KEY

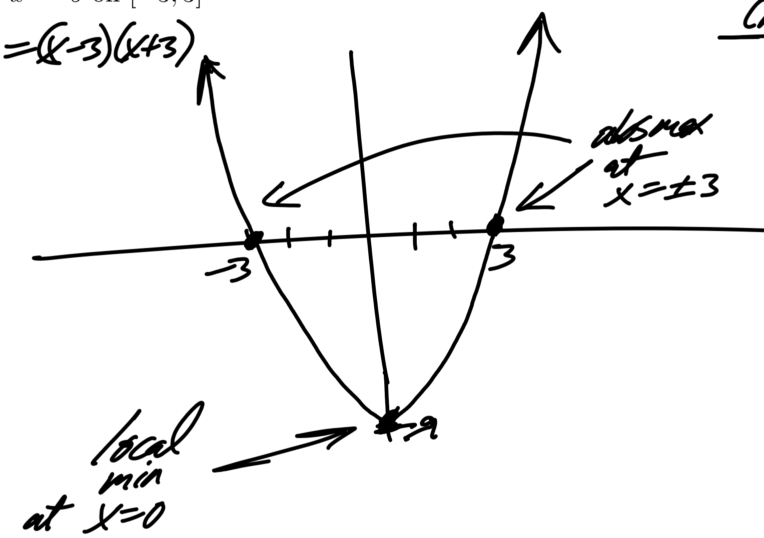
Unit 10 Group Work
PCHA 2021-22 / Dr. Kessner

No calculator! Have fun!

1. Graph the given function on the specified interval. Find all critical points. Identify any points where there is a local min/max, and verify with a derivative test. Identify the absolute max and min. If either fails to exist, state the condition of the Extreme Value Theorem that is *not* satisfied.

a. $g(x) = x^2 - 9$ on $[-3, 3]$

$g(x) = (x-3)(x+3)$



critical pts:

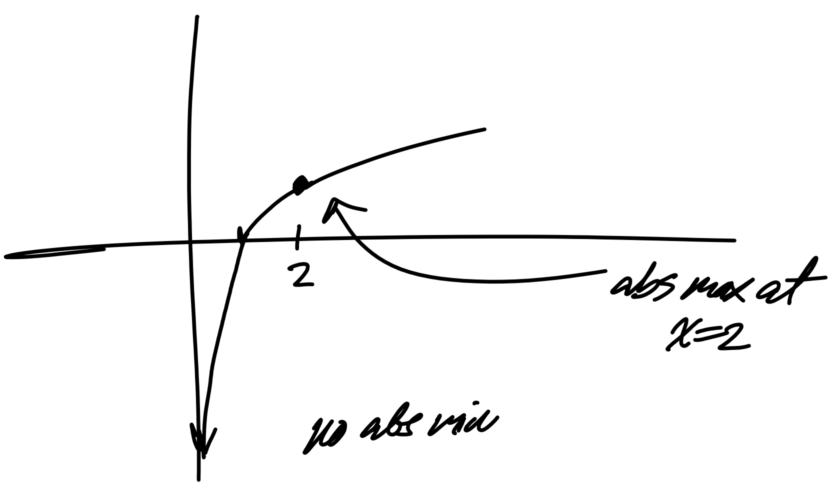
$g'(x) = 2x$

$g'(x) = 0 \Rightarrow x = 0$

$g''(x) = 2$

$g''(0) > 0 \Rightarrow$ local min.

b. $h(x) = \ln x$ on $(0, 2]$



$h'(x) = \frac{1}{x}$

$\Rightarrow$ no critical pts on $(0,2]$

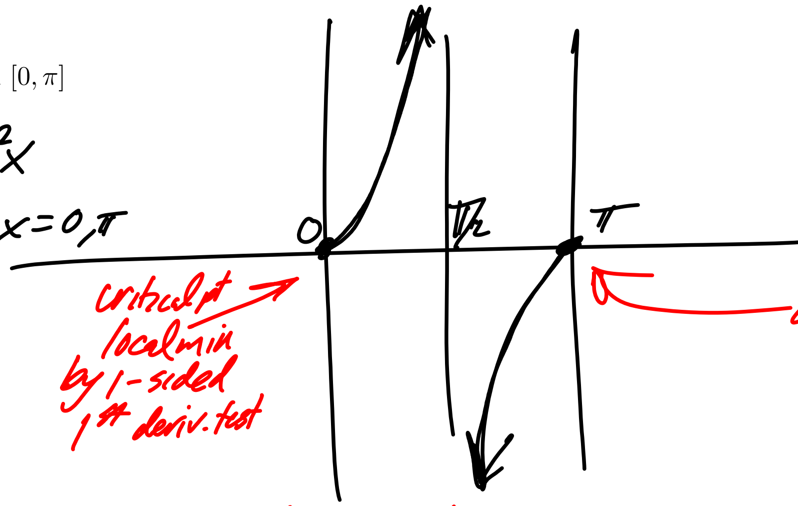
$(0,2]$ not closed

$\Rightarrow$ EVT does not apply

c. $m(x) = \tan x$ on $[0, \pi]$

$m'(x) = \sec^2 x$

$m'(x) = 0$ at $x = 0, \pi$



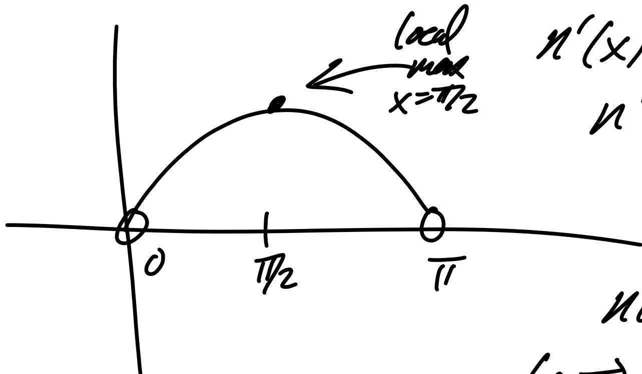
critical pt local min by 1-sided 1st deriv. test

critical pt & local max at $x = \pi$ by 1-sided 1st deriv. test

no absolute min/max

$m(x)$ not continuous on $[0, \pi]$ $\longrightarrow$ EVT does not apply

d. $n(x) = \sin x$ on $(0, \pi)$



local max $x = \pi/2$

$n'(x) = \cos x$

$n'(x) = 0$ at $x = \frac{\pi}{2}$

$n''(x) = -\sin(x)$

$n''(\pi/2) = -1 < 0$ local max (and absolute)

no local/absolute min

$(0, \pi)$ not closed $\longrightarrow$ EVT does not apply

2. For each of the given functions find all antiderivatives.

a. $p'(x) = x^4$

$p(x) = \frac{1}{5}x^5 + C$

b. $q'(x) = \cos 2x$

$q(x) = -\frac{1}{2}\sin 2x + C$

c. $r'(x) = \dfrac{1}{x}$

$r(x) = \ln x + C$

d. $s'(x) = \dfrac{1}{x^2}$

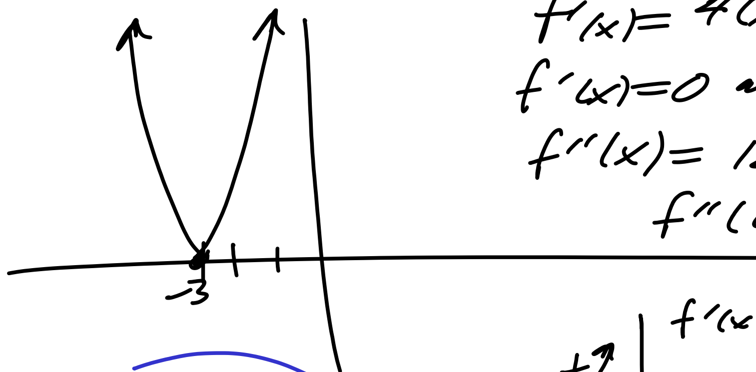
$s(x) = -\frac{1}{x} + C$

e. $t'(x) = e^{\frac{x}{3}}$

$t(x) = 3e^{x/3} + C$

3. For each of the following functions: sketch the graph, then find all local extrema and verify with a derivative test.

a. $f(x) = (x+3)^4$.

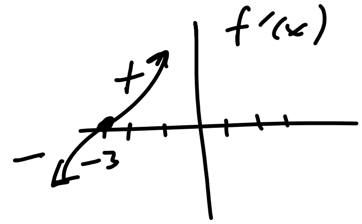


$f'(x) = 4(x+3)^3$

$f'(x) = 0$ at $x = -3$

$f''(x) = 12(x+3)^2$

$f''(0) = 0$ inconclusive



1st deriv. test $\Longrightarrow$ local min at $x=-3$

b. $g(x) = x^3 - 9x$. $= x(x^2-9)$

$g'(x) = 3x^2 - 9$

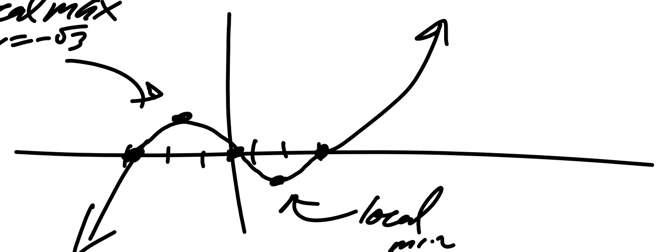
$g'(x) = 0 \Longrightarrow 3x^2 = 9$

$x^2 = 3$

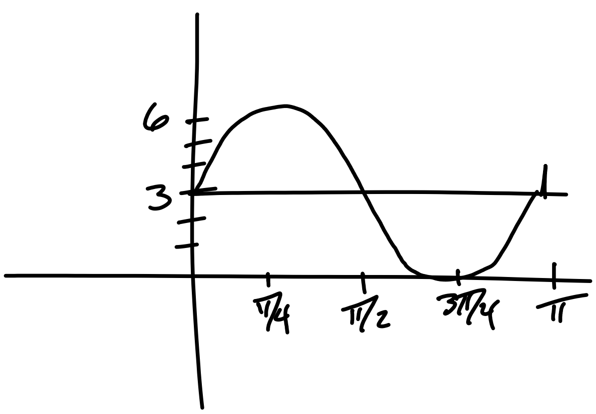
critical pts at $x = \pm\sqrt{3}$

$g''(x) = 6x$ $\quad g''(\sqrt{3}) > 0$ local min

$g''(-\sqrt{3}) < 0$ local max



c. $h(x) = 3 + 3\sin 2x$. You may restrict your attention to the first period of the function. But as an extra challenge, identify *all* local extrema (not just the first period), including derivative tests to show which are minima and which are maxima.

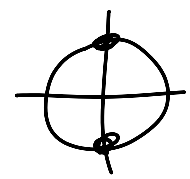


$h'(x) = 6\cos 2x$

$h'(x) = 0 \Longrightarrow \cos 2x = 0$

$2x = \frac{\pi}{2} + k\pi$

$x = \frac{\pi}{4} + \frac{k\pi}{2}$ critical pts

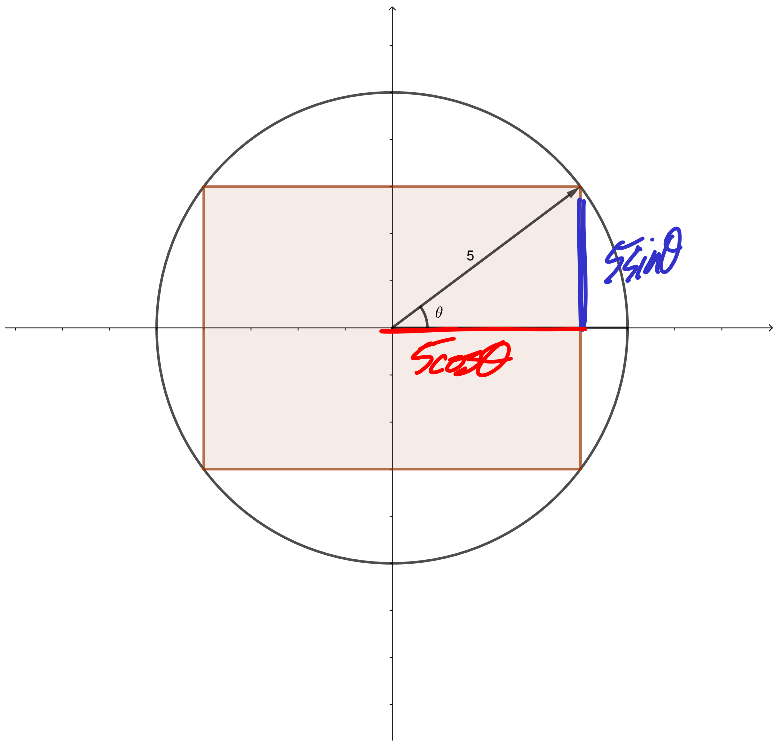


$h''(x) = -12\sin 2x$

$h''\left(\frac{\pi}{4} + k\pi\right) = -12\sin\frac{\pi}{2} = -12 < 0$ local max

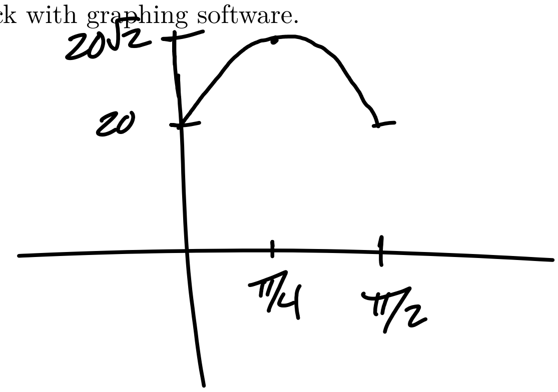
$h''\left(\frac{3\pi}{4} + k\pi\right) = -12\sin\frac{3\pi}{2} = 12 > 0$ local min

4. Consider the following rectangle inscribed in a circle of radius 5. Note that the perimeter of the rectangle changes as the angle θ changes.



a. Write an equation for the perimeter $P(\theta)$ of the rectangle as a function of θ. Challenge: draw a sketch of the graph of $P(\theta)$ on $[0, \frac{\pi}{2}]$ by hand, and then check with graphing software.

$$P(\theta) = 4\cdot 5\sin\theta + 4\cdot 5\cos\theta$$
$$= 20\sin\theta + 20\cos\theta$$



b. Find the absolute min and max of the perimeter, for θ in $[0, \frac{\pi}{2}]$. Why must there be an absolute minimum and maximum?

$$P'(\theta) = 20\cos\theta - 20\sin\theta$$
$$P'(\theta) = 0 \Rightarrow \cos\theta = \sin\theta$$
$$\tan\theta = 1$$
$$\theta = \pi/4$$

$$P'(\theta) = \sec^2\theta$$
$P'(\pi/4) > 0$ local max

$P(\theta)$ continuous on finite, closed interval $[0, \pi/2]$ $\Big\}$ EVT $\Rightarrow$ abs max/min exist in the interval