Intermediate February Problem Set

Due: ¡DATE¿

1.

2.

3. Find all $x_1, x_2..., x_{2021} \in \mathbb{R}$ such that

$$(2022 - x_1)^2 + (x_1 - x_2)^2 + (x_2 - x_3)^2 + ... + (x_{2020} - x_{2021})^2 + x_{2021}^2 = 2022$$

4. Let Γ be a circle with centre O and radius 1. Let A, B, C, D be points on Γ such that $AB \parallel CD$ and $\overline{AB}$ is a diameter of Γ. Let l be the line through A tangent to Γ and let BC and BD interscect l at X and Y respectively. Find $|\overline{AX}| \cdot |\overline{AY}|$.

5. For which $z \in \mathbb{Z}$ does there exist a polynomial P with integer coefficients such that $P(2021) = 2022$ and $P(z) = z$?

6. Liam and Dylan play a game, alternating turns with Liam playing first. At the start, there is a pile of $1 + 2 + ... + 2022$ stones. On his turn, a player chooses a positive integer n between 1 and 2022 inclusive such that neither player has previously chosen this n. He then removes n stones from the pile and keeps them next to him. Once the pile is empty, the winner is the player with an even number of stones next to him. Who has the winning strategy?

- Submit your solutions at https://forms.gle/Pv89v957obJMEAw26
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.

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