Q. Typically, $P=2M$ and $Q=2N$.
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7. Obtain the final processed result, $g(x,y)$, by extracting the MxN region from the top, left quadrant of $g_p(x,y)$

Note

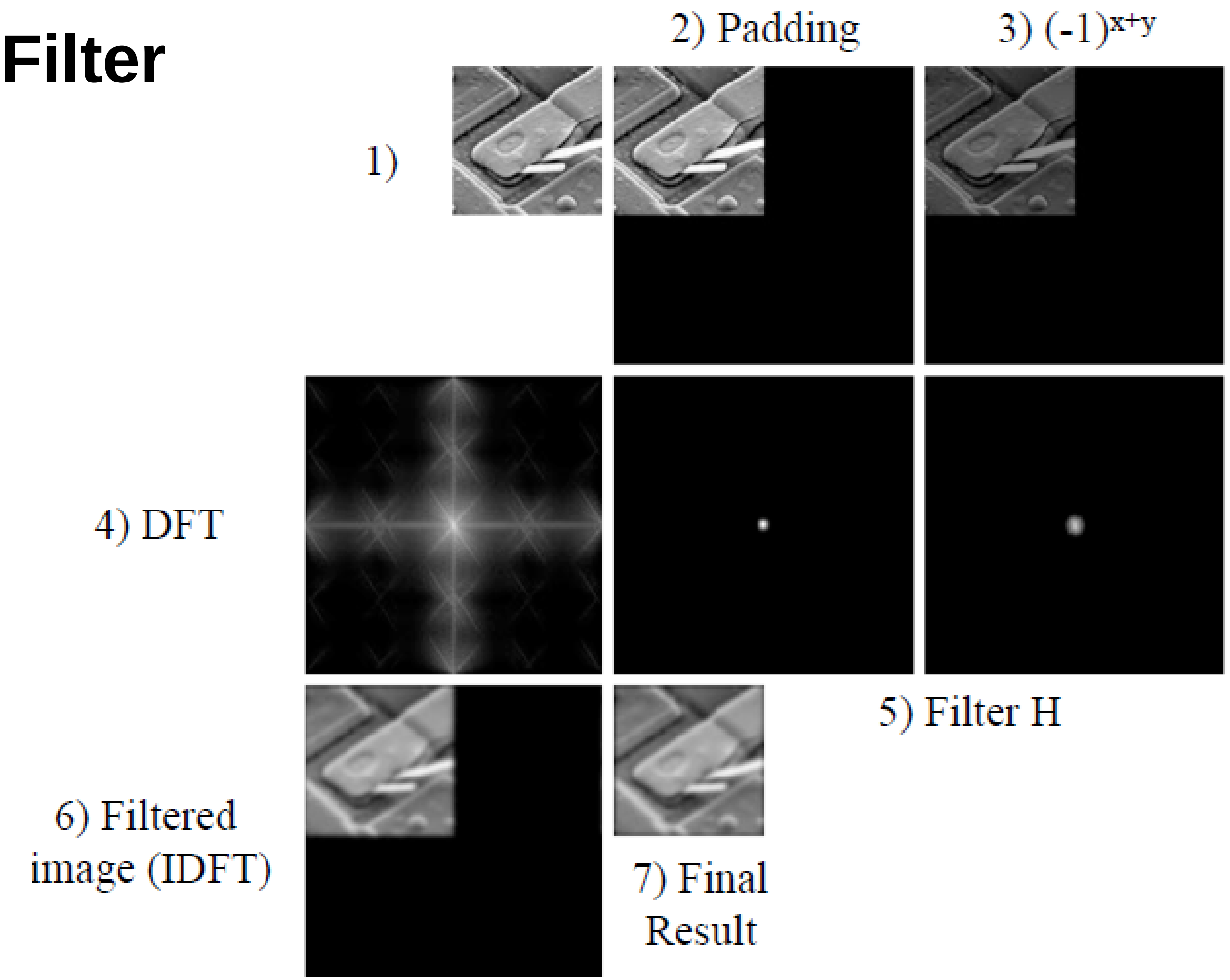
Multiplication in the frequency domain is a convolution in the spatial domain

Decentering

Remove the imaginary componente. Extract only the real component

Procedure for frequency domain

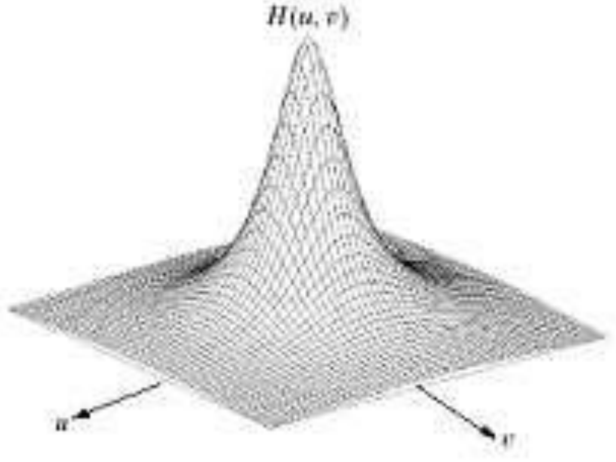
Example for Gaussian Filter

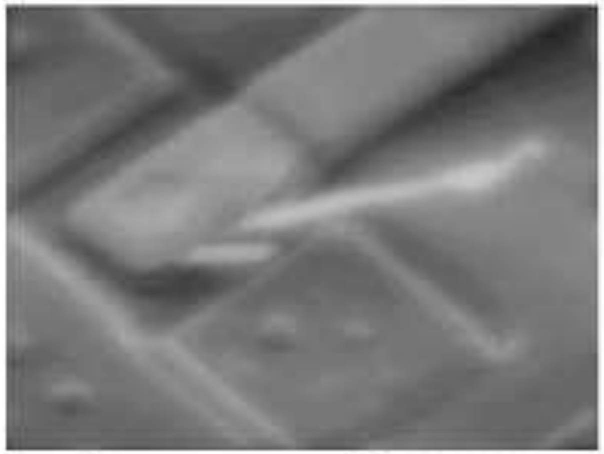


a b c
d e f
g h

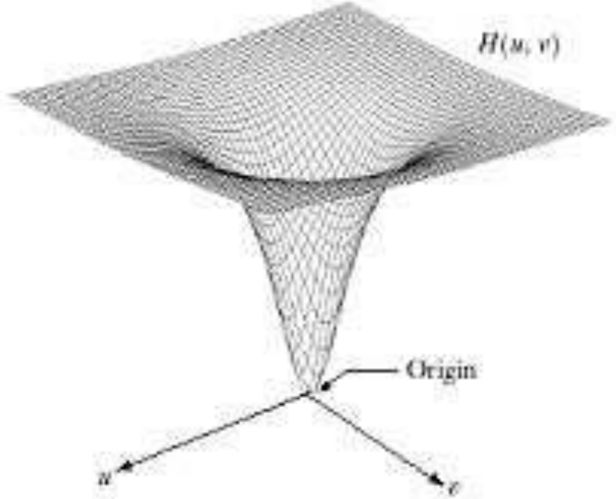
FIGURE 4.36 (a) An $M \times N$ image, f. (b) Padded image, f_p of size $P \times Q$. (c) Result of multiplying f_p by $(-1)^{x+y}$. (d) Spectrum of F_p. (e) Centered Gaussian lowpass filter, H, of size $P \times Q$. (f) Spectrum of the product HF_p. (g) g_p, the product of $(-1)^{x+y}$ and the real part of the IDFT of HF_p. (h) Final result, g, obtained by cropping the first M rows and N columns of g_p.

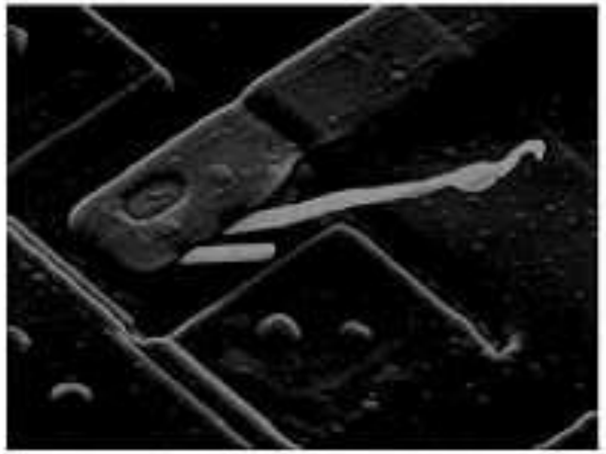
Low-Pass Filter and High-Pass





Low-Pass => Blur an image





High-Pass =>
enhance sharp detail
eliminates dc term
cause reduction in image contrast

Cutoff Frequency

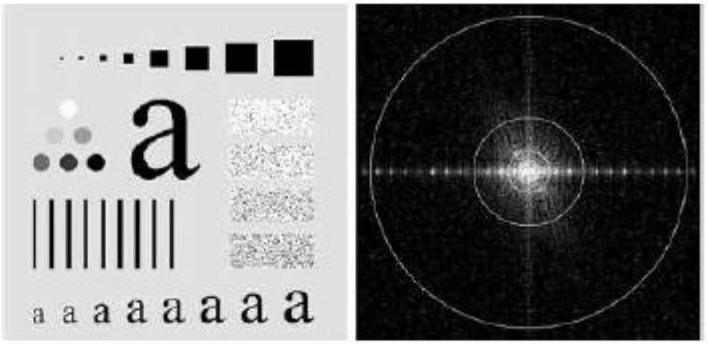
- One way to establish a set of cutoff frequencies, is to **compute circles that enclose specific amount of total image power** P_T

$$P_T = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} P(u,v)$$

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$

- As the **filter radius increases**, less and **less power is removed** (α % of enclose power), resulting in **less blurring**.

$$\alpha = \frac{100}{P_T} \sum_u \sum_v P(u,v)$$

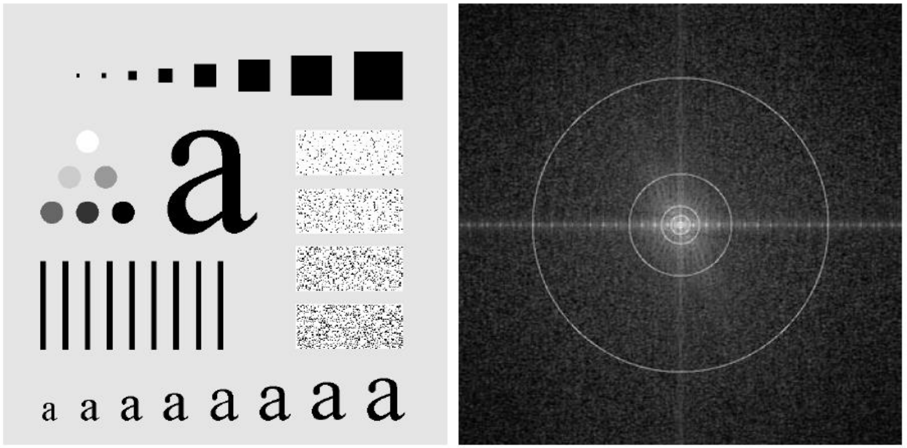


a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

Cutoff Frequency

- Variando o raio, altera-se a energia do sinal que passa, ou seja, controla-se o número de harmónicos que passam.
- **Quanto maior raio**, **mais energia passa** da imagem original.
- Quanto menor o raio, menor a energia, logo a suavização da imagem é maior.



a b

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Low-Pass Filtering

$$G(u,v) = H(u,v)\,F(u,v)$$

- **Smoothing:** attenuating specified range of high-frequency components
- **Three types of low-pass filter**:
  - **Ideal** (very sharp)
  - **Gaussian** (very smooth)
  - **Butterworth** (tunable according to the filter order - *n*). High orders behaviors as an ideal filter; lower order values behaviors as a Gaussian filter

Low-Pass Filtering: Ideal Low-Pass Filter

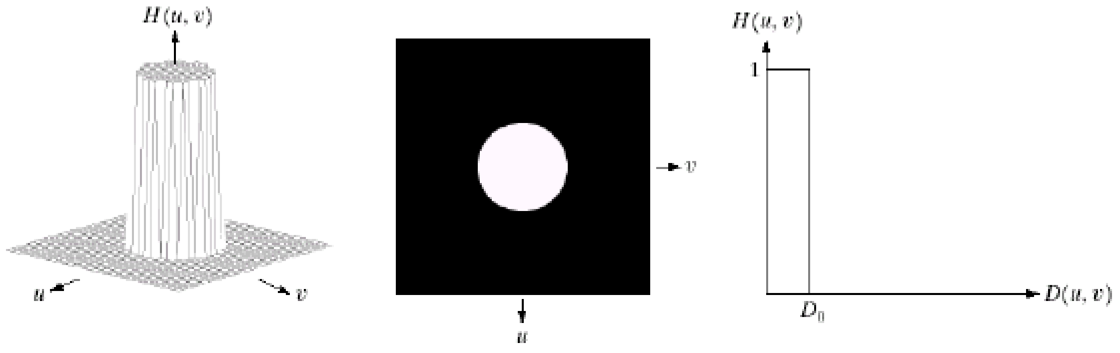
$$H(u,v) = \begin{cases} 1, se\ D(u,v) \leq D_0 \\ 0, se\ D(u,v) > D_0 \end{cases}$$

D_0= cutoff frequency, nonnegative quantity

$D(u,v) = [(u - P/2)^2 + (v - Q/2)^2]$ => Norm

$(P/2,\ Q/2)$ are the image dimensions

$D(u,v)$ is the distance between points (u,v) and $(P/2,\ Q/2)$, where the second is the center of the spectrum of the smoothed image



a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Low-Pass Filtering: Ideal Low-Pass Filter

$$H(u,v) = \begin{cases} 1, se\ D(u,v) \le D_0 \\ 0, se\ D(u,v) > D_0 \end{cases}$$

$$H(u,v) = \begin{cases} 1, se\ D(u,v) \le D_0 \\ 0, se\ D(u,v) > D_0 \end{cases}$$

- **Ideal Low-pass filter (ILPF)** = 2-D lowpass filter that **passes without attenuation** all frequencies within a circle of radius *D0* from the origin and **“cuts off” all frequencies outside this circle** (are completely attenuated)
- This effect **cannot be performed by electronic/physical filters**, but can simulated in a computer

Low-Pass Filtering: Ideal Low-Pass Filter

$$H(u,v) = \begin{cases} 1, se\ D(u,v) \leq D_0 \\ 0, se\ D(u,v) > D_0 \end{cases}$$

- **Ideal filter (ILPF):** all frequencies inside a circle of radius D_0 are passed with no attenuation
- In this case:
  - **zero-phase-shift filter** => real
  - **radially symmetric**

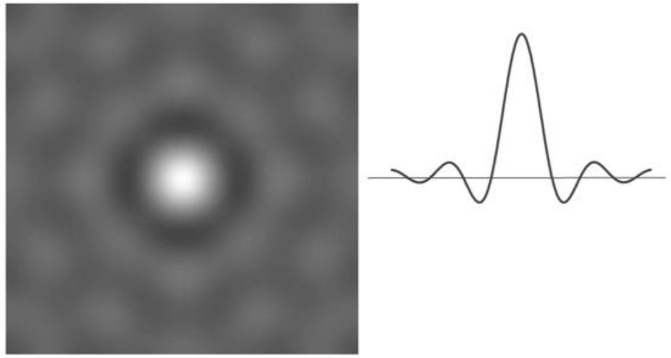
- **Ideal filter (ILPF):** all frequencies inside a circle of radius D_0 are passed with no attenuation
- In this case:
  - **zero-phase-shift filter** => real
  - **radially symmetric**

Funtion *(image, d),* where
d is diameter of filter

Low-Pass Filtering: Ideal Low-Pass Filter

- **Why appears ringing effect?**

Quanto maior raio, mais energia passa da imagem original. => Neste caso o filtro também introduz mais **artefactos**, os quais são **representados por ondulações**

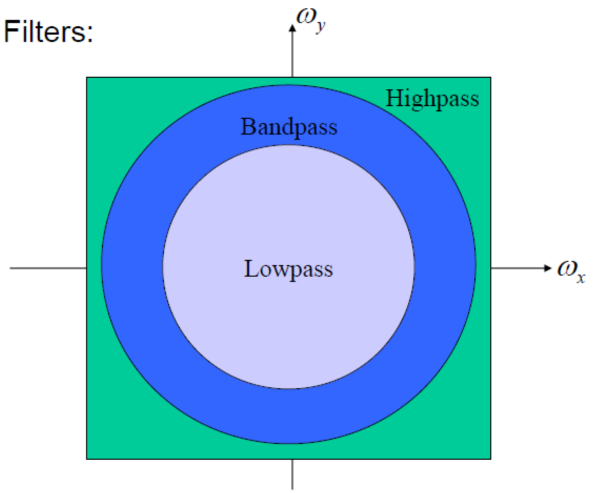


a b

FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000 × 1000. (b) Intensity profile of a horizontal line passing through the center of the image.

All ideal filters suffer from the ringing (Gibbs) phenomenon

Other Ideal Filters:



Low-Pass Filtering: Butterworth Filter

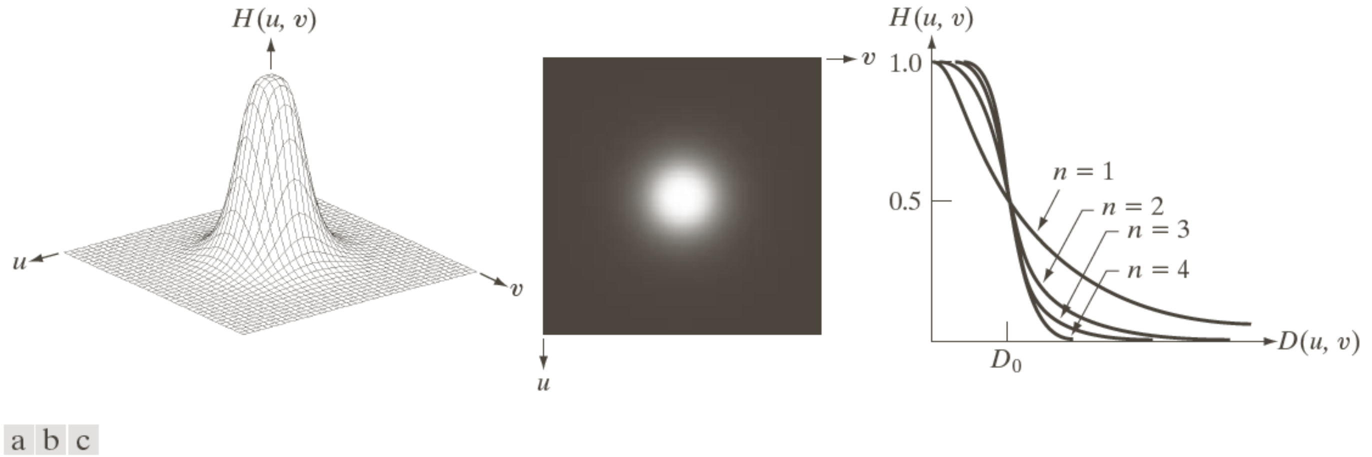


FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

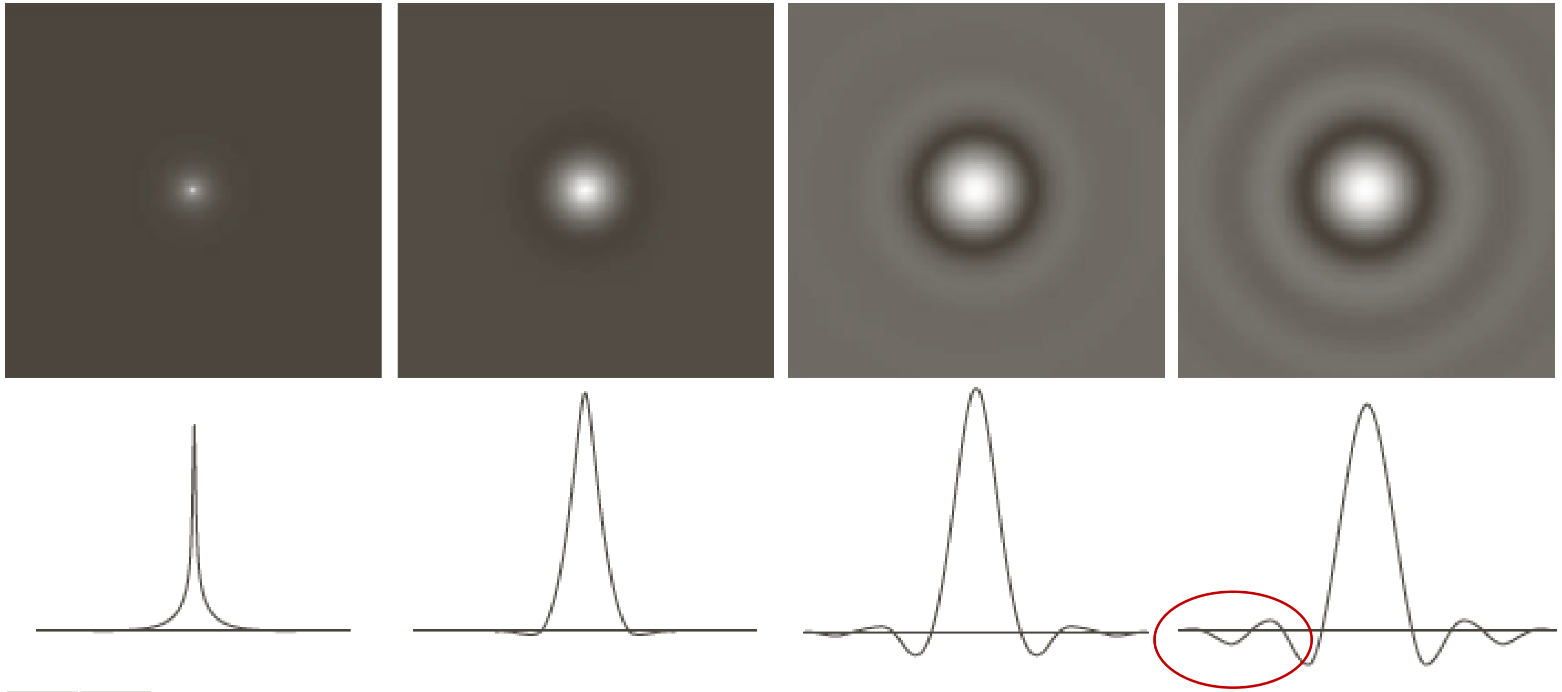
$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

cutoff frequency at a distance D_0 from the origin

- This filter **does not have a sharp discontinuity** that gives a clear cutoff between passed and filtered frequencies
- **Better than ideal filter**. However, still shows the ringing effect

Low-Pass Filtering: Butterworth Filter

- **Ringing effect appears with the increasing of the filter order, *n***



a b c d

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

Funtion *(image, d, n)*, where *d* is diameter of filter and *n*-filter order

Low-Pass Filtering: Gaussian Filter

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$ (σ = measure of spread about the centre)

$$\sigma = D_0 \;\Rightarrow\; H(u,v) = e^{-D^2(u,v)/2D_0^2}$$ (D_0 = cutoff frequency)

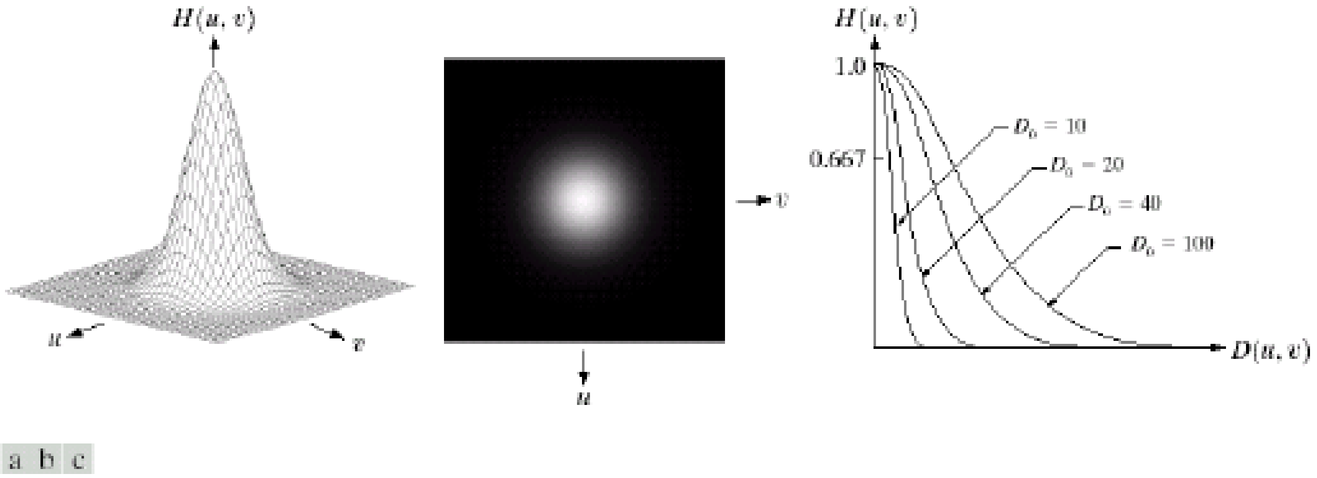


FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Low-Pass Filtering: Gaussian Filter

- The inverse Fourier transform of a GLPF is also a Gaussian. A spatial Gaussian filter will have no ringing
- **Gaussian filter does not introduce ringing effect** in the filter response => there is no undulation;
- Consequently, **it is the most used filter**
- By analyzing the shape of filter, **lower components pass more than the higher**

Low-Pass Filtering: Comparison

Ideal Filter

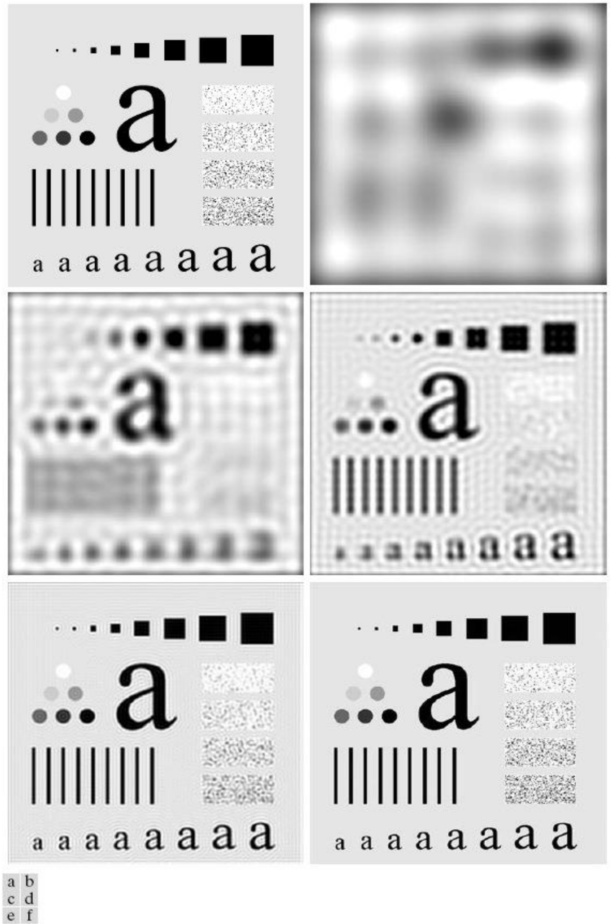


FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

Butterworth Filter

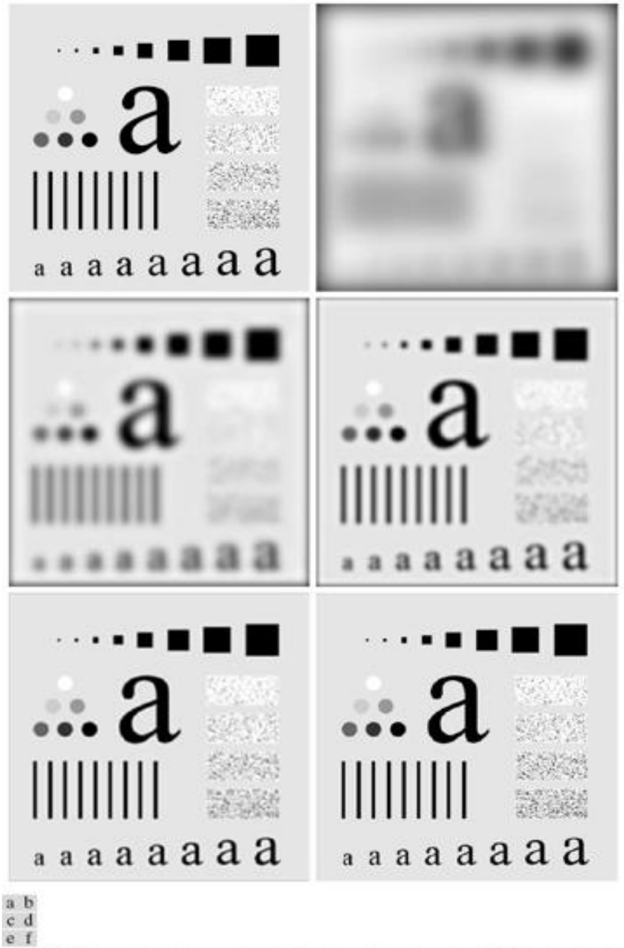


FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

Gaussian Filter

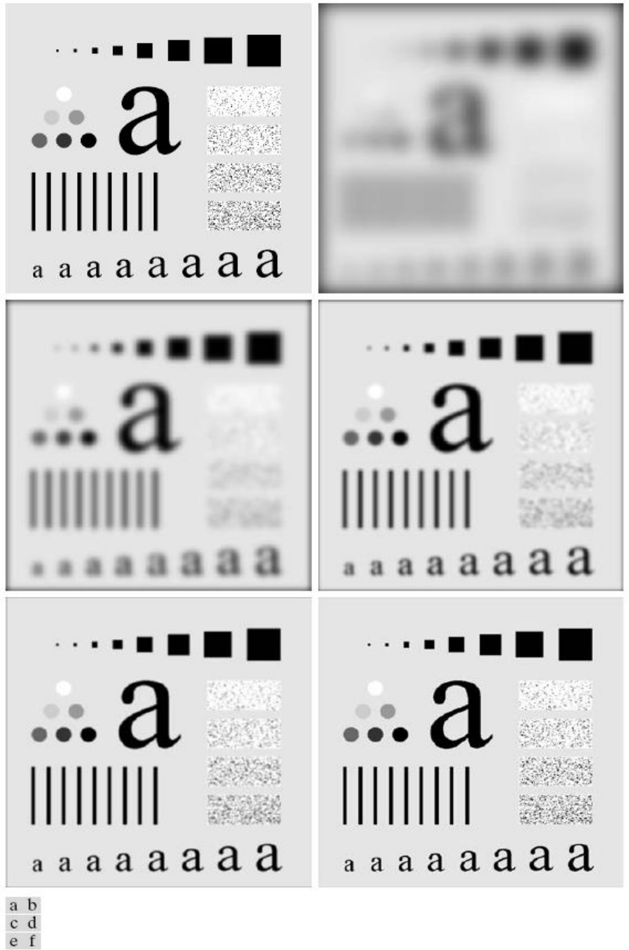
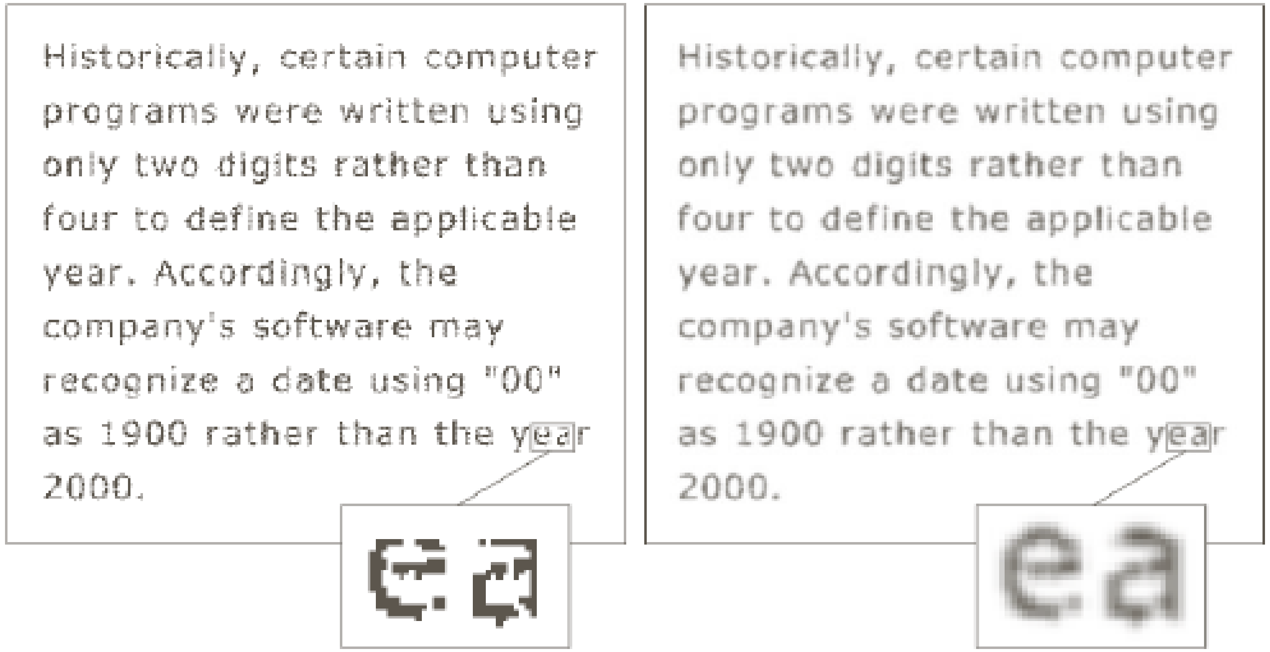


FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

Low-Pass Filtering: Applications

Character recognition (machine perception):



a b

FIGURE 4.49
(a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

Blurring to fill “visual gaps” => help reading broken characters

Low-Pass Filtering: Applications

Printing and publishing industry: “cosmetic” processing

Original Image

GLPF (Do=100)

GLPF (Do=80)

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

The lower the radius, higher is the smoothing of the image (image c)

High-Pass Filtering => Image Sharpening

Highpass filtering: attenuation of the low-frequency components of the Fourier transform of the image

Highpass filter: image sharpening (low-frequency attenuation)

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

- As before :
  - Radially symmetric filters
  - All filter functions are assumed to be discrete functions of size *P*x*Q*

High-Pass Filtering

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

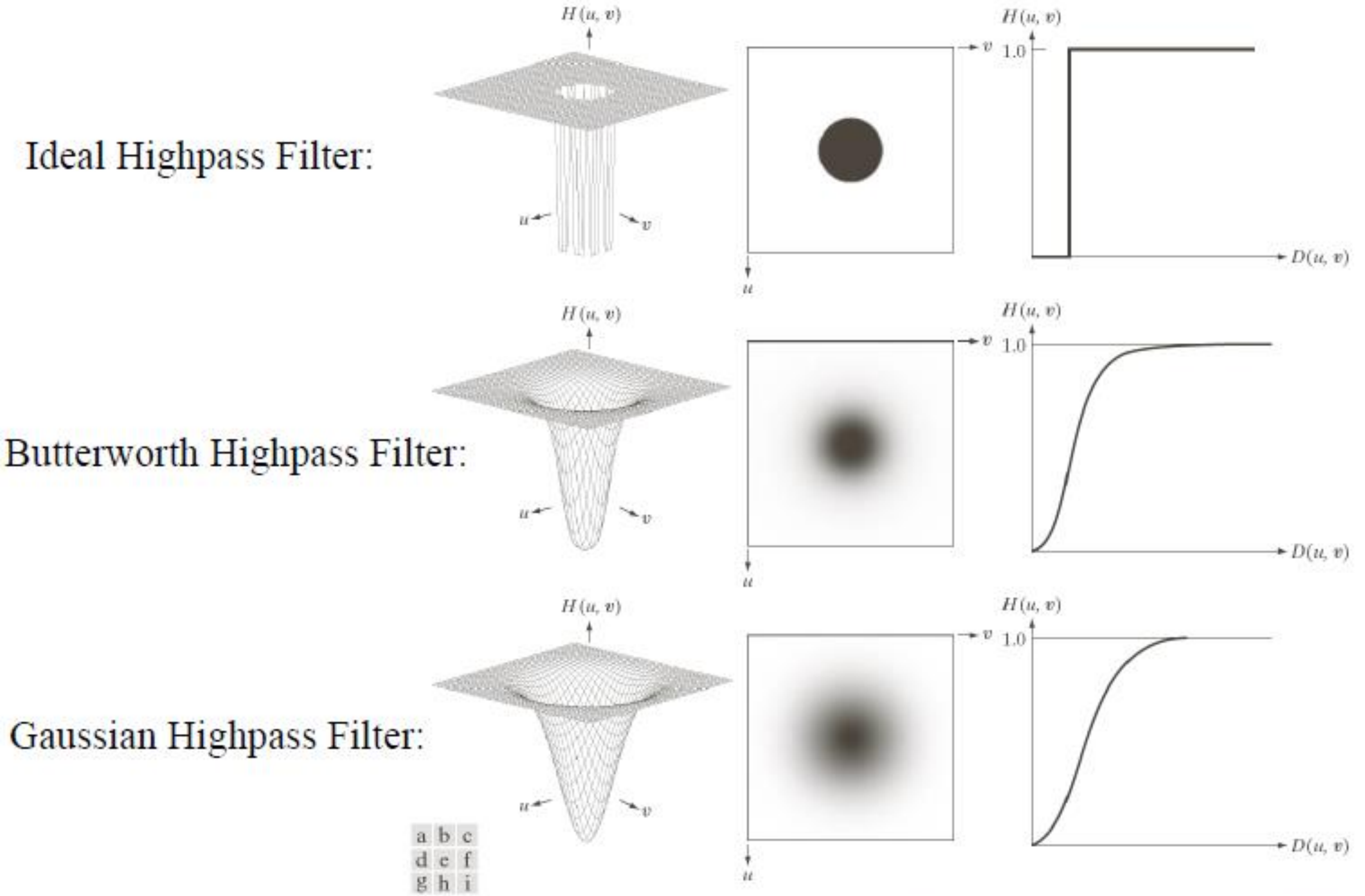
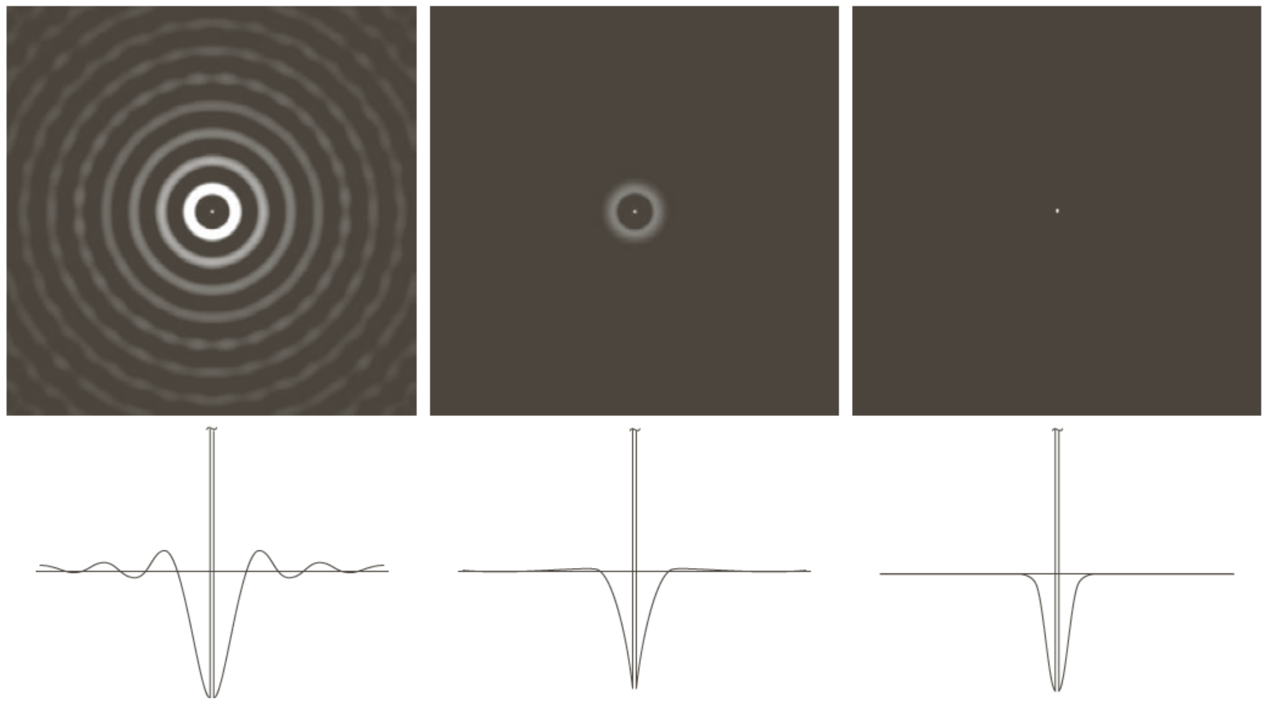


FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

High-Pass Filtering



a b c

FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

Comparison between the frequency responses of the following filters: **ideal, Butterworth and Gaussian**.

High-Pass Filtering: Ideal Filter

- Filtering using ideal filter $D_0 = 30, 60\ and\ 160$

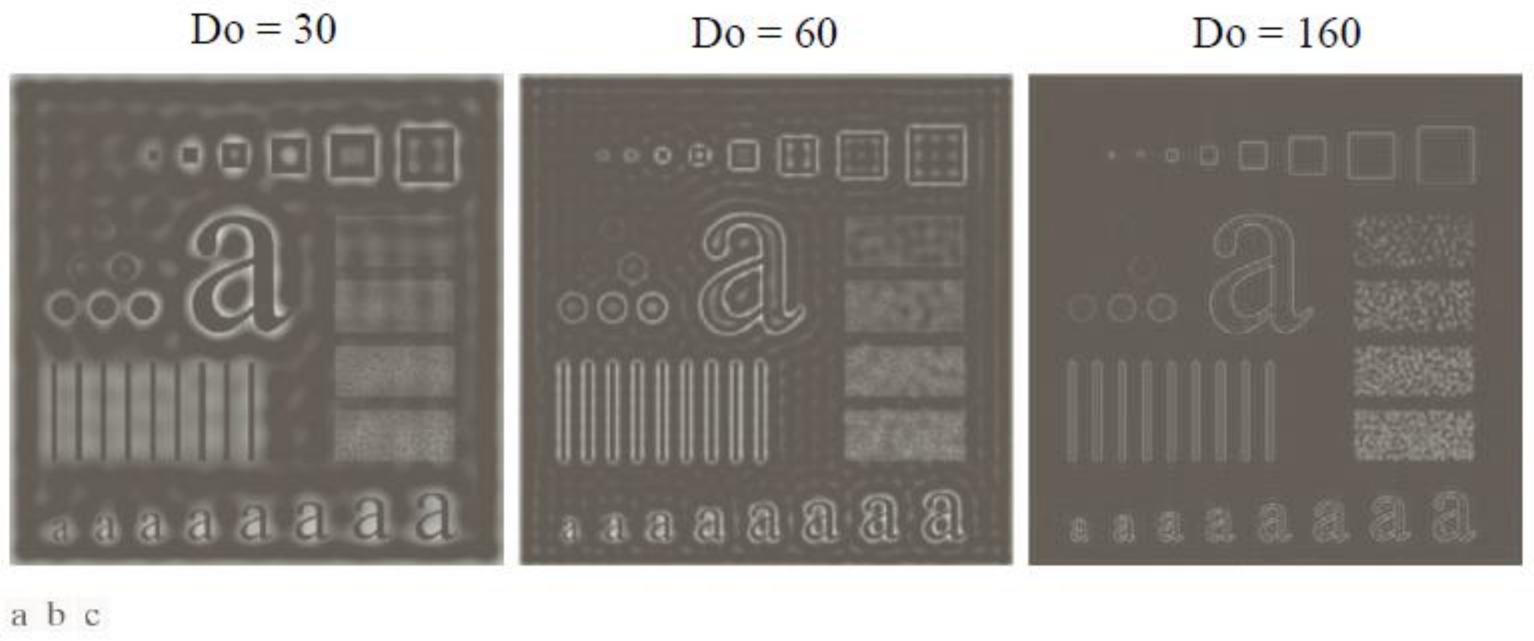


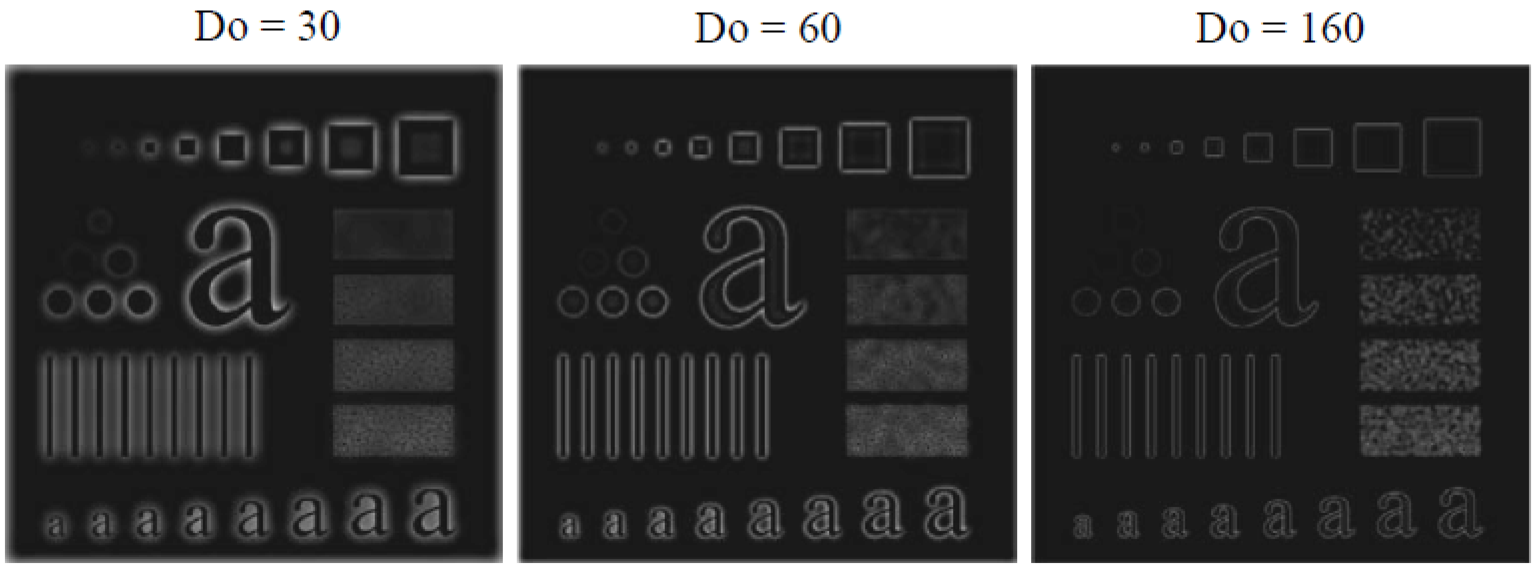
FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60$, and 160.

$$H(u,v) = \begin{cases} 1, se\ D(u,v) \leq D_0 \\ 0, se\ D(u,v) > D_0 \end{cases}$$

- **Quanto maior o raio**, menos a energia passa da imagem, logo a **suavização da imagem é maior e perde-se mais detalhes**.

High-Pass Filtering: Butterworth Filter

- Filtering using Butterworth filter of 2nd order $D_0 = 30, 60\ and\ 160$



$$H(u,v) = \frac{1}{1 + \left[\frac{D_0}{D(u,v)}\right]^{2n}}$$

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

High-Pass Filtering: Gaussian Filter

- Filtering using Gaussian filter $D_0 = 30, 60\ and\ 160$

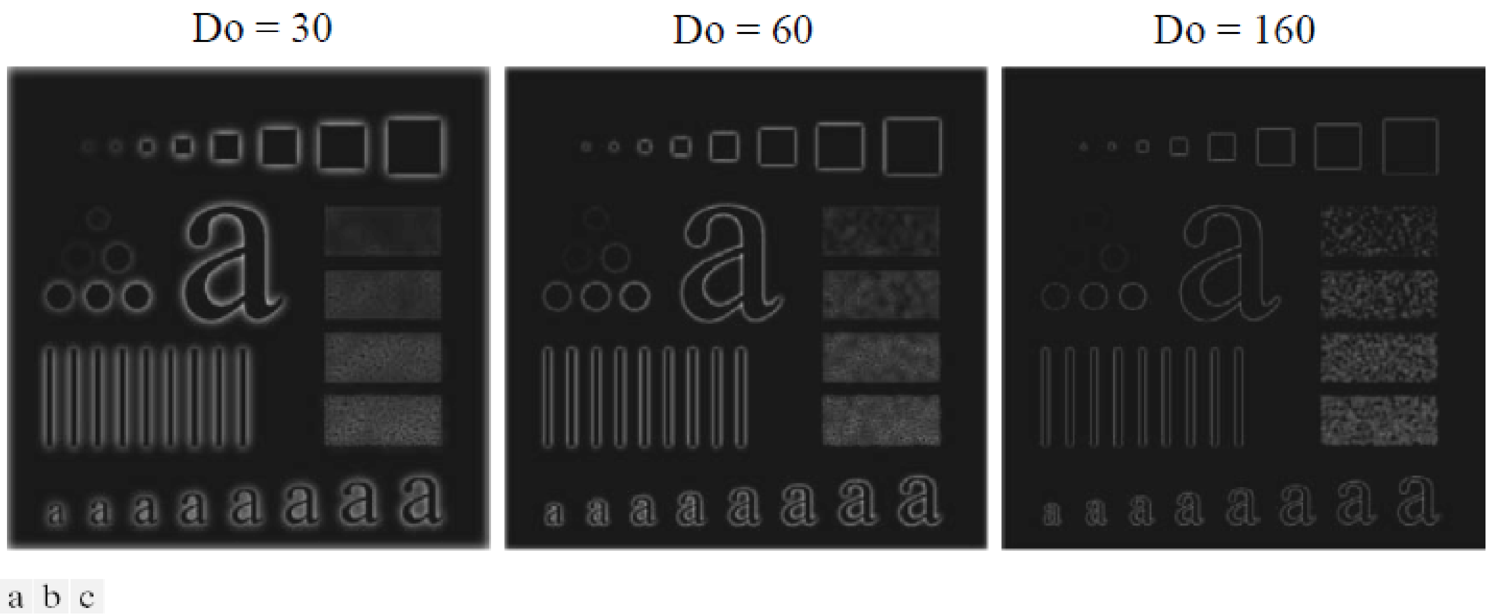


FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30$, 60, and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

The transfer function of the *Gaussian Highpass Filter* (GHPF) with cutoff frequency locus at a distance *D0* from the centre of the frequency rectangle is defined as:

- As bordas estão mais nítidas e sem sombreamento, comparativamente às filtragens anteriores

High-Pass Filtering: Laplacian

Note **In the spatial domain**

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- $$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$
- $$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

High-Pass Filtering: Laplacian

$$H(u,v) = -4\pi^2(u^2+v^2)$$

Laplacian filter in frequency domain

Or, with respect to the centre of the frequency rectangle:

$$H(u,v) = -4\pi^2 D^2(u,v)$$

The Laplacian image is obtained by:

$$\nabla^2 f(x,y) = IDFT\left[H(u,v)F(u,v)\right]$$

Enchancement of the image is achieved using (H(u,v) negative):

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

$$\begin{aligned} g(x,y) &= \Im^{-1}\left\{F(u,v) - H(u,v)F(u,v)\right\} \\ &= \Im^{-1}\left\{\left[1 - H(u,v)\right]F(u,v)\right\} \\ &= \Im^{-1}\left\{\left[1 + 4\pi^2D^2(u,v)\right]F(u,v)\right\} \end{aligned}$$

High-Pass Filtering: Laplacian

$$\nabla^2 f(x,y) = IDFT\left[H(u,v)F(u,v)\right]$$

⇒ Introduction of large scaling factors

⇒ Practical solution: normalize $f(x,y)$ to the range [0,1] before computing the DFT, and divide $\nabla^2 f(x,y)$ by its maximum value

a b

FIGURE 4.58 (a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

High-Pass Filtering: Applications

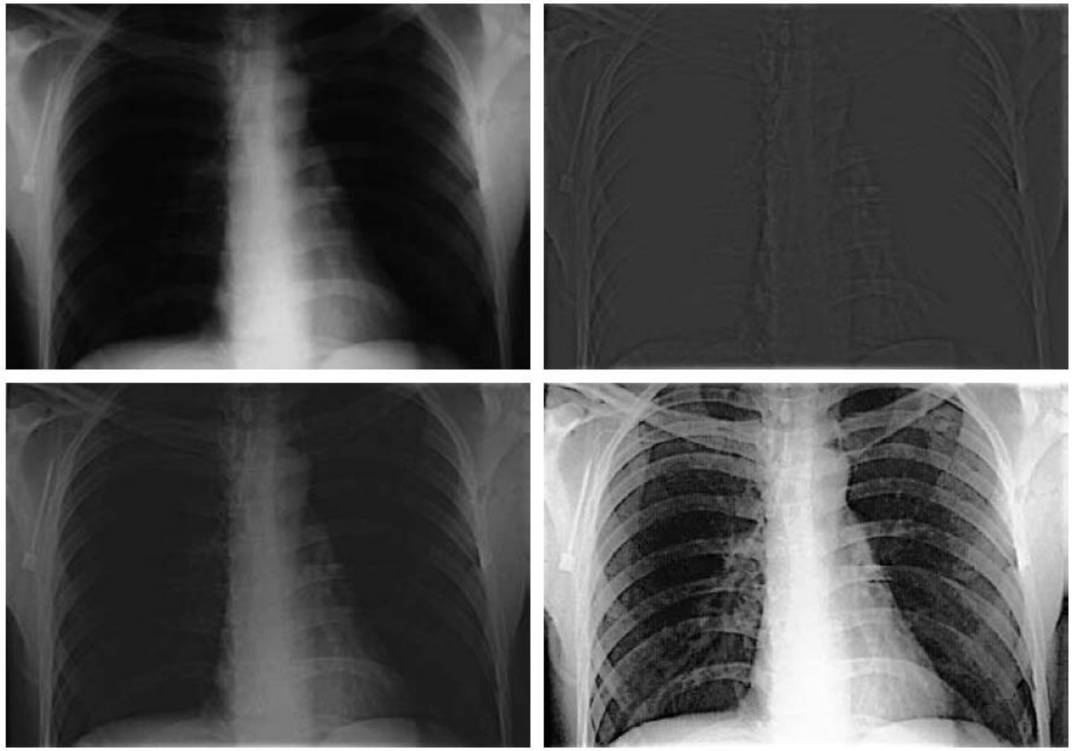


FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

Bandreject Filters

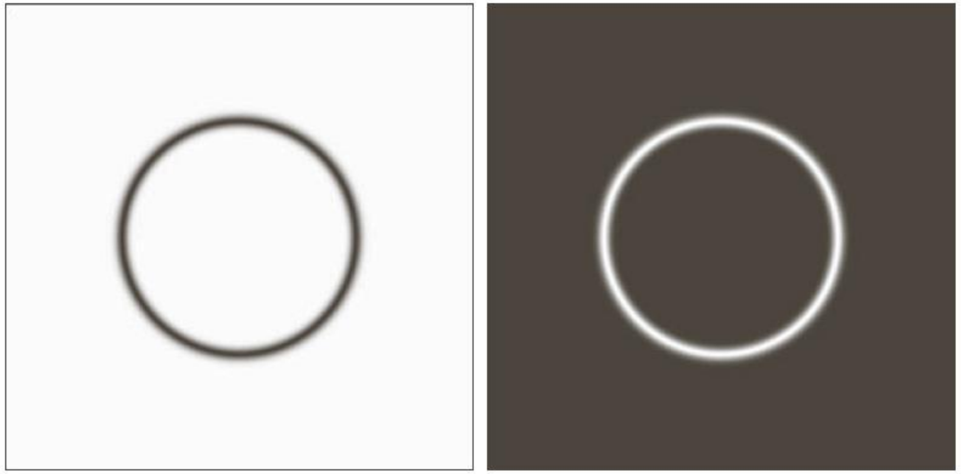
$$H_{Bandreject}(u, v) = 1 - H_{Band-pass}(u, v)$$

TABLE 4.6

Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \leq D \leq D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

Band-reject Filters: Applications



a b

FIGURE 4.63 (a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

- The **band-rejected and band-pass filters** select determined frequencies.
- In these filters, **it is not need image padding** since the process is directly applied on DFT of the image.
- Thus, **errors and noise is avoiding due to padding**.

Notch Filters

Example: Butterworth notch reject filter of order n, containing 3 notch pairs:

$$H_{NR}(u,v) = \prod_{k=1}^{3} \left[\frac{1}{1 + [D_{0k}/D_k(u,v)]^{2n}} \right] \left[\frac{1}{1 + [D_{0k}/D_{-k}(u,v)]^{2n}} \right]$$

A *Notch Pass filter* (NP) is obtained from a *Notch Reject filter* (NR) using:

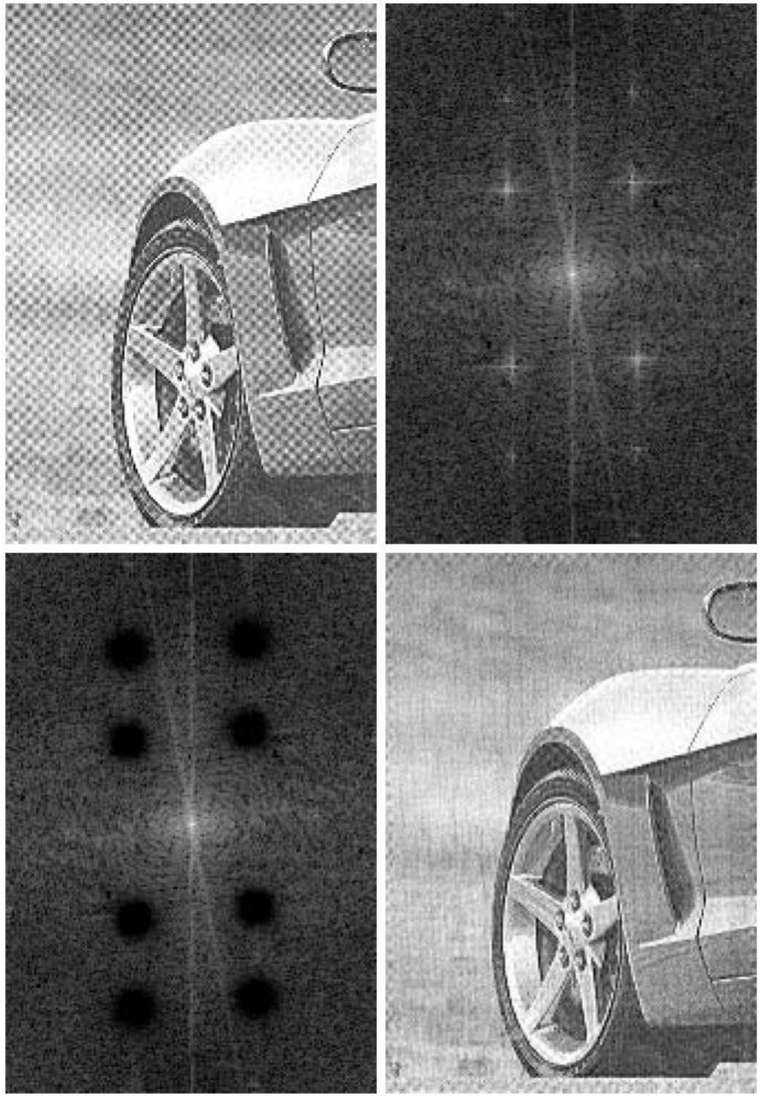
$$H_{NP}(u,v) = 1 - H_{NR}(u,v)$$

Band-reject Filters: Applications

- **It is important see the image spectrum to select the filter**. Note use module for the representation.

- Apply notch filter to remove the moiré pattern.

- **Image padding was not applied.** The filter location was defined by analyzing the spectrum of the image.



a b
c d

FIGURE 4.64 (a) Sampled newspaper image showing a moiré pattern. (b) Spectrum. (c) Butterworth notch reject filter multiplied by the Fourier transform. (d) Filtered image.

Summary

This lecture will cover:

- Filtering: Spatial domain *vs* Frequency domain
- Image filtering in frequency domain
  - Low-Pass filtering
  - High-Pass filtering
  - Band-reject filtering
  - Applications