Computing discrepancy with respect to non-Lebesgue measure

Daniel Miller

July 28, 2016

The basic idea is this. We have a set $\boldsymbol{x} = \{x_1, x_2, \dots\}$ in the unit square $[0,1] \times [0,1]$. Recall the classical *star discrepancy*:

$$D_N^*(\boldsymbol{x}) = \sup_{0 \leqslant \alpha_1, \alpha_2 < 1} \left| \frac{\#(\{x_n : n \leqslant N\} \cap [0, \alpha_1) \times [0, \alpha_2))}{N} - \alpha_1 \alpha_2 \right|.$$

I am interested in a variant of this using measures other than the Lebesgue.

In the one-dimensional case, I am interested in the following discrepancy for sequences $\boldsymbol{x}$ in $[0, \pi]$:

$$D_N^{(\mathrm{ST}),*}(\boldsymbol{x}) = \sup_{0 \leqslant \alpha < \pi} \left| \frac{\#(\{x_n : n \leqslant N\} \cap [0, \alpha))}{N} - \frac{2}{\pi} \int_0^\alpha \sin^2(\theta) \, \mathrm{d}\theta \right|,$$

or the obvious generalization to $[0, \pi]^2$ with respect to the measure

$$\frac{8}{\pi^2} (\cos\theta_1 - \cos\theta_2)^2 \sin^2\theta_1 \sin^2\theta_2 \, \mathrm{d}\theta_1 \mathrm{d}\theta_2.$$

The point is: the literature I've read on computing discrepancy deals exclusively with Lebesgue discrepancy, and can take advantage of the fact that the function:

$$(\alpha_1, \alpha_2) \mapsto \int_0^{\alpha_1} \int_0^{\alpha_2} \mathrm{d}\theta_2 \mathrm{d}\theta_1$$

is convex.

My question

Do you know of any computational resources and/or papers on computing $D_N^{(\mathrm{ST}),*}$ or its' two-dimensional analogue?