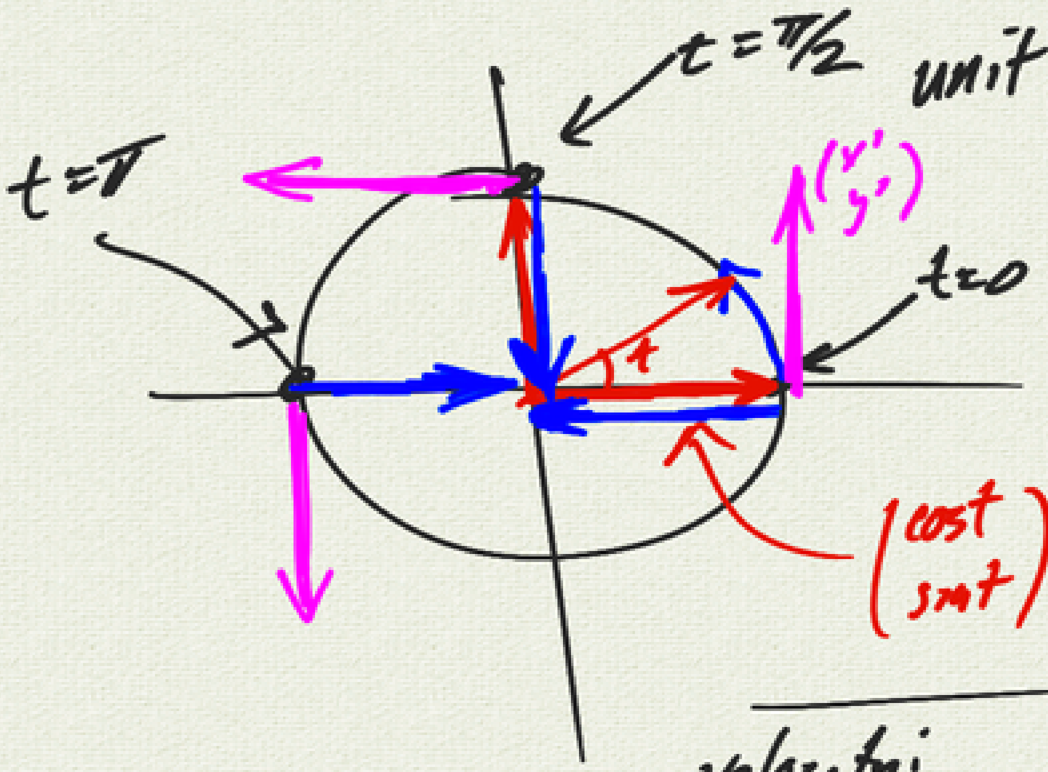
9.5 Applications

parametrize



$x(t) = \cos t$
$y(t) = \sin t$

velocity: $x'(t) = -\sin t$
$y'(t) = \cos t$
vector $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix}$

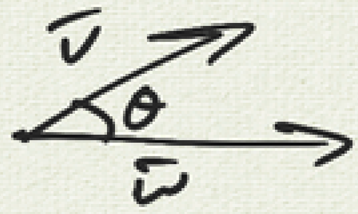
at $t=0$: $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

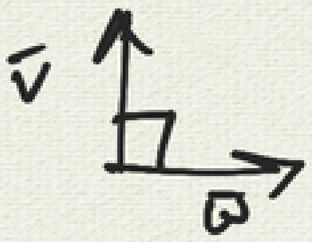
$t=\pi/2$: $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$t=\pi$ $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

velocity $\perp$ position:

$\bar{v} = \langle x_1, y_1 \rangle$
$\bar{w} = \langle x_2, y_2 \rangle$
$\bar{v} \cdot \bar{w} = x_1 x_2 + y_1 y_2$
$= |\bar{v}||\bar{w}| \cos\theta$
dot product





$\bar{v} \cdot \bar{w} = 0 \iff |\bar{v}| = 0$ or $|\bar{w}| = 0$ or
$\cos\theta = 0 \Longleftarrow \theta = \pi/2$

check: position $\cdot$ velocity

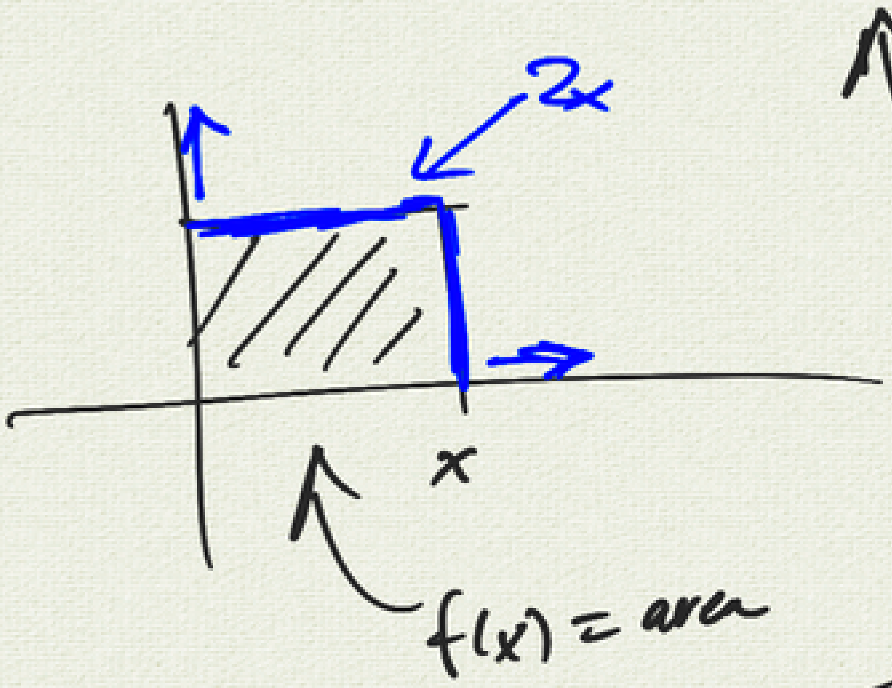
$$\begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \cdot \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} = \cos t(-\sin t) + \sin t(\cos t)$$
$$= 0$$

acceleration: $\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} -\cos t \\ -\sin t \end{pmatrix} = -\underbrace{\begin{pmatrix} \cos t \\ \sin t \end{pmatrix}}_{\text{position } \begin{pmatrix} x \\ y \end{pmatrix}}$

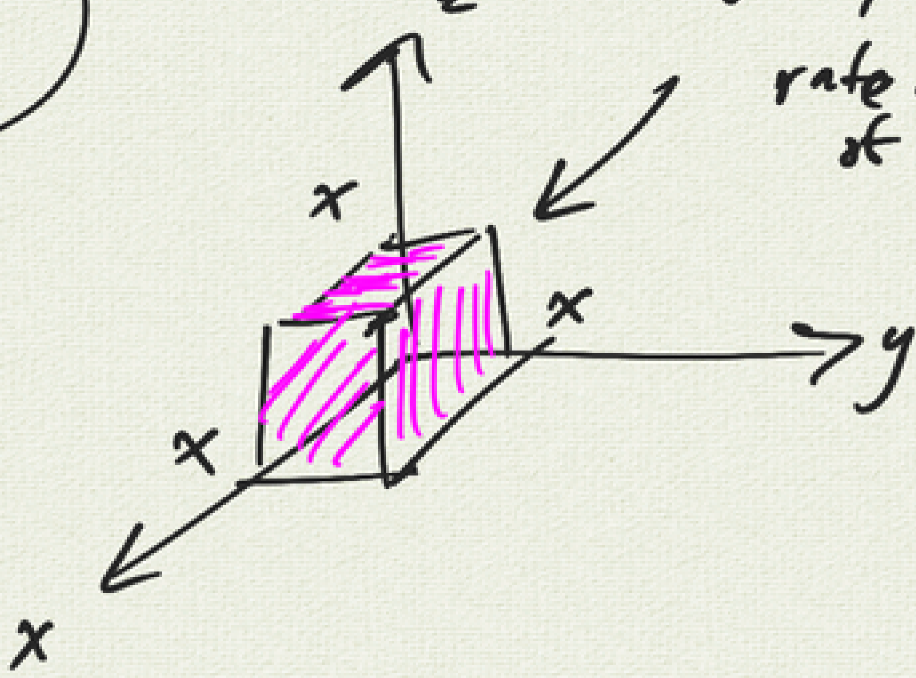
$f(x) = x^2 \implies f'(x) = 2x$

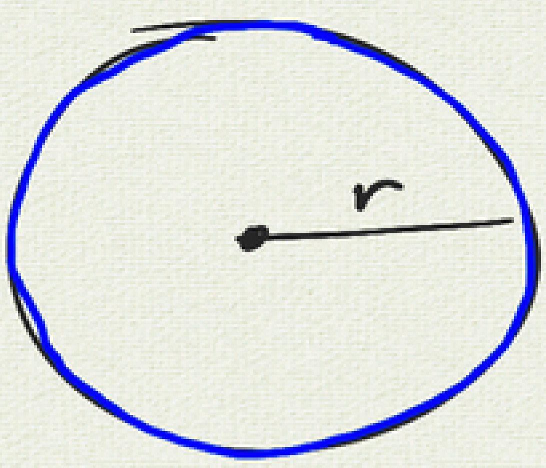
derivative
rate of change of area with respect to x
slope



$V(x) = x^3$

$V'(x) = 3x^2$
rate of change of volume

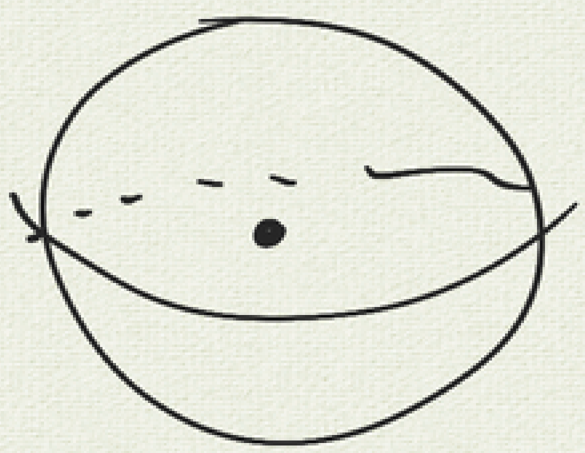




$A(r) = \pi r^2$ (area)

$A'(r) =$ rate of area as we change radius

$= 2\pi r$ (circumference)



sphere $V = \frac{4}{3}\pi r^3$ volume

$V'(r) = 4\pi r^2$ surface area

cylinder

$V = \pi r^2 h$

$\frac{dV}{dr} = 2\pi r h$ lateral surface area

$\frac{dV}{dh} = \pi r^2$ area of top

---

derivative = slope of tangent line
= rate of change