Stellenbosch Camp December 2017
Beginner Final Test
Additional Solutions

14. *Find all pairs (m,n) of positive integers which satisfy the equation*

$$mn^2 = 100(n+1).$$

Note that n and $n+1$ differ by 1, and so are coprime. Therefore n^2 must divide 100. Whichever value n takes, m can be found as $\frac{(n+1)\cdot 100}{n^2}$. So (m,n) can take only the following solutions: $(200,1)$; $(75,2)$; $(24,5)$; $(11,10)$.

15. *Point M and N are chosen on the sides BC and CD of a square $ABCD$, respectively, so that $\angle MAN = 45°$. Points P and Q are the intersections of the diagonal BD with AM and AN, respectively. Prove that P and Q lie on the circle with diameter MN.*

$\angle PAN = \angle PDN = 45°$ (diagonal BD), so $APND$ is a cyclic quadrilateral, since the angles subtended by chord PN are equal. Thus $\angle ANP = \angle PDA = 45°$ (AP common chord), and $\angle APN = 90°$ ($\triangle APN$). Then $\angle MPN = 90°$ (straight line), so P is subtended by diameter MN in circle MNP. By symmetry, the same must apply for point Q, using the cyclic quadrilateral $BMQA$. Therefore P and Q both lie on the circle with diameter MN.

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