Advanced March Problem Set

Due: 11 March 2022

1.

2. For which real numbers s and t does the polynomial $p(x) = X^3 - (2s+t)X^2 + (2s^2+t^2)X + 3st$ have three (not necessarily distinct) real roots?

3.

4. Let $ABCD$ be a parallelogram. Let M, N, O, and P be the midpoints of AB, BC, CD, and DA respectively. A circle Γ_1, centred at M, passes through A and B; a circle Γ_3, centred at O, passes through C and D; a circle Γ_2, centred at N, is tangent to Γ_1 and to Γ_3; and a circle Γ_4, centred at P, is tangent to Γ_1, to Γ_2, and to Γ_3. Find the ratio $AB : BC$.

5.

6.

- Submit your solutions at https://forms.gle/Pv89v957obJMEAw26
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.

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