Intermediate Test 4

Stellenbosch Camp 2017

Time: $2\frac{1}{2}$ hours

1. Determine the digits a and b if the number $\overline{a2017b}$ is divisible by 72.

2. A number written only with digits 2 and 3 is called *catty*. Therefore, the catty numbers are 2, 3, 22, 23, ... Determine the 2050th catty number.

3. Let x, y and z be nonnegative real numbers such that $x + y + z \leq 3$. Prove that

$$\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} \geq \frac{3}{2}.$$

4. Let Γ be a circle with two chords AB and CD which intersect at point X inside Γ. Let M and N be the midpoints of AB and CD respectively. Show that if MN is parallel to the angle bisector of $\angle AXC$, then $AB = CD$.

5. Does there exist a positive integer m such that $2^{m^2} - 4$ is divisible by 7?