UCF Local Contest — September 5, 2015

Turing's Challenge

filename: `turing`
(*Difficulty Level:* `Hard`)

Knuth was looking through some of Turing's memoirs and found a rather interesting challenge that Turing had left for one of his successors. Naturally, Knuth has slyly decided to ask you, his best student, to write a computer program to solve the challenge, but plans on taking credit for the work. Since you know that co-authoring a paper with Knuth is to computer scientists what co-authoring a paper with Erdos is to mathematicians, you've decided to take the bait. Help Knuth solve Turing's problem!

The Problem:

The challenge is as follows:

Given positive integer values for X and N, define the set T as follows:

$$T = \{T_i | 1 \leq i \leq N+1\}, \text{ where } T_i = \binom{N}{i-1} X^{i-1}$$

The goal of the challenge is to pick a set S of maximal sum with $S \subseteq \{i | 1 \leq i \leq N+1\}$, such that $\prod_{i \in S} T_i \equiv 2 (mod\ 4)$.

In other words, we seek a subset of terms in the binomial expansion of $(1+X)^N$ such that the product of the terms leaves a remainder of 2 when divided by 4 and the sum of the *indices* of those terms is maximal.

The goal of Turing's challenge is to determine this maximal sum.

As an example, consider X = 3 and N = 5. The corresponding terms are $T_1 = 1, T_2 = 15, T_3 = 90, T_4 = 270, T_5 = 405$, and $T_6 = 243$.

The product, $T_1 T_2 T_4 T_5 T_6 = 1 \times 15 \times 270 \times 405 \times 243 = 398580750 \equiv 2\ (mod\ 4)$, thus the solution to this specific challenge is $1 + 2 + 4 + 5 + 6 = 18$, since no other product of terms with a higher sum of indices is congruent to $2 \pmod 4$.

The Input:

The first input line contains a positive integer, q ($1 \leq q \leq 500$), indicating the number of queries. Each of the next q lines will contain a pair of space-separated integers, where the first integer is X ($1 \leq X < 2^{31}$), and the second integer is N ($1 \leq N < 2^{31}$), for that query.

The Output:

For each query, output on a line by itself, the desired maximal sum of indices. If no such subset of terms exists, output 0 instead.

Sample Input:

```
3
3 5
1 4
4 6
```

Sample Output:

```
18
9
0
```