Intermediate Test 1

Stellenbosch Camp 2021

Time: $2\frac{1}{2}$ hours

- 1. In rectangle ABCD, $|\overline{AB}| = 1$, $|\overline{BC}| = 2$, and points E, F, and G are midpoints of \overline{BC} , \overline{CD} , and \overline{AD} , respectively. Point H is the midpoint of \overline{GE} . Lines AF and DH intersect at point X, and lines BF and CH intersect at point Y. What is the area of rectangle XHYF?
- 2. A permutable prime is a prime number which remains prime when its digits are rearranged in all possible ways. Show that a permutable prime cannot be a four digit number where all the digits are distinct.
- 3. Show that, for all real numbers a, b, and c,

$$2021 \ge 4a + 18b + 88c - a^2 - b^2 - c^2.$$

- 4. How many ways are there to choose 24 black squares from a standard chess board (with 32 black and 32 white squares alternating) such that exactly three squares are chosen in each row and in each column?
- 5. Triangle ABC has AC = BC and is right-angled at C. Points K, L are chosen on side AC and points M, N on side BC. It turns out that K and L divide the side AC into three equal parts. Similarly, M and N divide BC into three equal parts. Prove that there exists precisely one point P interior to ABC such that $\angle KPL = \angle MPN = 45^{\circ}$.
 - Submit your solutions at https://forms.gle/T9HNgZgj8EhypBnR6
 - Submit each question in a single separate PDF file (with multiple pages if necessary).
 - If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
 - If you have multiple PDF files for a question, combine them using software such as PDFsam.

