KEY

Unit 6 Group Work PCHA 2021-22 / Dr. Kessner

No Calculator

1. Evaluate:

a.
$$\binom{7}{1}$$
 = \mathcal{I}

b.
$$\binom{7}{2} = \frac{7.6}{2} = 21$$

c.
$$\binom{7}{3} = \frac{7.6.5}{31} = 35$$

d.
$$\binom{7}{4} = \left(\frac{7}{3}\right) = \frac{35}{3}$$

e.
$$\binom{12}{2} = \frac{12 \cdot 11}{2} = 66$$

$$f_{0}\left(\frac{12}{3}\right) = \frac{12 \cdot 11 \cdot 10}{31} = 220$$

$$_{g.}\binom{12}{9} = \binom{12}{3} = 220$$

h.
$$\binom{12}{10} = \binom{72}{2} = 66$$

i.
$$\binom{100}{99} = \binom{100}{1} = 100$$

j.
$$\binom{2000}{2} = \frac{2000 \cdot 1999}{2} - \frac{1999000}{2}$$

- **2.** Let $\{a_k\}_{k=1}^{\infty} = \{\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \cdots\}.$
 - a. What type of sequence is this? Write recursive and explicit formulas for a_k .

geometric
$$a_{n+1} = -\frac{1}{z}a_n \quad recursive$$

$$a_n = a_1 \cdot r^{n-1}$$

$$= \left(\frac{1}{z}\right)\left(-\frac{1}{z}\right)^{n-1}$$

b. Let S_n be the n^{th} partial sum of the sequence $\{a_k\}$. Express S_n (for this particular sequence) in summation notation.

tion.

$$S_{n} = \sum_{k=1}^{n} a_{k}$$

$$= \sum_{k=1}^{n} \frac{1}{2} \left(-\frac{1}{2}\right)^{n-1}$$

c. Write a formula for the actual sum S_n (for this particular sequence).

$$S_{n} = \underbrace{\frac{u(1-r^{n})}{1-r}}_{1-r}$$

$$= \underbrace{\frac{z(1-(-\frac{z}{z})^{n})}{1-(-\frac{z}{z})}}_{1-(-\frac{z}{z})^{n}}$$

$$= \underbrace{\frac{z(1-(-\frac{z}{z})^{n})}{1-(-\frac{z}{z})^{n}}}_{\text{n-roo}} = \underbrace{\frac{z}{z}}_{\text{n-roo}}$$

$$= \underbrace{\frac{z(1-(-\frac{z}{z})^{n})}{1-(-\frac{z}{z})^{n}}}_{\text{n-roo}} = \underbrace{\frac{z}{z}}_{\text{n-roo}}$$

d. What is the sum of the infinite series $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots$? (Surprising?)

$$S_{0} = \frac{a_{1}}{1-r} = \frac{\frac{1}{2}}{1-(-\frac{1}{2})}$$

$$= \frac{a_{2}}{1-(-\frac{1}{2})}$$

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$$= \frac{a_{2}}{342}$$

$$= \frac{1}{3}$$

3. Expand $(2-x^2)^4$.

$$(2-x^2)^4 = 2^4 + 4 \cdot 2^3 (-x^2) + 6 \cdot 2^2 (-x^2)^2 + 4(2')(-x^2)^3 + (-x^2)^4$$

$$= (6 - 32x^2 + 24x^4 - 8x^6 + x^8)$$

Find the x^6 term in $(2-x^2)^5$.

$$(\frac{5}{2})(2)^{2}(-x^{2})^{3} = -10.4 \times 6$$

= -40×6

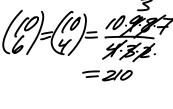
Find the x^8 term in $(2-x^2)^5$.

$$\binom{5}{1}(2)'(-\chi^2)^4 = 10\chi^8$$

lugar geometric

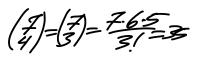
- 4. Suppose you have 7 red and 3 white marbles in a bag. You pick 6 of the marbles from the bag (without replacement).
 - a. What is the probability that you pick 6 red marbles?

$$P(6red) = \frac{\binom{7}{6}\binom{3}{6}}{\binom{10}{6}} = \frac{7}{210} = \frac{1}{30}$$



b. What is the probability that you pick 4 red (and 2 white marbles)?

$$P(4rel) = \frac{\binom{7}{4}\binom{3}{2}}{\binom{10}{6}} = \frac{35.3}{210} = \frac{105}{210} = \frac{1}{2}$$



c. What is the probability that you pick 2 red marbles?

5. Suppose you have 50 black and 50 white marbles in a bag. You sample 5 marbles with replacement (in other words, you pick a marble, look at it, and put it back, 5 times). Let B be the number of times you pick a black marble. Calculate all of the 6 probabilities P(B=0), P(B=1), ..., P(B=5). Hint: You only have to calculate half of these. Verify that $1 = \sum_{k=0}^{5} P(B=k)$.

$$P(B=0) = (5/2)(1)^{5} = \frac{1}{32} = P(B=5)$$

$$P(B=1) = (5/2)(1)^{4} = \frac{5}{32} = P(B=4)$$

$$P(B=2) = (5/2)(1)^{2}(1)^{3} = \frac{6}{32} = P(B=3)$$

$$P(B=2) = (5/2)(1)^{2}(1)^{3} = \frac{6}{32} = P(B=3)$$

$$Check: \sum_{k=0}^{5} P(B=k) = \frac{1}{32} + \frac{5}{32} + \frac{6}{32} + \frac{1}{32} + \frac{1}{32} = 1$$

Now suppose you have 75 black and 25 white marbles in the bag. You again sample 5 marbles with replacement. Calculate the probabilities P(B=0), P(B=1), ..., P(B=5) and again verify that $1 = \sum_{k=0}^{5} P(B=k)$.

$$P(B=0) = \binom{5}{3} \binom{3}{4} \binom{1}{4}^5 = \frac{1}{1024}$$

$$P(B=1) = \binom{5}{3} \binom{3}{4} \binom{1}{4}^4 = \frac{15}{1024}$$

$$P(B=2) = \binom{5}{3} \binom{3}{4}^2 \binom{1}{4}^3 = \frac{90}{1024}$$

$$P(B=3) = \binom{5}{3} \binom{3}{4}^3 \binom{4}{4}^2 = \frac{290}{1024}$$

$$P(B=4) = \binom{5}{3} \binom{3}{4}^4 \binom{1}{4}^4 = \frac{405}{1024}$$

$$P(B=5) = \binom{5}{3} \binom{3}{4}^4 \binom{1}{4}^6 = \frac{243}{1024}$$

$$P(B=5) = \binom{5}{3} \binom{3}{4}^5 \binom{4}{4}^6 = \frac{243}{1024}$$