Intermediate Test 3

Stellenbosch Camp 2022

Time: $2\frac{1}{2}$ hours

1. Given positive real numbers a, b, and c such that $a^2 + b^2 + c^2 = 26$ and $ab + bc + ca = 19$, find the value of $a + b + c$.

2. Find all positive integers a and b such that

$$a! + b! = 3^n - 1$$

where n is a positive integer.

3. Show that if every room in a house has an even number of doors, then the number of outside entrance doors must be even as well.

4. For acute triangle $\triangle ABC$, a point Z interior to $\triangle ABC$ satisfies $ZB = ZC$. Suppose points X and Y lie outside $\triangle ABC$ such that $\triangle XAB \,|||\, \triangle YCA \,|||\, \triangle ZBC$. Prove that A, X, Y, Z are the vertices of a parallelogram.

5. For nonzero real numbers a, b, and c, show that

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = \frac{a}{c} + \frac{c}{b} + \frac{b}{a}$$

if and only if two of a, b, and c are equal.