Reference: https://iitk.ac.in/math/index.php/2014-05-21-10-30-47/courses (MTH 302)

Syllabus: Pre-requisite: MTH 102, MTH 202 for M.Sc. 2 yr

Some basics of set theory: Relations, Partitions, Functions and sets of functions. Families of sets, Cartesian products of families. Principles of weak and strong mathematical induction and their equivalence. Schroder-Bernstein theorem. Countable and uncountable sets. Cantor's theorem. Classical propositional calculus (PC): Syntax. Valuations and truth tables, Truth functions, Logical equivalence relation. Semantic consequence and satisfiability. Compactness theorem with application. Adequacy of connectives. Normal forms. Applications to Circuit design. Axiomatic approach to PC: soundness, consistency, completeness. Other proof techniques: Sequent calculus, Computer assisted formal proofs: Tableaux. Decidability of PC, Boolean algebras: Order relations. Boolean algebras as partially ordered sets. Atoms, Homomorphism, sub-algebra. Filters. Stone's representation (sketch). Completeness of PC with respect to the class of all Boolean algebras. Classical first order logic (FOL) and first order theories, Syntax. Satisfaction, truth, validity in FOL. Axiomatic approach, soundness. Computer assisted formal proofs: Tableaux. Consistency of FOL and completeness (sketch). Equality. Examples of first order theories with equality. Peano's arithmetic. Zermelo-Fraenkel axioms of Set theory. Axiom of choice, Well-ordering theorem, Zorn's lemma and their equivalence; illustrations of their use. Well-ordering principle and its equivalence with principles of weak and strong induction. Elementary model theory: Compactness theorem, Löwenheim-Skolem theorems. Completeness of first order theories, Isomorphism of models, Categoricity-illustrations through theories such as those of finite Abelian groups, dense linear orders without end points and Peano's arithmetic. Statements of Gödel's incompleteness theorems and un-decidability of FOL.

Reference materials:

1. J. Bridge: Beginning Model Theory: The Completeness Theorem and Some Consequences. Oxford Logic Guides, 1977.
2. I. Chiswell and W. Hodges: Mathematical Logic. Oxford, 2007.
3. R. Cori and D. Lascar: Mathematical Logic, Oxford, 2001.
4. J. Goubalt-Larrecq and J. Mackie: Proof Theory and Automated Deduction, Kluwer, 1997.

5. P. R. Halmos: Naive Set Theory, Springer, 1974.
6. J. Kelly: The Essence of Logic, Pearson, 2011.
7. A. Margaris, First Order Mathematical Logic, Dover, 1990.

Credits: 11