Galois representations with specified Sato–Tate distributions

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Outline

Motivation

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Sato–Tate Conjecture

$E_{/\mathbf{Q}}$ non-CM elliptic curve, $\theta_p = \cos^{-1}\left(\frac{a_p}{2\sqrt{p}}\right)$.

Sato–Tate measure: $\mathrm{ST} = \frac{2}{\pi}\sin^2\theta\,\mathrm{d}\theta$ (Haar measure on SU(2)).

Theorem (Taylor et. al.)

The θ_p are equidistributed with respect to ST*.*

Generalized by Serre: conjecture for arbitrary motives.

Stick to elliptic curves and CM abelian varieties.

Quantify rate of convergence of $\frac{1}{\pi(N)}\sum_{p\leqslant N}\delta_{\theta_p}$ to ST.

Use discrepancy (Kolmogorov–Smirnov statistic).

Akiyama–Tanigawa Conjecture

$$D_N = \sup_{x\in[0,\pi]} \left| \frac{1}{\pi(N)} \sum_{p\leqslant N} 1_{[0,x)}(\theta_p) - \int_0^x \mathrm{d}\,\mathsf{ST} \right|.$$

Conjecture (Akiyama–Tanigawa)

$D_N \ll N^{-\frac{1}{2}+\epsilon}$.

There is a variant of this conjecture for CM elliptic curve.

Theorem (Akiyama–Tanigawa)

Akiyama–Tanigawa conjecture $\Rightarrow$ *Riemann Hypothesis for* E.

Theorem (Mazur)

Akiyama–Tanigawa conjecture $\Rightarrow$ *Riemann Hypothesis for* $\operatorname{sym}^k E$

Questions?