A strange class of L-functions

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Let $E_{/\mathbf{Q}}$ be an elliptic curve, and write $\theta_p = \cos^{-1}\left(\frac{a_p(E)}{2\sqrt{p}}\right)$. Then the k-th symmetric power (analytic) L-function of E has product formula:

$$L(\operatorname{sym}^k E, s) = \prod_p \prod_{j=0}^{k} \frac{1}{1 - e^{i(n-2j)\theta_p} p^{-s}}.$$

Rewrite this as

$$L(\operatorname{sym}^k E, s) = \prod_{j=0}^{k} \prod_p \frac{1}{1 - e^{i(n-2j)\theta_p} p^{-s}}.$$

This writes $L(\operatorname{sym}^k E, s)$ as a product of "strange L-functions."

Write $\boldsymbol{\lambda} = (\lambda_2, \lambda_3, \lambda_5, \dots)$ for a sequence of complex numbers indexed by the primes. Given such a $\boldsymbol{\lambda}$, we have

$$L(\boldsymbol{\lambda}, s) = \prod_p \frac{1}{1 - \lambda_p p^{-s}}.$$

Given this definition, we have

$$L(\operatorname{sym}^k E, s) = \prod_{j=0}^{k} L(e^{i(n-2j)\boldsymbol{\theta}}, s)$$

with $e^{i(n-2j)\boldsymbol{\theta}} = (e^{i(n-2j)\theta_2}, e^{i(n-2j)\theta_3}, \dots)$.

It is now known that $L(\operatorname{sym}^k E, s)$ has analytic continuation and functional equation. I'm wondering: does this imply anything about analytic continuation of the $L(e^{i(n-2j)\boldsymbol{\theta}}, s)$?