Computing G-star discrepancy

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The basic idea is this. Suppose we have two probability measures μ and ν on $\mathbf{R}$. These will be given by explicitly computable cumulative distribution functions cdf_μ, cdf_ν. For example, we might have

$$\mathrm{cdf}_\mu(x) = \begin{cases} 0 & x \leqslant 0 \\ \frac{x - \sin(x)\cos(x)}{\pi} & 0 \leqslant x \leqslant \pi \\ 1 & x \geqslant \pi \end{cases}$$

$$\mathrm{cdf}_\nu(x) = \frac{\#\{n : x_n < x\}}{N},$$

where $(x_1, \ldots, x_N)$ is a sequence of points in $[0, \pi]$. The *star discrepancy* of ν with respect to μ is just the supremum of the difference between their CDFs:

$$\mathrm{disc}(\mu, \nu) = \sup_x |\mathrm{cdf}_\mu(x) - \mathrm{cdf}_\nu(x)|.$$

The question I am interested in is: when cdf_μ is an explicit smooth function and cdf_ν is an explicit step function as above, how can we compute $\mathrm{disc}(\mu, \nu)$ quickly?