

# Counterexamples related to the Sato–Tate conjecture

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Motivation

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# Sato–Tate Conjecture

$E/\mathbb{Q}$  non-CM elliptic curve,  $\theta_p = \cos^{-1} \left( \frac{a_p}{2\sqrt{p}} \right)$ .

Sato–Tate measure:  $ST = \frac{2}{\pi} \sin^2 \theta \, d\theta$  (Haar measure on  $SU(2)$ ).

## **Theorem (Taylor et. al.)**

*The  $\theta_p$  are equidistributed with respect to ST.*

Generalized by Serre: conjecture for arbitrary motives.

Stick to elliptic curves and CM abelian varieties.

Quantify rate of convergence of  $\frac{1}{\pi(N)} \sum_{p \leq N} \delta_{\theta_p}$  to ST.

Use discrepancy (Kolmogorov–Smirnov statistic).

# Akiyama–Tanigawa Conjecture

$$D_N = \sup_{x \in [0, \pi]} \left| \frac{1}{\pi(N)} \sum_{p \leq N} 1_{[0, x)}(\theta_p) - \int_0^x dST \right|.$$

## Conjecture (Akiyama–Tanigawa)

$$D_N \ll N^{-\frac{1}{2} + \epsilon}.$$

There is a variant of this conjecture for CM elliptic curve.

## Theorem (Akiyama–Tanigawa)

*Akiyama–Tanigawa conjecture  $\Rightarrow$  Riemann Hypothesis for  $E$ .*

## Theorem (Mazur)

*Akiyama–Tanigawa conjecture  $\Rightarrow$  Riemann Hypothesis for  $\text{sym}^k E$*

# Questions

1. Does Riemann Hypothesis for all  $\text{sym}^k E$  imply Akiyama–Tanigawa conjecture?
2. Does functoriality imply the Akiyama–Tanigawa conjecture? For CM elliptic curves?
3. Is the Sato–Tate conjecture a Galois-theoretic result?
4. Does every Galois representation have equidistribution coming from a Sato–Tate group?
  1. Yes! (sort of)
  - 2–4. No!

Questions?