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CS3233

Competitive Programming

Dr. Steven Halim

Week 05 – Problem Solving Paradigms
(Dynamic Programming 2)

Outline

- Mini Contest #4 + Break + Discussion + Admins
- A simple DP problem to refresh our memory (Section 3.5.3)
- DP and its relationship with (implicit) DAG (Section 4.7.1)
 - These are CS2020/CS2010 materials
 - Those who have not taken either module must consult Steven separately
- DP on Math Problems (Section 5.4 and 5.6)
- DP on String Problems (Section 6.5)
- More DP techniques (Section 8.3)
- Pointers to other DP techniques in CP2.9

More DP Problems in Chapter 3-4-5-6-8-9 of CP2.9 Book

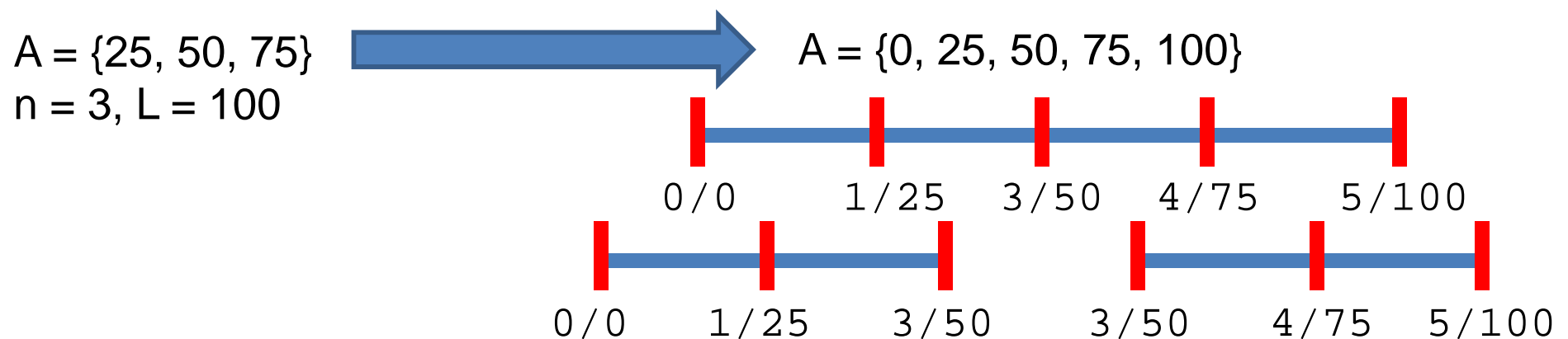
NON CLASSICAL DP PROBLEMS

Non Classical DP Problems

- **My definition:**
 - Not the pure form (or simple variant) of 1D/2D Max Sum, LIS, 0-1 Knapsack/Subset Sum, Coin Change, TSP where the DP **states** and **transitions** can be “memorized”
 - Requires **original*** **formulation** of DP states and transitions
 - Throughout this lecture, we will talk mostly in *DP terms*
 - **State** (to be precise: “*distinct* state”)
 - **Space Complexity** (i.e. the number of distinct states)
 - **Transition** (which entail overlapping sub problems)
 - **Time Complexity** (i.e. num of distinct states * time to fill one state)

Refresher: Cutting Sticks

- State: index (l, r) where $l, r \in [0..n+1]$ and $l < r$
 - Q: Why these two parameters?
- Space Complexity: $O(n^2)$ distinct states
- Transition: Try all possible cutting points i between l and r ,
 - i.e. cut (l, r) into (l, i) and (i, r) with cost $(A[r] - A[l])$
- Time Complexity: There are $O(n)$ possible cutting points, thus overall $O(n^2 * n) = O(n^3)$



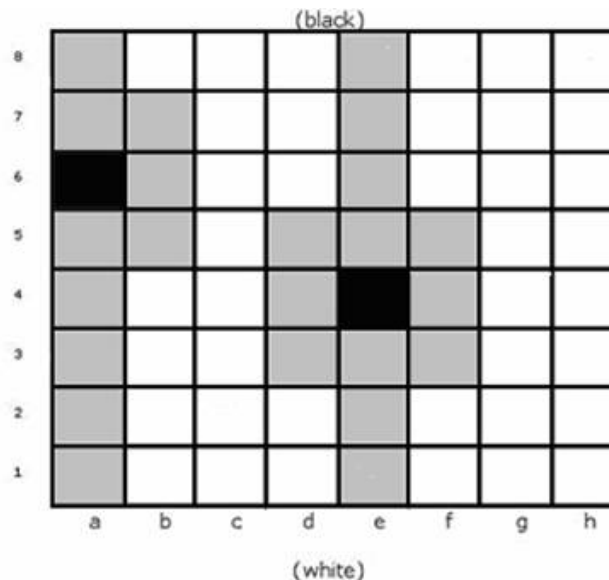
DP on DAG

Overview

- Dynamic Programming (DP) has a close relationship with (usually implicit) Directed Acyclic Graph (DAG)
 - The **states** are the **vertices** of the DAG
 - Space complexity: Number of vertices of the DAG
 - The **transitions** are the **edges** of the DAG
 - Logical, since a recurrence is always **acyclic**
 - Time complexity: Number of edges of the DAG
 - Top-down DP: Process each vertex just once via **memoization**
 - Bottom-up DP: Process the vertices in **topological order**
 - Sometimes, the topological order can be written by just using simple (nested) loops

The Injured Queen Problem

- Like N-queens problem, but the queens are “injured” (can only attack the current column but acts as king otherwise)
- With some of K ($0 \leq K \leq N$) injured queens positions have been predetermined, count how many possible arrangements of the other $(N-K)$ queens so that no two queens attack each other?



DP on Math Problems

- Some well-known mathematic problems involves DP
 - Some combinatorics problem have recursive formulas which entail overlapping subproblems
 - e.g. those involving Fibonacci number, $f(n) = f(n - 1) + f(n - 2)$
 - Some probability problems require us to search the entire search space to get the required answer
 - If some of the sub problems are overlapping, use DP, otherwise, use complete search
 - Mathematics problems involving **static** range sum/min/max!
 - Use dynamic tree DS for dynamic queries

Dice Throwing

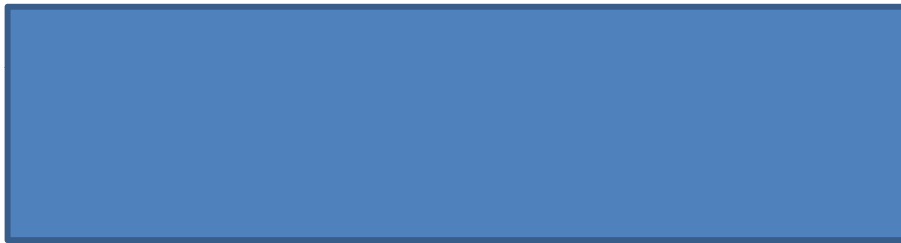


- n common cubic dice are thrown ($1 \leq n \leq 24$)
- What is the probability that the sum of all thrown dices is at least x ? ($0 \leq x \leq 150$)
- Basic probability = $\# \text{ events} / |\text{sample space}|$
- To compute the $|\text{sample space}|$ is easy: It is 6^n
- The $\# \text{ events}$ is harder to compute...



DP on String Problems

- Some string problems involves DP
 - Usually, we do not work with the string itself
 - But we work with the integer indices to represent suffix/prefix/substring



- Reason: Too costly to pass (sub)strings around as function parameters

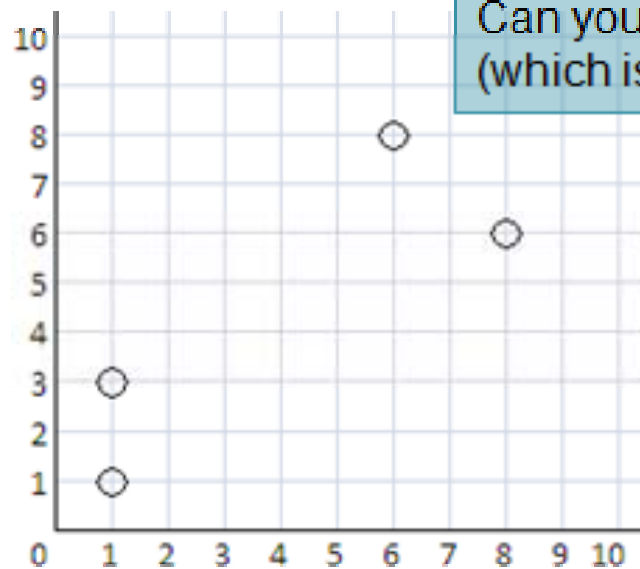
String Partition

- There are many ways to split a string of digits into a list of non-zero-leading (0 itself is allowed) 32-bit *signed* integers
 - What is the maximum sum of the resultant integers if the string is split appropriately? Examples:
 - 1234554321
 - $1234554321 < 2147483647$, so the answer is 1234554321 itself
 - 5432112345
 - $5432112345 > 2147483647$, thus 5432112345 must be partitioned
 - There are two ways to partition 5432112345
 - » $5 + 432112345 = 432112350$, or
 - » $543211234 + 5 = 543211239 \leftarrow$ the answer
 - 121212121212
 - $121212121212 > 2147483647$, thus 121212121212 must be partitioned
 - The answer is: $1 + 2121212121 + 2 = 2121212124$

DP with bitmask

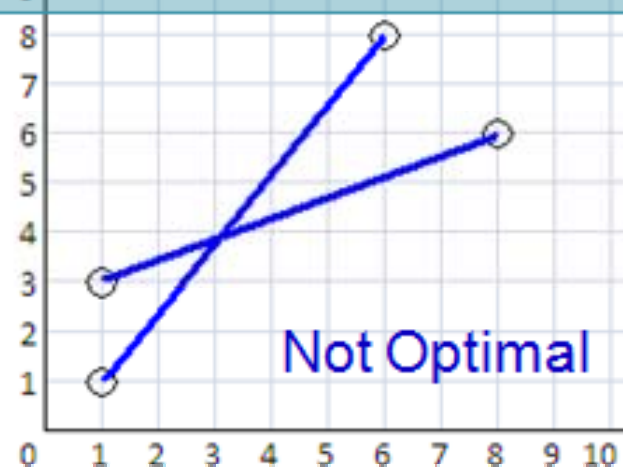
- Bitmask technique can be used to represent *lightweight set of Boolean* (up to 2^{64} if using unsigned long long)
- This is important if one of the DP parameter is a “small set”
- We have seen this form earlier in DP-TSP
- One other useful application (there are many others):
 - ***Finding min weighted perfect matching in small general graph***

Forming Quiz Teams

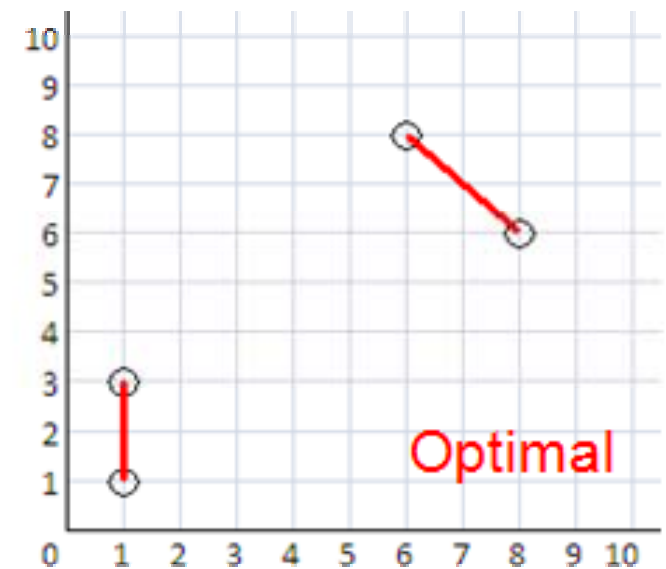


$N = 2$

Can you spot one more possible grouping?
(which is not optimal)



Cost = 8.60 + 7.61 = **16.21**



Cost = 2.00 + 2.83 = **4.83**

Common DP States (1)

- Position:
 - Original problem: $[x_1, x_2, \dots, x_n]$
 - Can be sequence (integer/double array), can be string (char array)
 - Sub problems, break the original problem into
 - Sub problem and Prefix: $[x_1, x_2, \dots, x_{n-1}] + x_n$
 - Suffix and sub problem: $x_1 + [x_2, x_3, \dots, x_n]$
 - Two sub problems: $[x_1, x_2, \dots, x_i] + [x_{i+1}, x_{i+2}, \dots, x_n]$
 - Example: 1D Max Sum, LIS, etc

Common DP States (2)

- Positions:
 - This is similar to the previous slide
 - Original problem: $[x_1, x_2, \dots, x_n]$ and $[y_1, y_2, \dots, y_n]$
 - Can be two sequences/strings
 - Sub problems, break the original problem into
 - Sub problem and prefix: $[x_1, x_2, \dots, x_{n-1}] + x_n$ and $[y_1, y_2, \dots, y_{n-1}] + y_n$
 - Suffix and sub problem: $x_1 + [x_2, x_3, \dots, x_n]$ and $y_1 + [y_2, y_3, \dots, y_n]$
 - Two sub problems: $[x_1, x_2, \dots, x_i] + [x_{i+1}, x_{i+2}, \dots, x_n]$ and $[y_1, y_2, \dots, y_i] + [y_{i+1}, y_{i+2}, \dots, y_n]$
 - Example: String Alignment/Edit Distance, LCS, Matrix Chain Multiplication (MCM), etc
 - PS: Can also be applied on 2D matrix, like 2D Max Sum, etc

Tips: When to Choose DP

- Default Rule:
 - If the given problem is an **optimization** (max/min) or **counting** problem
 - Problem exhibits optimal sub structures
 - Problem has overlapping sub problems
- In ICPC/IOI:
 - If actual solutions are not needed (only final values asked)
 - If we must compute the solutions too, a more complicated DP which stores *predecessor information* and *some backtracking* are necessary
 - The number of distinct sub problems is small enough ($< 1M$) and you are not sure whether greedy algorithm works (why gamble?)
 - Obvious overlapping sub problems detected :O

Dynamic Programming Issues (1)

- Potential issues with DP problems:
 - They may be disguised as (or looks like) non DP
 - It looks like greedy can work but some cases fails...
 - e.g. problem looks like a shortest path with some constraints on graph, but the constraints fail *greedy* SSSP algorithm!
 - They may have subproblems but not overlapping
 - DP does not work if overlapping subproblems not exist
 - Anyway, this is still a good news as perhaps Divide and Conquer technique can be applied

Dynamic Programming Issues (2)

- Optimal substructures may not be obvious
 1. Find correct “states” that describe problem
 - Perhaps extra parameters must be introduced?
 2. Reduce a problem to (smaller) sub problems (with the same states) until we reach base cases
- There can be more than one possible formulation
 - Pick the one that works!

DP Problems in ICPC (1)

- The number of problems in ICPC that must be solved using DP are growing!
 - At least one, likely two, maybe three per contest...
- These new problems are **not** the classical DP!
 - They require deep thinking...
 - Or those that look solvable using other (simpler) algorithms but actually must be solved using DP
 - Do not think that you have “mastered” DP by only memorizing the classical DP solutions!

DP Problems in ICPC (2)

- In 1990ies, mastering DP can make you “king” of programming contests...
 - Today, it is a must-have knowledge...
 - So, get familiar with DP techniques!
- By mastering DP, your ICPC rank is probably:
 - from top \sim [25-30] (solving 1-2 problems out of 10)
 - Only easy problems
 - to top \sim [15-20] (solving 3-4 problems out of 10)
 - Easy problems + brute force + DP problems

For Week 07 homework 😊

(You can do this over recess week too)

BE A PROBLEM SETTER

Be a Problem Setter

- Problem Solver:
 - A. Read the problem
 - B. Think of a good algorithm
 - C. Write 'solution'
 - D. Create tricky I/O
 - E. If WA, go to A/B/C/D
 - F. If TLE/MLE, go to A/B/C/D
 - G. If AC, stop 😊
- Problem Setter:
 - A. Write a good problem
 - B. Write good solutions
 - The correct/best one
 - The incorrect/slower ones
 - C. Set a good secret I/O
 - D. Set problem settings
- A problem setter must think from a different angle!
 - By setting good problems, you will simultaneously be a better problem solver!!

Problem Setter Tasks (1)

- Write a good problem
 - Options:
 - Pick an algorithm, then find problem/story, or
 - Find a problem/story, then identify a good algorithm for it (harder)
 - Problem description must not be ambiguous
 - Specify input constraints
 - Good English!
 - Easy one: longer, Hard one: shorter!
- Write good solutions
 - Must be able to solve your own problem!
 - To set hard problem, one must increase his own programming skill!
 - Use the best possible algorithm with lowest time complexity
 - Use the inferior ones 'that barely works' to set the WA/TLE/MLE settings...

Problem Setter Tasks (2)

- Set a good secret I/O
 - Tricky test cases to check AC vs WA
 - Usually 'boundary case'
 - Large test cases to check AC vs TLE/MLE
 - Perhaps use input generator to generate large test case, then pass this large input to our correct solution
- Set problem settings
 - Time Limit:
 - Usually 2 or 3 times the timings of your own best solutions
 - Java is slower than C++!
 - Memory Limit:
 - Check OJ setting^
 - Problem Name:
 - Avoid revealing the algorithm in the problem name

FYI: Be A Contest Organizer

- Contest Organizer Tasks:
 - Set problems of *various* topic
 - Better set by >1 problem setter
 - Must balance the difficulty of the problem set
 - Try to make it fun
 - Each team solves some problems
 - Each problem is solved by some teams
 - No team solve all problems
 - Every teams must work until the end of contest

More References

- **Competitive Programming 2.9**
 - Section 3.5, 4.7.1, 5.4, 5.6, 6.5, 8.3, and parts of Ch9
- **Introduction to Algorithms**, p323-369, Ch 15
- **Algorithm Design**, p251-336, Ch 6
- **Programming Challenges**, p245-267, Ch 11
- <http://www.topcoder.com/tc?module=Static&d1=tutorials&d2=dynProg>
- Best is practice & more practice!