

1 Elementary Cellular Automata as Multiplicative
2 Automata

3 Daniel W. McKinley

DOI: 10.xxxxxx/draft

Software

- Review
Repository
Archive

Editor: Open Journals

Reviewers:

- @openjournals

Submitted: 01 January 1970

Published: unpublished

License

Authors of papers retain copyright
and release the work under a
Creative Commons Attribution 4.0
International License (CC BY 4.0)

4 Summary

5 Elementary cellular automata (ECA) are a set of simple binary programs in the form of truth
6 tables called Wolfram codes that produce complex output when done repeatedly in parallel, and
7 quaternions are frequently used to represent 3D space and its rotations in computer graphics.
8 Both are well-studied subjects, this Java library puts them together in a new way. This project
9 changes classical additive cellular automata into multiplicative automata (Wolfram, 2002, p.
10 861) via permutations, hypercomplex numbers, and pointer arrays. Valid solutions extend the
11 binary ECA to complex numbers, produce a vector field, make an algebraic polynomial, and
12 generate some very interesting fractals.

13 Statement of Need

14 The main algorithm produces several multiplicative versions of any given standard additive
15 binary Wolfram code up to 32 bits and is written to support user supplied complex input at
16 row 0 with choice of type of multiplication tables and partial product tables among other
17 parameters. An algebraic polynomial of the automata that works with real and complex
18 numbers is produced, and the hypercomplex 5-factor identity solution allows for the complex
19 extension of any binary cellular automata. The GUI, though not required, allows for visual
20 exploration of solutions with easy access to various parameters. The Java this is written in is
21 designed to integrate well in other programs, such as Mathematica's JLink or Matlab, and
22 is documented with Javadoc. The Cayley-Dickson and Fano construction libraries may be of
23 value to the open source community as well. The link to the code repository is in the references
24 at the bottom.

25 Functions

26 Hypercomplex unit vector implementation

Table with 16 columns (0-15) and 4 rows (Complex, Quaternions, Octonions, Negative sign bit). It lists unit vectors and their corresponding values for different hypercomplex systems.

27
28 The Cayley-Dickson (CD) and Fano support classes are discussed in greater detail in the
29 readme and the documentation, they along with the Galois class provide sets of multiplication
30 tables to be compared with cellular automata. The CD multiplication implementation permutes
31 the steps of splitting and recombining hypercomplex numbers to increase the scope of the CD
32 equation, (a, b)x(c, d) = (ac - d * b, da + bc*), where * is the conjugate. It verifies itself by
33 producing the symmetric group of its degree when interacting with other CD multiplications.
34 The Fano library octonions produce a triplet that is a linear match to the CD octonions as

triplets{0} when the up and down recursion factoradics are equal, and produce the triplet set of John Baez's Fano plane as triplets{10}. (Baez, 2001).

The main algorithm uses a set of permutations operating on cellular automata input, each permutation permuting the neighborhood, becoming a factor, with four kinds of multiplications. The multiplication tables are input as 2D but used as N-D, where N=numFactors.

	Multiplications A	Multiplications B	Multiplications C	Multiplications D
Type	Hypercomplex or finite, brute-force of all the permutations of that number of factors	Cartesian product summed by a hypercomplex or finite partial product table	Complex product	Permutation composition
Size	Wolfram code length = L	Size of neighborhood, log2(L)	Size of neighborhood, log2(L)	Size of neighborhood, log2(L)
Function	Validates permutation group, reproducing the Wolfram code as a pointer array	Applies a valid solution to a user given complex neighborhood	Like B, but does the normalization before the multiplication	Orders the cell's neighborhood vector from (B), post multiplication, pre normalization
Scope	Entire Wolfram code, every possible binary neighborhood	Single given input neighborhood	Single given input neighborhood	Single given input neighborhood
Produces	Set of permutations that changes the additive automata to multiplicative, with the given multiplication table	Polynomial	Output visually similar to B	Vector
Data type	Binary	Complex	Complex	Discrete permutation
Base 2 sum of neighborhood	Construction of factors, pre multiplication	Normalization, post multiplication	Construction of factors, pre multiplication	n/a
N-th root in normalization	n/a	N = size of neighborhood	N = number of factors	n/a

The first set of multiplications, column A, brute forces all possible sets of permutations on all possible binary neighborhoods of the Wolfram code. A permutation in the set rearranges the columns of the input neighborhood, these become a set of factors. A valid set of permutations is one that, for all possible input neighborhoods, the set of constructed factors using the permuted neighborhoods always multiplies out to a value that points to an equal value within the Wolfram code. The set of multiplication results is a pointer array that reproduces the original Wolfram code for every possible binary neighborhood.

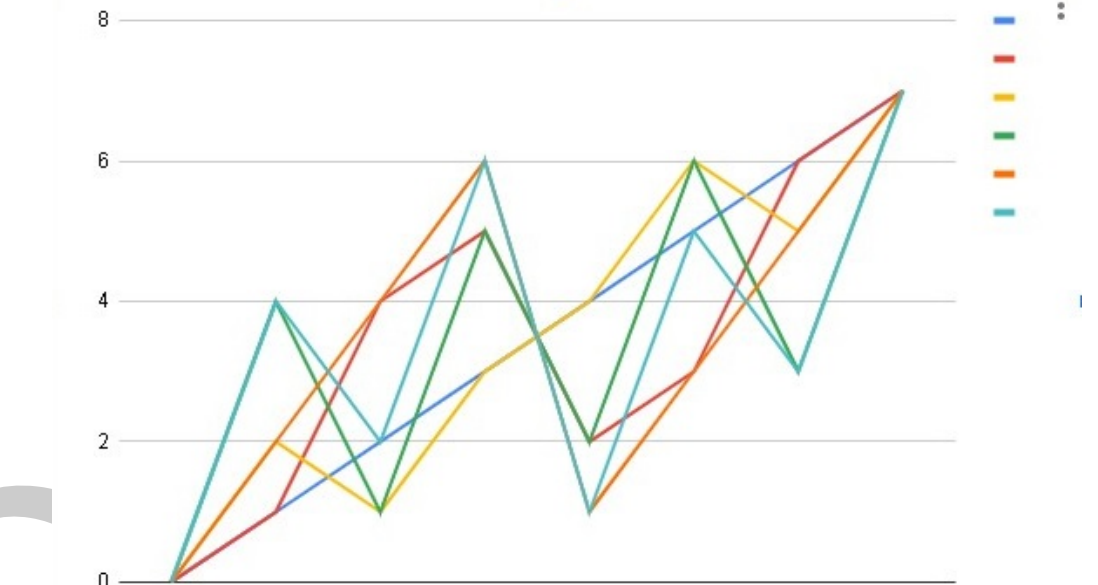
49 Identity solutions of 5 factors using all zero permutations exist for Wolfram codes up to 32 bits
50 in this library using hypercomplex numbers and Galois addition. Galois multiplication takes a
51 mix of numbers of factors to get the identity multiplication result array, there is a function in
52 the GaloisField class that provides it. The factors constructed are a loose diagonal through the
53 multidimensional multiplication table, starting at the origin and ending at the opposite corner
54 while zig-zagging. The path lengths of each factor and the result are included in ValidSolution
55 results.

56 Permutations of 3 bit neighborhoods

Permutation: 0, [0, 1, 2, 3, 4, 5, 6, 7]
Permutation: 1, [0, 1, 4, 5, 2, 3, 6, 7]
Permutation: 2, [0, 2, 1, 3, 4, 6, 5, 7]
Permutation: 3, [0, 4, 1, 5, 2, 6, 3, 7]
Permutation: 4, [0, 2, 4, 6, 1, 3, 5, 7]
57 Permutation: 5, [0, 4, 2, 6, 1, 5, 3, 7]

58 Flattened path through a six dimensional multiplication table

59 Six factors, permutation set = {0,1,2,3,4,5}



60
61
62 Multiplications B and C apply a valid solution from the first set of multiplications to any given
63 individual neighborhood with binary, non-negative real, and complex values. Multiplication B is
64 the Cartesian product of the permuted neighborhoods, using a closed partial product table to
65 generate a polynomial. Multiplication C does the binary sum of complex neighborhood, then
66 multiplies as complex. Both B and C take the n-th root of the result, with $n = \text{numColumns}$
67 and $n = \text{numFactors}$, respectively. Multiplications B and C both include a binary weighted
68 sum of the neighborhood, same as the construction of the factors from A, though B and C
69 use complex. B, as part of the normalization and C as the construction. Multiplication C is
70 the permutation composition product. B, just before the normalization is a neighborhood of
71 multiplication results, with each column of it being a unit vector coefficient. This multiplication
72 result neighborhood is permuted by the inverse of the permutation composition product to
73 properly order the output vector.

74 Control Panel

ECA rule

Multiplication Table to use

Specific solution to use

Degree: 2 = quaternions, 3 = octonions, etc., if applicable

Number of factors to use

Number of rows in the ECA, 1 row = 3 bit neighborhood, 2 rows = 5 bit neighborhood

Partial product table, size = places x places

Keeps functions from running longer than the user want, in seconds

the calculate button produces all solutions for the chosen parameters

this button re-randomizes and displays the ECA rule with the particular solution number chosen

Deep search using above parameters

Width of random input 200

Number of factors in logic gate search

Logic gate, AND = 8, OR = 14, XOR = 6, etc

Logic gate solution:

Which multiplication table to use

Partial product table

Refresh logic gate solutions

Display specific logic gate solution

Search all logic gates for solutions and crossreference gates that have solutions in common

Table Display Degree, 2 = quaternions, 3 = octonions, 4 = sedonions

Cayley-Dickson permutation number, (cdz, __), down in recursion

Cayley-Dickson permutation= number, (__ , cdo), up in recursion

Fano plane octonions

Galois Field, Prime

Galois Field, Power

Length of permutations

Refresh permuted Cayley-Dickson solutions

Display tables with above parameters

Compare Fano-generated octonions with permuted Cayley-Dickson octonions

Compare permuted CD with permuted CD

Picks a random Wolfram code with 5 factors, identify solution

54

Permuted Cayley-Dickson

0

2

5

1

Galois addition, XOR, 3x3

30

Refresh

Display specific solution

Start deep search

5

6: XOR

XOR

Galois addition, XOR, 2x2

Refresh

Display specific solution

Deep logic gate search

2

0

0

0

2

1

4

Refresh

Display specific tables

Fano/CD Compare

CD v CD

Random Wolfram Code

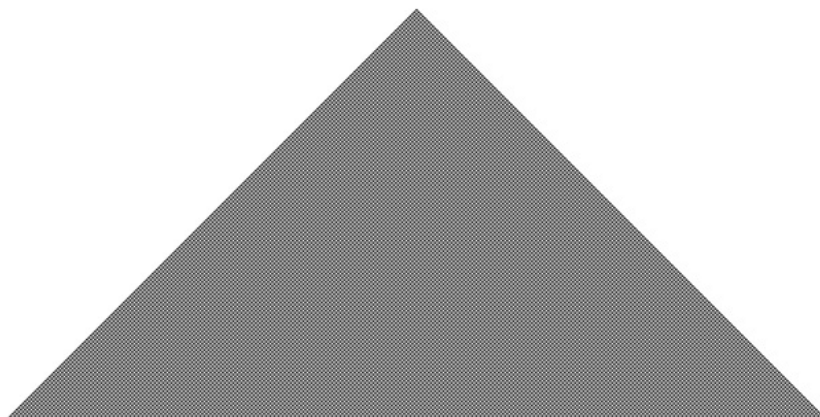
75

76

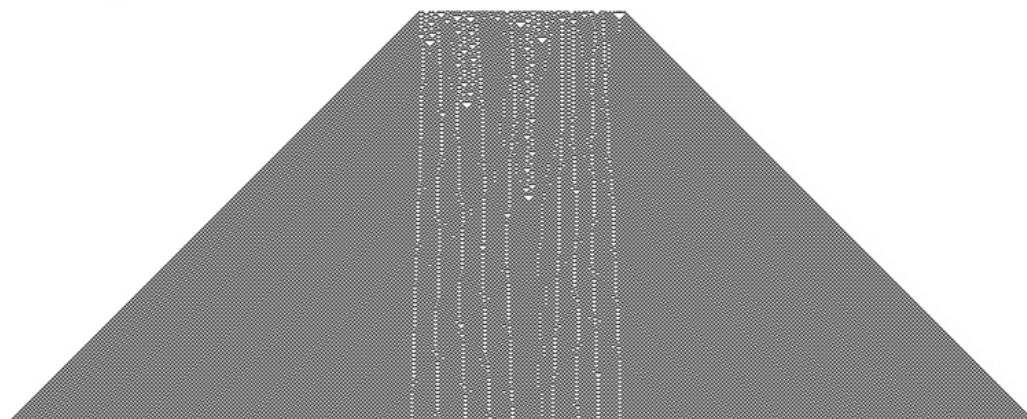
77

ECA 54, binary and non negative real

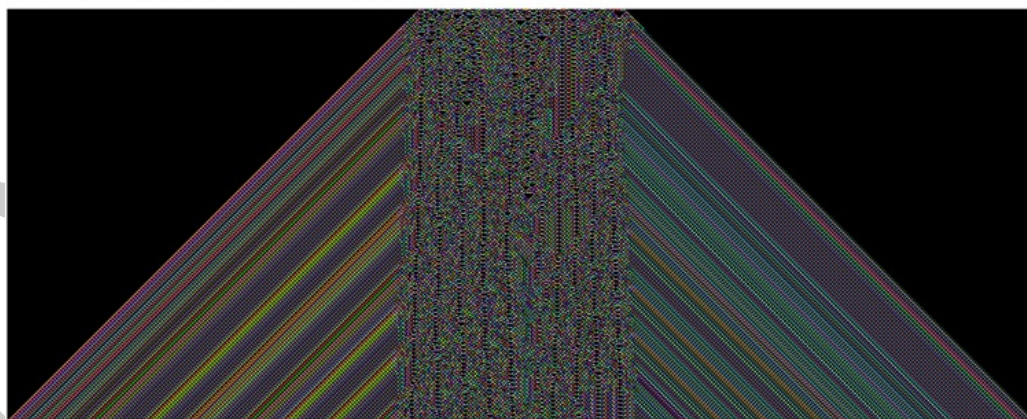
Single bit initial input:



Random initial input:



Same random initial input with solution applied to random $(0,1)$ non negative real:



78

79

80

ECA 54, solution parameters, including polynomial

```
ValidSolution
Wolfram code: [0, 1, 1, 0, 1, 1, 0, 0]
Permutation: 0 Permuted Axis: [0, 1, 2, 3, 4, 5, 6, 7]
times
Permutation: 0 Permuted Axis: [0, 1, 2, 3, 4, 5, 6, 7]
times
Permutation: 0 Permuted Axis: [0, 1, 2, 3, 4, 5, 6, 7]
times
Permutation: 0 Permuted Axis: [0, 1, 2, 3, 4, 5, 6, 7]
times
Permutation: 0 Permuted Axis: [0, 1, 2, 3, 4, 5, 6, 7]
times
-----
Equals: [0, 1, 2, 3, 4, 5, 6, 7]
Apply Wolfram code to multiplication result
Equals: [0, 1, 1, 0, 1, 1, 0, 0]
Original Wolfram code: [0, 1, 1, 0, 1, 1, 0, 0]

Permutation composition product: 0, inverse: 0

Multiplication table type: 0
2D multiplication table used:
[0, 1, 2, 3, 4, 5, 6, 7]
[1, 4, 7, 2, 5, 0, 3, 6]
[2, 3, 4, 5, 6, 7, 0, 1]
[3, 6, 1, 4, 7, 2, 5, 0]
[4, 5, 6, 7, 0, 1, 2, 3]
[5, 0, 3, 6, 1, 4, 7, 2]
[6, 7, 0, 1, 2, 3, 4, 5]
[7, 2, 5, 0, 3, 6, 1, 4]

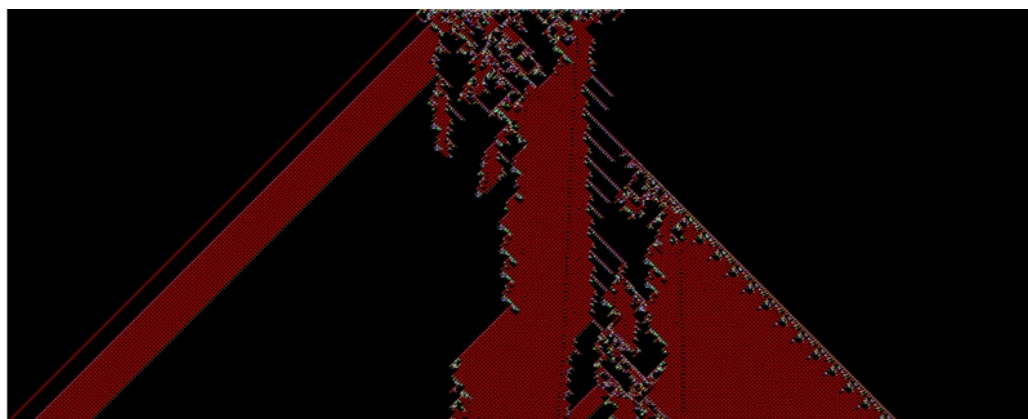
numFactors: 5 numBits: 3

1*((a^5)*(b^0)*(c^0)) + 20*((a^3)*(b^1)*(c^1)) + 10*((a^2)*(b^3)*(c^0)) + 10*((a^2)*(b^0)*(c^3)) + 30*((a^1)*(b^2)*(c^2)) +
5*((a^0)*(b^4)*(c^1)) + 5*((a^0)*(b^1)*(c^4))

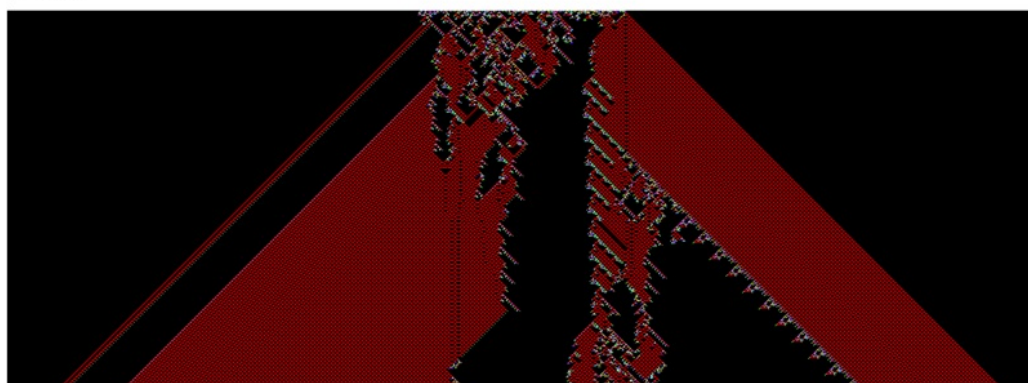
5*((a^4)*(b^1)*(c^0)) + 10*((a^3)*(b^0)*(c^2)) + 30*((a^2)*(b^2)*(c^1)) + 5*((a^1)*(b^4)*(c^0)) + 20*((a^1)*(b^1)*(c^3)) +
10*((a^0)*(b^3)*(c^2)) + 1*((a^0)*(b^0)*(c^5))

5*((a^4)*(b^0)*(c^1)) + 10*((a^3)*(b^2)*(c^0)) + 30*((a^2)*(b^1)*(c^2)) + 20*((a^1)*(b^3)*(c^1)) + 5*((a^1)*(b^0)*(c^4)) +
1*((a^0)*(b^5)*(c^0)) + 10*((a^0)*(b^2)*(c^3))
```

ECA 54, solution output, complex



Complex part



84

85

86 References

- 87 Baez, J. (2001). *The octonions* (10.1090/S0273-0979-01-00934-X). Bulletin of the American
88 Mathematical Society.
- 89 Wolfram, S. (2002). *A new kind of science*. Wolfram Media. ISBN: 1579550088