This course material is now made available for public usage.
Special acknowledgement to School of Computing, National University of Singapore
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CS3233
Competitive Programming

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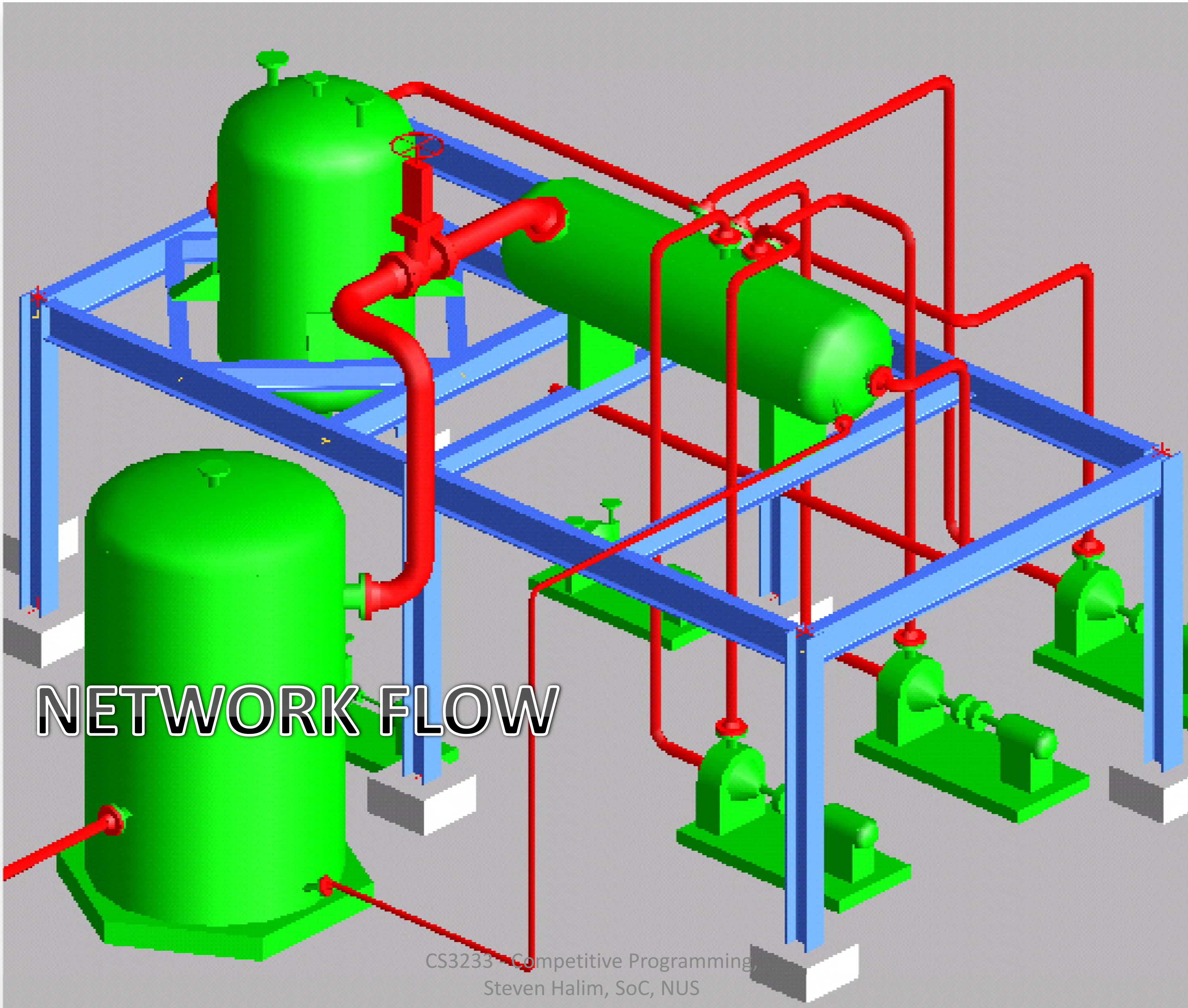
Week 07 – How to Prevent Floods?

Outline

- Mini Contest #5 + Break + Discussion + Admins
- Very Quick CS2010/2020 Review
- Network Flow (not in IOI 2009 syllabus)
  - Overview & Motivation (yes, to prevent floods :D)
  - Focus on Max Flow
  - Ford Fulkerson's Method
  - Edmonds Karp's Algorithm
- Flow Graph Modeling (*several* examples)
  - Bipartite Matching Variant, Min Cut, Multi Sources/Sink, Vertex Capacity, Independent Path, Edge-Disjoint Path, Min Cost (Max) Flow

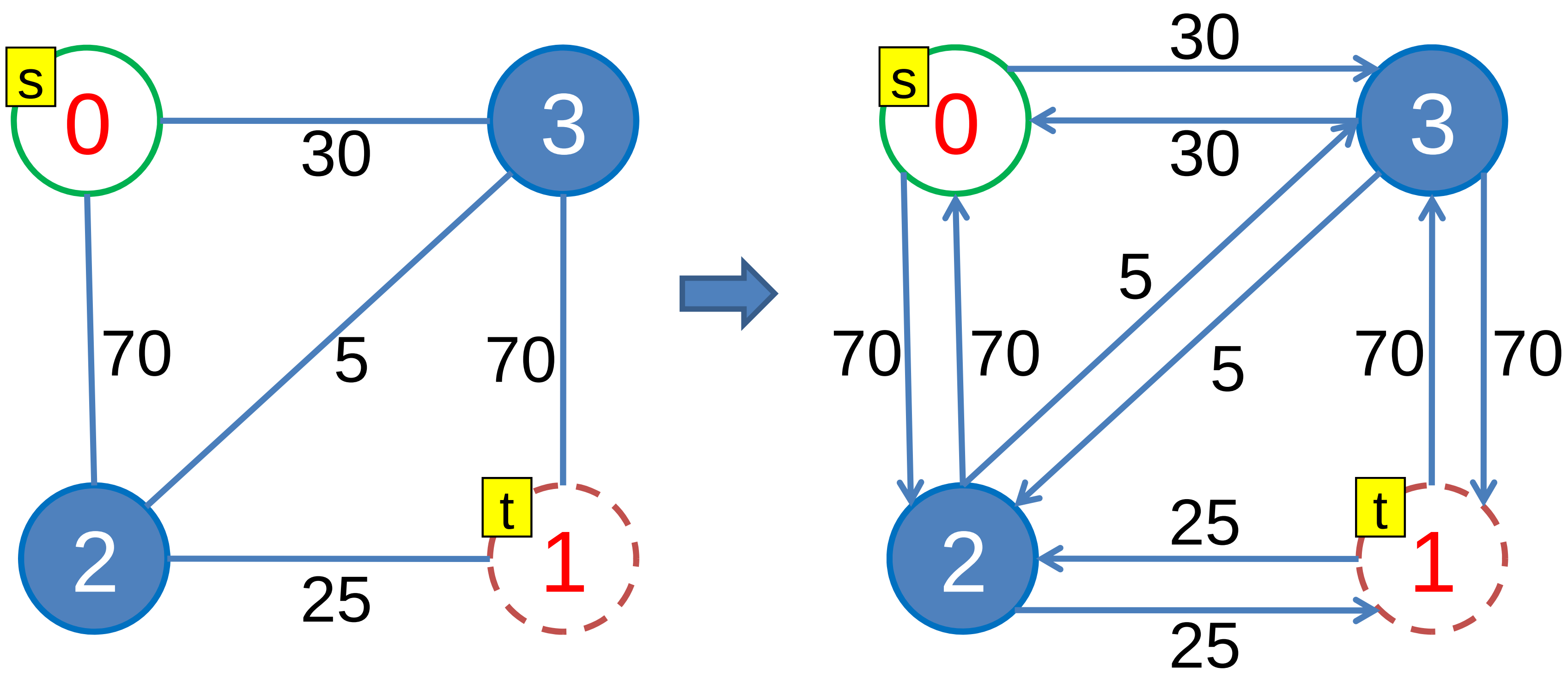
Graph in CS2010/2020…

- These have been revisited in Week2/5 additional classes
- Graph Data Structures
  - Adjacency Matrix, Adjacency List, Edge List, Implicit Graph…
- Graph Traversal: DFS/BFS
  - Various applications: Connected Component, Topological Sort
- Minimum Spanning Tree: Prim’s/Kruskal’s
  - Various applications
- Single-Source and All-Pairs Shortest Paths
  - Unweighted, weighted, negative cycle
  - Various applications



NETWORK FLOW

Max Flow in a Network (1)

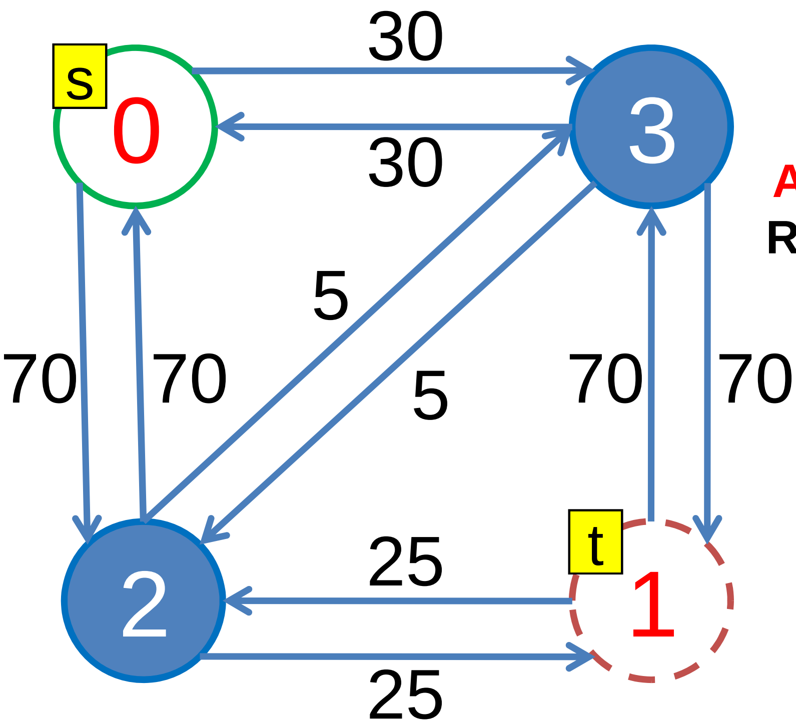


Max Flow in a Network (2)

- One Solution: Ford Fulkerson's Method

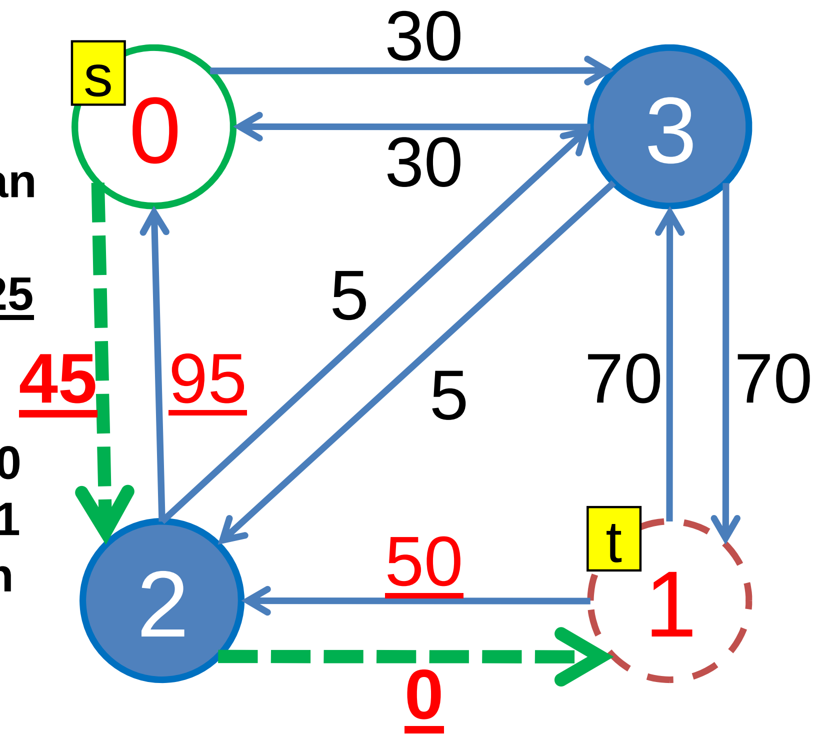
  - A surprisingly **simple** *iterative* algorithm

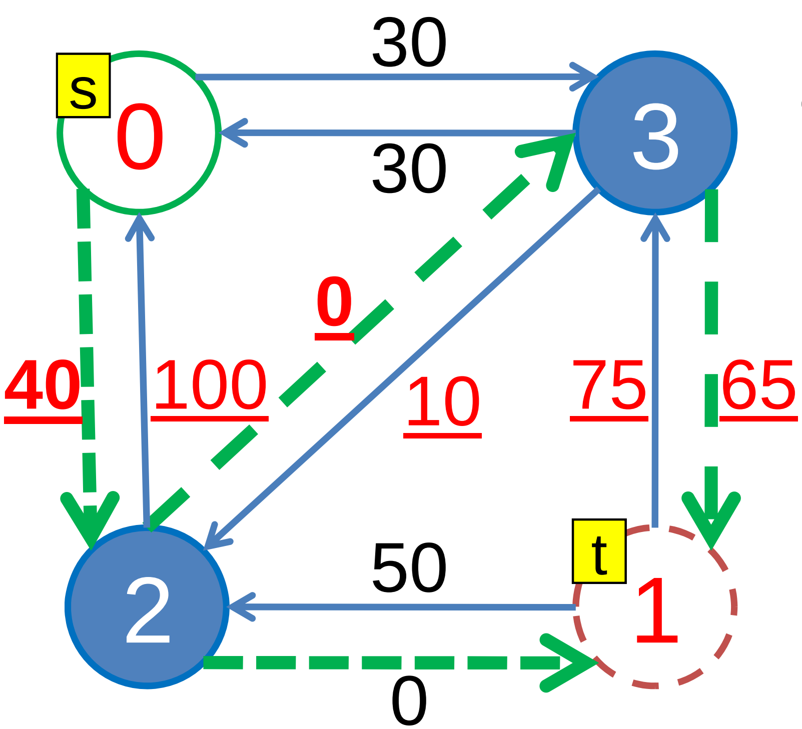
Send a flow *f* through path *p* whenever there exists an **augmenting path** *p* from *s* to *t*



A. **Initial Residual Graph**

B. **We can pass a flow of __25 units__ from source 0 to sink 1 via path 0-2-1**

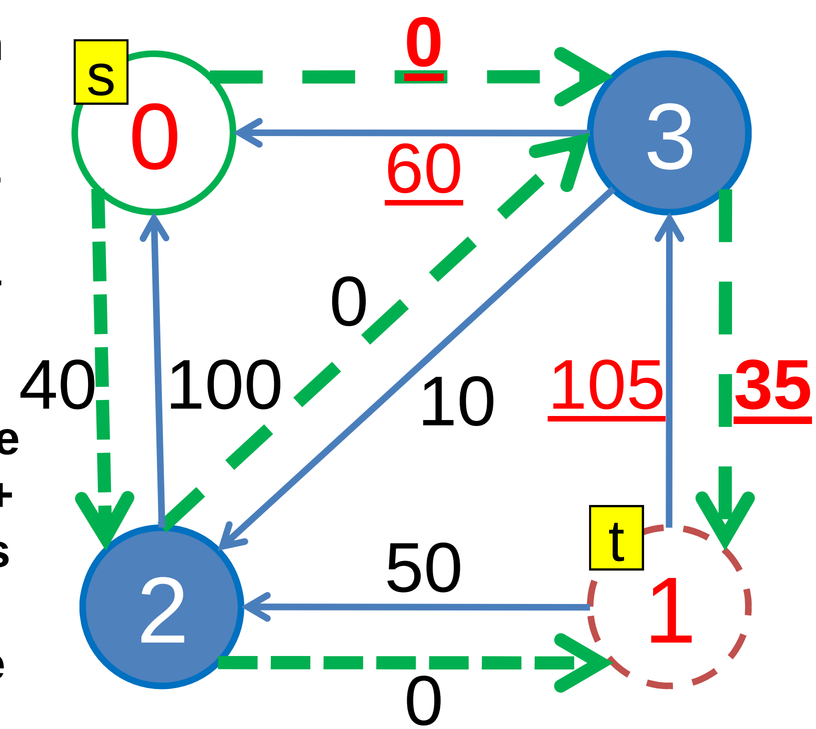




C. **We can pass another flow of __5 units__ via path 0-2-3-1**

D. **We can pass last flow of __30 units__ via path 0-3-1**

No more flow can be added. 25+ 5+30=60 is the __max flow__ value



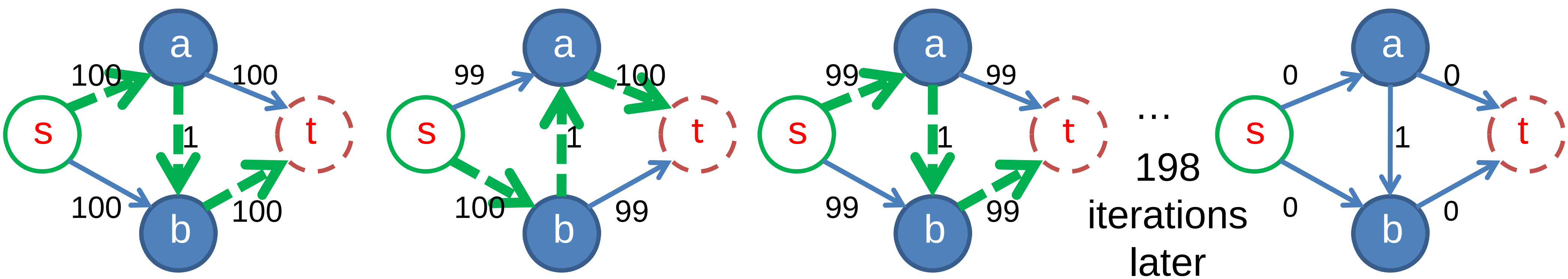
http://www.comp.nus.edu.sg/~stevenha/visualization/maxflow.html

Ford Fulkerson's Method

```
setup directed residual graph
each edge has the same weight with the original graph

mf = 0 // this is an iterative algorithm, mf stands for max_flow
while (there exists an augmenting path p from s to t) {
  // p is a path from s to t that pass through positive edges in residual graph
  augment/send flow f along the path p (s -> ... -> i -> j -> ... t)
   1. find f, the min edge weight along the path p
   2. decrease the weight of forward edges (e.g. i -> j) along path p by f
      reason: obvious, we use the capacities of those forward edges
   3. increase the weight of backward edges (e.g. j -> i) along path p by f
      reason: not so obvious, but this is important for the correctness of Ford
      Fulkerson's method;
  mf += f // we can send a flow of size f from s to t, increase mf
}
output mf
```

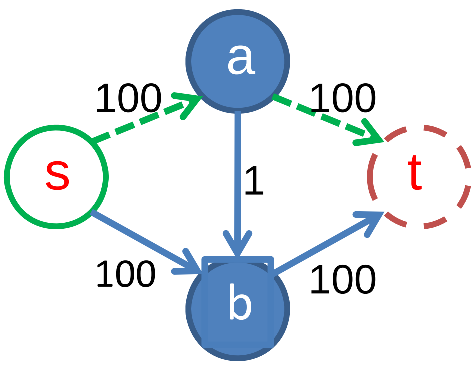
DFS Implementation

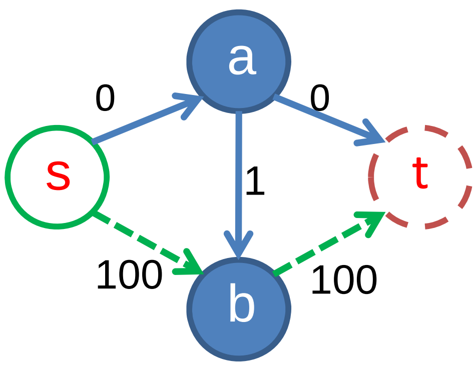
- DFS implementation of Ford Fulkerson's method runs in O(|f*|E) and can be very slow on graph like this:
  - Notice the presence of backward edges (only drawn for edge a→b or b→a this time)
    - Q: What if we do not use backward edges?



BFS Implementation

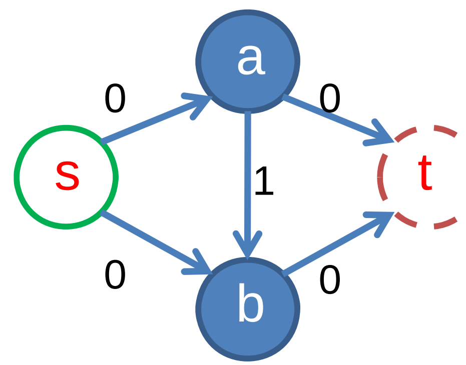
- BFS implementation of Ford Fulkerson's method (called Edmonds Karp's algorithm) runs in $O(VE^2)$





...

After just 2 iterations



Edmonds Karp's (using STL) (1)

```
int res[MAX_V][MAX_V], mf, f, s, t; // global variables
vi p; // note that vi is our shortcut for vector<int>

// traverse the BFS spanning tree as in print_path (section 4.3)
void augment(int v, int minEdge) {
  // reach the source, record minEdge in a global variable 'f'
  if (v == s) { f = minEdge; return; }
  // recursive call
  else if (p[v] != -1) { augment(p[v], min(minEdge, res[p[v]][v]));
  // alter residual capacities
                         res[p[v]][v] -= f; res[v][p[v]] += f; }
}

// in int main()
  // set up the 2d AdjMatrix 'res', 's', and 't' with appropriate values
```

Edmonds Karp's (using STL) (2)

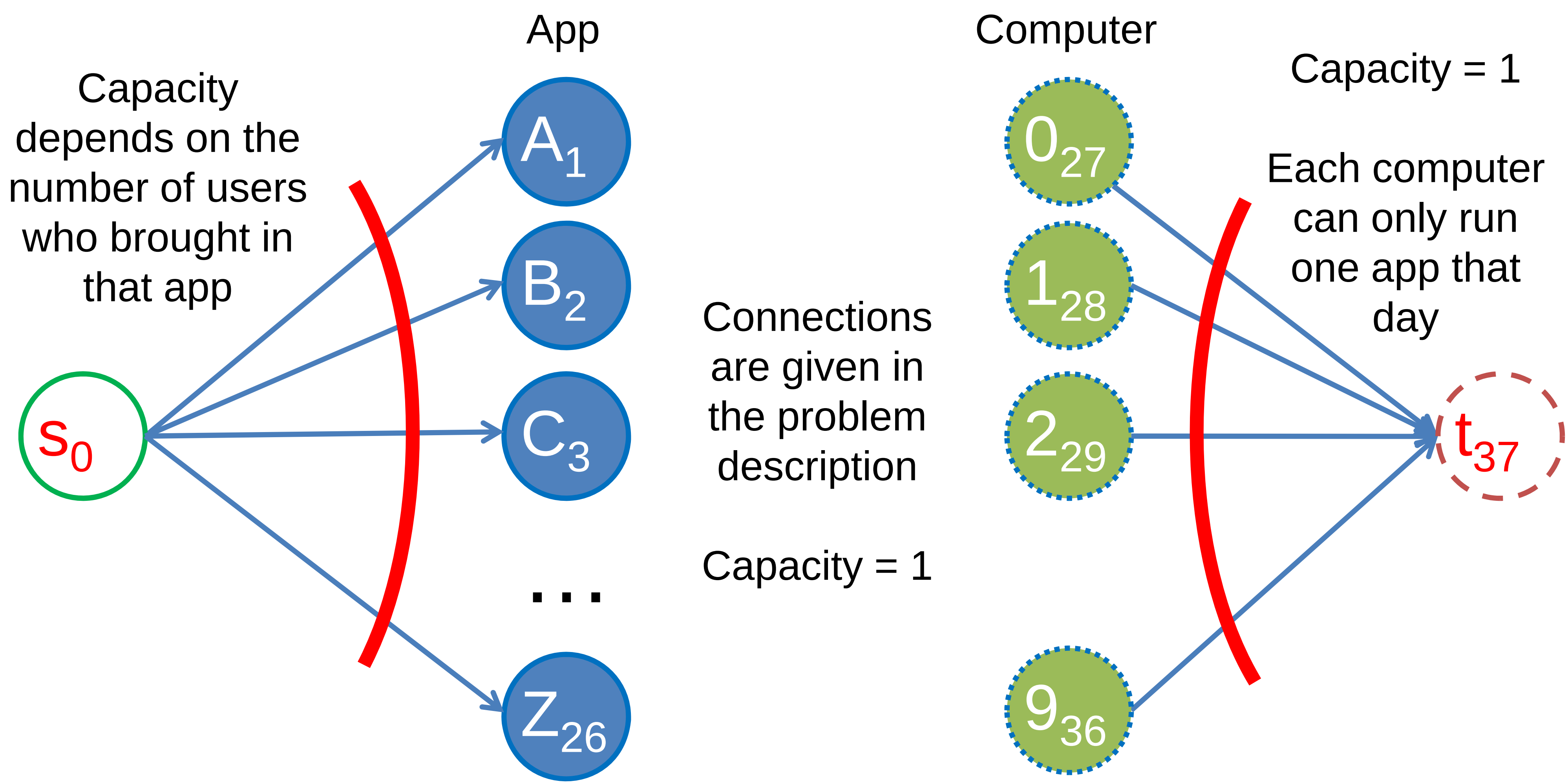
```
mf = 0;
while (1) { // run O(VE * V^2 = V^3*E) Edmonds Karp to solve the Max Flow problem
  f = 0;

  // run BFS, please examine parts of the BFS code that is different than in Section 4.2.23
  vi dist(MAX_V, INF); dist[s] = 0; // #define INF 2000000000
  queue<int> q; q.push(s);
  p.assign(MAX_V, -1); // (we have to record the BFS spanning tree)
  while (!q.empty()) { // (we need the shortest path from s to t!)
    int u = q.front(); q.pop();
    if (u == t) break; // immediately stop BFS if we already reach sink t
    for (int v = 0; v < MAX_V; v++) // note: enumerating neighbors with AdjMatrix is 'slow'
      if (res[u][v] > 0 && dist[v] == INF) dist[v] = dist[u] + 1, q.push(v), p[v] = u;
  }

  augment(t, INF); // find the min edge weight 'f' along this path, if any
  if (f == 0) break; // if we cannot send any more flow ('f' = 0), terminate the loop
  mf += f; // we can still send a flow, increase the max flow!
}

printf("%d\n", mf); // this is the max flow value of this flow graph
```

FLOW GRAPH MODELING

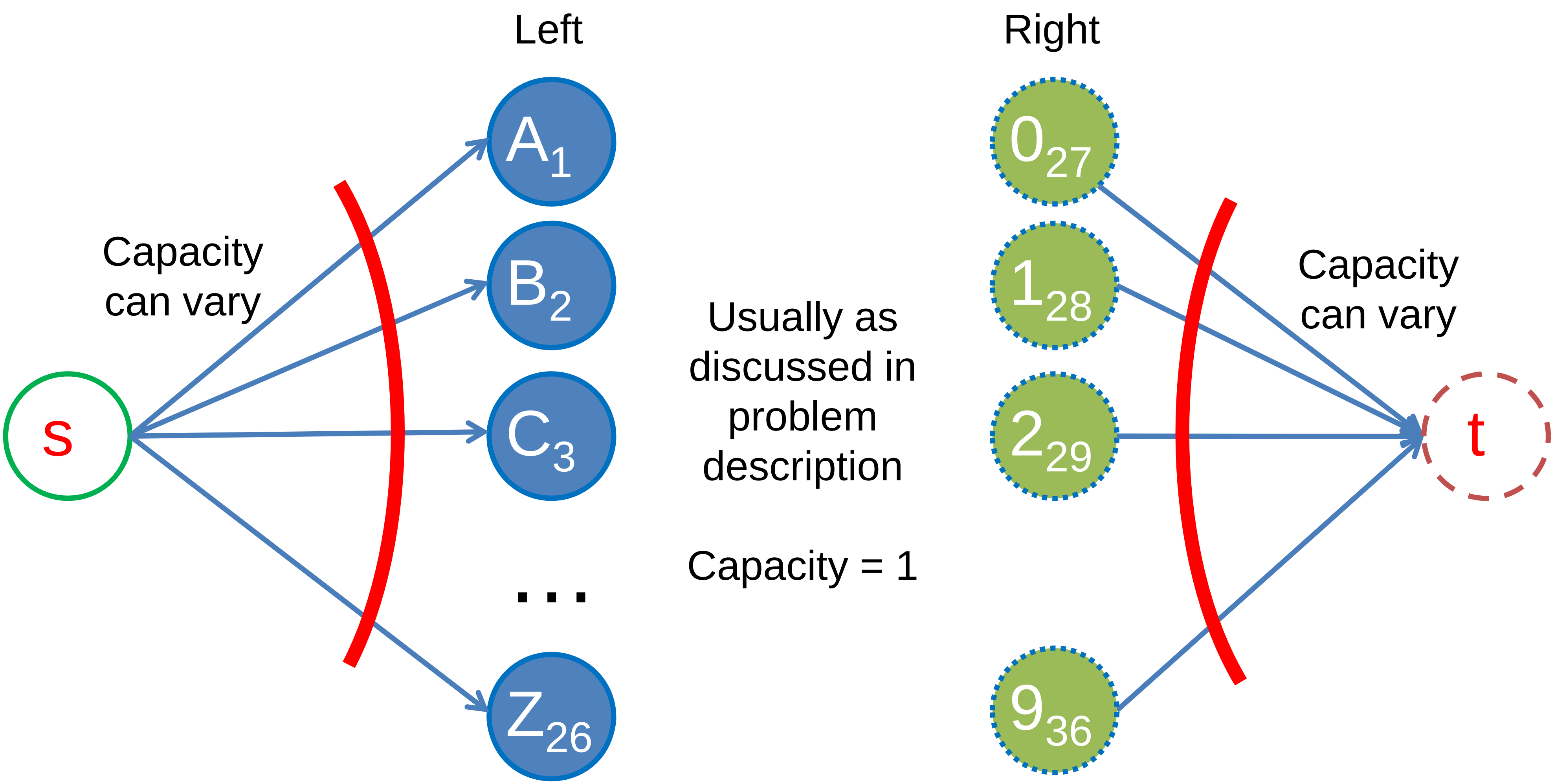
Network Flow Variants

- **Bipartite Matching Variant (more details next week)**
- **Min Cut**
- Multi-source Multi-sink Max Flow
  - The “super source and super sink” technique
- Max Flow with Vertex Capacities
  - The “vertex splitting” technique
- Max Independent Path
- Max Edge-Disjoint Path
- Min Cost Max Flow (MCMF)

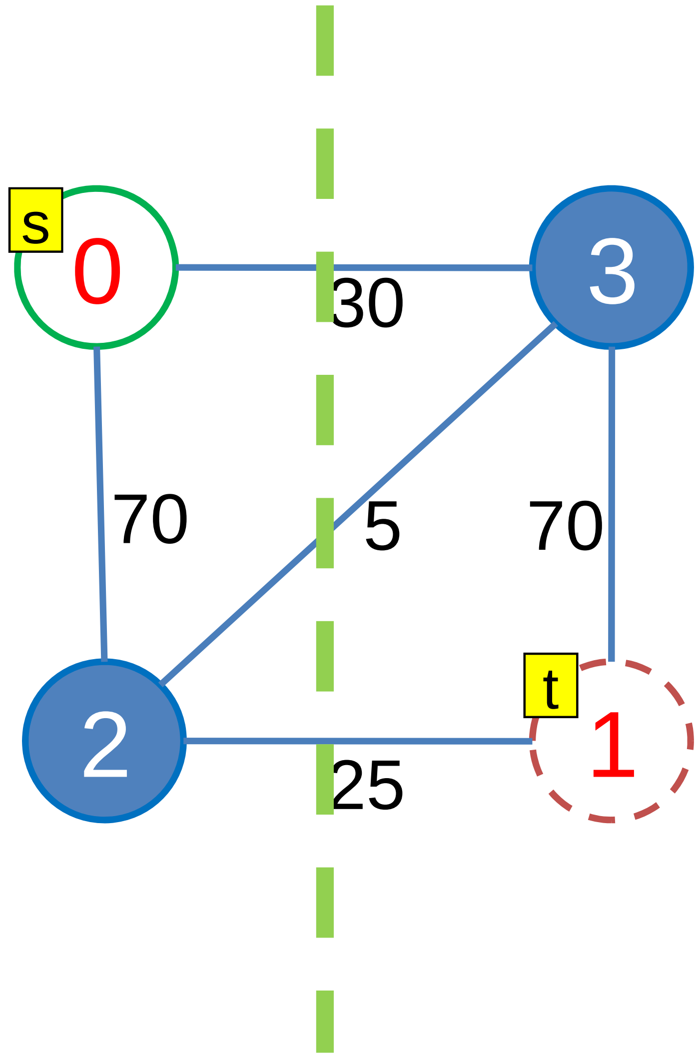
UVa 259 – Software Allocation

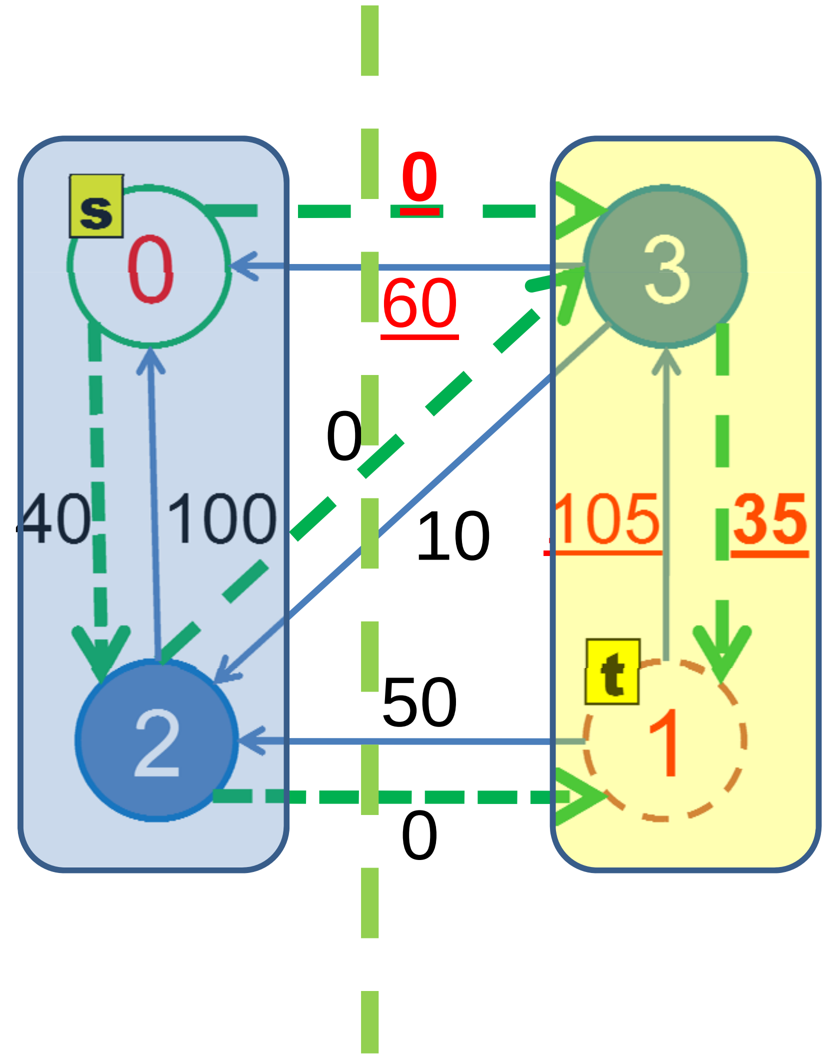


Bipartite Matching Variant



Min Cut





Graph Theory in ICPC

- Graph problems appear several times in ICPC!
  - Min 1, normally 2, can be 3 out of 10
  - Master all known solutions for classical graph problems
  - Or perhaps combined with DP/Greedy style
- This can move your team nearer to top 10
  - Perhaps rank [11-20] out of 60 now ☺
  - Solving 3-5 problems out of 10
- For IOI trainees… all these Network Flow stuffs…
  - **ARE NOT IN THE SYLLABUS…**

References

- **CP2.9, Section 4.6, 9.6, 9.13, 9.14 ☺**
- Introduction to Algorithms, Ch 22,23,24,25,26 (p643-698)
- Algorithm Design, Ch 3,4,6,7 (p337-450)
- Algorithms (Dasgupta et al), Ch 6 & Ch 7
- Algorithms (Sedgewick), Ch 33 & Ch 34
- Algorithms (Alsuwaiyel), Ch 16 & Ch 17
- Programming Challenges, p227-230, Ch 10
- http://www.topcoder.com/tc?module=Static&d1=tutorials&d2=standardTemplateLibrary2
- Internet: TopCoder Max-Flow tutorial, UVa Live Archive, UVa main judge, Felix's blog, Suhendry's blog, Dhaka 2005 solutions, other Max Flow lecture notes, etc…