Advanced Test 2

Stellenbosch Camp 2021

Time: $2\frac{1}{2}$ hours

1. Prove that
$$\frac{1}{(\sqrt{1}+\sqrt{2})(\sqrt[4]{1}+\sqrt[4]{2})}+\frac{1}{(\sqrt{2}+\sqrt{3})(\sqrt[4]{2}+\sqrt[4]{3})}+\cdots+\frac{1}{(\sqrt{9999}+\sqrt{10000})(\sqrt[4]{9999}+\sqrt[4]{10000})}=9.$$

2. Each of the 19999 species of plants in Cape Town are assigned a label from 2 to 20000. Liam's computer stores a list showing each plant species and the label each corresponds to. Dylan's computer stores a list showing each pair of plant species and the greatest common divisor of their labels. The data on Liam's computer was lost after load shedding surged the computer's hard drive. Dylan, however, was responsible and backed up his data. Is it possible to recover Liam's data from Dylan's data?

3. A group of 2021 students write a test that consists of 2021 questions. Each question is solved by at least 1011 students. Prove that there is a group of 10 students such that each question was solved by at least one member of the group.

4. Let $ABCDE$ be a regular pentagon inscribed in a circle. Let X be a point on the minor arc AE. Prove that
$$|AX|-|BX|+|CX|-|DX|+|EX|=0.$$

5. Let $\mathbb{N}_0$ denote the set of nonnegative integers. Find all functions $f:\mathbb{N}_0\to\mathbb{N}_0$ such that for all $x,y\in\mathbb{N}_0$ we have that $x\cdot 3^{f(y)}$ divides $f(x)\cdot 3^y$.

- Submit your solutions at https://forms.gle/T9HNgZgj8EhypBnR6
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.

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