Intermediate Test 5

Stellenbosch Camp 2021

Time: $4\frac{1}{2}$ hours

1. Let n be a k digit number. n is said to be a *Stellenbosch prime* if every segment of consecutive digits from n (this segment could have $1, 2, ...$ or k digits) is prime. Find all Stellenbosch primes.

2. Square $ABCD$ has centre K. If point P is different from K such that $\angle APB = 90°$, prove that KP bisects one of the angles formed by lines AP and AP.

3. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $\forall x, y \in \mathbb{R}$:

$$f(y) = f(x + 2y) + f(x^2)$$

4. A maths test written by s students is comprised of q questions. A question in the test is called *easy* if more than half of the students solve it. A student fails the test if they do not solve at least half of the questions. Find all possible pairs (s, q) such that it is possible for:

   (a) All the questions to be easy when all the students fail the exam

   (b) None of the questions to be easy when none of the students fail the exam

5. Let ABC be a triangle with $AB = AC$, and let D be the midpoint of BC. Let E be the reflection of D across AC. Let F be the point on AB such that $FE||BC$. Prove that $AB \perp FC$.

6. Let A be a natural number with 2^n digits, all of which are equal. Show that A has at least n distinct prime factors.

- Submit your solutions at https://forms.gle/T9HNgZgj8EhypBnR6
- Submit each question in a single separate PDF file (with multiple pages if necessary).
- If you take photographs of your work, use a document scanner such as Office Lens to convert to PDF.
- If you have multiple PDF files for a question, combine them using software such as PDFsam.