

Counterexamples related to the Sato–Tate conjecture

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Motivation

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Sato–Tate Conjecture

E/\mathbb{Q} non-CM elliptic curve, $\theta_p = \cos^{-1} \left(\frac{a_p}{2\sqrt{p}} \right)$.

Sato–Tate measure $ST = \frac{2}{\pi} \sin^2 \theta \, d\theta$ (Haar measure on $SU(2)$).

Theorem (Taylor et. al.)

The θ_p are equidistributed with respect to ST.

Serre generalized conjecture to arbitrary motives.

Stick to elliptic curves and CM abelian varieties.

Quantify rate of convergence of $\frac{1}{\pi(N)} \sum_{p \leq N} \delta_{\theta_p}$ to ST.

Use discrepancy (Kolmogorov–Smirnov statistic).

Akiyama–Tanigawa Conjecture

Let

$$D_N = \sup_{x \in [0, \pi]} \left| \frac{1}{\pi(N)} \sum_{p \leq N} 1_{[0, x)}(\theta_p) - \int_0^x dST \right|.$$

Conjecture (A–T): $D_N \ll N^{-\frac{1}{2} + \epsilon}$.

Akiyama–Tanigawa Conjecture makes sense for CM elliptic curve.

Theorem (A–T)

A–T implies $RH(E)$.

Questions?