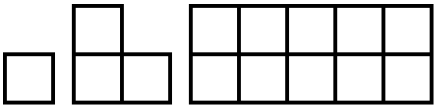
Intermediate Test 1

Stellenbosch Camp 2022

Time: $2\frac{1}{2}$ hours

1. Ian is placing 3 **L**-shaped tiles and a single square tile on a 2×5 chessboard. How many ways can Ian tile this board by placing all the tiles such that no tiles overlap and all squares are covered?



2. Given a triangle ABC, with $AB = BC = 1$, what is the maximal area that can be achieved by varying the length of the third side CA?

3. Given an acute angled triangle with sides of lengths a, b and c. Prove that:

$$a^2 + b^2 > c^2.$$

4. Where p is a prime and $n \in \mathbb{N}$, find all solutions to the equation

$$p^2 = 2^n + 1.$$

5. There are at least 3 people at a party. All of them have an even number of friends, where friendship is mutual. Show that there are 3 of them who each have the same number of friends.