# Intermediate Test 4

## Stellenbosch Camp 2022

## Time: $2\frac{1}{2}$ hours

1. At a certain maths camp, there are ten coaches to be allocated to four groups: two for beginner, four for intermediate and four for advanced. However, two of the coaches, Tim and Emile, insist that they not be allocated to the same group. How many ways can we allocate the coaches to the three different groups?

2. Find all polynomials $P$ such that $P(x+1) - P(x) = 8$ for $x \in \mathbb{R}$.

3. Let $\Gamma$ be the incircle of acute triangle $\triangle ABC$ with tangency points $D$ and $E$ on sides $BC$ and $CA$ respectively. Let $X$ be a point on $AC$ such that $BX \parallel DE$. Let $Y$ be the point where the angle bisector of $C$ meets $AB$. Prove that $XY$ is tangent to $\Gamma$.

4. Show that $2022 \times 2^n$ can be written as the sum of three distinct non-zero squares for every natural number $n$.

5. You have an $n \times n$ chessboard which starts with all its squares white. An operation involves choosing a row or column and flipping the colour of every square in this row or column between white and black. Is it possible that for all $0 \leq k \leq n^2$, you can do some sequence of operations and end up with $k$ black squares?

```
     (()__(()
     /       \
    ( /    \  \
     \ o o    /
     (_()_)__/ \
    / _,==.____ \
   (   |--|      )
   /\_.|__|'-.__/\_
  / (        /     \
  \  \      (      /
   )  '._____)    /
(((____.--(((____/
```