Intermediate Test 3

Stellenbosch Camp 2021

Time: $2\frac{1}{2}$ hours

1. Sandra and Phil play a game, alternating turns with Sandra moving first. They begin with the numbers 1, 2, 3,...,2021 arranged in a straight line. On their turn, a player may highlight between 1 and 1000 (inclusive) consecutive numbers, as long as none of them have been highlighted before. The player who highlights the last number wins. Who has a winning strategy?

2. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that:

$$f(xy) + yf(x) + xf(y) = 3xy$$

   for all $x, y \in \mathbb{R}$.

3. Find all integer solutions to the equation

$$x^3 + y^3 = 10000.$$

4. Point O is in the interior of an equilateral triangle ABC. Prove that there exists a triangle with edge lengths equal to OA, OB and OC.

5. Aaron attempts all six questions on the SAMO senior paper. For each question, his mark is an integer from 0 to 7. He never scores more points on a later question than on any earlier question. How many different possible sequences of six marks can he achieve?

- Submit your solutions at https://forms.gle/T9HNgZgj8EhypBnR6
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.