Intermediate Test 2

Stellenbosch Camp 2021

Time: $2\frac{1}{2}$ hours

1. A positive integer is written on each face of a cube. Each vertex is the assigned the product of the numbers written on the faces intersecting the vertex. The sum of the numbers assigned to all the vertices is equal to 1001. Find the sum of the numbers written on the faces of the cube.

2. Let $m \in N$. How many subsets S are there of $\{2m+1, 2m+2, ...4m\}$ such that S does not contain 2 elements which sum to $6m+1$?

3. Let $ABCD$ be a rectangle and E be a point on the line DC such that $DC = CE$ and $E \neq D$. Let M be the midpoint of AB. Also let BC intersect AE at F, and let AE intersect CM at P. Show that

$$\frac{area(APD)}{area(PFD)} = 2.$$

4. Prove that

$$\frac{1}{(\sqrt{1}+\sqrt{2})(\sqrt[4]{1}+\sqrt[4]{2})} + \frac{1}{(\sqrt{2}+\sqrt{3})(\sqrt[4]{2}+\sqrt[4]{3})} + \cdots + \frac{1}{(\sqrt{9999}+\sqrt{10000})(\sqrt[4]{9999}+\sqrt[4]{10000})} = 9.$$

5. Each of the 19999 species of plants in Cape Town are assigned a label from 2 to 20000. Liam's computer stores a list showing each plant species and the label each corresponds to. Dylan's computer stores a list showing each pair of plant species and the greatest common divisor of their labels. The data on Liam's computer was lost after load shedding surged the computer's hard drive. Dylan, however, was responsible and backed up his data. Is it possible to recover Liam's data from Dylan's data?

- Submit your solutions at https://forms.gle/T9HNgZgj8EhypBnR6
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.