

Test 1
April Camp 2023
Time: $4\frac{1}{2}$ hours

1. A 6×6 table is given.
 - (a) If any 9 fields of the table are labelled, prove that it is possible to select three rows and three columns that together contain all of the labelled fields.
 - (b) Label 10 fields of the table so that for any three rows and three columns that we select there is at least one labelled field that is contained neither in the selected rows nor the columns.
2. Let ABC be a triangle and I its incentre. Let D , E , and F be the tangency points of the incircle of ABC with BC , CA , and AB respectively. The line BI cuts the line EF at the point B' and the line CI cuts the line FD at the point C' . The line AC' cuts again the circumscribed circle of the triangle $B'IC'$ at the point X . Let T be the foot of the altitude from the point C onto the line XI . Prove that TX bisects the angle $B'TC'$.
3. A number is called *Norwegian* if it has three distinct positive divisors whose sum is equal to 2022. Determine the smallest Norwegian number. (Note: The total number of positive divisors of a Norwegian number is allowed to be larger than 3.)
4. Find all positive integers $n \geq 2$ for which there exist n real numbers $a_1 < a_2 < \dots < a_n$ and a real number $r > 0$ such that the $\frac{1}{2}n(n-1)$ differences $a_j - a_i$ for $1 \leq i < j \leq n$ are equal, in some order, to the numbers $r^1, r^2, \dots, r^{\frac{1}{2}n(n-1)}$.

