

**Test 5: IMO**  
**April Camp 2023**  
**Time:  $4\frac{1}{2}$  hours**

1. Find all positive integers  $n$  such that

$$n! \mid \prod_{\substack{p < q \leq n \\ p, q \text{ primes}}} (p + q).$$

2. Let  $n \geq 3$  be an integer, and let  $x_1, x_2, \dots, x_n$  be real numbers in the interval  $[0, 1]$ . Let  $s = x_1 + x_2 + \dots + x_n$ , and assume that  $s \geq 3$ . Prove that there exist integers  $i$  and  $j$  with  $1 \leq i < j \leq n$  such that

$$2^{j-i} x_i x_j > 2^{s-3}.$$

3. Let  $n$  be a positive integer. We start with  $n$  piles of pebbles, each initially containing a single pebble. One can perform moves of the following form: choose two piles, take an equal number of pebbles from each pile and form a new pile out of these pebbles. For each positive integer  $n$ , find the smallest number of non-empty piles that one can obtain by performing a finite sequence of moves of this form.

