

January Intermediate Monthly Problem Set

Due: 20 January 2023

1. Find the last two digits of 6^{2023} ?
2. What is the area of the triangle with side lengths 10, 17, and 21?
3. Find the natural number a for which $N = (a^2 + 85)^2 - (18a + 3)^2$ is prime.
4. King Hiero has 11 metal pieces indistinguishable in appearance. The king knows that their weights in some order are 1, 2, ..., 11 kg. The king also has a bag that breaks if it contains more than 11 kg (it will not break if it contains exactly 11 kg). Archimedes knows the weight of each piece and he wants to convince King Hiero beyond a shadow of a doubt about which piece has the 1 kg weight. By a 'move', Archimedes can put several pieces into the king bag to demonstrate that the bag is still not broken (He may not break the bag!). Find the least number of moves Archimedes needs to convince King Hiero.
5. Points K , and L are marked on side AB of triangle ABC so that $KL = BC$ and $AK = LB$. Given that O is the midpoint of side AC , prove that $\angle KOL = 90^\circ$.
6. We say that a non-empty set A consisting of positive integers is *complete* if for any positive integers a and b such that $a + b \in A$, the number ab also lies in A (the numbers a and b are not required to be distinct or to belong to A). Find all complete sets.

- Submit your solutions at <https://forms.gle/9EBuvypU7ppDmprt8>.
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.

