Stellenbosch Camp December 2017: Test 1
Senior
Time: $2\frac{1}{2}$ hours

1. The side BC of the triangle ABC is extended beyond C to D so that $CD = BC$. The side CA is extended beyond A to E so that $AE = 2CA$. Prove that, if $AD = BE$, then the triangle ABC is right-angled.

2. Determine all integers m for which the $m \times m$ square can be dissected into five rectangles, the side lengths of which are the integers $1, 2, 3, \ldots, 10$ in some order.

3. Let $a \in \mathbb{R}^+, n \in \mathbb{N}$. Prove that

$$(a^n + 1)^{n+1} \geq (a^{n+1} + 1)^n.$$

4. Find all positive integers a and b for which there are three consecutive integers at which the polynomial

$$P(n) = \frac{n^5 + a}{b}$$

takes integer values.

5. Let ABC be an acute angled triangle. Let point B' be the reflection of B across the line AC, and point C' the reflection of C across the line AB. Circles circumscribed to triangles ABB' and ACC' intersect at points A and P. Prove that the circumcentre of triangle ABC lies on the line AP.