Small UTMs

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Universal Turing Machines :

It is a turing machine which when given, an arbitrary turing machine and an input string, as it's input simulates the given input string on the turing machine.

It was first proposed by Alan Turing.

The Idea Of Small Turing Machines:

- Posed by Shannon in 1956.
- Symbols can always be interchanged by states.(2 symbols were sufficient so long as enough states are used)
- He proposed to measure the complexity of an m-state, n-symbol turing machine as the product of its states and the symbols i.e. mn.
- Marvin Minsky discovered 7-state 4-symbol universal Turing machine in 1962 using 2-tag systems. Other small universal Turing machines have since been found by Yurii Rogozhin and others by extending this approach of tag system simulation.

Post Tag Machine :

Post tag machine is a finite state machine :

- Input tape is a FIFO queue of unbounded length.
- Such that in each transition the machine :
  - Reads the symbol at the head of the queue
  - Deletes a fixed number of symbols from the head
  - Appends a symbol-string preassigned to the deleted symbol at the tail.
- Because all of the indicated operations are performed in each transition, a tag machine strictly has only one state.

Tag System:

Formally, a tag system is a triplet (m, A, P), where :

- m = deletion number, m > 0.
- A is a finite alphabet of symbols
  - one of which is a special halting symbol.
  - All finite (possibly empty) strings on A are called words.
- P is a set of production rules, assigning a word P(x) (called a production) to each symbol x in A. The production (say P(H)) assigned to the halting symbol is seen below to play no role in computations, but for convenience is taken to be P(H) = 'H'.
- The term m-tag system is often used to emphasise the deletion number.

Rogozhin's Tag System Definition :

- A halting word is a word that either begins with the halting symbol or whose length is less than m.
- A transformation t (called the tag operation) is defined on the set of non-halting words, such that if x denotes the leftmost symbol of a word S, then t(S) is the result of deleting the leftmost m symbols of S and appending the word P(x) on the right.
- A computation by a tag system is a finite sequence of words produced by iterating the transformation t, starting with an initially given word and halting when a halting word is produced.

2-Tag System (Sample Run):

- 2-tag system

  Alphabet: {a,b,c,H}

  Production rules:

  a --> ccbaH ; b --> cca ; c --> cc

  Computation:

  Initial word: baa {delete ba and append cca }

  acca {delete ac and append ccbaH }

  caccbaH {delete ca and append cc }

  ccbaHcc {delete cc and append cc }

  baHcccc {delete ba and append cca }

  Hcccccca (halt). { Since, leftmost characters is “H”.}

Categories Of Small Turing Machines :

Tag-System Simulation:

This is the class of small turing machines which were found by extending the approach of 2-tag based systems, the one which Marvin Minsky used to discover the 7-state 4-symbol universal Turing machine.

Yurii Rogozhin and others have discovered many small turing machines extending the idea of 2-tag based systems, including (15, 2), (9, 3), (6, 4), (5, 5), (4, 6), (3, 9), and (2, 18), where (m, n) => m-state, n-symbol turing machine.

The (4,6) turing machine has only 22 instructions in its transition function and no standard UTM of lesser descriptional complexity is known

4-State 6-Symbol UTM

Categories Of Small Turing Machines :

"Semi-weak" or "Weak" UTMs :

Generalizing the standard turing machine model in such a way that it is allowed to have infinitely repeated word on one or both sides of the Turing machine input, extends the idea of universality to "Semi-weak" or "Weak" Universality respectively.

Small weakly universal Turing machines that simulate the Rule 110 cellular automaton have been found for the (6, 2), (3, 3), and (2, 4) state-symbol.

Categories Of Small Turing Machines :

Wolfram's (2,3) Universal Turing Machine :

Alex Smith proved the universality of the (2,3) Turing Machine, by showing that the machine was equivalent to a variant of a tag system already known to be universal.

Problems:

- Has no halting state.
- Requires an infinite non-repeating input.
- Contradicts the fact that the linear bounded automata are not universal Turing machines as proved by Chomsky(as claimed by Vaughan Pratt).

Categories Of Small Turing Machines :

Wolfram's (2,3) Universal Turing Machine :

The proof for universality is still under dispute.

In 1969 Minsky showed that (2,2) turing machine is not possible, thus, the (2,3) turing machine might be the smallest turing machine possible.

But Minsky in his proof assumed the turing machines to be halting, thus, (2,3) UTM being universal and/or smallest is still under debate.

References :

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Thank You