

Stellenbosch Camp December 2017: Test 1
Intermediate
Time: $2\frac{1}{2}$ hours

1. How many five letter words are there which contain at least one letter a (non-sensical words also count, for example $aaazz$ is a word)?
2. If $8a^2 + 1 = b^2$, show that ab is divisible by 3.
3. Let a , b and c be positive real numbers. Show that

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} \geq 3.$$

4. The side BC of the triangle ABC is extended beyond C to D so that $CD = BC$. The side CA is extended beyond A to E so that $AE = 2CA$. Prove that, if $AD = BE$, then the triangle ABC is right-angled.
5. Determine all integers m for which the $m \times m$ square can be dissected into five rectangles, the side lengths of which are the integers $1, 2, 3, \dots, 10$ in some order.