

Discrete Math Practice
PCHA 2021-22 / Dr. Kessner

No calculator unless its an emergency. Have fun!

1. Evaluate:

a. $\binom{8}{0}$

b. $\binom{8}{1}$

c. $\binom{8}{2}$

d. $\binom{8}{3}$

e. $\binom{8}{6}$

f. You have 8 dogs and 5 cats. How many ways can you pick a committee of 3 dogs and 2 cats?

g. You still have 8 dogs and 5 cats. How many ways can you rank your top 3 cats and top 3 dogs?

2. Phone numbers have the format $(xxx)yyy-zzzz$. For both the area code xxx and exchange yyy , the first digit is not allowed to be 0 or 1.

a. How many possible phone numbers are there?

b. Some phone numbers are not available for standard telephones. In particular, the numbers 555-0100 through 555-0199 (in any area code) are reserved for fictional use. In addition, the N11 numbers (e.g. 911, 411) are also reserved, so these are not usable for the exchange yyy . Considering these two restrictions, how many phone numbers are available for telephones?

3. You have 11 possible toppings you could put on your pizza.
- How many different pizzas could you make?
 - Your favorite number is 9. How many different pizzas can you make using exactly 9 of the 11 toppings?
 - You pick a random pizza from the set of all possible different pizzas. What is the probability that it has exactly 9 toppings?
 - You pick a random pizza from the set of all possible different pizzas. What is the probability that it has exactly 10 toppings?

5. a. Expand and write in standard form: $(-3x + y)^4$.

b. Find the coefficient of the x^2y^6 term in the expansion of $(-2y + x)^8$.

5. a. How many ways can you choose a random 5-letter word? Assume that the alphabet has the usual 26 letters, and a word is any sequence of letters, whether you can pronounce it or not.

b. Suppose you pick a random 5 letter word. What is the probability of picking the word **PROOF**?

c. Assume that the letter **y** is a vowel, so there are 6 vowels and 20 consonants in the alphabet. What is the probability of picking a word having the form **XX00X**, where **X** is a consonant and **0** is a vowel?

d. What is the probability that a random 5-letter word has exactly 2 vowels? *Hint:* In part (c) you calculate the probability of picking a word with vowels in the 3rd and 4th positions. But a random word may have exactly 2 vowels in some other positions. How many ways can you pick 2 positions to be vowels from the 5 positions?