Discrete Math Practice
PCHA 2021-22 / Dr. Kessner

No calculator unless its an emergency. Have fun!

1. Evaluate:

a. $\binom{8}{0}$

b. $\binom{8}{1}$

c. $\binom{8}{2}$

d. $\binom{8}{3}$

e. $\binom{8}{6}$

f. You have 8 dogs and 5 cats. How many ways can you pick a committee of 3 dogs and 2 cats?

g. You still have 8 dogs and 5 cats. How many ways can you rank your top 3 cats and top 3 dogs?

2. Phone numbers have the format `(xxx)yyy-zzzz`. For both the area code `xxx` and exchange `yyy`, the first digit is not allowed to be 0 or 1.

a. How many possible phone numbers are there?

b. Some phone numbers are not available for standard telephones. In particular, the numbers 555-0100 through 555-0199 (in any area code) are reserved for fictional use. In addition, the N11 numbers (e.g. 911, 411) are also reserved, so these are not usable for the exchange `yyy`. Considering these two restrictions, how many phone numbers are available for telephones?

3. You have 11 possible toppings you could put on your pizza.

a. How many different pizzas could you make?

b. Your favorite number is 9. How many different pizzas can you make using exactly 9 of the 11 toppings?

c. You pick a random pizza from the set of all possible different pizzas. What is the probability that it has exactly 9 toppings?

d. You pick a random pizza from the set of all possible different pizzas. What is the probability that it has exactly 10 toppings?

4. a. Expand and write in standard form: $(-3x + y)^4$.

b. Find the coefficient of the x^2y^6 term in the expansion of $(-2y + x)^8$.

5. a. How many ways can you choose a random 5-letter word? Assume that the alphabet has the usual 26 letters, and a word is any sequence of letters, whether you can pronounce it or not.

b. Suppose you pick a random 5 letter word. What is the probability of picking the word `PROOF`?

c. Assume that the letter `y` is a vowel, so there are 6 vowels and 20 consonants in the alphabet. What is the probability of picking a word having the form `XXOOX`, where `X` is a consonant and `O` is a vowel?

d. What is the probability that a random 5-letter word has exactly 2 vowels? *Hint:* In part (c) you calculate the probability of picking a word with vowels in the 3rd and 4th positions. But a random word may have exactly 2 vowels in some other positions. How many ways can you pick 2 positions to be vowels from the 5 positions?